Key sectors in Brazilian production networks

Área 3: Economia Regional e Urbana

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Abstract

This work aims to characterize the Brazilian economy productive structure as an activity network. Using networks analysis methods, 2010 national input-output matrix data from the Brazilian statistics bureau (IBGE) and 2011 subnational matrices from Haddad, Júnior and Nascimento (2017), we were able to describe the interdependence among resources employed by sectors in the production of intermediate and finished goods, and also verify how shocks in final demand and gross production vectors impact the whole economy. Results show that retail and wholesale trade, financial services and land transportation are key sector in the Brazilian economy. In the subnational approach, activities linked to the food industry and skilled labor were found to be key transmitters of economic fluctuations.

Keywords: networks, production, input-output.

JEL Codes: C67, D57, C45.

Resumo

O presente trabalho tem como objetivo caracterizar a estrutura produtiva e identificar setores centrais na economia brasileira como uma rede de atividades. Combinando a metodologia de redes complexas com dados da matriz insumo-produto de 2010 do Instituto Brasileiro de Geografia e Estatísticas e dados da matriz insumo-produto inter-regional de Haddad, Júnior and Nascimento (2017) para 2011, é possível descrever as interdependências entre recursos e usos empregados pelos setores da economia na produção de bens intermediários e finais, além de verificar como choques nos vetores de demanda final e valor bruto da produção se propagam em toda a economia. Os resultados apontam para o comércio por atacado e varejo, serviços financeiros e transportes terrestres como setores centrais na análise nacional, além de enfatizar a importância das atividades ligadas ao setor alimentício e científicas/técnicas nas unidades federativas.

Palavras chave: redes, produção, insumo-produto.

Códigos JEL: C67, D57, C45.

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1 Introduction

This work aims to describe the Brazilian economy as a complex network of economic activities, starting from national and subnational input-output matrices, as well as the impact of shocks emerging from a specific sector to the whole economy. The work revolves around an emerging literature that focuses on the propagation of economic shocks from disaggregated levels. This discussion gained relevance since the 2008 financial crisis, as central bankers went to the rescue of big commercial banks, justified by fears of bank runs and greater economic collapse, through a cascade effect. To quantify this effects is to better understand how connected the many sectors of modern economies are. As recalled by Carvalho (2014), at the time of the 2008 financial crisis, many U.S. automakers such as Ford, General Motors and Chrysler plead for financial aid, not only to themselves but also to their competitors, arguing that production lines would be quickly halted in the event of bankruptcy of competitors. Through cascade effects, it may not be the case that a sector is the greatest of its economy because it moves most resources or generates the greatest value added. Else, there may exist a cluster of economic activities that can rapidly propagate economic fluctuations.

To analyze economic sectors as integrated entities, we note the increasing popularity of the complex network methods, to be further defined in the next section. Complex networks derive from graph theory, that proposes a mathematical foundation to represent relationships between *vertices* and *edges* (or *nodes*). Using a network analysis, we can understand economic relationships through topology, where a graphic visualization allows for detection of economic linkages that are not obvious when using statistical methods. In addition, we can find shortest paths and strong linkages between sectors. In general, networks are directed, that is, edges have a clear direction from a vertex to another (or from a vertex to itself, forming a feedback loop). Finally, we can identify not only strategic sectors of an economy, but also economic clusters, determined by the strongest linkages.

The usual starting point in the analysis of economies as complex networks are input-output matrices, that synthesize relationships between economic sectors or activities. A sector can obtain their own inputs or buy from other sectors to produce goods. When representing an economic network, we define sectors as the *edges*, and their flows of intermediate inputs as *vertices*. Using this framework, we can evaluate the impacts of fluctuations in specific sectors, define measures of centrality (objective measures of economic complexity), the intensity of economic flows and the formation of economic clusters. In the next section, we will bring a brief overview of the literature on the topic of economic networks.

2 Literature review

The literature on the description of global economies as networks is fast growing. As of 2018, a web search using the keyword *"economic networks"* in the Elsevier's *ScienceDirect* platform, in the field of Economic, reveals more that 100 results, and around 90 results in the *JSTOR* platform. McNerney, Fath and Silverberg (2013) comments that there are applications of complex networks in finance, international commerce and innovation. There are a number of network applications in input-output analysis (AROCHE-REYES, 2003; ACEMOGLU et al., 2012; CARVALHO, 2014). Reuniting multi-country data, Blöchl et al. (2011) discuss several measures of edge importance and the degree of economic interconnection. The author's main contribution was to provide metrics that take into account the fact that sectors can reuse their outputs back into the productive process. Acemoglu et al. (2012) points out that economic cycles can be recalled from productivity shocks and inter-sectorial relationships within an economy. The authors show how the organization of the economy (as vertical or horizontal production chains) affect how economic shocks are propagated, in terms of speed and magnitude, generating cascade effects.

Now, we will further discuss applications of complex networks in Economics. Bargigli and Gallegati (2011) develop a methodology for systemic risk analysis in financial markets, with the concept of multiple graphs (networks with more than one class of edges). In the same line, Trancoso (2014) evaluates the degree of interdependence among global economies, to test the hypothesis of the recent decoupling of growth rates between emerging and developed economies. To reach its objectives, the author combine econometric and network metrics, using their dynamic correlations as the strength of economic links. One advantage of this technique is to identify economic clusters, by applying multivariate analysis. Researchers are also curious as how technical coefficients of input-output matrices are distributed. In this sense, Liang et al. (2016) finds that they are approximately follow the log-normal or Weibull probability distributions.

