Optimal Fiscal Policy: A Dynamic Analysis for Brazil

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Resumo

A política fiscal é um dos mais importantes instrumentos de política macroeconômica, especialmente em países em desenvolvimento como o Brasil. Este estudo pretende contribuir para a literatura através da apresentação de um modelo textit Dynamic Stochastic General Equilibrium - DSGE que capta o efeito de um conjunto de políticas fiscais sobre o crescimento econômico, em particular com aplicação à economia brasileira. Primeiro identificar qual o tipo de tributação distorcionária tem um impacto maior no crescimento econômico. Em segundo lugar, calibrar um modelo DSGE para a economia brasileira. Terceiro verificar como diferentes combinações de impostos podem afetar o crescimento econômico. A solução analítica do modelo proposto mostra que, considerando o problema de Ramsey, é ótimo não tributar a renda proveniente de capital no estado estacionário. Em geral, um choque nas despesas públicas tem um efeito que dura um número maior de períodos do que num choque na produtividade. Embora os efeitos do choque na produtividade sejam maiores em magnitude dos efeitos de um choque nos gastos do governo.

KeyWords: Política Fiscal, Problema de Ramsey, Crescimento Econômico.

Abstract

Fiscal policy is one of the most important instruments of macroeconomic policy, especially in developing countries such as Brazil. This study intends to contribute to the literature by presenting a *Dynamic Stochastic General Equilibrium - DSGE* model that captures the effect of a set of fiscal policies on economic growth, in particular with application to the Brazilian economy. First we identify what type of distortionary taxation has a lager impact on economic growth. Second, we calibrate a DSGE model for the Brazilian economy. Third, we verify how different combinations of taxes affect economic growth. The analytical solution of the proposed model shows that, given the Ramsey problem, it is optimal not to tax the capital income in the steady state. In general, a government spending shock has an effect for a greater number of periods than a productivity or technology shock. However, the effects of a productivity shock are greater in magnitude than the effects of a government spending shock.

KeyWords: Fiscal Policy, Ramsey Problem, Economic Growth.

Código JEL: D63, E62, O40

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1 Introduction

Since the 1990s, Brazil has experienced great changes in economic policy, and these transformations have brought economic stability and the possibility of increases in real income. But these changes were not enough to change the income concentration process and increased on fiscal policy. Therefore, understanding the dynamics of fiscal policy is of great importance to Brazilian economic policy.

In Brazil, the discussion about fiscal policy and economic growth it is linked to the inequality because this has been the main form of redistribution of income in Brazil. The literature in Brazil began in the 1950s however, the first empirical studies only came in the 1990s, mainly driven by computational advances. The majority of these studies utilize the methodology proposed by Barro & Sala-i-Martin (1992) to verify the existence of *beta* convergence of per capita income. As an example of such studies from this period, we can mention for example Ferreira & Diniz (1995), Azzoni (1997) and Ferreira (1998).

In Brazil, the focus of research on inequality has been mainly to analyze the convergence of income, based on the methodology used by Baumol (1986). This approach shows that economies with different initial endowments can converge to the same level of per capita income (BARRO; SALA-I-MARTIN, 2004). This theory implies that economies with low income levels grow at a higher rate than economies with high income levels, such that inequality decreases over time.

Another current of research in Brazil is related to the theoretical line of Kuznets (1955), such as; Figueiredo, Junior & Jacinto (2011), Ferreira & Lledó (1997), Bagolin, Gabe & Ribeiro (2004), Taques & Mazzutti (2010). Ferreira & Lledó (1997), for instance, estimate a panel data model to verify these relations for the Brazilian states, showing that fiscal policy and economic growth have a nonlinear long-term relationship. This result confirms the Kuznets s hypothesis for the model made by this authors.

Brazilian regional inequality and income distribution is investigated in several studies, in which the northeastern states of Brazil are considered low income (CHIARINI, 2008; JUSTO, 2009), whereas the states in the South of Brazil have a higher level of income per capita. Studies focusing on income convergence have been conducted in Brazil mostly at regional levels, for example, Rio Grande do Sul (MONASTERIO; AVILA, 2004), São Paulo and Paraná (KAMITANI; WASQUES; PARRÉ, 2011) and Espírito Santo (LEITE; MAGALHAES, 2012).

Based on a spatial econometric model, Chiarini (2008) presents evidence of income convergence between the municipalities of Ceará, thereby proving the existence of great inequality in this state. A similar study is performed by Justo (2009), who, based on the same methodology, shows that the growth dynamics of the municipalities pertaining to the Mesoregion of Araripe depends on their initial condition of income, on their stock of human capital and the effect of social policies, and, in addition, on the dynamics of neighboring municipalities. Monasterio & Avila (2004) presents evidence of income convergence in Rio Grande do Sul. The study applies spatial econometrics to analyze the economic growth of 58 statistically comparable areas between 1939 and 2001.

In a study at the aggregate level, adopting a spatial multilevel analysis, Guimaraes (2004) notes the influence of exogenous variables infrastructure at the convergence of income, and reduction of inequality in Brazil. The results showed that road infrastructure influences most in determining the municipal income convergence.

An extensive study was conducted by Oliveira & Monasterio (2011), which intended to show the dynamics of the Gross Domestic Product (GDP) per capita in municipalities of the states Alagoas, Ceará, Paraíba, Espirito Santo, Paraná and Pará, from 2002 to 2007. The authors shows the economic development of the respective states and their current socio-economic structure, using a set of socioeconomic indicators: labor income, skills, infrastructure and economic performance. Their results indicate that local income distribution depends largely on the initial endowments of each region.

