Modeling of Banks' Optimal Behavior in the Brazilian Funds Market

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Abstract

This paper uses numerical methods to solve banks' cost minimization problem. Following Clouse and Dow (2002), we model bank's behavior using stochastic dynamic programming. We take account of institutional aspects of the Brazilian regime that influence bank's choice of reserves at the end of the day. The overlap of the accounting and accomplishment periods increases the demand of banks for reserves on the first three days of the accomplishment period. This result is different from the American market where the demand for reserves has an upward trend over the accomplishment period, except on Fridays. We show that demand for reserves in Brazil would present smaller variance during the accomplishment period if the uncertainty about reserve requirement on the first three days were abolished.

Keywords: Bank Reserves, Reserve Management, Demand Deposits, Reserve Requirements
JEL Classification: E52

O presente trabalho utiliza métodos numéricos para resolver o problema de minimização de custos dos bancos na administração dos saldos da conta Reservas Bancárias. A sobreposição dos períodos de cálculo e de cumprimento faz com que os bancos demandem mais reservas no começo do período de cumprimento. Tal fato faz com que o caso brasileiro difira do americano, onde a demanda por reservas é crescente ao longo do período de cumprimento, exceto nas sextas-feiras. Mostramos que a demanda por reservas apresentaria menor variância ao longo do período, caso a incerteza sobre a exigibilidade nos três primeiros dias do período de cumprimento fosse retirada.

1 Introduction

From 1999, Brazil adopted an inflation-targeting regime with a floating exchange rate. In this regime, the major economic policy tool is the Selic interbank interest rate. Thus, the study on how this market operates has become very important so that the Central Bank can better control the interest rate and make expectations converge to the rate established at the meeting of the Monetary Policy Committee (MPC).

In this study, we analyze the optimal behavior of banks towards the management of their reserves. Supposedly, they seek to minimize the present value discounted from their expected costs, given the current normative rules of the federal funds market in Brazil.

Banks are subjected to different restrictions imposed by the Cen-

⋆ The ideas and opinions expressed in this paper are those of the authors and not necessarily that of the Central Bank or its members. *Email address*: julio.cesar@bcb.gov.br (Julio Cesar Costa Pinto).

tral Bank on the management of their funds. In Brazil, compulsory reserve is an important monetary policy tool used by the Central Bank. In the case of reserve requirement on demand deposits, banks are obliged to maintain, for two weeks, a certain reserve requirement in the daily ending balance of their reserve account. If banks do not fulfill this requirement, financial costs will be incurred. Therefore, this norm should be taken into account at the moment in which the banks determine the ending balance of their account. The second restriction is the requirement of a minimum balance. If banks do not have sufficient balance in their account at the end of the day, they will incur financial costs. The Central Bank improved the normative framework when the new Brazilian payment system was implemented. This new system does not allow for overdrafts in the reserve account, then, besides the gain obtained for the Brazilian society, which does not have to pay for the loss if the bank goes bankrupt, it demands that banks improve the management of their daily flow of payments.

As observed, banks aim at a given ending balance for their reserve account. To do that, they weigh the financial costs imposed by the normative framework and the opportunity cost to keep reserves "idle" in the reserve account. By determining the banks' optimal behavior, we may have an idea of how the seasonal behavior of the demand for reserves is like on the days of accomplishment of reserve requirements on demand deposits. This way, to a certain extent, it is possible to predict such seasonality. Another interesting aspect is that we may change the compulsory reserve rules and assess the effect of this change on the behavior of banks. We may change a certain normative characteristic to know whether the demand for reserves becomes more or less variable over the period. The less variable the demand for reserves, the easier it will be to predict it and, consequently, the easier the control over the daily interbank interest rate by the

Central Bank.

The present paper seeks to assess how the use of monetary policy tools affects the behavior of private agents. In Brazil, there is a paucity of studies evaluating the effects of compulsory reserves on the behavior of banks. In terms of empirical evidence, a study was developed by Queiroz (2004). As we will show further ahead, the theoretical model, to a certain extent, manages to reproduce the empirical evidence provided in that study. Therefore, the current study attempts to contribute to the literature by establishing a theoretical benchmark for the analysis of the effect of compulsory reserves on the behavior of Brazilian banks.

The paper is organized as follows: Section 2 describes how the Brazilian funds market works. Section 3 describes the model used to find the optimal solution to the banks' problems. Section 4 presents and analyzes the results. Section 5 concludes.