Some works are interested in the spatial properties of economic networks. We highlight two articles that study the Chinese economy, known for its recent fast economic growth pace but also for the geographical dispersion of activities. Sun, An and Liu (2018) identified strategical activities spread across Chinese provincies, by applying network analysis techniques to regional input-output data. The authors are able to find the protagonist activities in each region. They also find that, in the Chinese economy, a great deal of value added is produced within the sector's own production chain. In another approach, Sen et al. (2013) focus on the geographical aspects of the Chinese production network, with a finding that, while intuitive in the economic sense, is nonetheless interesting: financial activities are linked with distant regions, while labor-intensive activities are contained in regional clusters.

Hossu et al. (2009) dive into theoretical aspects of the topic. Precisely, they argue that the phenomenon of globalization resulted in greater economic complexity, increasing uncertainty and the volatility of economic forecasts. The closer to the reality we want to represent an economic network, the more agents and links we need to specify. Following the same reasoning, Schweitzer et al. (2009) point out that to identify patterns and behaviors when lots of information are available, one must use tools such as time series analysis, simulations and elements from the theory of complexity. The authors argue that applications of complex systems in the natural sciences showed that large shocks are needed in order to obtain significant changes in the system as a whole. Hence, in an economic system, a large number of shocks at microeconomic level (as in vertices distant from the center of the network) can propagate and generate instability, depending of the strength of economic relationships between agents. This sheds light into an observation from behavioral economics: as exemplified by many economic crises, even solid institutions can fall as a result of bank runs, which are events caused by a large number of small agents. An example can be found in Lux and Kiel (2014): the authors build a model where entrepreneurs and banks interact in a financial market. Through simulations and stability analysis, they find that there is no stable "intermediate" result: or the financial market is barely affected, or a cascade effect deeply affects both groups of agents. In this case, there is exist a threshold value in some of the fundamentals of the modelled financial market, such as bank leverage.

Blöchl et al. (2011) characterize how shocks in economic networks would propagate at the macroeconomic level. With the ideas of Black (2010) as a starting point, the authors show that supply shocks at intermediate sectors have strong impacts in the whole economy, as trade between sectors is endogenous in the input-output accounting, while other factors such as firm distribution, profits and technology shocks are exogenous. Excess supply in intermediate sectors is absorbed by a large part of other sectors, as all absorbed demand represents new expenditures in some other sector. However, the authors consider that – individually – shocks have low magnitude, and should not have disruptive effects in the economy as a whole (BLÖCHL et al., 2011, p. 3). This is justified as the Leontief model admits fixed proportions of each input in the sector productive process.

In terms of the production networks, which are our object of study, the proposed methodology allows us to go a step beyond conventional input-output analysis (i.e., the monetary impact of shifts in final demand of sectors). By defining a production network, we can identify key sectors, clusters and fragile spots, according to the number and strength of linkages between the Brazilian economic activities. As will be further discussed, the inter-sectoral technical coefficients of input-output matrices will be used as this measure of economic interconnection.

2.1 Input-output matrices

Tan et al. (2019) consider that input-output models provide a powerful and versatile structure for the analysis and optimization of linear networks. Most methods are used with the aim of analyzing economic structure, while input-output analysis also applies to other analog network structures. Input-output modelling emphasizes the interdependence of relationships between raw materials and finished goods in the productive process (MILLER; BLAIR, 2009). Starting with the basic model from Leontief (1941), the input-output matrix identifies the sectors are linked to each other and reveal value aggregation to the final products, from primary to high-tecnology products and services in a given point in time (GUILHOTO, 2011). The matrix incorporates the macroeconomic identities in the final demand vector, that aggregates household and government consumption, investment and net exports (GUILHOTO, 2011). Goods are characterized by distinct goals, such as primary, intermediate and final goods. Among these, there are a variety of specific characteristics, and even when sectors do not buy and sell from each other, an indirect relationship emerges: some capital goods indirectly buy from the extractive sector (i.e. iron ore), and they are linked by other activities such as steelworks (LIMA, 2018).

The economic flows between sectors can be represented in the input-output matrix. These values are registered in rows and columns and depict our core of analysis. Every sector presents their final demand and total product, as well as the ratios of used inputs, specifically from each sector. In the input-output analysis, a sector increases its production in response to a positive shift in its final demand, because they need to buy more inputs. Sectors that supply such inputs also increase their demands, resulting in the growth of the supply chain (MILLER; BLAIR, 2009).

The complexity measure of Hausmann et al. (2011) is based on diversity and distinct features of products. Goods embed the knowledge and technology required to produce them, revealing how complex is the productive process of a country. So, comparative advantages of any country are exposed. However, as of today, studies rely heavily in data relative to exports. Thus, Lima (2018) consider that to overcome this limitation the input-output theory is more appropriate instead of data on trade to build complex networks. The Leontief matrix show the connectedness between sectors that buy and sell primary and intermediate inputs, where final demand signals increase or decrease of production, generating impacts that spread to all sectors in the economy, according to their degree of connection. Hence, it is possible to find how much chain effects can be generated in total production with respect to an upward shift in the demand for some final good. The technical coefficient is the ratio that reveal how much inputs of a given sector are required to produce a unit of some other final good (MILLER; BLAIR, 2009).