Several studies verify the hypothesis of income convergence for Brazilian regions, as shown by Ribeiro & Almeida (2012). However, the persistence of income inequality is still observed among regions. In this context, using the methodology of *Weighted Regression Geographically*, the hypothesis of multiple equilibria was confirmed, where each region would be converging to their own specific steady state.

Amorim, Scalco & Braga (2008) examine production functions of the simple Solow standard growth model for Brazilian states and for the agricultural, industrial and services sectors, for the period of 1980-2000. The model that best fits the data was the extended Solow growth, although it appears that a Cobb-Douglas function of this type is not appropriate to describe production in these sectors. The authors checked for absolute convergence in the general case and for the industrial and services sectors. In the case of conditional convergence, only the industry sector presents a trend to convergence. In another study, Taques & Mazzutti (2010) present evidence that there is a positive correlation in the short run between income inequality and economic growth, which would be reversed in the long run, establishing a U inverse relationship.

However, these studies of Brazilian income inequality focus mainly on the analysis of the conditions of income distribution, they show whether the income gap between rich and poor regions are increasing or decreasing. These studies do not investigate the causes of inequality, or ways of how reduce it. The studies about the impacts of fiscal policy on inequality are still incipient, for instance: Fochezatto & Bagolin (2006), Costa-Junior, Sampaio & Gonçalves (2012), Carvalho & Valli (2011) and Cavalcanti & Vereda (2014).

Using a post-Keynesian theoretical framework, Neto (2006), and Neto & Oreiro (2008) examine the impact of long-run changes in fiscal policy and income distribution with a focus on government borrowing. They show that the economy does not respond linearly to fiscal shocks, so that an expansionary fiscal policy can have positive or negative effects on the level of activity, depending on economy's capital accumulation level.

Using an applied general equilibrium model to analyze the effects of fiscal policies on growth and income distribution in Brazil, Neto (2006), Neto & Oreiro (2008) show that there exist fiscal policies which will simultaneously lead to higher growth and lower inequality. The main fiscal policies mentioned by these authors are: I) Reduction of income tax on the poorest households offset by increased tax on the wealthiest families, II) Government transfers, III) An increase in public spending financed by an increase in direct taxes.

An analysis of the effects of fiscal policy on economic growth and income distribution in Brazil was also conducted by Fochezatto & Bagolin (2006). Using a static computable general equilibrium model, these authors show that there are several alternative policies that lead, simultaneously, to higher growth and to an improvement in income distribution. Among the simulations performed, fiscal polices that resulted in a distributive growth were: reducing the burden of income tax on the poorest families compensated by an increased burden of this tax on the wealthiest families, redirection of government transfers with an increase in the amount for the poorest families and a decrease for the richest families, and an increase in public spending financed by an increase in direct tax burden. These authors also show that tax policies have lower impact on the income distribution than government transfers.

Using a dynamic stochastic model approach, Costa-Junior, Sampaio & Gonçalves (2012) studied how Brazilian economic growth is influenced by an income transfer program. The estimation of the parameters was performed using a Bayesian methodology. This model assumes two types of consumers: Ricardian individuals and non-Ricardian individuals. The first agent maximizes its inter temporal utility function, while the second type of agents is limited to consume the amount received through income transfer. The results show that income transfers have returns, except for Ricardian individuals. This result is a direct consequence of the assumption of the existence of a non-Ricardian agent who can not save.

An open economy dynamic stochastic model using fiscal policy, calibrated using Brazilian data, was made by Carvalho & Valli (2011). The model incorporates primary surplus targets, cyclical expenditures and social programs in the form of public transfers, public investment and distortive taxation. There are both Ricardian and non-Ricardian agents. The presence of non-Ricardian households allows fiscal policy shocks to affect real economy aggregates and distribution. In general, the responses of real variables, including GDP, to shocks to the primary surplus or to public transfers fade out after a short period. The authors also present evidence that recent fiscal policy in Brazil has exerted significant inflationary pressures.

Although the convergence studies prove the existence of a large income disparity in Brazil, at the micro and macro levels, and present strong evidence of relationships between economic growth and inequality, these models neglect the impact of exogenous variables such as fiscal and monetary policy. There are few studies that analyze the impact of fiscal policy on economic growth in Brazil, using dynamic models, however these models have a great limitation when non-Ricardian individuals are included, which consume all their income and do not save. A natural consequence of this assumption is that the individual remains poor, regardless of the economic policy adopted,so that the inequality is not affected.

Therefore, there are few studies in Brazil showing the relationship between fiscal policy on these two macroeconomic aggregates and economic growth. Thus the international literature, and in particular the national, has a lack of studies addressing the issue of fiscal policy and economic growth in a more realistic and direct way.

The Brazilian economic growth is one of the main obstacles to economic development. Understanding the dynamics of this economic and social phenomenon is central to understanding the Brazilian economy. It is also critical to investigate policies that can promote growth.

Therefore, this work intends to contribute to the literature by presenting a *Dynamic Stochastic General Equilibrium - DSGE* model that captures the effect of a set of fiscal policies on economic growth, in particular with application to the Brazilian parameters. DSGE models are widely used for this type of analysis because they capture the key interactions between agents and the economic system. DSGE models aim to describe the behavior for the economy as a whole by analyzing the interaction of many microeconomic decisions.