2 The Brazilian Funds Market

This section describes the rules that control the operation of the Brazilian funds market. These rules are established by normative documents that regulate the reserve requirements on demand deposits. This compulsory reserve is fulfilled by the mean daily ending balance of the reserve account organizations have at the Central Bank of Brazil. Circular notes 3.094/2002, 3.169/2002 and 3.199/2003 [1] constitute the current normative basis for the reserve requirements on demand deposits. The subsequent subsections summarize the systematics in these circular notes.

[1] These circular notes and other normative documents related to this issue can be obtained from *www.bcb.gov.br*.

2.1 Calculation of liabilities

Multiple and investment banks, reserve account holders, commercial banks and savings banks are subject to reserve requirement on demand deposits.

The tax basis for the reserve requirement on demand deposits corresponds to the arithmetic mean of the values registered in different entries, known as values subject to reserve requirements (VSRR) [2], computed in the accounting period, discounting the fixed value of R$ 44 million. A 45% rate is applied to this tax basis. Currently, liabilities amount to approximately R$ 30 billion.

Up to 15% of the tax basis (or one third of the liabilities) can be paid by the average cash balance of banks during the accounting period. As this form of accomplishment is computed in the accounting period, it causes the ending balance of the reserve account to be equal to the liabilities subtracted from the mean cash balance. Te remainder must be accomplished by the arithmetic mean of ending balances of the reserve account.

Banks with liability equal to or less than R$ 10,000 are exempt from paying the reserve requirements on demand deposits.

2.2 Accounting and accomplishment periods

In order to decrease excess demand and supply, banks were categorized into two groups: A and B. The accounting and accomplishment periods of one group have a one-week lag in relation

[2] Circular note 3.169/2002 defines the accounting principles that constitute the VSRR in case of reserve requirements on demand deposits.

to the banks in the other group. This way, if there is some seasonality within the turnover period, it will be minimized by the other group, which is in a different moment during the period.

The accounting period consists of two consecutive week, starting on the Monday of the first weeks and ending on Friday of the subsequent week. The accomplishment period starts on Wednesday of the second week of the accounting period and ends on Tuesday of the second subsequent week. There occurs an overlap of three days in the accounting and accomplishment periods. This overlap makes the bank start to accomplish the reserve requirement on demand deposits three days before the end of the accounting period, i.e., when its liabilities are still uncertain.

2.3 Uncertainties about reserve management

As emphasized in the previous subsection, the reserve manager starts its accomplishment period with uncertainty about the level of its required reserves.

As it could be seen in the last section, a bank funding manager starts its maintenance period with uncertainty about the level of its required reserves. Since it is the average end-of-day Reserves Account balance that is considered in the computation and all settlements and payments ultimately have an impact on this account, the bank funding manager faces another kind of uncertainty: which will be the exact end-of-day balance of this account? The new system of payments, implemented in April 2002, decreased this uncertainty by creating clearing and settlement systems, either of payment or of assets, and by setting the rule thatonly the owner of the Reserves Account can command a withdrawal. This means that the Central Bank, the Brazilian National Treasury or any other system cannot directly command

an account withdrawal. However, the large number of transactions made on a daily basis, especially credit orders managed by clients or for clients, which usually are not previously known, and the margin of maneuver the reserve pilot [3] is provided with so that he can operate in real time make the ending balance of the reserve account differ from that initially planned by the reserve manager.

2.4 Financial costs resulting from deficits

The uncertainties described in the previous subsection are of great concern to reserve managers, since the impossibility to carry out operations to cover the credit orders of their clients does not allow the targeted ending balance of the reserve account to be achieved, which may cause banks to incur costs in case of deficits or opportunity cost if they have an excess balance in the account, not paid by the Central Bank. Currently, there are two financial costs from the deficit of reserve requirement on demand deposits: excess deficit in the daily balance of the reserve account and excess deficit in the payment of liabilities during the period.

Banks may have a minimum daily balance in the reserve account equivalent to 80% of the liability in the period. If the balance is smaller than the limit, there are financial costs, calculated by multiplying the value of deficiency by the Selic rate on the date, and adding 14% [4]

The cost due to deficiency in the accomplishment of liability

[3] Reserve pilot is the person designated by the bank to control the balance of intradaily reserves in real time.

