(Non) true inflation targets and (in) effectiveness of monetary policy

Celso José Costa Junior *

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Abstract

This paper analyzes whether the new classical proposition of monetary policy ineffectiveness is sustained on two different assumptions of bounded rationality: with complete memory; and with bounded memory. To do so, a canonical new keynesian model with central bank communication is used. The hypothesis is that the monetary authority, which can be committed or strategic, after observing its private signal, sends a message about the inflation target, and subsequently, the private sector infers on the "type" of that agent and defines its expectation on the inflation rate. So, given the expectations of the private sector, the central bank establishes a monetary policy action that will determine the inflation rate and the output. The results show that the values of the output gap and of the inflation rate in the assumption with bounded rationality and complete memory tend to the values of rational expectations, that is, for the ineffectiveness of monetary policy. On the other hand, in the assumption of bounded rationality and bounded memory, monetary policy becomes effective because the private agent is not able to identify the strategic type of the central bank.

keywords: inflationary bias, bounded rationality, bounded memory, ineffectiveness of monetary policy.

JEL classification: E58, E30, C11.

Resumo

Este artigo analisa se a proposição novo-clássica da ineficácia da política monetária se sustenta em dois pressupostos diferentes de racionalidade limitada: com memória completa; e com memória limitada. Para isso, é utilizado um modelo novo-keynesiano canônico com comunicação do banco central. A hipótese é que a autoridade monetária, que pode ser comprometida ou estratégica, após observar seu sinal privado, envia uma mensagem sobre a meta de inflação e, posteriormente, o setor privado infere sobre o "tipo" desse agente e define sua expectativa sobre a taxa de inflação. Assim, dadas as expectativas do setor privado, o banco central estabelece uma ação de política monetária que determinará a taxa de inflação e o produto. Os resultados mostram que os valores do hiato do produto e da taxa de inflação no pressuposto com racionalidade limitada e memória completa tendem aos valores das expectativas racionais, ou seja, para a ineficácia da política monetária. Por outro lado, no pressuposto da racionalidade limitada e da memória limitada, a política monetária torna-se efetiva porque o agente privado não é capaz de identificar o tipo estratégico do banco central.

Palavra(s)-chave: VIÉS INFLACIONÁRIO / RACIONALIDADE LIMITADA / MEMÓRIA LIMITADA / INEFICÁCIA DA POLÍTICA MONETÁRIA

*Department of Economics – State University of Ponta Grossa. Address: Praça Santos Andrade, n1, Centro, 84010790 - Ponta Grossa, PR - Brasil. E-mail: cjcostaj@yahoo.com.br and cjcjunior@uepg.br

1 Introduction

Many studies in the macroeconomic literature seek to explain that employment and output can be influenced by unanticipated inflation and, therefore, by unanticipated monetary growth. If monetary policymakers consider the natural level of employment to be very low, they may be tempted to create monetary surprises in order to boost employment beyond its natural level, even at the cost of some inflation. Therefore, the literature provides several explanations for the dissatisfaction of policymakers with the current natural level of employment. One approach treats this agent as a benevolent social planner who conducts monetary policy to maximize a well-defined social welfare function (BARRO AND GORDON, 1983). Another assumes that the policymaker is responsible for the political pressures that arise from distributional issues (WOOLLEY, 1984). Obviously, these two approaches are not necessarily mutually exclusive.

A basic assumption of many economic models is that agents have complete memory and update their beliefs using Bayes' rule. In long-term relationships, players condition their strategies to the entire history of the game. However, complete memory is an exaggerated assumption, because in the "real world" agents ignore part of the information and perform updates less frequently than the general assumption of the models. So bounded memory could be used to explain some phenomena linked to "reputation building" (MONTE, 2007).

Given the previous ideas that the monetary authority has incentives to expand the level of output and that the public could have bounded memory, the aim of this paper is to discuss the behavior of providing an "intentional" wrong signal about the true inflation target, waiting for an inflationary bias to occur. For this, a canonical new keynesian model will be used with communication of inflation targets on three different assumptions of rationality.

Thus, it is assumed that the public (receiver) is not sure about the preferences of the monetary authority (sender). In each period, the central bank knows the true state of the world and sends a message about this state (inflation target) to the public. However, the sender of the message does not have to "tell the truth", his strategy will depend on his preferences. The sender can be "committed" (always tells the truth) or it can be "strategic" (looking for the best result given your preferences). After the public observes the message from the sender, an action is taken and the payoffs are reached. At the end of the period, the state of the world is checked and the receiver knows whether the sender lied or not. Then, the receiver – having bounded rationality – updates his beliefs regarding the type of the sender through "categorization" (committed or strategic) instead of using the Bayes rule. Therefore, the main innovations of this work are to study the inflationary bias with a different framework incorporating the categorization of the monetary authority by the level of confidence in determining inflation rate expectations.

For studies on monetary policy, the most important result of the new classical school is the existence of the inflationary bias. Sargent and Wallace (1975 and 1976) developed the so-called ineffective proposition of monetary policy. For these authors, any monetary policy rule would be perfectly incorporated into the agents' expectations, making it incapable of affecting real variables, even in the short term. In other words, the only way for the monetary authority to affect the level of employment (or output) is by surprising economic agents, who, being rational, do not make systematic mistakes.

In the Barro and Gordon (1983), although the monetary authority is encouraged to inflate the economy to raise the level of social welfare in the short term - due to the temporary reduction in unemployment - it has to consider the costs futures associated with the implementation of the inflationary bias, since, in the long run, the economy is at a point of equilibrium in which unemployment is equal to its natural rate, and the corresponding reduction in its reputation.

In this way, the concern with the loss of reputation would act as an incentive mechanism that would induce it to practice a responsible monetary policy. Canzoneri (1985) works on an extension of the static monetary policy game of Barro and Gordon (1983) and argues that standard solutions for temporal inconsistency are inadequate when the central bank has private information on the state of the economy. Furthermore, their results indicate that the timing of events makes cheap talk communication redundant. Stein (1989) shows, in a cheap talk monetary policy game, that temporal inconsistency induces the sender to lie in the same way as a genuine preference bias. And this author's contribution is to identify the beneficial effects of competence on the credibility of the monetary authority.

Moscarini (2007) also works on this idea of competence and credibility, following Cazoneri (1985), but expanding a static version of Barro and Gordon (1983) to allow private information from the central bank on the state of the economy. Its main innovation is to make an observation of the monetary authority on the state of the economy with noise, and to study the effects of its precision, which it referred to as the competence of the central bank. This agent, after observing his private signal, sends a message to the public about the inflation target in a cheap talk game in the style of Crawford and Sobel (1982). The public then defines its rational expectation for inflation based on information that can be extracted from the central bank's announcements. Finally, given the expectations of the public, the central bank takes monetary policy action that determines the rates of inflation and growth of the economy's output. Their results indicate that, in equilibrium, a true message would be impossible and communication would be "opaque" if, and only if, the monetary authority has an inflationary bias. And that, the amount of communication between public and private agents increases both by refining the message space and by the ability of the private sector to predict the future actions of the central bank, that is, competence implies credibility.

In the credibility models of Sobel (1985) and Benabou and Laroque (1992), in which an uninformed policymaker makes decisions based on the advice of a specialist, the specialist being a "friend" or a " enemy ", as defined by Sobel (1985). In this context, if the policymaker is restricted to bounded memory, the enemy can exploit the policymaker indefinitely, without ever damaging his reputation. A popular result of the reputation literature on repeated games with incomplete information is that the game converges asymptotically to the complete information game (CRIPPS *ET AL*, 2004). This means that the player can profit from a "false" reputation only in the short term.