Guilhoto (2011) consider that specific sectors, individually, have a relatively small quantity of relationships in terms of suppliers and clients. However, there are important indirect linkages that result from their chain of dependence. The return from labor, capital and land, in example, generate the income used in the economy for household consumption and investment. On the other hand, government consumption is generated by taxes levied. The generated demand induce economic activities to increase or reduce their production, and this interactions create a powerful multiplier effect in production chains. Miller and Blair (2009) consider two types of multipliers: the *type I multiplier* occurs when multiplying effects from intermediate goods also raises the demand for raw materials and intermediate goods. The *type II multipliers* consider households endogenously: demand for labor increases purchasing power, and by induction productive sectors augment their supply – that consequently increase the demand for intermediate inputs, until equilibrium is reached.

Still concerning impact analysis, the Rasmussen-Hirschman index (RASMUSSEN, 1956; HIRSCHMAN, 1958) allows to evaluate what are the sectors with greatest linkage power within an economy. The index calculates *backward linkages* (how much a sector demand inputs from other sectors) and *forward linkages* (how much a sector supplies inputs to other sectors). Some other methods also apply: field of influence (SONIS; HEWINGS, 1989, 1995), intensity matrix (SONIS et al. 1997; SONIS;

HEWINGS,1999) and the GHS method (GUILHOTO; SONIS; HEWINGS, 1996). Guilhoto (2004) and Tan et al. (2019) point out that it is important, given the evolution in the exploration of input-output matrix data, to develop techniques that bring a better overview of the economic "topography" of a region. Authors such as Acemoglu et al. (2012), that analyzed the network of intersectoral relationships of the U.S. from 1972 to 2002, and Lima (2018), that studied the properties of the world input-output network in 2000 and 2014 (comparing Brazil and the U.S.), have been employing the economic network approach.

3 Methodology

3.1 National analysis

For the national empirical analysis, we use the inter-sectoral technical coefficient matrix from *Instituto Brasileiro de Geografia e Estatística (IBGE)*, the Brazilian statistics bureau[4], corresponding to the year 2010. The data derives from the Brazilian make and use tables, that shows how domestic and foreign products are used by sectors. Data is accounted at basic current prices, free of taxes/subsidies and commerce/transportation markups.

It is assumed that the demand for products is proportional to the sector's market share. Consider a model with i products and j activities:

$$\mathbf{V} = \mathbf{D}\mathbf{q}^{-1} \tag{1}$$

where $\mathbf{V}_{i\times n}$ is the production matrix, that determines the value of production for each product, $\mathbf{D})i\times n$ is the market share matrix and $\mathbf{q}_{i\times 1}$ is the vector of gross value of production (henceforth abbreviated as GVP) for each product. Then the technical coefficient matrix is calculated for each activity $\mathbf{Bn}_{n\times n}$, that derives from the intermediate consumption matrix $\mathbf{Un}_{n\times n}$ and the vector of GVP per activity $\mathbf{g}_{n\times 1}$:

$$\mathbf{Bn} = \mathbf{Un}\cdot\mathbf{g}^{-1} \tag{2}$$

The value of production per product is given by:

$$\mathbf{q} = \mathbf{Un}\cdot\mathbf{i}+\mathbf{Fn} \tag{3}$$

where $\mathbf{Fn}_{n\times n}$ is the matrix of final demand for products and $\mathbf{i}_{n\times 1}$ is a column vector with all entries equal to 1. Plugging (2) in (3) we have that:

$$\mathbf{q} = \mathbf{Bn}\cdot\mathbf{g}\cdot\mathbf{i}+\mathbf{Fn} \tag{4}$$

$$\mathbf{q} = \mathbf{Bn}\cdot\mathbf{g}+\mathbf{Fn} \tag{5}$$

Inserting 5 in 1:

$$\mathbf{V}\cdot\mathbf{i} = \mathbf{D}\cdot\mathbf{q}^{-1}\cdot\mathbf{i} \tag{6}$$

$$\mathbf{g} = \mathbf{D}\cdot\mathbf{q} \tag{7}$$

Inserting 7 in 5:

$$\mathbf{q} = \mathbf{Bn}\cdot\mathbf{D}\cdot\mathbf{q}+\mathbf{Fn} \tag{8}$$

[4]While the technical coefficients matrix links products with sectors, the inter-sectoral matrix links sectors with sectors. The Brazilian input-output matrix contains 67 sectors and 127 products.

Finally, plugging 5 in 7 we have the input-output model for sectors:

$$\mathbf{g} = \mathbf{D} \cdot (\mathbf{Bn} \cdot \mathbf{g} + \mathbf{Fn}) \tag{9}$$

$$\mathbf{g} = (\mathbf{I} - \mathbf{D} \cdot \mathbf{Bn})^{-1} \cdot (\mathbf{D} \cdot \mathbf{Fn}) \tag{10}$$

The entries of the matrix $\mathbf{D} \cdot \mathbf{Bn}$ are the direct technical coefficients between activities, that represent inter-sectoral relationships. These are the entries used in our complex network model. The technical coefficient informs how each sector contributes to the GVP of all sectors, including his own[5]. Using the input-output system, we can predict shifts in the demand for inputs of all sectors, in response to shocks in the GVP of a specific sector, with all production technologies held constant. Still, in the equation above, the Leontief matrix is represented by $(\mathbf{I} - \mathbf{D} \cdot \mathbf{Bn})^{-1}$, henceforth defined as $\mathbf{L}$. The entries of $\mathbf{L}$ are the partial multipliers that show the overall impact of the increase in R$ 1 in the GVP of a sector (or all sectors, obtained by the column sums of $\mathbf{L}$).