[4] The financial cost on daily deficit is calculated using the following

occurs when the average daily balance (bank reserves + cash value) is smaller than the respective liability for the period. The cost is calculated by multiplying the deficit value by the Selic rate of the last day of the period, adding 14%, by the number of days of the period [5] . In order to avoid charging twice for the same deficiency, we consider an 80% liability, on the days on which the minimum daily balance was deficient, as the daily ending balance of the reserve account.

If the deficiency in the average daily balances is equal to or less than 3% of the respective liability and provided that there is an excess reserve in the immediately previous period equal to or greater than that of the deficiency, the cost will not be charged. This excess, which may be used in the subsequent period, is known henceforth as excess reserves [6] .

In Brazil, contrary to what occurs in the United States, where excess or insufficient reserves are allowed, it is only possible to use the positive excess reserves from the previous period. Thus, the banks' freedom of management is decreased, making them adopt a more conservative behavior towards the management of their reserves than the U.S. banks.

formula:

$$Cost = \left\{ \left[(1 + Selic)^{\frac{1}{252.}} (1,14)^{\frac{1}{252}} \right] - 1 \right\} .Insufficiency.$$

[5] The financial cost on deficiency in mean daily balances is calculated using the following formula:

$$Cost = \{[(1 + Selic)^{\frac{n}{252}} .(1,14)^{\frac{n}{252}}] - 1\}.Deficiency,$$

where n is the number of working days in the accomplishment period. In this study n will always be equal to 10.

[6] In the literature, these excess reserves are known as carryover.

3 Modeling of Banks' Behavior for Brazil

3.1 General description of the banks' cost minimization problem[7]

In the previous section, we described the accounting and accomplishment of reserve requirements on demand deposits in Brazil, and showed the costs to which banks are subjected to if they fail to accomplish the minimum limit of reserves at the end of the day and the required average reserves in the accomplishment period. In this section, we will describe how to model the behavior of banks in the Brazilian case.

Following Clouse and Dow (2002), we will use stochastic dynamic programming to determine the optimal behavior for the banks, considering the Brazilian normative rules for reserve requirements. The use of dynamic programming is appropriate in this case due to the possibility of carryover from one period to the next. Dynamic programming usage in this case is appropppriate due to the existence of carry-over rules since a period decision has impacts on reserve requirements of all subsequent periods. The Bellman equation for this problem is:

$$V(C_t) = \min_{\{R^*_{1j} \ldots R^*_{10j}\}_{j=t}^{\infty}} E_t \left[\varphi_t + \beta V\left(C_{t+1}\right)\right] \tag{1}$$

[7] Implicitly, we will suppose that banks are risk-neutral, since they are only concerned with the expected value of costs.

Where C_t is the excess reserve carried over by the bank from the previous period that may be used to reduce the reserve requirements to be accomplished in this period [8] and φ_t is the sum of costs incurred during the period. Besides the two costs described in the previous section, established by regulation, there exists the opportunity cost incurred by the banks in a given period for maintaining reserves "idle" at the Central Bank.

The problem is stochastic because the bank has two types of uncertainty. We suppose that, after the bank chooses a target value for the ending balance of the reserve account, there is a shock to this value that may make the status of reserves at the end of the day different from that which had been planned. The reason for this hypothesis is the uncertainty about the flow of payments in the reserve account.

The second uncertainty is concerned with the overlap of accounting and accomplishment periods. Since there is intersection between the accounting and accomplishment periods, on the first three days of the accomplishment period, the banks do not know what the liability to be accomplished will be and, therefore, they are uncertain about the limit to be imposed on daily deficit. This overlap makes banks adopt a more conservative behavior towards the management of their reserves on these days, until they have the real VSRR available and, consequently, their liability for the current period. This uncertainty will be modeled as a shock that affects the level of liability on the first three days of the accomplishment period [9].

[8] We are following the notation of Clouse and Dow (2002). In the USA, this excess reserve transferred by the bank from the previous period, as mentioned in note (6), is known as carryover. Hence the selected nomenclature.

[9] Note that shocks on the first three days should also affect the deficit cost on the mean, since this cost depends on the level of liability.