Monte (2013) compares his results with those of Cripps *et al* (2004) assuming that the receiver under bounded memory has only a fixed number of available memory states, in such a way that he knows the information of the current period, but it is forgotten between periods, that is, everything you know about the history of the game is in its current state of memory. Its results seek to answer how the severity of learning and memory restrictions affect the building of reputation, that is, under bounded memory, one may not have learning, even in the long run. Therefore, the receiver's bounded memory can explain how the long-term reputation can be sustained. In fact, it shows that players can learn almost nothing if their memory is small enough compared to the noise of the game.

As mentioned, under rational expectations, an important result of the new classical theory is that monetary policy is ineffective. The results of this paper show that this same event occurs in the assumption of bounded rationality and complete memory, given that private agents are able to learn from the behavior of the central bank, thus converging their expectations to the value of rational expectations. However, when private agents suffer from bounded rationality and bounded memory, this agent is unable to learn the type of central bank, and the reputation of this agent remains, even in the long run. Thus, under this assumption, monetary policy is effective.

In addition to this introduction, this paper is structured as follows: section two presented the inflationary bias of monetary policy; and the third demonstrates the results.

2 Inflationary bias of monetary policy

This section develops the idea of the inflationary bias and its equilibriums in three rationality assumptions that will be analyzed in the next section. So first, it is convenient to present a closed definition for the inflationary bias.

Definition 2.1 (Inflationary bias). Incentive that the monetary authority has to create surprise inflation, with the objective of increasing the level of employment (or output) for beyond its natural level.

Now, consider a central bank that is concerned with both price stability and a high level of employment, more precisely, assume that the objective of this agent is to maximize the following quadratic loss function:

$$-\left[\frac{\lambda}{2}(N^*-N)^2+\frac{\pi^2}{2}\right], \quad N^*-N\geq 0 \tag{1}$$

where N and N^* are the levels of current and desired employment by the monetary authority, respectively. π is the inflation rate, and λ is a positive parameter that reflects the central bank's relative concern for a high level of employment and price stability. The higher value for this parameter reflects a greater concern of the policy maker to keep employment close to the desired level N^*.

Then, it is supposed the following Phillips relationship:

$$N-N_n=s(\pi-\pi^e) \tag{2}$$

where $s>0$ measures the sensitivity of the difference between the current employment level (N) and the natural employment level (N_n) given the difference between current inflation (π) and expected inflation (π^e), that is, the level of nominal rigidity of the economy.

Ignoring real shocks and changes in the speed of money circulation, the rate of inflation will be equal to the rate of monetary growth, $\pi=m$. Then, the inflationary expectation will be equal to the monetary growth expectation, $\pi^e=m^e$. Thus, the Phillips relationship can be rewritten as follows:

$$N-N_n=s(m-m^e) \tag{3}$$

And the problem of monetary authority will be:

$$\max_m -\left\{\frac{\lambda}{2}[N^*-N_n-s(m-m^e)]^2+\frac{m^2}{2}\right\} \tag{4}$$

This agent selects m with m^e given. Thus, the first order condition for this problem is:

$$m=\left(\frac{s\lambda}{1+s^2\lambda}\right)(N^*-N_n)+\left(\frac{s^2\lambda}{1+s^2\lambda}\right)m^e=\phi(m^e) \tag{5}$$

2.1 Rational expectations

The rational expectations hypothesis establishes that economic agents maximize the use of all available information when forming their expectations. In its strongest version, this hypothesis postulates that the subjective expectation of economic agents with respect to a given variable coincides with the objective mathematical expectation of that variable. Formally, inflation expectations in period t, π_t^e, is equal to the mathematical expectation of inflation in t, conditioned by the set of information available in period t (I_t):

$$\pi_t^e = E(\pi_t|I_t)$$

This equation does not imply that the inflation expectation is always equal to realized inflation, but that the inflation expectation is not biased, that is, agents do not systematically make mistakes when forming their expectations.

2.1.1 Lying about the inflation target on the assumption of rational expectations ($m = m^e$)

Since the desired employment is higher than its natural level, the central bank will choose a positive rate of money growth. There is no uncertainty here, due to rational expectations. In particular, the public is perfectly informed about the objectives of the central bank, and it can calculate, using equation (5), the monetary growth rate chosen by the monetary authority at each level of m^e. So any expectation that is not reproduced by equation (5) is not rational from the public's point of view, since it implies that the public believes that inflation will be m^e despite the knowledge that for such expectation, the best response for the central bank is a rate of monetary growth that differs from m^e. Therefore, by equation (5), as long as the desired employment is greater than the natural one, the equilibrium of the monetary growth rate will be positive (and positive current inflation rate):

$$m = s\lambda (N^* - N_n) \tag{6}$$

Figure 1 presents the rise of the inflationary bias for this game between the monetary authority and the private agent. The ideal point for the central bank is $m = 0$ and $N = N^*$. Given expectations and the short-term Phillips curve, this agent chooses the corresponding point on the indifference curve that is closest to the ideal point $(N^*, 0)$. This occurs at a tangent point on the short-term tradeoff curve and the map of indifference. When $m^e = 0$, the tangent is at point B, and the rate of monetary expansion is m_0, which is positive. If $P(m^e = m_0)$ the monetary expansion would be m_1, and this would be greater than m_0.

Of all points of tangency in panel a) of the figure 1, there is only one to which the action taken by the central bank validates the expected rate of monetary expansion of the private agent. This occurs at point D on the figure 1. Panel b) emphasizes the reason that any expected rate of monetary expansion that is different from m_2 is not a rational expectations equilibrium. Given that current and expected inflation rates are the same, the central bank has no influence on employment, which is at its natural level, but with a positive inflation rate (result of equation (6)).

Panel a) of the figure 1 suggests that the incentive to inflate will be present regardless of whether the private agent believes whether monetary growth will be zero or positive. So, choosing a positive monetary expansion rate is a dominant strategy for the central bank. The private agent, being aware in advance of this incentive framework, predicts that the inflation

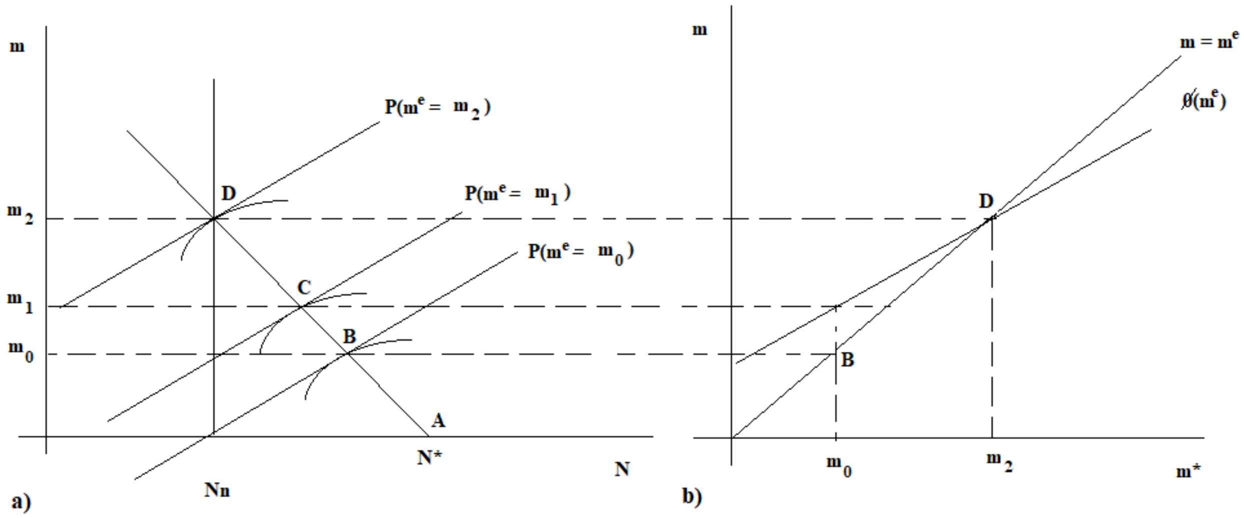

Figure 1: Inflationary bias of monetary policy. Point D represents the ineffectiveness of monetary policy, that is, the equilibrium with rational expectations, but it also represents the convergence equilibrium of bounded rationality with complete memory. At point B, it is possible to have an effective monetary policy, and this is the equilibrium point of bounded rationality with bounded memory. Source: Cukierman (1992).

rate will be m_2, since this is the only rate that, if expected, induces the policymaker to reproduce it. Thus, even if the monetary authority announces a monetary expansion of less than m_2, private agents anticipate the fact that the central bank is encouraged not to implement the announced policy and produce a positive monetary shock, in order to generate inflation-surprise in exchange for an increase in current employment. Consequently, messages from a monetary authority with discretionary powers will not be credible.