The use of DSGE models has spread over the last decade especially among research institutions and central banks in developed countries, as an important tool in the analysis of macroeconomic issues. Cavalcanti & Vereda (2011) highlights the historical evolution of macroeconomic forecasting models, making clear the trend towards more consistent models from a theoretical point of view. The attractiveness of DSGE models can be attributed due to its rigorous theoretical framework, which facilitates the accomplishment of consistent economic policy simulations of the model with microfoundations, and its flexibility to explain several stylized facts of macroeconomics.

In fact, traditional models for macroeconomic forecasting are quite sensitive to what is known in economic theory as the Lucas critique. The Lucas critique argues that the effect of macroeconomic policy can not be predicted using historical data from a period in which the policy was not in place. While a model with micro fundamentals is based on the preferences of decision makers, DSGE models feature a natural benchmark for evaluating the welfare effects of policy changes – in this case, fiscal policy changes.

The objective of this study is to analyze the impact of fiscal policy on economic growth in Brazil. Specifically, first identify what type of distortionary taxation has a lager impact on economic growth. Second, calibrate a DSGE model for the Brazilian economy. Third verify how different combinations of taxes can affect the economic growth.

2 Model of Fiscal Policy

The infinitely lived household receives income from providing labor and capital, and chooses a path of consumption and capital investment to maximize their utility given by:

$$\max_{C_t,H_t,K_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t). \tag{1}$$

Instantaneous utility, U, is increasing, strictly concave, and three times continuously differentiable in their arguments, whereas C_t and H_t are household consumption and labor provided at time t, respectively. The household is endowed with one unit of time that can be used for leisure $l_t = 1 - H_t$ and labor H_t. We assume in this work that the utility function has the following functional form:

$$U(C_t, H_t) = ln(C_t) + \gamma ln(1 - H_t). \tag{2}$$

The single good is produced by a representative firm with labor H_t and capital K_t provided by the representative household. The output (Y_t) of the economy can be consumed by households, used by the government, or used to augment the capital stock. Thus, the resource constraint of this economy is given by:

$$C_t + G_t + K_{t+1} = F(K_t, H_t) + (1 - \delta)K_t. \tag{3}$$

A representative household chooses $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize expression (1) subject to the following sequence of budget constraints:

$$C_t + K_{t+1} = (1 - \tau^H)W_t H_t + (1 - \tau^K)R_t K_t + (1 - \delta)K_t, \tag{4}$$

where K_t is the capital stock, W_t is the wage rate and R_t the capital rental rate as described below, and δ is the capital depreciation rate. The capital stock evolves according to the law of motion:

$$K_{t+1} = I_t + (1 - \delta)K_t. \tag{5}$$

This equation shows how the stock of capital is accumulated from one period to another, that is, how the investment is determined in the economy.For $t \geq 0$, with $\beta^t \lambda_t$ as the Lagrange multiplier on the period t budget constraint, the first order conditions are:

$$\begin{aligned}
C_t &: \beta^t U_{C_t}(C_t, 1 - H_t) - \lambda_t = 0 && (6)\\
H_t &: -\beta^t U_{H_t}(C_t, 1 - H_t) + \lambda_t(1 - \tau_t^H)W_t = 0 && (7)\\
K_{t+1} &: -\lambda_t + \lambda_{t+1}\Big[1 + (1 - \tau_t^K)R_t - \delta)\Big] = 0 && (8)\\
\lambda_t &: C_t + (1 - \tau_t^H)W_t H_t + (1 - \tau_t^K)R_t K_t = 0 && (9)
\end{aligned}$$

From the first order conditions we can find the labor supply and Euler equations.Using equations (6) and (7) we can find the labor supply:

$$(1 - \tau^H)W_t = \gamma \frac{C_t}{1 - H_t}. \tag{10}$$

From the equations (6), (7) and (8) we can find the Euler equation that shows the inter-temporal relation of consumption between the period t and $t+1$ which in the present case assumes the following form:

$$\frac{1}{C_t} = \beta E\left\{\left(1 + (1-\tau^K)R_t - \delta\right)\frac{1}{C_{t+1}}\right\}. \tag{11}$$

The Euler equation (11) is a optimality condition that show the inter-temporal variant of a well-known result in consumption theory, which present the price ratio of consumption in both periods. According to the Euler equation, there are two opposing forces that affect the inter-temporal choices of the agent. The stronger the degree of time preference, the smaller is β, and thus the less attractive is C_{t+1} to the representative agent. On the other hand, the higher the interest rate, the more attractive it is to save.

A representative firm rents capital and employs labor to produce a single consumption good. The firm maximizes profit, we assume a Cobb-Douglas production function such as:

$$F(H_t, K_t) = (e^{a_t}H_t)^{\alpha}K_t^{1-\alpha}, \tag{12}$$

where $F(H_t, K_t)$ is the product (Y) of the economy, from the condition of profit maximization we can find the first order conditions of the firm, which correspond to zero profits, such that capital and labor are paid their marginal products:

$$W_t = \alpha(e^{a_t})^{\alpha}\left(\frac{H_t}{K_t}\right)^{1-\alpha} \tag{13}$$

$$R_t = (1-\alpha)(e^{a_t})^{\alpha}\left(\frac{H_t}{K_t}\right)^{\alpha}, \tag{14}$$

where W_t, it's the wage, R_t is capital rental rate, e^{a_t} represents the labor augmented technical progress with a following an AR(1) process to allow the analysis of the effects of productivity or supply shocks $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$.