3.2 Subnational analysis

This subsection evolves from the efforts of Haddad, Júnior and Nascimento (2017) in constructing a Brazilian input-output system disaggregated by all states in the federation. In this system, the matrix coefficients derive from state make and use tables and tax data from *Conselho Nacional de Política Fazendária (CONFAZ)*, the Brazilian tax policy council. We use the tables estimated by the authors for the year 2011. This subnational system allows not only to determine key sectors in the Brazilian economy, but also to characterize geographical relationships in the Brazilian productive structure – although, for simplicity's sake, this work does not explore interconnections between states, treating them in isolation.

Consider the simple input-output model with S sectors:

$$\mathbf{x} = \mathbf{Ld} \tag{11}$$

where $\mathbf{x}_{s\times 1}$ is the GVP of each sector, $\mathbf{L}_{s\times s}$ is the inverse Leontief matrix and $\mathbf{d}_{s\times 1}$ is the final demand vector. The Leontief matrix derive from the inter-sectorial coefficient matrix, $\mathbf{Z}$ (also defined as $\mathbf{D} \cdot \mathbf{Bn}$ in the national model), such that $\mathbf{L} = (\mathbf{I} - \mathbf{Z})^{-1}$.

3.3 Network metrics

In the national analysis, the Brazilian network of activities is represented as a complex, directed and weighted network, containing $i, j = 1, 2, \ldots, S,\ S = 67$ sectors (vertices). The representation of the subnational network is analogous. Relationships between sectors can be defined by an adjacency matrix $Z_{S\times S}$. The simplest adjacency matrix can take values $z_{i,j} = 1$ if there is and edge from vertex j to vertex i, and 0 otherwise. However, we will input the inter-sectoral technical coefficients in the adjacency matrix, to determine the intensity of links among sectors:

$$z_{i,j} = w_{i,j} \tag{12}$$

where $w_{i,j}$ are the entries of the $\mathbf{D} \cdot \mathbf{Bn}$ matrix. In other terms, to produce intermediate or final goods, a sector can combine other goods to feed as inputs, or feed back outputs into the production technology. Hence, we characterize the economy as the list of inputs to be used in the productive process of each sector.

[5] In the input-output analysis, the GVP derives from intermediate consumption and final demand. Then, we can calculate the impact in production, in all sectors, of a variation in the final demand in some activity. This will be given by the technical coefficients, that link what is consumed to what is produced.

Next, we present metrics that aim to summarize characteristics of the production networks. The simples metric is the weighted in-degrees and out-degrees that, respectively, are the sums of links that reach or depart from sector i:

$$g_i^e = \sum_j z_{i,j}, \quad g_i^s = \sum_j z_{j,i} \tag{13}$$

Note that the weighted degrees are simple row and column sums of the adjacency matrix $\mathbf{Z}$. Here the use of technical coefficients become clear: sectors that exhibit higher in-degrees are the ones whose contribution from other sectors are the greatest. Analogously, sectors with high out-degrees are the ones that contribute with inputs to other sectors. Hence, shocks in the final demand or relative prices of these sectors generates powerful economic fluctuations along production chains.

Now, we move on to centrality measures. Centrality is a key concept in the study of complex networks, as they aim to find influent vertices (sectors) in the set. The first measure, *vertex betweenness*, can be calculated for a sector i using the formula:

$$vc(i) = \sum_{i \neq j \neq k} \frac{\gamma_{j,k}(i)}{\gamma_{j,k}} \tag{14}$$

where $\gamma_{j,k}$ is the count of shortest paths between vertices j and k, and $\gamma_{j,k}(i)$ is the count of shortest paths between vertices j and k that visit vertex i.

We will also calculate *eigenvector centrality*. This is an interesting metric as it weighs not only the number of links between sectors, but also their magnitude: e.g., sectors that have few but important buyers of inputs can appear as central, in opposition to an economy where relationships are pulverized. Using eigenvector centrality scores, we can find sectors which buy (or sell) the largest quantities of inputs. Following Contreras and Fagiolo (2014), we calculate the scores using the formula:

$$\mathbf{X} = (\mathbf{I} - \lambda \mathbf{Z}'\mathbf{Z})^{-1}\mathbf{1} \tag{15}$$

where $\mathbf{X}$ is the vector of eigenvector centrality for all sectors that provide inputs, λ is the largest eigenvalue and $\mathbf{Z}$ is our inter-sectoral technical coefficients matrix.

Besides centrality measures, there are also methods to find clusters in complex networks. We employ the algorithm proposed by Girvan and Newman (2002). The algorithm can be described by the following steps: (i) edge centrality for all edges in the network are calculated; (ii) the edge with greatest centrality is removed; (iii) the edge centrality for all edges affected by the previous removal are recalculated; (iv) the algorithm repeats until no more edges can be removed. The algorithm can identify groups with strong linkages among its members, but weak linkages with other groups.