In addition to the dynamic programming problem between accomplishment periods, we have the finite intraperiod dynamic programming problem. How much the bank decides to accomplish on a given day influences how much it will demand for reserves on all subsequent days in that accomplishment period. The state variable that links the days in the accomplishment period is the average accomplishment up to that day, as this average shows to what extent the bank is below or above the average reserve requirement and, therefore, will determine the deficit cost on the average to be incurred by the bank on the last day of the accomplishment period. We define the state variable for the intraperiod problem as:

$$A_{it} = \frac{1}{(i-1)} \sum_{k=1}^{i-1} \left(R_{kt}^{*} + z_{kt}\right) \tag{2}$$

Where A_{it} is the average balance of reserves between the first day of accomplishment and the day prior to the ith day of accomplishment. This will be the state variable for the intraperiod problem. R_{kt}^{*} is the target chosen by the bank for the ending balance of reserves for day k and $z_{kt} \sim N(0, \sigma_z^2)$ is the shock suffered by the bank on its ending balance after the selection of R_{kt}^{*}. When the bank chooses R_{kt}^{*}, it does not observe z_{kt}. The previous equation can be rewritten as:

$$A_{it} = \gamma_i \left(R_{i-1t}^{*} + z_{i-1t}\right) + \left(1 - \gamma_i\right) A_{i-1t} \tag{3}$$

We will abstract from this fact, since this effect is expected to be small, considering that the bank has a reasonable time (seven days) to adjust its reserves to these shocks. This characteristic could be easily modeled, but the computational time required to process the model would increase considerably.

Where:

$$\gamma_i = \frac{1}{(i-1)}$$

From days 2 to 9, the intraperiod dynamic programming problem will have a sequence of value functions that will depend on the excess reserve carried over from the previous period (C_t) and on the state variable for the intraperiod problem corresponding to that day (A_{it}). These functions will be described for day i as:

$$W_i(C_t, A_{it}) = \min_{R^*_{it}} E[i_{co}(R^*_{it} + z_{it}) + o(R^*_{it} + z_{it}) + W_{i+1}(C_t, A_{i+1t})] \quad (4)$$

Where i_{co} is the interest rate the bank does not gain for maintaining its reserves "idle" in its account at the Central Bank, herein referred to as Selic rate.

$o(R^*_{it} + z_{it})$ is the function that describes the daily deficit cost on day i.

On the tenth day, the shape of the function changes a little, since besides the costs that arise from the previous days, the bank may also have to pay for the deficit cost on the accomplishment of the mean. Note that the value function on the tenth day depends on the current value expected from all future costs. The shape of the value function on the last day of the accomplishment period is:

$$W_{10}(C_t, A_{10t}) = \min_{R^*_{10t}} E[i_{co}(R^*_{10t} + z_{10t}) + o(R^*_{10t} + z_{10t}) + d(C_t, R^*_{10t} + z_{10t}, A_{10t}) + \beta V(C_{t+1})] \quad (5)$$

Where:

$d\left(C_t, R^*_{10t} + z_{10t}, A_{10t}\right)$ is the function that describes the deficit cost for the accomplishment of the mean requirement.

The value function on the first day of accomplishment coincides with the value function of the interperiod problem and depends only on the excess reserves from the previous period (C_t). We can describe this function as:

$$W_1(C_t) = V(C_t) = \min_{R^*_{1t}} E[i_{co}(R^*_{1t} + z_{1t}) + o(R^*_{1t} + z_{1t}) + W_2(C_t, A_{1t})] \tag{6}$$

3.2 Description of cost functions for the banks

In this section, we will describe the shape of cost functions. It is interesting to note that these functions are non-differentiable and, therefore, it is not possible to find analytical solutions to the problem. Thus, the use of the numerical method to solve this problem becomes essential [10].

3.2.1 Daily deficit cost

From the fourth to the last day of the accomplishment period, the daily deficit cost function can be described as follows:

$$\begin{aligned} o(R^*_{it} + z_{it}) &= 0 && \text{if } R^*_{it} + z_{it} \geq 0,8Exig \\ o(R^*_{it} + z_{it}) &= -i_{od}(R^*_{it} + z_{it} - 0,8Exig) && \text{if } R^*_{it} + z_{it} < 0,8Exig \end{aligned} \tag{7}$$

[10] The details of the computational procedures used to solve this problem are described in the appendix.

Where $Exig$ is the liability to be accomplished by the bank in the period, after discounting the rate corresponding to the cash value, and i_{od} is the rate levied on daily deficit.