The government imposes taxes on household income from labor and capital to finance exogenously given consumption. It is assumed that the government must operate with a balanced budget. The budget constraint can then be expressed as

$$e^{g_t}G_t = \tau^H W_t H_t + \tau^K R_t K_t, \tag{15}$$

where G_t is the government spending and is exogenously given, and g_t is used to represent exogenous shocks to government spending. Similarly to a_t, g_t evolves according to the AR(1) process $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$.

2.1 Ramsey Problem

Optimal taxation theory aims to show how a tax system should be chosen to maximize a social welfare function, subject to a set of constraints. Usually the social planner is modeled as a utilitarian, that is, the social welfare function is based on the utilities of individuals in the society.

Definition 2.1. The government solves the Ramsey problem by optimizing the social welfare function accounting the household utility, the firm behavior and a set of fiscal policies under commitment. Thus, a competitive equilibrium of government policy, allocations and prices is defined such that the following conditions hold. First, households maximize their utility (1) subject to (4) such that (10) and (11) hold. Second, firms maximize profits such that (13) and (14) hold. Third, the government budget is balanced (15).

The government solves the Ramsey problem by optimizing the social welfare function, accounting for firm and household behavior, and setting fiscal policy under commitment.

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U_t(C_t, H_t) \;\; &+ \;\; \lambda_t \Big[F(K_t, H_t) - C_t - e^{g_t} G_t - K_{t+1} + (1-\delta) K_t \Big] \\ &+ \;\; \phi_t \Big[F(K_t, H_t) - (1-\tau_t^K) R_t K_t - (1-\tau_t^H) W_t H_t - e^{g_t} G_t \Big] \\ &+ \;\; \psi_t \Big[U_{H_t} - U_{C_t} (1-\tau_t^H) W_t \Big] \\ &+ \;\; \Phi_t \Big[U_{C_t} - \beta U_{C_{t+1}} \Big((1-\tau_t^K) R_t + 1 - \delta \Big) \Big] \Bigg\} \end{aligned} \tag{16}$$

The derivation of the consumer first order conditions and the constraints used to construct the Lagrangian can be found in appendix A.

Proposition 1. *If there exists a steady-state, the associated limiting tax rate on capital income should be always zero,* $\tau^K = 0$.

Note that the household's budget constraint is not explicitly included, because it is redundant when the government satisfies its budget constraint and the resource constraint holds. Taking the first order condition with respect to K_{t+1} to find the optimal tax on capital income:

$$\lambda_t = \beta \Big[\lambda_{t+1} \big[F_{K_{t+1}} + (1-\delta) \big] + \phi_{t+1} \big[F_{K_{t+1}} - (1-\tau_{t+1}^K) R_{t+1} \big] \Big]. \tag{17}$$

Assuming that there is a steady state in which the conditions are valid, $C_{t+1} = C_t$, and the first-order conditions of the firm, we must have $F_K = R$ then we have that equation (17) which in steady state could be written:

$$\lambda = \beta \Big[\lambda \big[R + 1 - \delta \big] + \phi \big[R - (1-\tau^K) R \big] \Big]. \tag{18}$$

This condition can be simplify taking the Euler equation (11) in steady state, which is $1/\beta = [1 + (1-\tau^K)R - \delta]$ and subbing into this condition (18) leads to:

$$\lambda = \lambda \beta (R + 1 - \delta) + \phi \beta \Big[R - (1-\tau^K) R \Big]. \tag{19}$$

We can further simplify this condition, which then simplifies to:

$$(\phi + \lambda)[R - (1-\tau^K)R] = 0 \tag{20}$$

From the first-order conditions of the households we can interpret λ as the marginal social value of goods, and is strictly positive. From the Ramsey problem we can define ϕ as the marginal value of reducing taxes, which is nonnegative. Thus, given $\lambda > 0$ and $\phi \geq 0$, equation (20) implies that $\tau^K = 0$ in steady state.

This analysis is in agreement with some previous studies, for instance those by Chamley (1986) and Judd (1985).

3 Calibration

The strategy used for the calibration of the model is similar to that used by for instance Sargent & Ljungvist (2000) and Swarbrick (2012). The model will be implemented in the *Dynare* software. The model will be calibrated to match Brazilian data; the values of the labor share, discount factor and depreciation rate will be taken from earlier studies. The procedure will be similar to that indicated by Kydland & Prescott (1982), namely:

- Use microeconomic studies or theory to find values for all of the parameters.
- Solve the model numerically, and simulate the economy.
- Compare the moments: standard deviations, correlations, *etc*. of the simulated economy with those of the Brazilian economy.
- If the moments are matched the model is correct.
- If not, the moments which do not match up suggest areas of potential improvement.

Running the model in *Dynare* will show how government expenditure will be financed by taxing labor and consumption. This analysis will help define how changes in fiscal policy impact economic fluctuations.

A fundamental point in the calibration of the model is the definition of the parameters that appear in the equations that characterize the behavior of the economy in the steady state. In particular, the calibration of DSGE models can be useful not only as a way to analyze the dynamic properties of the models, but also as a way of pointing out parameters of the model that are particularly important to the determination of the equilibrium paths of the variables in response to shocks.

The implementation of DGSE models requires us to attach numerical values to the parameters, which can be done by using econometric estimations and calibration proceedings, such as those by Cavalcanti & Vereda (2011) and Bender-Filho (2011). The interaction between the two methodologies can help with the validation of our model. We follow the methodology proposed by Cavalcanti & Vereda (2011), who analyze the dynamic properties of a DSGE model for Brazil. Parameter values are based on a careful review of the literature, attempting to identify the best sources for the each of the parameter values, and acceptable ranges of values for the main parameters for the model of the Brazilian economy.