Finally, measures of diffusion are also proposed. We consider a diffuse sector as having a large number of relevant interactions with other members. Sectors with low diffusion, in general, are the ones that reinvest their outputs back into their productive process. To evaluate the diffusion of sectors, we implement a measure proposed by Contreras and Fagiolo (2014), based on the Herfindahl-Hirschmann market concentration index:

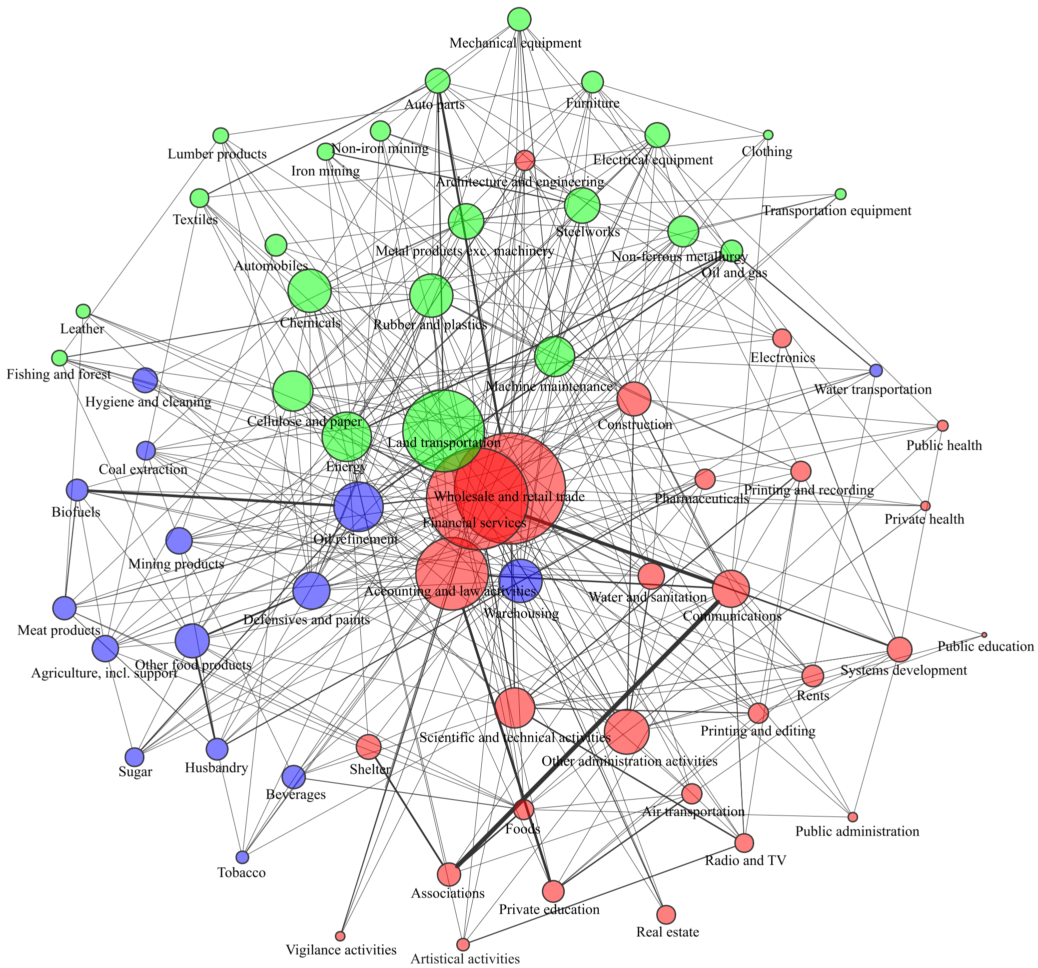
$$H_t = \sum_{s=1}^{S} \left(\frac{l_{st}}{\sum_{s=1}^{S} l_{st}} \right)^2 \tag{16}$$

where $l_{s,t}$ is the s,t entry of the Leontief matrix $\mathbf{L}$. Lower values of H_t means higher diffusion of sector t in our model economy. Applied to all sectors, we can approximately define the diversity of the economy as a whole. In this work, we will report for each state the sector with the highest diffusion index (presented as $1 - H_t$ for the ease of reading), with the aim to identify the most diffuse sectors in the subnational economies, that is, what are the sectors with the greater number of relevant interactions, as buyers and sellers of inputs. As the calculation derives from Leontief coefficients, indexes are comparable.

4 Results

Figure 1 represents the Brazilian input-output structure as a complex network, starting from the intersectoral coefficients[6]. Vertices are positioned on the graph according to the algorithm of Kamada and Kawai (1989), which aims to place the more important vertices in the center. Vertex size is defined according to their *weighted degree*, which is a measure of centrality. The thickness of edges are proportional to the technical coefficients[7].

Figure 1: The Brazilian production network



Source: The authors.

In Figure 1, vertices are colored according to the clustering algorithm of Girvan and Newman (2002). The result is analogous of a dendogram, a common tool in exploratory factor analysis, a branch of multivariate analysis. Three clusters are identified, and they seem to group activities of similar nature: (i) sectors that extract raw materials; (ii) manufactures and (iii) activities linked to services. We can also identify strong linkages between sectors, such as financial services, communications and the supply chains of oil and iron ore.