2.2 Bounded rationality

In general, when economists use the term "rationality", they do not only require that an action of choice is the best possible, given the knowledge of the decision maker, but also that the knowledge employed is derived from coherent inferences. In contrast, models of bounded rationality are designed to allow talking about agents who systematically do not make inferences correctly. In this work, two types of bounded rationality are assumed. The first is less restrictive, that is, at least the agent has complete memory, enabling learning. In the second assumption of bounded rationality, agents are supposed to face a problem of bounded memory, and with this hypothesis, learning is not allowed.

2.2.1 Complete memory

A relevant point of this work is how the private agent forms its expectations about m_t^e (or, more specifically, about the inflation target) using a rationality different from the assumption of rational expectations. The assumption of bounded rationality and complete memory is able to induce the private agent to learn. In general, learning is often used to refer to literature in which agents do not use Bayes' law to form their expectations. An example is adaptive learning, where agents behave like "econometrists", trying to discover the optimal linear forecasting rule for the state of the next period. Following this line, one possibility would be that this variable had the following characteristics:

$$m_t^e = A + p_t, \quad A > 0$$

$$p_t = \rho p_{t-1} + \upsilon_i, \quad 0 < \rho < 1$$

$$\upsilon_t \sim N\left(0, \sigma_\upsilon^2\right)$$

where A is the publicly known average of m and p_t is the stochastic part of m_t^e. In period t, the realization of A is known to the policy maker but not to the private agent. The private agent knows the stochastic structure of the m. In other words, this agent knows σ_υ^2 and ρ, and uses this knowledge in conjunction with observations of past monetary growth to forecast future inflation (CUKIERMAN, 1992).

2.2.2 Bounded memory

Before talking about bounded memory, it is necessary to explain what an information structure means, that is, it is a pair (Ω, P) where Ω is the space states representing the "complete description of the world" or at least the relevant facts of the world (to make decisions), and that these states are mutually exclusive; and P is a function that you assign to each state $\omega \in \Omega$ a non-empty set of states, $P(\omega)$. The main interpretation of P is that in the state ω, the inference agent excludes all states that do not belong to $P(\omega)$, and does not exclude all states in $P(\omega)$ (RUBINSTEIN, 1998).

It is also necessary to define what would be the "knowledge" set. Let (Ω, P) be an information structure, an event E is said known in ω if the decision maker in ω is able to delete all states that are not in E, that is, $P(\omega) \subseteq E$. The statement: "the decision maker knows E" is identical to all states in which E is known, that is, with the set $K(E) := \{\omega : P(\omega) \subseteq E\}$. If P is a partition, then $K(E)$ is the union of the cells contained in E. Memory is a type of knowledge in which the decision maker has at a certain date about what he understood at some previous date. Bounded memory is a particular case of imperfection of knowledge, and it would be the situation in which an individual has to carry out several successive actions, but faces a limit of memory. To explain this idea of bounded memory an example will be used (figure 2). In this example, an individual chooses between right and left with equal probability, and the optimal strategies would be: side (S) on d1; center (C) in d2; and right (R) in d3. In the set $d3 = \{(L, C), (R, C)\}$, the decision maker just knows that he played center (C) in the first or second stages. In other words, he doesn't remember whether it was $d1$ or $d2$ (PICCIONE AND RUBINSTEIN, 1997).

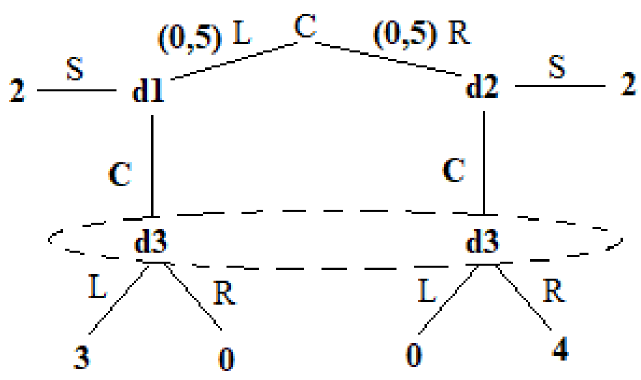

Figure 2: Game with bounded memory. Source: Piccione and Rubinstein (1997).

It is now possible to talk about the problems of complete and bounded memories. These concepts refer to the knowledge expressed only by sets of information. Defining $exp(h)$ as the decision maker's experience throughout its history ($h \in H$), that is, the list of information sets it has found throughout history and actions that you took in each set of information. Therefore, a decision problem with complete memory is one in which $exp(h) = exp(h')$ for two stories that belong to the same set of information. Then, in a decision problem with complete memory, the decision maker "remembers" the actions he has taken in the past whenever information has been obtained about the opportunities for movement. Therefore, a decision problem that does not satisfy the previous condition is called a problem with bounded memory.

In the previous example (figure 2), the individual perceives some information of value at a certain stage, information that may be useful at a later stage, but he will not remember this information later. Specifically, the individual can obtain payoff 2 in stage 1 (d1), or make a choice in stage 2 between L and R, which can result in a payoff less than or greater than 2. In the first stage, he receives information about the action he should take in stage 2, but when he reaches that future date, he will not remember the content said. Formally, the decision problem is one of bounded memory, because both stories, (L, C) and (R, C), are in $d3$, but $exp(L, C) = (\oslash, \{L\}, C)$, while $exp(R, C) = (\oslash, \{R\}, C)$ (RUBINSTEIN, 1998).

2.2.3 Lying about the inflation target on the assumptions of bounded rationality ($m > m^e = 0$)

It was seen that the monetary authority having an objective of $N^* > N_n$, the value of m corresponds to that of equation (6), point D in figure 1, and this would be the equilibrium of the inflationary bias with the assumption of rational expectations. Also, in a discretionary structure, but with bounded rationality, the central bank may announce that it will pursue a zero inflation target, assuming that this authority would have sufficient credibility to induce private agents to infer an expectation of monetary expansion compatible with the announced target. Thus, as long as the desired employment is higher than its natural level, the central bank will choose a monetary growth rate greater than zero, even when the expected inflation rate is zero ($m = \pi > \pi^e = m^e = 0$):

$$m = \left(\frac{s\lambda}{1 + s^2\lambda}\right)(N^* - N_n) \tag{7}$$

Therefore, equation (7) is related to point B of figure 1. Under complete memory, through learning, the private agent converges his expectation on m in such a way that there is a convergence from point B towards point D (figure 3). On the other hand, under bounded memory, the private agent learns nothing about the lie of the central bank, so his expectations remain at point B. Therefore, of the three assumptions of rationality, the only one in which monetary policy is effective is the assumption of bounded rationality and bounded memory.

3 Quantitative Results

In this section, first, it will be introduced the repeated games that support the idea of categorization used in the new Keynesian model. Afterwards, this model is presented with its results. Finally, two robustness analysis will be produced.