Tabela 1: Model Parameter Calibrations

Parameter	Value	Description	Source
β	0.9506	Discount Factor	IBGE
α	0.65	Capital Share	National Accounts
δ	0.025	Depreciation Rate	Literature
G	0.305	Government Spending	STN
ρ_{g_t}	0.9831	Autocorrelation of government shock	STN
σ_{g_t}	0.0193	Variance of shocks government	STN
ρ_{a_t}	0.8061	Autocorrelation of techology shock	IBGE
σ_{a_t}	0.0369	Variance of techology shock	IBGE

Source: Own elaboration.

Calibration of the discount factor β is done in such a way that the expression $(1/\beta) - 1 = R^{SS}$ is valid, where the discount factor equals the assumed value of the average real interest rate observed over a sufficiently long time interval. The calculated value for the inter-temporal discount rate measures the impatience of agents, and is the determining factor for inter-temporal choices. Thus, discount rate relates the decisions between present and future consumption, so that an increase in β will increase the weight of future consumption in the utility of the economic agents, stimulating present savings.

This parameter has presented a great variation in studies on the Brazilian economy. This can be attributed to two factors: First, the wide range of interest rates in Brazil, and, second, and most importantly, the period of time selected in the sample. In the national literature, this value varies between 0.89 and 0.98 (for more details see Cavalcanti & Vereda (2011)). We chose to use the value of the discount rate as determined by Bender-Filho (2011).

Usually, in studies involving DSGE models, the rate of capital depreciation in the firm's production function is presumed to be 10% annually. (or 2.5% quarterly). In Brazil, there is no consensus on this value, due to the absence of a suitable data series for this parameter. Because of this consideration, we adopted the standard value in the literature.

For the parameter related to the participation of the remuneration of capital in the product, α, we calculated the ratio between Gross Operating Surplus and GDP, obtained from the data series of the IBGE National Accounts System. However, remuneration of the self-employed was deducted from the Gross Operating Surplus.

In the national accounts, the Gross Operating Surplus is the residual value of the income, which comprises the remuneration of the capital, interest, profits and rents. We find the value of $\alpha = 0.65$, which seems to be largely in agreement with other studies on Brazil, such as, for instance, that by Bender-Filho (2011).

To calibrate the parameter G, we use the series of tax burden as a proportion of GDP for the period from 1947 to 2011, published by IBGE. The average for this period was 30.5%.

4 Empirical Results

First, we present the steady-state values of the variables, obtained from the mathematical expressions in the section 2. The results are shown in Table 2. An important result that emerges from the values found for the steady state, is that of the interest rate (R), which corresponds to 7.697% per year which is in accordance with the Brazilian economy. When we consider the period from 1990 to 2016, the real interest rate in the Brazilian economy is 7.83% per year.

Tabela 2: Estimated steady state values

Y	C	τ^K	τ^H	K	I	H	W	R
0.7714	0.3782	0.00	0.6092	3.5077	0.0877	0.3413	1.4692	0.07697

Source: Research results.

As we can see, the numerical solution of the model corroborates the result found analytically in the section, that is, the optimal tax on capital at steady state should be zero ($\tau^K = 0$). Therefore, the government should finance its spending through a distortionary tax on labor income $\tau^H \geq 0$.

4.1 Effects of a Shock in Government Spending

The analysis of both of the shocks proposed in the theoretical model give some insight into how fiscal policy should be conducted over the business cycle. We begin our analysis by presenting results for a tax on capital and labor income. Figure 1 shows the behavior of taxes after a shock in government spending.

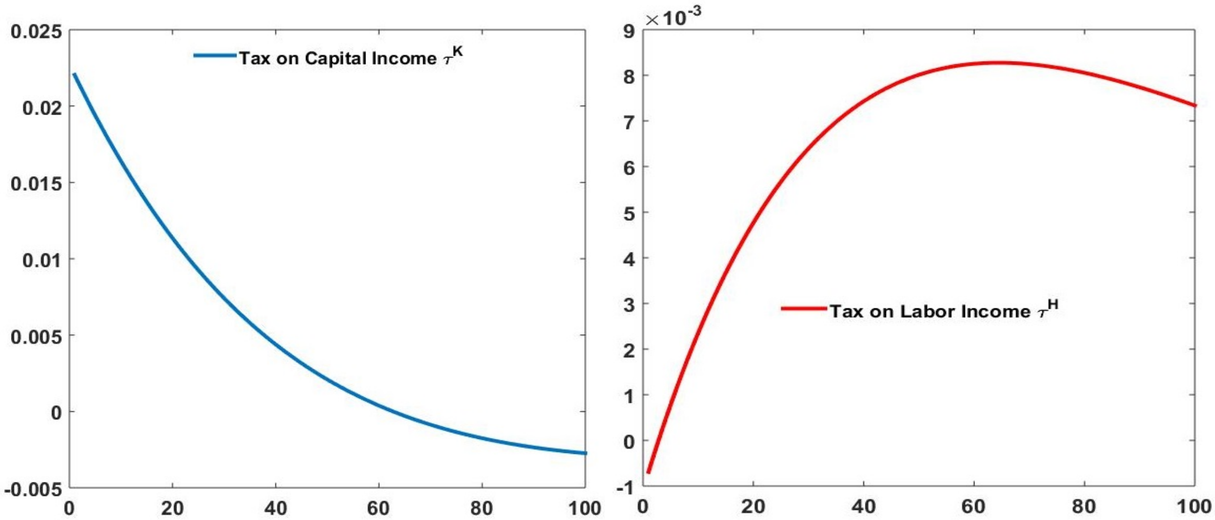

Figura 1: Reactions of the optimal tax rates to shocks in government spending.