On the first three days of accomplishment, there are shocks to the liability in such a way that the daily deficit cost function is dependent on these shocks. We can describe this function for the three first days as:

$$\begin{array}{ll} o(R^*_{it} + z_{it}) = 0 & \text{if} R^*_{it} + z_{it} \geq 0,8(Exig + q_{it}) \\ o(R^*_{it} + z_{it}) = -i_{od}(R^*_{it} + z_{it} - 0,8(Exig + q_{it})) & \text{if} R^*_{it} + z_{it} < 0,8(Exig + q_{it}) \end{array} \quad (8)$$

Where $q_{it} \sim N(0, \sigma^2_q)$ is the shock that affects liability on the first three days of accomplishment.

3.2.2 Deficit cost for the accomplishment of the average reserve requirement in the period

The deficiency cost for the accomplishment of the average requirement will also depend on the excess reserves from the previous period. We can describe this function as:

$$\begin{array}{l} d(C_t, R^*_{10t} + z_{10t}, A_{10t}) = 0 \\ \quad \text{if } \frac{1}{10}(R^*_{10t} + z_{10t}) + \frac{9}{10}A_{10t} \geq Exig - C_t \\ d(C_t, R^*_{10t} + z_{10t}, A_{10t}) = -i_d[\frac{1}{10}(R^*_{10t} + z_{10t}) + \frac{9}{10}A_{10t} - Exig + C_t] \\ \quad \text{if } \frac{1}{10}(R^*_{10t} + z_{10t}) + \frac{9}{10}A_{10t} < Exig - C_t \end{array} \quad (9)$$

3.3 Description of excess reserves for Brazil[11]

In Brazil, banks can only carry over positive excess reserves to the immediately subsequent period[12]. As described in the previous section, there is an upper limit for the excess reserves. This function can be described as:

$$\begin{aligned} C_{t+1} &= 0 && \text{if } RM_t \leq Exig \\ C_{t+1} &= \min(0,03Exig, RM_t - Exig) && \text{if } RM_t > Exig \end{aligned} \tag{10}$$

Where: RM_t is the average reserve maintained by the banks in the reserve account in the Central Bank, that is:

$$RM_t = \frac{1}{10}\sum_{k=1}^{10}(R^*_{kt} + z_{kt}) \tag{11}$$

4 Results

Graph 1 shows the result found for the demand for reserves from Brazilian banks. The three first days of the accomplishment period, due to the uncertainty over the value of the liability to be accomplished, have a relatively higher mean than the other days.

[11] As negative excess reserves are not allowed in Brazil, C_{t+1} indirectly depends on C_t, since the selection of the average balance maintained in the accomplishment period depends on Ct. In addition, note that φ_t in (1) also depends on the state variable C_t, which strengthens the interconnection between the accomplishment periods.

[12] In the U.S., as mentioned in the previous section, banks can also carry over negative excess reserves, i.e., they can be deficient in a period and compensate for this deficiency in the subsequent period.

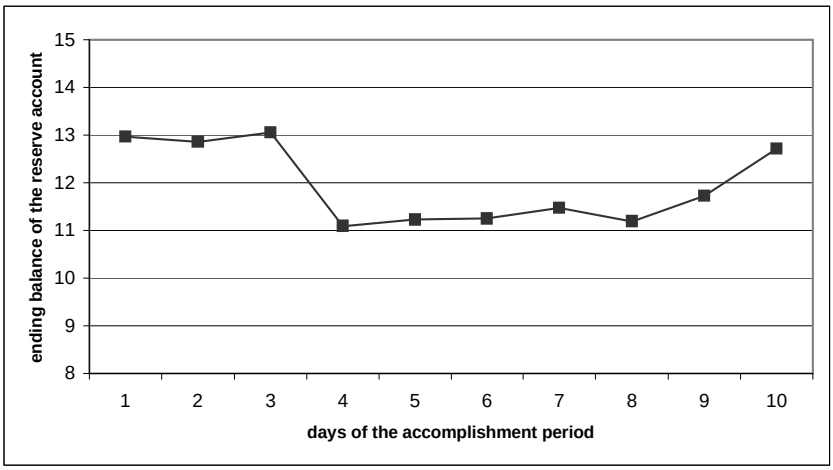

Fig. 1. Demand for reserves with $\sigma_q = 6$, $\sigma_z = 1,5$ and interest rate = 16% per year

From the fourth day on, we may note that the ending balance has an upward trend due to the accumulation of uncertainties over the flow of payments. This occurs, because in the selection of the ending balance on the eighth day, all shocks that occurred from the first to the seventh day should be considered[13]. As shown in Queiroz (2004), contrary to the U.S. case, the demand for reserves in Brazil has a downward trend in the accomplishment period. In our model, this behavior is observed only for the first four days.