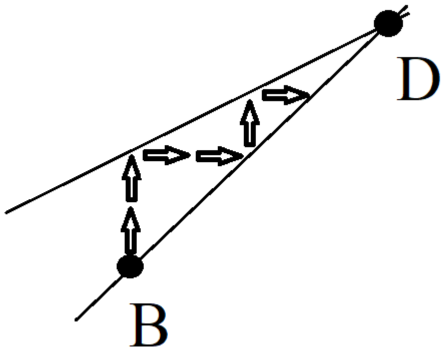

Figure 3: This is a zoom of figure 1 showing the convergence of m from the assumption of bounded rationality with complete memory to the assumption of rational expectations, (from point B to point D). Source: Prepared by the author.

3.1 Repeated games with complete memory and bounded memory

Here, it is discussed repeated games with the receiver having incomplete sender type information, which will serve as theoretical motivation for the analysis of the next subsection. Thus, it is assumed that before the first stage of the game, the nature assigns one of two possible types for the sender with probability ρ committed for the type (C) – which always tells a true sign - and $(1-\rho)$ for strategic type (S) – which outputs the signal according to your interest.

The timing of each stage game is as follows: nature defines a state of the world in each period, $\omega_t \in \Omega$, and the type of sender (compromised or strategic). The sender watches ω_t and sends a π_t^{target} message to the receiver, the sender tells the truth if $\pi_t^{\text{target}} = \omega_t$. The receiver watches the message and performs an action, then payoffs are reached and states are verified. At this point, the receiver can know whether or not the sender has lied. Based on this information, the receiver updates his beliefs about the type of sender that maximizes his goal when performing an action that corresponds to the state of the world, and will be worse off when his action is "distant" from the true state of the world.

As mentioned, a committed central bank always tells the truth and the public always believes in it. On the other hand, a strategic central bank has the tradeoff of building reputation or revealing itself. Therefore, this agent may imitate the compromised type, telling the truth, or may lie. The figure 4 shows the game tree with the public having complete memory. In this figure, the green and blue channels represent the committed and strategic central banks that builds a reputation, respectively.

Nature (N) in defining whether monetary authority is compromised (C) or strategic (S) has a strategy $q : H \to \{T, L\}$, where H is the history in this game[1], that is, the central bank sends a true signal (T) or a lying signal (L). When the memory of the public (P) is empty or when the signal is true and this is verified, P always believes in the sign of the monetary authority. On the other hand, once the lie is perceived, the public loses confidence, and no longer believes in the central bank signal, meaning that P ignores the sender's message. Thus, $\pi_t^e : \{T, L\} \to \{\pi_t^{\text{target}}, \omega_{t-1}\}$, that is, if P believes in the message of the central bank, it sets its expectation by following the target, otherwise it sets its expectation using the state of the world from the previous period. This receiver behavior is best represented by the following definition.

[1]History in this game is defined as the choice of nature over the correct type of central bank, the sequence of action profiles and the states of the world.

Definition 3.1 (Trigger strategy)**.** The player who adopts the trigger strategy will follow a course of action as long as a certain condition is met, and if that condition at any time is no longer satisfied, will follow another course of action for the rest of the game. In other words, if the private agent realizes that the central bank has sended a true signal (T), he believes in signal (B). On the other hand, if the signal is perceived to be a liar (L), it does not believe (NB). And once the private agent has chosen this strategy, it will keep this choice for the rest of the game.

In base structure under complete memory (figure 4) only in its first decision moment (stage P2a and P2b) the public has an empty information structure – they believe any sign of the central bank. Subsequently, in forming its information structure, the public always believes whether the signal is true, and no longer believes it, if the central bank lies at least once.

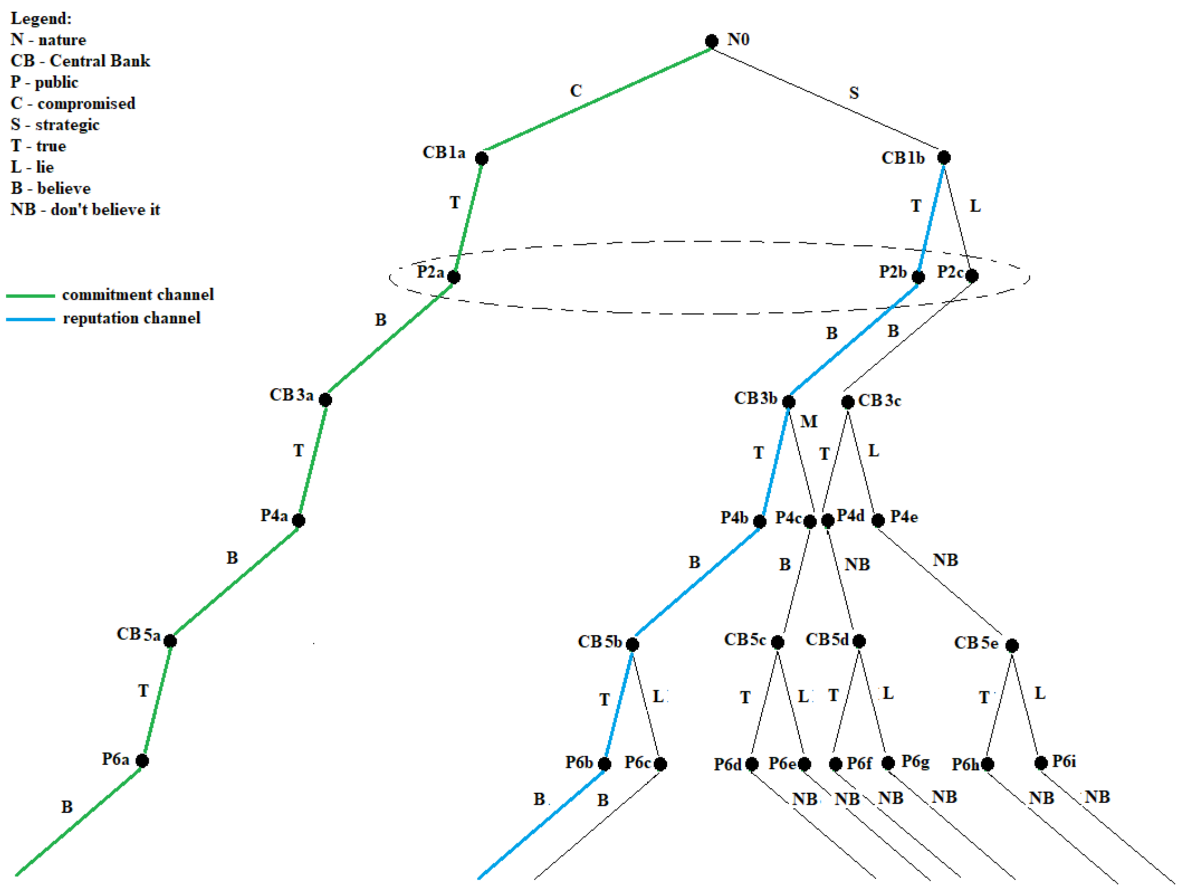

Figure 4: Game tree under complete memory. Source: Prepared by the author.

To study public behavior with bounded memory, you can think of a structure in which P takes an action using your memory just a stage, and forgets following (figure 5). This tree is the same as the previous one up to stage CB5 (nodes CB5a-e). Then the public is supposed to forget about the relationship with the central bank, and come back to believe that agent's message. So it's like the game has a fresh start from time to time. By counterpointing the base structure, the results of this configuration would be in line with the results of Monte (2013): receiver's bounded memory can explain how long-term reputation can be sustained.