Figure 1 shows that, in response to a shock in government spending, the optimal tax on capital income increases at first, then slowly returns to the steady-state value. The optimal tax on labor income, however, decreases slightly at first and then slowly increases. This is natural, because since the capital stock only changes slowly, an increase in the tax rate of capital income has a lower instantaneous effect on output than would, for instance, an increase in the tax rate of labor income. The capital stock takes time to disaccumulate and has no alternative uses, whereas the labor force of the representative household may immediately be diverted to leisure instead of work in response to an increase in the income tax rate. The shock in government spending, therefore, is initially covered by increased tax on capital income rather than labor income.

Figure 2 shows the impulse-response functions for capital stock K_t, hours worked H_t, rental price of capital R_t and wage W_t, in response to a shock in government spending. Notably, the impulse-response function of the capital stock decreases immediately following the shock, then continues to decrease, presumably because the increased tax rate on capital income reduces the incentives to own capital. Immediately following the shock, the capital stock is above the desired capital stock, and this also results in a reduction in the rental price of capital.

The impact of the government spending shock on hours worked and the wage rate is opposite to that of capital. Due to the lower tax on labor income immediately following the shock, the incentives for the households to work are suddenly increased, and the hours worked naturally increases in response to this incentive. Since wage is inversely related to capital stock (equation 13), the drop in capital stock also helps to increase wage and incentivize an increase in the number of hours worked.

4.2 Effects of a Shock in Productivity

We now turn our attention to the effects of a productivity or technological shock. Figure 3 shows the effect of the shock on the optimal capital income tax rate and the optimal labor income tax rate. Immediately following the shock, the tax on capital income drops. In other words, if the economy is originally in the steady state – in which the optimal capital income tax rate is zero – the capital income tax rate becomes negative. The technological shock already provides an incentive to increase capital stock, yet the decrease in the capital income tax rate permits capital stock to accumulate faster than

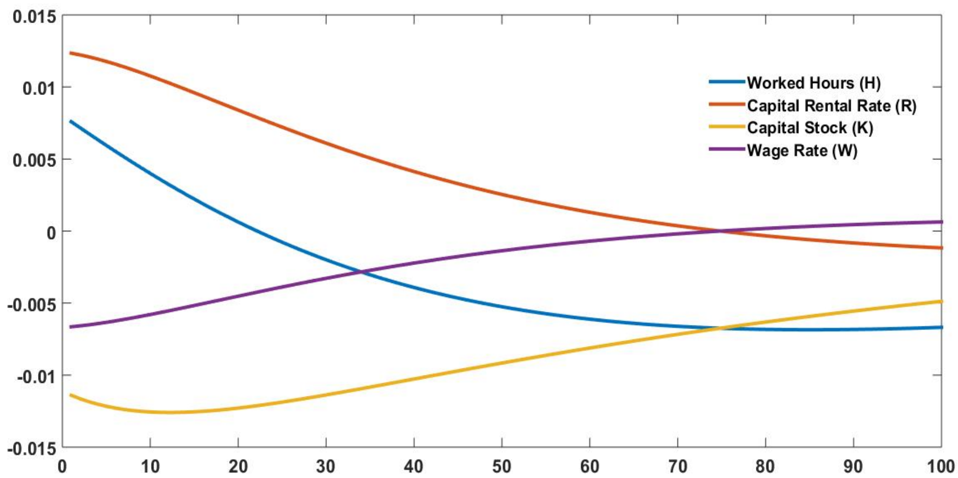

Figura 2: Reactions to shocks on government spending.

otherwise. In a sense, the government wishes to incentivize a more rapid increase in the capital stock to take advantage of the increased productivity caused by the technological shock. Conversely, the tax rate on labor income must be increased immediately following the shock, since the government must still pay its bills. However, a few periods after the initial shock, the labor income tax rate also falls below its steady state value, presumably as the rapid increase in capital and the positive effects of the technological shock allow the government to more easily balance its budget.

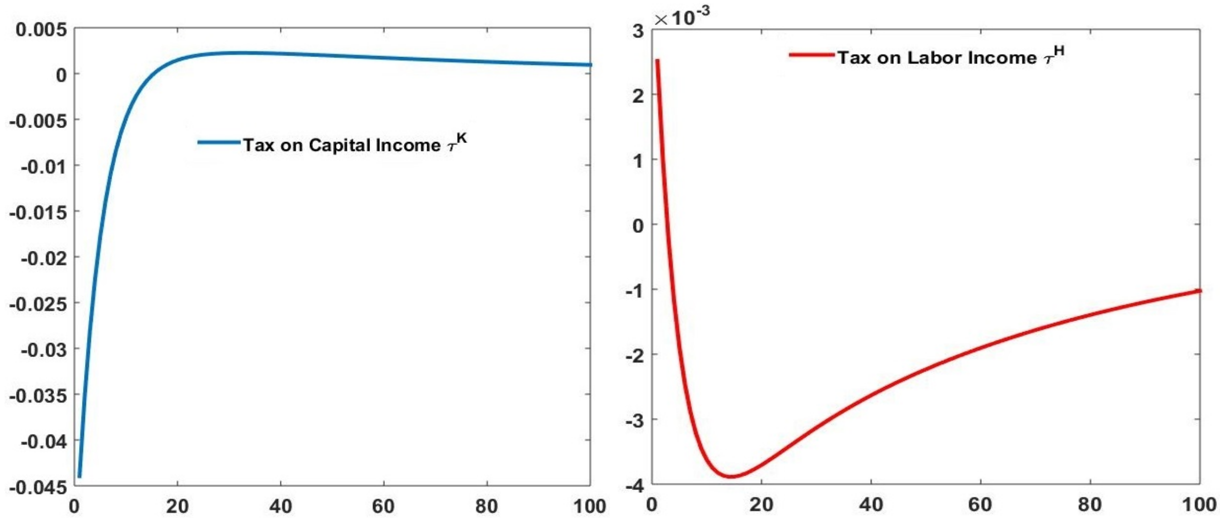

Figura 3: Reactions to shocks on productivity.