[13] For further details, see Clouse and Dow (2002), page 1798.

Graph 2 shows how the demand for reserves would be like if an overlap were not observed between the accounting and accomplishment periods. As the reserve manager is not uncertain about liability on the first three days, the ending balance on these days shows a decrease compared to the rule in force. In order to accomplish the average reserve requirement, from the fourth day, the ending balance shows an increase in relation to the case in which periods overlap. When the bank is certain about its liability before the start of the accomplishment period, the variance between the daily ending balances of the reserve account is smaller. This is an interesting result in terms of monetary policy, since in a system in which the Central Bank has the interest rate as a tool, a greater predictability of reserves demand makes market easier to clear. However, it is important to underscore that the current normative rules consider seasonal aspects of the balances of demand deposits. Removing the current overlap of periods may cause the accounting period, for instance, to occur at a time at which banks have greater liquidity (large volume of demand deposits), increasing liability, and the accomplishment period to occur in periods of low liquidity (low volume of demand deposits), making it difficult to accomplish the mean.

Graph 3 shows the case in which the bank knows that there will be a 10% increase or decrease in the interest rate after the seventh day of the accomplishment period. In case of an increase intended to minimize the opportunity cost and accomplish the mean, the bank increases its demand for reserves on the days that precede this change and has a lower balance on the last four days. Otherwise, the bank takes advantage of the relatively higher interest rate at the beginning of the period (first six days) by demanding fewer reserves in order to compensate for the mean and maintain a higher balance on the last four days.

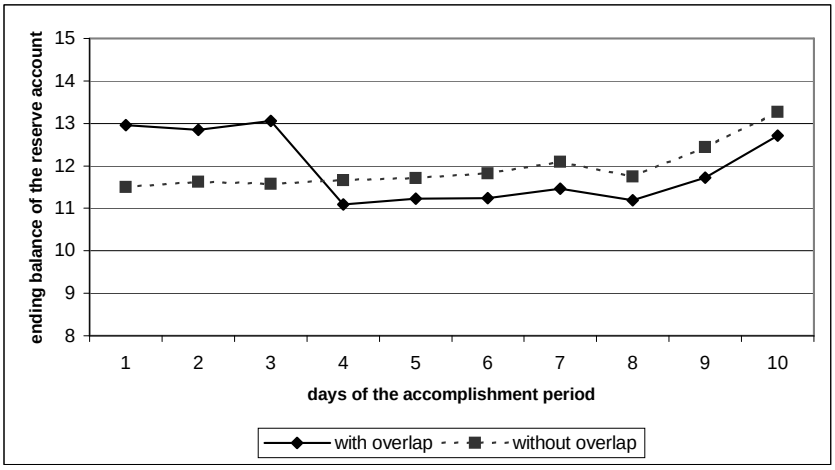

Fig. 2. Comparison between demand for reserves with and without overlap between accounting and accomplishment periods

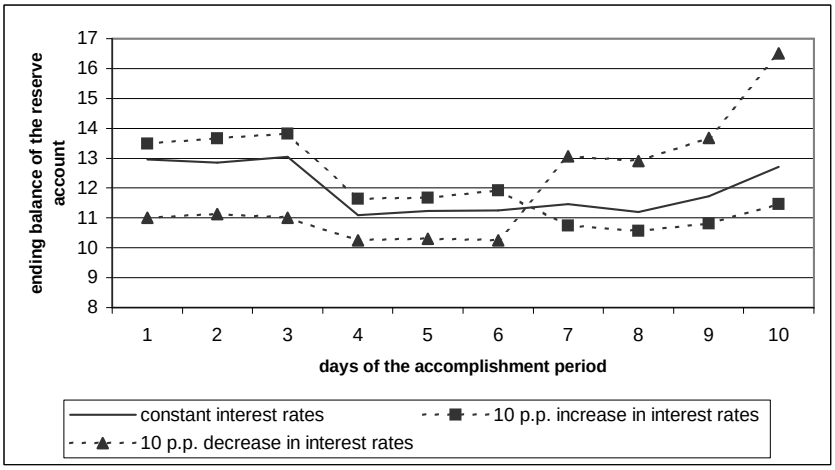

Fig. 3. Demand for reserves in the case in which the bank deals with constant interest rate and in the cases in which the bank is certain about a 10% increase or reduction in the interest rate