Summarizing the results of the game trees with complete memory and bounded memory. In the first case, it is noted that, given the private agent's ability to learn the type of the sender,

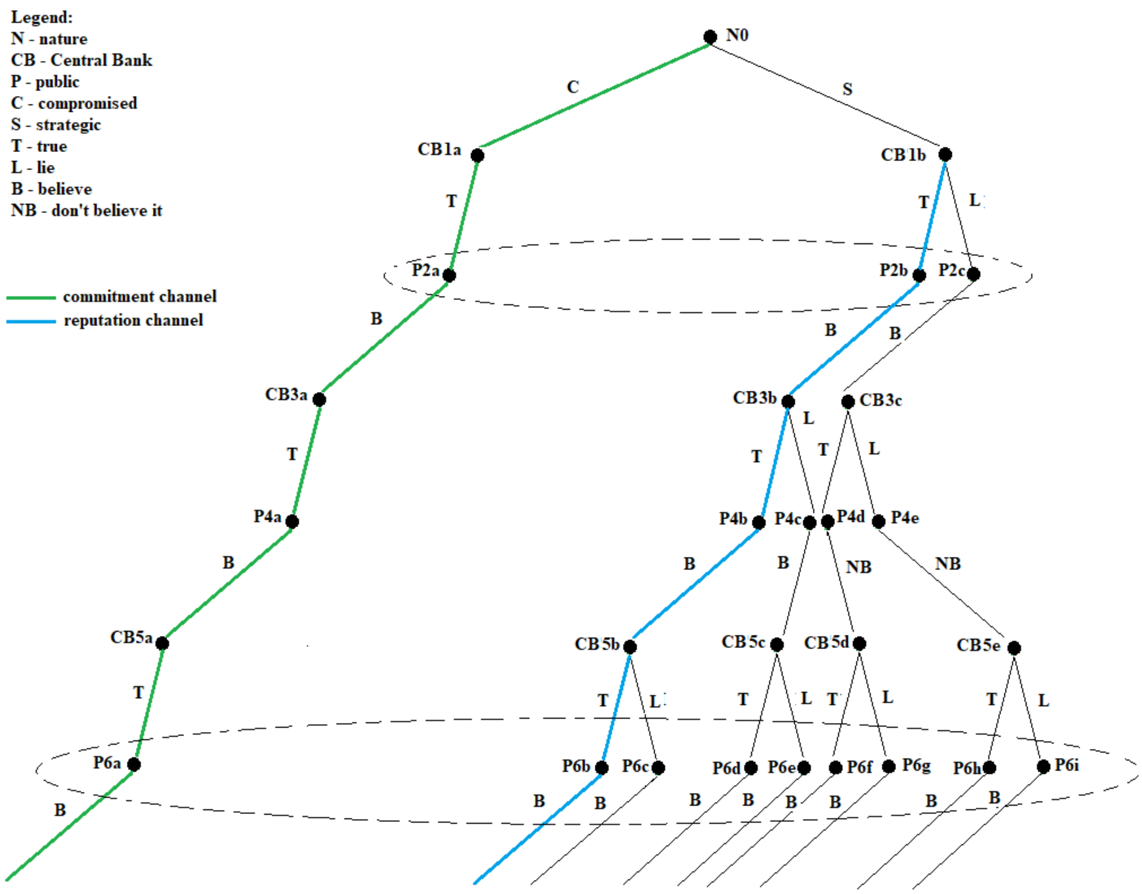

Figure 5: Game tree under bounded memory. Source: Prepared by the author.

he ceases to believe the message sent. However, when this agent faces a bounded memory problem, he forgets the lie of the sender, so he is not able to learn the type of the sender, so he continues to believe the message sent.

3.2 Canonical new keynesian model with communication of inflation targets

This study uses a canonical new keynesian model with central bank communication. Thus, there is a continuum of households indexed by $j \in [0,1]$. This representative household maximizes its intertemporal utility by choosing consumption, savings and leisure to maximize a discounted flow of expected utility. Firms are of two types: firms producing final goods, in perfect competition, aggregate intermediate goods into a single final good, which will be provided to families; and firms producing intermediate goods, in monopolistic competition, acquire labor from families to produce an input that will be supplied to the final goods sector. In addition, firms producing intermediate goods must decide the price of their product following a Calvo rule. In addition, there is a Taylor rule to maintain price stability. Finally, the categorization rule, basically, is to say whether the agent believes in the inflation target, provided by the monetary authority, or if the agent does not believe in the target. Solving this model, three equations are arrived at: IS curve; Phillips curve; and Taylor's rule. In addition, it is necessary to have a rule for determining inflation rate expectations.

The demand side of the economy is given by the following IS curve:

$$y_t = \bar{E}_t y_{t+1} - \left(\frac{1}{\sigma}\right)\left(R_t - \bar{E}_t \pi_{t+1}\right) \tag{8}$$

where y is the output gap, R is the nominal interest rate, $\bar{E}_t \pi_{t+1}$ and $\bar{E}_t y_{t+1}$ are the non-rational expectations for inflation and for the output gap, respectively, σ is the aversion to relative risk.

The supply side of the model, Phillips curve, is represented by:

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa y_t \tag{9}$$

where $\kappa = \lambda\left[\sigma + \left(\frac{\phi+\alpha}{1-\alpha}\right)\right]$ and $\lambda = \omega\left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right]$, π is the current inflation rate, β is the intertemporal discount, ϕ is the marginal disutility of labor, α is the sensitivity of labor in production, θ is the price rigidity and ψ is the elasticity of substitution for intermediate goods.

The price level stability is given by the Taylor rule:

$$R_t = \gamma_y y_t + \gamma_\pi \left(\pi_t - \pi^{\text{target}}_{\text{true},t}\right) \tag{10}$$

where $\pi^{\text{target}}_{\text{true}}$ is the inflation target that is truly pursued by the central bank, γ_y is the sensitivity of the interest rate in relation to the output gap and γ_π is the sensitivity of the interest rate in relation to the difference between current inflation and the true inflation target.

The definition of expectations formation rule on the inflation rate follows Bomfim and Rudebusch (2000). At the beginning of each period, the central bank announces its inflation target. The private sector, which is a continuum of agents $i \in [0,1]$, should assess the future reliability of these targets by judging the credibility of the monetary authority's intention, that is, whether the target represents the real objective of this authority. Thus, it is assumed that inflation expectations for the period t + 1 is the weighted average of the target announced by the central bank[2], $\pi^{\text{target}}_{\text{informed,t+1}}$, and the inflation rate of the previous period, π_t:

[2]Since the private sector is made up of a continuum of agents $i \in [0.1]$, the formation of inflation rate expectations

$$\bar{E}_t \pi_{t+1} = conf_t \pi^{\text{target}}_{\text{informed},t+1} + (1 - conf_t)\pi_t \tag{11}$$

where $0 \leq conf \leq 1$ it is the index of confidence in the inflation target of the monetary authority. If $conf = 1$ ($conf = 0$), there is perfect confidence (null confidence), and the inflation expectations of the private sector will be equal to the targets announced by the policymaker (equal to the inflation rates of previous periods). So, the idea of categorization is used here to assign values of 0 or 1 to the confidence level, based on the idea of repeated games presented in the previous subsection.

For the following equilibrium analysis, it is important to present the definitions of the categorization of confidence relationship presented in equation (11) and the expected values of the model parameters.

Definition 3.2 (Categorization)**.** Assumptions of bounded rationality and complete memory and of bounded rationality and bounded memory are those in which the private agent's confidence levels are null, $conf = 0$, and maximum, $conf = 1$, respectively, in equation (11).