The impulse-response functions of capital stock K_t, hours worked H_t, rental price of capital R_t, and wage W_t, with respect to technology shocks, are shown in figure 4. The behavior of capital stock K_t largely corroborates the narrative: capital stock accumulates rapidly after the initial shock, reaches a peak, and slowly returns to the steady state value as the technological shock fades.

The rental price of capital R_t increases dramatically immediately following the shock, simply due to the technological shock. As capital accumulates much more rapidly than would otherwise be the case, due to additional government incentives, the rental price of capital quickly turns negative. At a point, a minimum is reached and the rental price of capital starts climbing towards the steady state value again as the shock fades and capital must be disaccumulated.

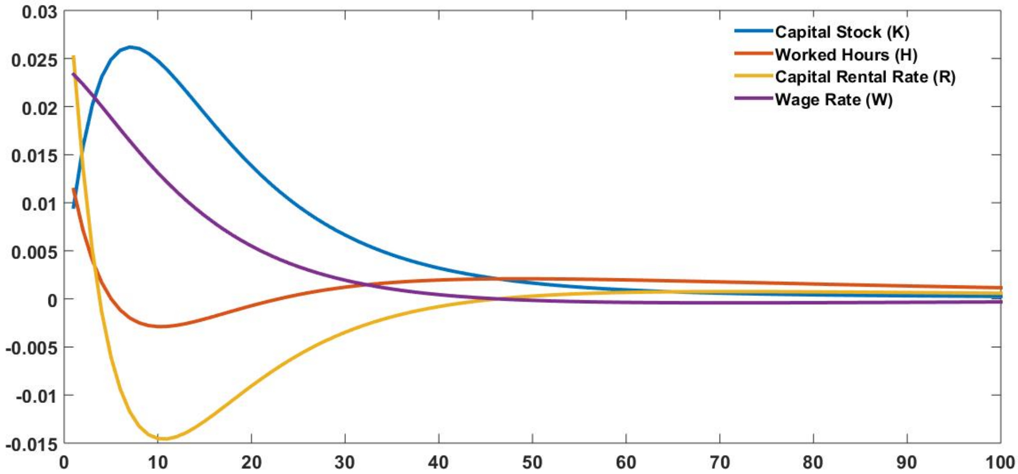

Figura 4: Reactions to shocks on productivity.

The technological shock increases labor productivity, and hence salaries. Since hours worked can adjust much more rapidly than capital stock to take advantage of the technological shock, hours worked rises immediately following the shock. As capital gradually accumulates and the effect of the technological shock fades, wages gradually decline. In response to the drop in wages, hours worked also declines, and seems to experience some minor oscillations as it approaches the steady state value.

5 Conclusions and Remarks

The purpose of this paper is to show the optimum level of taxation for the Brazilian economy over the real business cycles. To achieve this goal we calibrated a DSGE model for the Brazilian economy.

The analytical solution of the proposed model shows that, given the Ramsey problem, it is optimal not to tax the capital at steady state. The numerical solution confirms this result, indicating that in the long run this result is valid theoretically and empirically.

The results of the simulations show that optimal tax on capital income and labor income respond to a shock in government spending in opposite ways. While the optimal tax on capital income increases at first above the steady-state rate and then subsequently decreases slowly, the optimal tax on labor income first decreases and then increases above the steady-state rate before approaching the steady-state rate once again. As a consequence of the change in composition of tax, the stock of capital diminishes and the hours worked increases.

When we analyze the effects of a productivity or technological shock, we can see that the optimal tax on capital income becomes negative, indicating that governments should create an additional incentive to increase the capital stock, which, in response, increases quickly after a few periods. Thus, to maintain the balanced budget, the government must increase the tax on labor income, although this increase lasts for a few periods, after that falls in order to maintain balanced the government budget.

In general, the supply shock from government spending has an effect for a greater number of periods than the productivity shock, although the impacts of the productivity shock are greater in magnitude than those of a government spending shock.

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A Household Solution

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} U_t(C_t, H_t) + \lambda_t \Big[F(K_t, H_t) - C_t - e^{g_t} G_t - K_{t+1} + (1-\delta) K_t \Big] \\ +\phi_t \Big[F(K_t, H_t) - (1-\tau_t^K) R_t K_t - (1-\tau_t^H) W_t H_t - e^{g_t} G_t \Big] \\ +\psi_t \Big[U_{H_t} - U_{C_t}(1-\tau_t^H) W_t \Big] \\ +\Phi_t \Big[U_{C_t} - \beta U_{C_{t+1}} \Big((1-\tau_t^K) R_t + 1 - \delta \Big) \Big] \end{aligned} \right\} \tag{21}$$