The main cluster that emerges from the analysis is the cluster of activities linked to services, with three key activities: wholesale and retail trade, financial services, and legal/accounting services. Within the cluster of extractive activities, we highlight oil refinement and storage (silos and warehouses). In the manufactures cluster, the key activites are land transportation and energy.

[6]Drawn on `R` statistical software, with the aid of package `igraph`.

[7]For easier reading, edges representing self-loops are omitted

4.1 Network topology

How our network data compares to other measures of closeness of the economy? Table 1 compares the ten most relevant sectors, according to our measure of weighted outdegree with the Rasmussen-Hirschmann backward linkage index. The latter is defined as:

$$U_j = \left(\sum_{j=1}^{i} l_{ij}/S \right) /\mathbf{L}^* \tag{17}$$

where $l_{i,j}$ are the entries of the Leontief matrix, S is the number of sectors and $\mathbf{L}^*$ is the arithmetic mean of all the entries of the Leontief matrix. In simple terms, the backward index shows the sectors that demand most inputs in comparison with the whole economy.

Table 1: Ranking of sectors according to weighted out-degree and linkage index

Rank	Weighted out-degree	Backward linkage R-H index
1	Retail and wholesale trade	Meat processing and products
2	Financial services	Other food products
3	Land transportation	Biofuels
4	Law and accounting activities	Oil refinemet
5	Energy and natural gas	Sugar products
6	Oil refinement	Steelworks
7	Adminstrative activities	Tobacco products
8	Storage services	Fabricação de veículos, exc. peças
9	Rubber and plastic products	Iron and steel tubes production
10	Chemicals	Beverages

Source: The authors.

The comparison shows that results are diverse, but this happens mostly because of the elements used in each metric: while the weighted outdegree uses the elements of the technical coefficients matrix, the backward linkages index use the Leontief multipliers. In consequence, the weighted out-degree reveals sectors that contribute for the GVP, while the backward linkage index explores sectors that respond to shifts in the final demand vector of the economy. Second, the weighted outdegree emphasizes the number of links between sectors, in order to find sectors that are in fact central to the Brazilian economy.

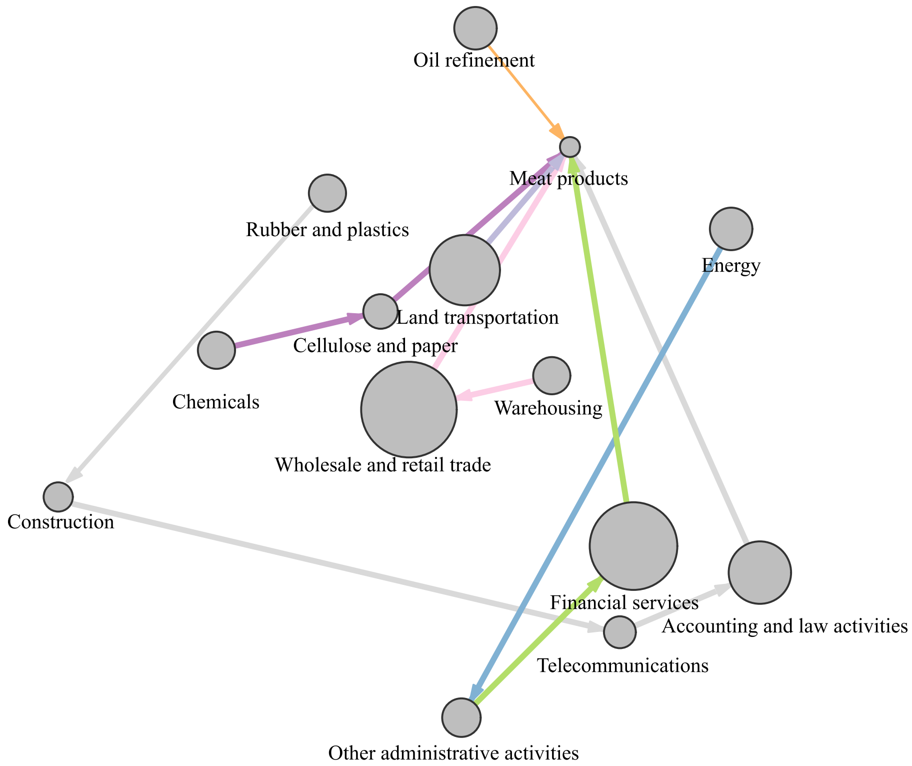
Now, based on the linkages between sectors, we can determine how perturbations to the technical coefficients (i.e. shifts in the relative prices of inputs) of the sectors identified earlier as central propagate throughout the economy. This can be achieved by analyzing the *geodesics* starting from a vertex. The extent to which shocks will be propagated is sensitive to the strength of sector linkages, defined by the technical coefficients. The economic intuition is that the shortest paths show which sectors will be first impacted by shifts in the GVP of a given sector, and so on until shocks dissipate. Figure 2 shows the geodesics that start on the sectors with the highest weighted outdegrees:

The meat products sector appears as the main final destination of shocks incoming from other sectors, including ones that are seemingly unrelated to the foods production chain. This reinforces the hypothesis of economies as production networks as it reveals interconnections and cascade effects, that may affect prices, wages and welfare.