Definition 3.3 (Expected values of model parameters)**.** The parameters of this paper follow the literature, that is: $\sigma \geq 0$, the closer to zero this parameter is, $\sigma \rightarrow 0$, greater the intertemporal elasticity of consumption; $0 < \beta < 1$, the closer to one this parameter is, $\beta \rightarrow 1$, the more the individual is patient in relation to the present consumption; $\phi > 0$; $0 < \alpha < 1$; $0 < \theta < 1$; $\psi \geq 0$, the closer to zero this parameter is, $\psi \rightarrow 0$, less elastic is the substitution between intermediate goods; $\gamma_y \geq 0$, $\gamma_\pi > 1 >> \gamma_y$ and $\kappa = \left(\frac{1-\alpha}{1-\alpha+\alpha\psi}\right)\left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right]\left[\sigma + \left(\frac{\phi+\alpha}{1-\alpha}\right)\right] > 0$.

Still, it is necessary to have a definition (hypothesis) about the behavior of the monetary authority.

Definition 3.4 (Behavior of the strategic central bank)**.** A strategic central bank with an inflationary bias always lies on the inflation target (Mostarini, 2007). Thus, $\pi^{target}_{informed} < \pi^{target}_{true}$ to $\forall t$.

To facilitate the presentation of the equilibrium analysis results, it uses two lemmas common in the study of numerical sequences.

Lemma 3.5. Let (a_n) be a sequence of real numbers, given $a = \lim a_n$, if $b < a$ with $a, b \in \mathbb{R}$, then for every sufficiently large n, it has $b < a_n$.

Proof. Taking $\varepsilon = a - b$, it has $\varepsilon > 0$ and $b = a - \varepsilon$, by the definition of limit, there is $n_0 \in \mathbb{N}$ such that $n > n_0 \Rightarrow a - \varepsilon < a_n < a + \varepsilon \Rightarrow b < a_n$. □

Lemma 3.6. Let (a_n) and (b_n) be sequences of real numbers, $\lim a_n = a$ and $\lim b_n = b \Rightarrow \lim \frac{a_n}{b_n} = \frac{a}{b}$, with $a, b \in \mathbb{R}$ and if $b \neq 0$.

Proof. The aim is to prove that $\lim \frac{a_n}{b_n} = \frac{a}{b}$ or $\lim \frac{a_n}{b_n} - \frac{a}{b} = \lim \frac{ba_n - b_n a}{b_n b} = 0$. So first, it will be proven that $\lim ba_n = \lim b_n a = ba$. Thus, let be $\varepsilon_1 = \frac{\varepsilon}{|b|}$, and $\forall \varepsilon_1 > 0$, $\exists n_0 \in \mathbb{N}$; $n > n_0 \Rightarrow |ba_n - ba| = |b||a_n - a| < |b|\frac{\varepsilon}{|b|} = \varepsilon$. Thus, by the definition of limit, $\lim ba_n = ab$, similarly, $\lim ab_n = ab$.
Now, it's needed to prove that $\frac{1}{b_n b}$ is bounded. Choosing $c = \frac{b^2}{2}$, it has $0 < c < b^2$. Since $\lim b_n b = b^2$, it follows from the lemma 3.5 that, for all sufficiently large n, it has $c < b_n b$, therefore, $\frac{1}{b_n b} < \frac{1}{c}$. □

follows: $\bar{E}_t \pi_{t+1} = \int_0^{conf_t} \pi^{\text{target}}_{\text{informed},t+1} di + \int_{conf_t}^1 \pi_t di = conf_t \pi^{\text{target}}_{\text{informed},t+1} + (1 - conf_t)\pi_t$

To study the effectiveness of monetary policy in the three proposed rationality assumptions, an analysis of steady state equilibrium[3] will be made. And the proposition below presents the first important result of this paper, the relationship between bounded rationality and complete memory with rational expectations.

Proposition 3.7. The output gap and the inflation rate, in a assumption of bounded rationality and complete memory, converge to the values of the assumption of rational expectations.

Proof. Assuming that $\pi_{RE,t}$ and $\pi_{CM,t}$ are the sequences of real numbers for inflation rates in assumptions of rational expectations and of bounded rationality and complete memory, respectively. Thus, given $\lim \pi_{RE,t} = \pi_{RE}$ and $\lim \pi_{CM,t} = \pi_{CM}$, where π_{RE} and π_{CM} are the steady states of these two rationality assumptions. The hypothesis is that $\pi_{RE} = \pi_{MC}$, and using Lemma 3.6, it has that $\lim \frac{\pi_{RE,t}}{\pi_{CM,t}} = \frac{\pi_{RE}}{\pi_{CM}} = 1$. Given in the steady state,

$$\pi_{RE} = \left[\frac{\kappa \gamma_y}{(1-\beta)\gamma_y + \kappa(\gamma_\pi - 1)} \right] \pi_{true}^{target} \tag{12}$$

and,

$$\pi_{CM} = \left[\frac{\kappa \gamma_y}{(1-\beta)(\sigma + \gamma_y) + \kappa(\gamma_\pi - 1)} \right] \pi_{true}^{target} \tag{13}$$

Combining equations (12) and (13) and assuming $\beta \to 1$, thus:

$$\frac{\pi_{RE}}{\pi_{CM}} = 1 + \left[\frac{\sigma(1-\beta)}{(1-\beta)\gamma_y + \kappa(\gamma_\pi - 1)} \right] \to 1$$

The same reasoning applies to the output gap when $\beta \to 1$. So,

$$\frac{y_{RE}}{y_{CM}} \to 1$$

□

Proposition 3.7 says that an agent facing bounded rationality, but having the capacity to learn from the states of nature over time, that is, capable of learning from the behavior of the central bank, converges his expectations into the assumption of rational expectations. And this would be the result of Cripps et al (2004) – reputation is a short-term phenomenon. On the other hand, proposition 3.8 presents the result when the agent does not have the capacity to learn over time, that is, the agent suffers from bounded memory.

Proposition 3.8. The output gap and the inflation rate, in a assumption of bounded rationality and bounded memory, do not converge to the values of the assumption of rational expectations.

Proof. Assuming that $\pi_{BM,t}$ is the sequence of real numbers for the inflation rate in the assumption of bounded rationality and bounded memory. Thus, given $\lim \pi_{BM,t} = \pi_{BM}$, where π_{BM} it is the steady state of this rationality assumption. The hypothesis is that $\pi_{BM} \neq \pi_{RE}$, and using Lemma 3.6, it has that $\lim \frac{\pi_{BM,t}}{\pi_{RE,t}} = \frac{\pi_{BM}}{\pi_{RE}} \neq 1$. Given in the steady state,

$$\pi_{BM} = \left[\frac{\beta(+\gamma_y) + \kappa}{\sigma + \gamma_y + \gamma_\pi \kappa} \right] \pi_{informed}^{target} + \left(\frac{\gamma_\pi \kappa}{\sigma + \gamma_y + \gamma_\pi \kappa} \right) \pi_{true}^{target} \tag{14}$$

Combining equations (12) and (14) and assuming $\beta \to 1$ and $\pi_{informed}^{target} = 0$ (even if $\pi_{informed}^{target} \neq 0$, this value can be subtracted from the stationary states of π to the three rationality assump-

[3]The steady state for the three rationality assumptions is shown in the appendix.

tions and thus have $\pi^{target}_{informed} = 0$), it has:

$$\frac{\pi_{BM}}{\pi_{RE}} = \frac{\kappa(\gamma_\pi - 1)}{\sigma + \gamma_y + \gamma_\pi \kappa} \tag{15}$$

Assuming, by contradiction, that $\frac{\pi_{BM}}{\pi_{RE}} = 1$, so from equation (15):
$\frac{\kappa(\gamma_\pi - 1)}{\sigma+\gamma_y+\gamma_\pi\kappa} = 1 \Rightarrow \kappa(\gamma_\pi - 1) = \sigma + \gamma_y + \gamma_\pi\kappa \Leftrightarrow \kappa\gamma_\pi - \kappa = \sigma + \gamma_y + \gamma_\pi\kappa \Leftrightarrow \sigma + \gamma_y + \kappa = 0$, absurd, because by definition 3.3, $\sigma \geq 0, \gamma_y \geq 0, \kappa > 0$. So, $\frac{\pi_{BM}}{\pi_{RE}} \neq 1$.