It is unnecessary to include the household budget constraint as this holds when the government and resource constraints hold. The first order conditions are then:

$$\frac{\partial \mathcal{L}}{\partial C_0} = U_{C_0} + \lambda_0 - \phi_0 \Big[U_{CC_0}(1-\tau_0^H) W_0 \Big] + \psi_0 U_{CC_0} + \psi_1 \beta^2 \Big[U_{CC_1} \Big((1-\tau_0^K) R_0 + 1 - \delta) \Big) \Big] = 0 \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial H_0} = -\gamma \frac{1}{(1-H_0)} - \lambda_0 F_{H_0} + \psi_0 \Big[F_{H_0} - (1-\tau_0^H) F_{H_0} \Big] + \phi_0 \Big(\gamma \frac{1}{(1-H_0)^2} \Big) = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial K_1} = \lambda_1 \beta \Big[F_{K_1} + (1-\delta) \Big] + \lambda_0 + \psi_1 \beta \Big[F_{K_1} - (1-\tau_1^K) F_{K_1} \Big] = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_0^H} = \psi_0 (F_{H_0} H_0) + \phi_0 (U_{C_0} F_{H_0}) = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_0^K} = \psi_0 (F_{K_0} K_0) + \phi_0 (\beta U_{C_1} R_0) = 0 \tag{26}$$

Now we analyze for all t.

$$\frac{\partial \mathcal{L}}{\partial C_t} = U_{C_t} - \lambda_t - \phi_t \Big[U_{CC_t}(1-\tau_t^H) W_t \Big] + \psi_t U_{CC_t} + \psi_{t+1} \beta^2 \Big[U_{CC_{t+1}} [(1-\tau_t^K) R_t + 1 - \delta] \Big] = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = -\gamma \frac{1}{(1-H_t)} - \lambda_t F_{H_t} + \phi_t \Big[F_{H_t} - (1-\tau_t^H) F_{H_t} \Big] + \phi_t \Big(\gamma \frac{1}{(1-H_t)^2} \Big) = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \lambda_{t+1} \beta \Big[F_{K_{t+1}} + (1-\delta) \Big] + \lambda_t + \psi_{t+1} \beta \Big[F_{K_{t+1}} - (1-\tau_{t+1}^K) F_{K_{t+1}} \Big] = 0 \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_t^H} = \psi_0 (F_{H_t} H_t) + \phi_t (U_{C_t} F_{H_t}) = 0 \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_t^K} = \psi_t (F_{K_t} K_t) + \phi_t (\beta U_{C_{t+1}} R_t) = 0 \tag{31}$$

B Code of Dynare

```
//------------------------------------------------
// 1. Declaration of Endogenous and Exogenous Variables
//------------------------------------------------
var y c kk hh rr ww tauK tauH k i h a w g r;
varexo epsA epsG;
//------------------------------------------------
// 2. Declaration of Parameters
//------------------------------------------------
parameters
G
beta
delta
alpha
rhoA
rhoG
psi
theta
sigmaA
sigmaG
chi;
//------------------------------------------------
// 3. Parameter Calibration
//------------------------------------------------
G = 0.305460; // exogenous government spending
alpha = 0.65; // labour (wage) share
beta = 0.9506; // discount factor
delta = 0.025; // depreciation rate
rhoA = 0.8061; // tech-shock persistence
rhoG = 0.9831; // gov-shock persistence
sigmaA = 0.0369; // tech-shock standard deviation
sigmaG = 0.0193; // gov-shock standard deviation
chi = 1; // utility weight of leisure
// parameters used for initial value calculation
psi = ((1+beta*delta-beta)/(beta*(1-tauK)*(1-alpha)))^(1/alpha);
theta = ((1-tauH)*alpha*psi^(alpha-1)+chi*G)/((psi^alpha)*chi
+ (1-tauH)*alpha*(psi^alpha)-delta*chi);
//------------------------------------------------
// 4. The MODEL
//------------------------------------------------
model;
(1/(c)) = beta*(1/(c(+1)))*(1+(1-tauK)*r-delta);
chi*(c)/(1-h) = (1-tauH)*w;
c+i+exp(g)*G = y;
y = (k^(1-alpha))*(exp(a)*h)^(alpha);
i = k(+1)-(1-delta)*k;
exp(g)*G = w*tauH*h + r*tauK*k;
a = rhoA*a(-1)+epsA;
g = rhoG*g(-1)+epsG;
```

```
w = alpha*((exp(a))^(alpha))*h^(alpha-1)*k^(1-alpha);
r = (1-alpha)*((exp(a))^(alpha))*k^(-alpha)*h^alpha;
// Deviation from Steady State
ww=w/STEADY_STATE(w);
rr=(r)/(STEADY_STATE(r));
hh=(h)/(STEADY_STATE(h));
kk=(k)/(STEADY_STATE(k));
end;
//------------------------------------------------
// 5. Initial guesses for steady-state computation
//------------------------------------------------
initval;
tauK = 0;
tauH = 0;
k = theta;
h = psi*k;
c = (1-tauH)*alpha*psi^(alpha-1)*chi^(-1)
-(1-tauH)*alpha*psi^alpha*k*chi^-1;
y = h^(alpha)*k^(1-alpha);
w = alpha*psi^(alpha-1);
r = (1-alpha)*psi^alpha;
a = 0;
epsA = 0;
end;
//------------------------------------------------
// 6. Specification of shocks
//------------------------------------------------
shocks;
var epsA = sigmaA^2;
var epsG = sigmaG^2;
end;
ramsey_policy (periods=2000,order=1,planner_discount=0.9506);
planner_objective ln(c)+chi*ln(1-h);
stoch_simul(periods=2000,irf=100);
```