4.2 Subnational analysis

Table 2 presents the sector with greater vertex centrality score in each Brazilian state. The third and last column of the Table shows what is the impact of the increase in R$ 1 in the respective sector

Figure 2: Shortest paths departing from key sectors



Source: The authors.

final demand in the GVP of the state[8]:

Interesting patterns emerge from the results of the subnational analysis. As shown by the shares of sectors in the state GVP, centrality measures do not identify key sectors in terms of value added (i.e., retail and wholesale trade), but sectors whose shifts in technical coefficients affect the most activities in the subnational economies. Notwithstanding, vertex centralities detect sectors known for its relevance in each state (i.e., public administration and social security in Distrito Federal, oil and gas extraction in Rio de Janeiro, meat processing in Mato Grosso and Mato Grosso do Sul). In addition, we can identify the importance of the food production chain in Brazil, in 13 of 27 states. Here, a counterpoint emerges, due to the fact that – as observed in Table 1 – foods and meat processing sectors appear at the top in Rasmussen-Hirschmann backward linkage indexes. Hence, the food sector is able to propagate significant economic shocks, emerging from the supply of other inputs.

Table 3 present sectors identified as central in every state according to eigenvector centralities. As opposed to observed in Table 2, there may be more than one sector identified as central. This happens because we can have multiple eigenvalues satisfying Equation (15). Here, the sectors more commonly found differ from those found in Table 2, respectively: (i) retail and wholesale trade; (ii) non-metallic mineral products; (iii) meat processing, including dairy and fishing products; (iv) architecture and engineering services.

How both centrality measures are correlated? Figure 3 shows the estimated scores for each combination of state and sector[9]. The Figure shows that the methods differ slightly, and the overall correlation isn't strong (.187). In fact, while vertex centrality focuses on the quantity of linkages in input-output relationships in each state, eigenvector centrality – by design – is more sensible to the

[8]This measure is analogous to the weighted in-degree of the production network, calculated as the row sum of a sector s in matrix $\mathbf{Z}$ of each state $\sum_s z_{s,t}$.

[9]With 27 states and 68 sectors we have 1836 data points.

Table 2: Key sectors according to vertex centrality, by state

State	Key sector	Part. GVP
Acre	Meat processing and production	0.14
Alagoas	Other food products	0.22
Amapá	Wholesale and retail trade	2.64
Amazonas	Rubber and plastic products	0.57
Bahia	Organic and non-organic chemical products	0.72
Ceará	Meat processing and production	0.03
Distrito Federal	Public administering, defense and social security	0.16
Espírito Santo	Paper and cellulose products	0.19
Goiás	Architecture, engineering and R&D	0.18
Maranhão	Other food products	0.08
Mato Grosso	Meat processing and production	0.18
Mato Grosso do Sul	Meat processing and production	0.14
Minas Gerais	Meat processing and production	0.14
Pará	Meat processing and production	0.18
Paraíba	Construction	0.35
Paraná	Non-metallic mineral products	0.15
Pernambuco	Non-metallic mineral products	0.18
Piauí	Other food products	0.20
Rio de Janeiro	Oil and gas extraction, and support activities	0.25
Rio Grande do Norte	Other food products	0.03
Rio Grande do Sul	Non-metallic mineral products	0.18
Rondônia	Meat processing and production	0.18
Roraima	Construction	0.37
Santa Catarina	Non-metallic mineral products	0.17
São Paulo	Meat processing and production	0.09
Sergipe	Organic and non-organic chemical products	0.52
Tocantins	Meat processing and production	0.16

Source: The authors.

variability of intersectoral technical coefficients. Note that, in many points, vertex centrality is null, while eigenvalue centrality is not.

Table 4 present the diffusion indexes for sectors, calculated according to Equation (16). Recalling the discussion in the Methodology section, the higher the index, the more diffuse is the usage of inputs for a sector. In the same manner, a sector with low diffusion index buys or sells inputs to a reduced number of other sectors. We report the values for the most diffuse sector, in each state.

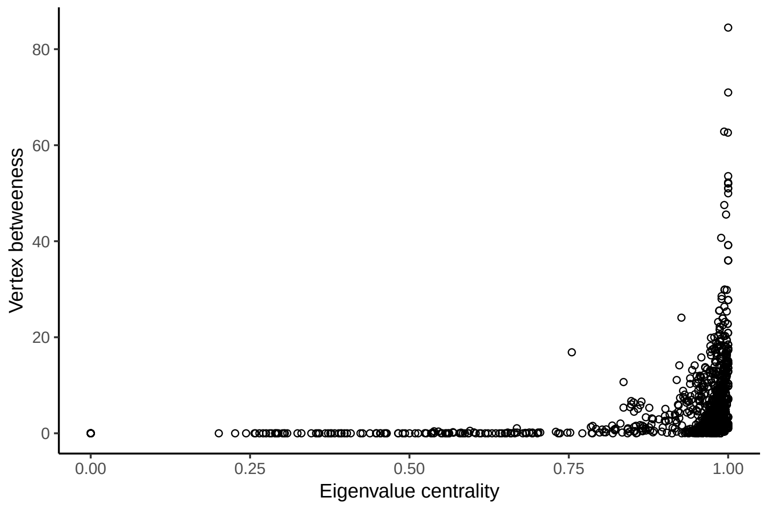
Among the results, the sector of professional, scientific and technical activities prevail, followed by meat processing, dairy and fishing products. The index seems to capture the fact that professional activities is a potential supplier of inputs such as skilled labor. By construction, the indexes are comparable, and the highest value is found in Rio de Janeiro (.684). We found that the diffusion index is a useful tool to build a ranking of key sectors in states, by the supply side. However, there seems to exist no empirical link between the characteristics of a sector and the state GVP when evaluating for the diffusion index, nor emerges any geographical aspect.