The same reasoning applies to the output gap, with $\pi^{target}_{informed} = 0$. It has that,

$$\frac{y_{BM}}{y_{RE}} = \frac{1}{\sigma + \gamma_y + \gamma_\pi \kappa} + \frac{\kappa(\gamma_\pi - 1)}{(1-\beta)(\sigma + \gamma_y + \gamma_\pi \kappa)} \tag{16}$$

With $\beta \to 1$, the second term of the equation (16) $\to \infty$, so, evidently, $\frac{y_{BM}}{y_{RE}} \neq 1$.

$\square$

Proposition 3.8 says that a private agent facing a friction of bounded rationality and bounded memory would not be able to learn the strategic type of the central bank. Thus, their expectations follow the values disclosed by the monetary authority, that is, they do not come close to the values of rational expectations. And this result agrees with Monte (2013), that is, the receiver's bounded memory can explain how the reputation can be sustained in the long-term.

From propositions 3.7 and 3.8, the results of the assumpiton of bounded rationality and complete memory converge to the values of rational expectations, while the values of bounded rationality and bounded memory do not converge. Given that, under rational expectations, monetary policy is ineffective, it is known that this phenomenon will also occur in the assumption of bounded rationality and complete memory, that is, in this assumption, monetary policy is ineffective. Even knowing that the values of the assumption of bounded rationality and bounded memory do not converge with those of rational expectations, it is still not possible to say that monetary policy is effective in this case. But the next proposition seeks to fill that gap.

Proposition 3.9. Monetary policy in the assumption of bounded rationality and bounded memory is effective.

Proof. Basically, it is necessary to prove that there is an ε-neighborhood of zero such that: $\pi_{BM} \in \varepsilon$-neighborhood of zero; $\pi_{RE}, \pi_{CM} \notin \varepsilon$-neighborhood of zero; $y_{RE}, y_{CM} \in \varepsilon$-neighborhood of zero and $y_{BM} \notin \varepsilon$-neighborhood of zero. That is, it is necessary to obtain: $\pi_{BM} < \varepsilon_1 < \pi_{RE} - \pi_{BM}$ and $y_{RE} < \varepsilon_2 < y_{BM} - y_{RE}$. In short, it is necessary to prove the existence of ε_1 and ε_2.

So, for the first case (ε_1), using the steady states of π for RE and BM, and assuming $\beta \to 1$ and $\pi^{target}_{informed} = 0$, it has:

$$\pi_{RE} - \pi_{BM} = \kappa\gamma_\pi\pi^{target}_{true}\left[\frac{\sigma + \gamma_y + \kappa}{\sigma(\gamma_\pi - 1)(\sigma + \gamma_y + \kappa\gamma_\pi)}\right] > \pi_{BM} > 0 \tag{17}$$

And, for the second case (ε_2), using the steady states of y for RE and BM, and assuming $\beta \to 1$ and $\pi^{target}_{informed} = 0$, it has:

$$y_{BM} - y_{RE} = \left(\frac{1 - \gamma_\pi\kappa}{\sigma + \gamma_y + \gamma_\pi\kappa}\right)\pi^{target}_{informed} + \left(\frac{\gamma_y}{\sigma + \gamma_y + \gamma_\pi\kappa}\right)\pi^{target}_{true}$$

$$-\left[\frac{(1-\beta)\gamma_y}{(1-\beta)\gamma_y+\kappa(\gamma_\pi-1)}\right]\pi_{true}^{target} > y_{RE} > 0 \tag{18}$$

□

Given the result of proposition 3.9, the contribution of this paper can be summarized, that is, the ineffectiveness of monetary policy under rational expectations is one of the pillars of the new classical theory. By proposition 3.7, it was obtained that this same event occurs in the assumption of bounded rationality and complete memory, given that private agents are able to learn from the behavior of the central bank, thus converging their expectations to the value of rational expectations. In other words, monetary policy remains ineffective. On the other hand, when private agents suffer from bounded rationality and bounded memory, this agent is unable to learn the type of central bank, and the reputation of this agent remains, even in the long run. Thus, under this assumption, monetary policy is effective.

The proofs for propositions 3.7, 3.8 and 3.9 require one observation, in which, $\beta \to 1$ was assumed, that at first, it may seem something ad-hoc, but the purpose of this hypothesis is to restrict the β value to the values used by the literature – which follow the actual data values. These two observations are analyzed in the following robustness test.

3.3 Robustness tests

In this subsection, the robustness tests for the β parameter and for the proportion of optimists in determining expectations about the output gap are presented. Thus, it is assumed that agents use simple rules to design the output gap following De Grauwe (2011). So, given that agents do not fully understand how the output gap is determined, their projections are biased. Thus, the hypothesis is that some agents are optimistic and systematically have a positive bias in the output gap, $\bar{E}_t^{optim} y_{t+1} = g$, while others are pessimistic and, systematically, they have a negative bias, $\bar{E}_t^{pess} y_{t+1} = -g$, where $g > 0$ is the level of bias in estimating the output gap. And the market projection is obtained as a weighted average of these two projections:

$$\bar{E}_t y_{t+1} = \alpha_{optim}\bar{E}_t^{optim} y_{t+1} + \alpha_{pess}\bar{E}_t^{pess} y_{t+1}, \quad \text{with} \quad \alpha_{optim} + \alpha_{pess} = 1 \tag{19}$$

or,

$$\bar{E}_t y_{t+1} = (2\alpha_{optim} - 1)g$$

where α_{optim} and α_{pess} represent the proportions of optimistic and pessimistic agents, respectively. And the values of the parameters used in the simulation are: $\alpha_{optim} = 0.5$; $\alpha = 0.4$; $\theta = 0.75$; $\psi = 10$; $\sigma = 2$; $\varphi = 1.5$; $\rho_A = 0.5$; $\pi_{true}^{target} = 0.005$; $g = 0.02$; $\gamma_y = 0.1$; and $\gamma_\pi = 2$.

3.3.1 β testing

Table 1 presents the results of the robustness test for the β parameter assuming values: $1/1.04^{0.25}$; $1/1.02^{0.25}$ and $1/1.005^{0.25}$, corresponding to the following annual interest rates: 4%; 2% and 0.5%, respectively. Note that as $\beta \to 1$, the output gap and inflation rate ratios approach 1 for complete memory/rational expectations, because, $\frac{y_{CM,\beta=1/1.04^{0.25},}}{y_{RE,\beta=1/1.04^{0.25}}} = \frac{0.125}{0.169} = 0.74$; $\frac{y_{CM,\beta=1/1.005^{0.25},}}{y_{RE,\beta=1/1.005^{0.25}}} = \frac{0.021}{0.025} = 0.84$, and $\frac{\pi_{CM,\beta=1/1.04^{0.25},}}{\pi_{RE,\beta=1/1.04^{0.25}}} = \frac{0.741}{0.984} = 0.75$; $\frac{\pi_{CM,\beta=1/1.005^{0.25},}}{\pi_{RE,\beta=1/1.005^{0.25}}} = \frac{0.959}{0.998} = 0.96$. On the other hand, as $\beta \to 1$, the output gap ratio $\to \infty$ and the inflation rate ratio $\to 0$ for bounded memory/rational expectations, $\frac{y_{BM,\beta=1/1.04^{0.25},}}{y_{RE,\beta=1/1.04^{0.25}}} = \frac{0.4528}{0.169} = 2.68$; $\frac{y_{BM,\beta=1/1.005^{0.25},}}{y_{RE,\beta=1/1.005^{0.25}}} = \frac{0.4534}{0.025} = 18.14$, and $\frac{\pi_{CM,\beta=1/1.04^{0.25},}}{\pi_{RE,\beta=1/1.04^{0.25}}} = \frac{0.0262}{0.984} = 0.027$; $\frac{\pi_{CM,\beta=1/1.005^{0.25},}}{\pi_{RE,\beta=1/1.005^{0.25}}} = \frac{0.0256}{0.998} = 0.026$. In other words, the values of the assumption of bounded rationality and bounded memory do not alter much with the changes in β, however, it is noticed that with $\beta \to 1$ the assumption of bounded rationality and complete memory approaches of the value of the assumption of rational expectations, the same does not happen with the assumption of bounded rationality and bounded memory, and this corroborates the results obtained in propositions 3.7, 3.8 and 3.9.

Table 1: Result of the robustness test for the β parameter (%). Source: Prepared by the author.

variables	analyzed values	Rational expectations	Complete memory	Bounded memory
output gap (y)	$\beta = 1/1.04^{0.25}$	0.169	0.125	0.4528
	$\beta = 1/1.02^{0.25}$	0.088	0.074	0.4531
	$\beta = 1/1.005^{0.25}$	0.025	0.021	0.4534
inflation rate (π)	$\beta = 1/1.04^{0.25}$	0.984	0.741	0.0262
	$\beta = 1/1.02^{0.25}$	0.992	0.849	0.0258
	$\beta = 1/1.005^{0.25}$	0.998	0.959	0.0256

3.3.2 α_{optim} testing

Table 2 presents the results of the robustness test for the proportion of optimists that determine the expectation for the output gap, 0.25; 0.5; and 0.75. It is noted that the relations between the output gap and the inflation rate between the assumptions of bounded rationality and bounded memory and bounded rationality and complete memory do not change with changes in the proportion of optimists, that is, $\frac{y_{BM,\alpha_{optim}=0.25}}{y_{CM,\alpha_{optim}=0.25}} = \frac{-0.3416}{-0.071} = 4.81$; $\frac{y_{BM,\alpha_{optim}=0.75}}{y_{CM,\alpha_{optim}=0.75}} = \frac{1.4061}{0.2923} = 4.81$, and $\frac{\pi_{BM,\alpha_{optim}=0.25}}{\pi_{CM,\alpha_{optim}=0.25}} = \frac{-0.0181}{-0.4394} = 0.041$; $\frac{\pi_{BM,\alpha_{optim}=0.75}}{\pi_{CM,\alpha_{optim}=0.75}} = \frac{0.0745}{1.809} = 0.041$. In short, despite the assumption about the share of optimists in the economy affect the level of the variables, it does not change the relationship between rationality assumptions, and the results of the propositions 3.7, 3.8 and 3.9 still apply.

Table 2: Result of the robustness test for the α_{optim} parameter (%). Source: Prepared by the author.

variables	analyzed values	Complete memory	Bounded memory
output gap	$\alpha_{optim} = 0.25$	-0.071	-0.3416
	$\alpha_{optim} = 0.5$	0.1107	0.5323
	$\alpha_{optim} = 0.75$	0.2923	1.4061
inflation rate	$\alpha_{optim} = 0.25$	-0.4394	-0.0181
	$\alpha_{optim} = 0.5$	0.6848	0.0282
	$\alpha_{optim} = 0.75$	1.809	0.0745

4 Conclusions

One of the main results of the new classical theory is the ineffectiveness of monetary policy. Thus, the aim of this paper was to discuss the central bank's behavior of providing an "intentional" wrong signal about the true inflation target, waiting for an inflationary bias to occur. For this, a canonical new keynesian model was used with communication of inflation targets on three different assumptions of rationality.

Given that rational expectations, monetary policy is ineffective. By proposition 3.7, under the assumption of bounded rationality and complete memory, the private agent is able to learn the type of strategic central bank, and thus converge on the result under the assumption of rational expectations. This is similar to the convergence of point B towards point D in figure 2. And it coincides with the result of Cripps et al (2004) that reputation is a short-term phenomenon. But, most importantly, monetary policy remains ineffective under this assumption. On the other hand, when private agents suffer from bounded rationality and bounded memory, this agent is unable to learn the type of central bank, and the reputation of this agent remains, even in the long run, and this result coincides with Monte (2013). In figure 2, this result is the one located at point B. Thus, under this assumption, monetary policy is effective.

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Appendix

Steady state

To calculate the steady state[4] for the assumption of rational expectations, the system of equations of 8-10 is assumed, resulting in:

$$R = \pi \tag{20}$$

$$\pi = \frac{\kappa}{1-\beta} y \tag{21}$$

$$R = \gamma_y y + \gamma_\pi \left(\pi - \pi_{true}^{target}\right) \tag{22}$$

From this system of equations, it is obtained:

$$\pi_{RE} = \left[\frac{\kappa\gamma_y}{(1-\beta)\gamma_y + \kappa(\gamma_\pi - 1)}\right] \pi_{true}^{target} \tag{23}$$

and,

$$y_{RE} = \left[\frac{(1-\beta)\gamma_y}{(1-\beta)\gamma_y + \kappa(\gamma_\pi - 1)}\right] \pi_{true}^{target} \tag{24}$$

To calculate the steady state for the assumption of bounded rationality and complete memory, it is assumed $conf = 0$, $\bar{E}y = 0$ and $\bar{E}\pi = \pi_{informed}^{target}$, thus, from the system of equations of 8-10, it is obtained:

$$y = -\frac{1}{\sigma}(R - \pi) \tag{25}$$

$$\pi = \frac{\kappa}{1-\beta} y \tag{26}$$

$$R = \gamma_y y + \gamma_\pi \left(\pi - \pi_{true}^{target}\right) \tag{27}$$

From this system of equations, it is obtained:

$$\pi_{CM} = \left[\frac{\kappa\gamma_y}{(1-\beta)(\sigma + \gamma_y) + \kappa(\gamma_\pi - 1)}\right] \pi_{true}^{target} \tag{28}$$

and,

[4] Without loss of generality, it was assumed that $\alpha_{optim} = 0.5$, in equation (19), such that $\bar{E}_t y_{t+1} = 0$.

$$y_{CM} = \left[\frac{(1-\beta)\gamma_y}{(1-\beta)(\sigma+\gamma_y)+\kappa(\gamma_\pi - 1)}\right]\pi_{true}^{target} \tag{29}$$

Finally, to calculate the steady state for the assumption of bounded rationality and bounded memory, it is assumed $conf = 1$, $\bar{E}y = 0$ and $\bar{E}\pi = \pi_{informed}^{target}$, thus, from the system of equations of 8-10, it is obtained:

$$y = -\frac{1}{\sigma}\left(R - \pi_{informed}^{target}\right) \tag{30}$$

$$\pi = \beta\pi_{informed}^{target} + \kappa y \tag{31}$$

$$R = \gamma_y y + \gamma_\pi\left(\pi - \pi_{true}^{target}\right) \tag{32}$$

From this system of equations is obtained:

$$\pi_{BM} = \left[\frac{\beta(+\gamma_y)+\kappa}{\sigma+\gamma_y+\gamma_\pi\kappa}\right]\pi_{informed}^{target} + \left(\frac{\gamma_\pi\kappa}{\sigma+\gamma_y+\gamma_\pi\kappa}\right)\pi_{true}^{target} \tag{33}$$

and,

$$y_{BM} = \left(\frac{1-\gamma_\pi\kappa}{\sigma+\gamma_y+\gamma_\pi\kappa}\right)\pi_{informed}^{target} + \left(\frac{\gamma_y}{\sigma+\gamma_y+\gamma_\pi\kappa}\right)\pi_{true}^{target} \tag{34}$$