Table 3: Key sectors according to eigenvector centrality, by state

State	Key sectors	Part. GVP
Acre	Wholesale and retail trade	2.47
Alagoas	Wholesale and retail trade	2.23
Amapá	Architecture, engineering and R&D	0.35
Amazonas	Electrical machinery and equipment production	0.24
Bahia	Non-ferrous metallurgy	0.23
	Architecture, engineering and R&D	0.31
	Mechanical machinery and equipment production	0.05
	Rubber and plastic products	0.16
Distrito Federal	Public administering, defense and social security	0.16
Espírito Santo	Paper and cellulose products	0.19
Goiás	Architecture, engineering and R&D	0.18
Maranhão	Construction	0.39
Mato Grosso	Meat processing and production	0.18
Mato Grosso do Sul	Paper and cellulose products	0.27
Minas Gerais	Hygiene and cleaning products	0.01
	Machinery and equipment repairs	0.46
	Architecture, engineering and R&D	0.35
Pará	Chemical products	0.06
Paraná	Architecture, engineering and R&D	0.22
Paraíba	Construction	0.33
	Architecture, engineering and R&D	0.22
Pernambuco	Non-metallic mineral products	0.15
Piauí	Wholesale and retail trade	2.95
Rio de Janeiro	Paper and cellulose products	0.07
Rio Grande do Norte	Wholesale and retail trade	2.77
Rio Grande do Sul	Non-metallic mineral products	0.19
Rondônia	Construction	0.45
Roraima	Energy, natural gas and other utilities	0.86
Santa Catarina	Steelworks	0.26
São Paulo	Mechanical machinery and equipment production	0.18
Sergipe	Construction	0.37
Tocantins	Meat processing and production	0.16

Source: The authors.

Figure 3: Correlation between centrality measures



Source: The authors.

Table 4: Key sectors according to the diffusion index, by state

State	Sector with greatest diffusion index	Index
Acre	Professional, scientific and technical activities	0.574
Alagoas	Professional, scientific and technical activities	0.601
Amapá	Water and sanitation activities	0.630
Amazonas	Professional, scientific and technical activities	0.607
Bahia	Meat processing and production	0.601
Ceará	Meat processing and production	0.620
Distrito Federal	Professional, scientific and technical activities	0.603
Espírito Santo	Professional, scientific and technical activities	0.606
Goiás	Professional, scientific and technical activities	0.591
Maranhão	Professional, scientific and technical activities	0.608
Mato Grosso	Meat processing and production	0.615
Mato Grosso do Sul	Professional, scientific and technical activities	0.645
Minas Gerais	Meat processing and production	0.570
Pará	Professional, scientific and technical activities	0.622
Paraíba	Professional, scientific and technical activities	0.619
Paraná	Water transportation	0.574
Pernambuco	Meat processing and production	0.592
Piauí	Meat processing and production	0.570
Rio de Janeiro	Professional, scientific and technical activities	0.684
Rio Grande do Norte	Meat processing and production	0.594
Rio Grande do Sul	Professional, scientific and technical activities	0.646
Rondônia	Professional, scientific and technical activities	0.589
Roraima	Professional, scientific and technical activities	0.551
Santa Catarina	Professional, scientific and technical activities	0.633
São Paulo	Professional, scientific and technical activities	0.625
Sergipe	Meat processing and production	0.573
Tocantins	Professional, scientific and technical activities	0.587

Source: The authors.

5 Concluding remarks

This work aimed to characterize the productive structure of Brazil as a complex network and to identify key sectors using network-based metrices. Using the 2010 national input-output matrix, we found that retail and wholesale trade, financial services and land transportation are the most central sectors in the Brazilian economy. Using 2011 subnational input-output developed by Haddad, Júnior and Nascimento (2017), we reveal heterogeneities and geographical aspects of the Brazilian production network which are undetectable using national data, officially provided by IBGE.

Overall, the results showed that in the Brazilian production network, the central sectors are influent not only because the generate the most added value, but also because they handle a great number of production factors and are strongly connected with the whole economy. Searching for measures of importance of sectors and inter-sectoral relationships, we explored centrality, diffusion and clustering techniques.

Obviously, a number of limitations emerge, most of them due to the nature of input-output matrices. For example, we should assume that technical coefficients remain constant, all sectors use a constant returns-to-scale technology, and inputs are always used in the same ratios. Additional constraints were imposed for the sake of simplicity, such as the absence of linkages between states in the subnational analysis, although such a model can be thought of. Future works can explore complex network methods drawing from the present literature, such as welfare-maximizing resource allocations, tests for economic resilience in response to exogenous shocks, and inter-regional input-output systems. We also hope that our findings can provide additions subsidies for policy making, with respect to economic planning and development.

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