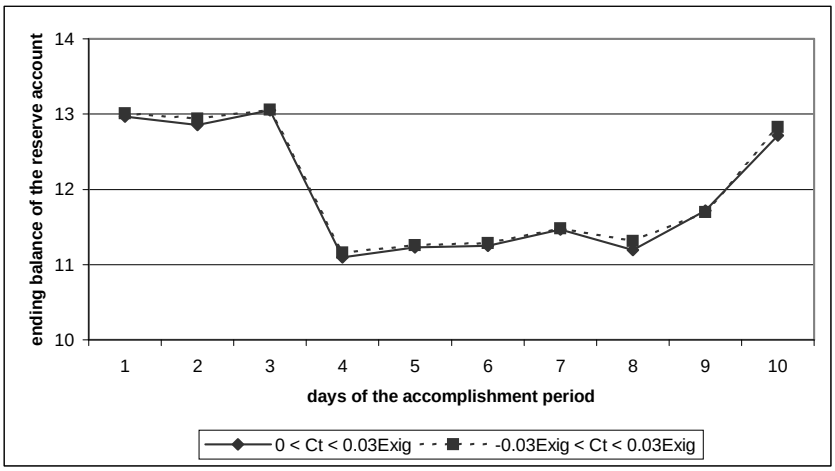

Fig. 4. Comparison between demand for reserves in the case in which $0 < C_t < 0,03Exig$ and $-0,03Exig < C_t < 0,03Exig$

Another experiment was carried out to check the behavior of the demand for reserves if excess negative reserves were allowed, as in the U.S. case [14] . Graph 4 shows that the behavior of banks is minimally changed. Since the amount of excess reserves is small in the Brazilian case, this experiment virtually does not change the behavior of banks.

As explained in the appendix, the variance and magnitude of the shock on the Brazilian flow of payments is lower than in the U.S. case, due to the large volume of reserve requirements on demand deposits. With the development of the financial market, we expect an increase in the ratio between reserve turnover and the volume of compulsory reserves [15] .

If the Central Bank of Brazil reduces the current rate of compul-

[14] In this case, $_{t+1}$ is a direct function of C_t. For further details, see Clouse and Dow (2002), page 1811.

[15] In the literature, the ratio between reserve turnover and the balance of reserve account is known as turnover ratio.

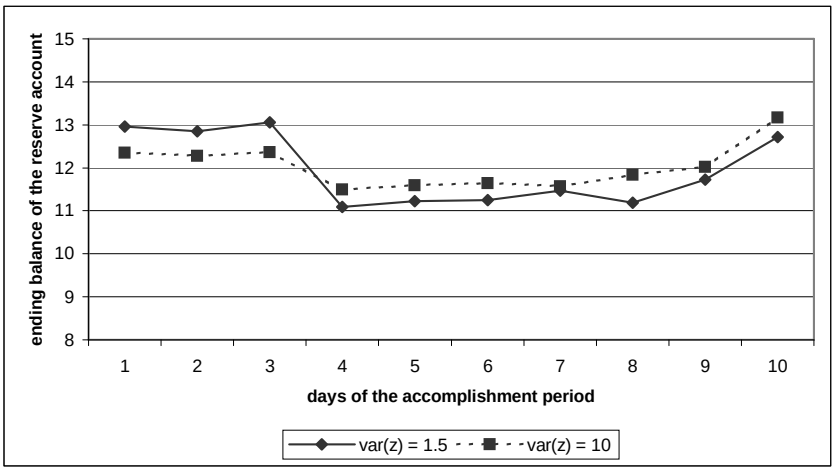

Fig. 5. Comparison between the demand for reserves in the case in which $var(z) = 1,5$ and $var(z) = 10$

sory reserves, the reserve turnover in relation to the aggregate balance of reserve accounts will increase. This increases the variance and magnitude of shock z. Graph 5 shows the demand for reserves in cases in which the variance of shock on the flow of payments is equal to 1.5 and 10. As expected, with the increase in uncertainty, the reserve manager assumes a more conservative behavior between the fourth and tenth days. The reduction in the ending balance of the reserve account on the first three days occurs due to the minor relative importance of shock q.

Graph 6 shows the behavior expected from banks if the daily deficit limit were reduced to 40% of the liability. In this case, the demand for reserves will decrease on the first six days of the accomplishment period due to the greater freedom banks have to manage liquidity. After the eighth day, the demand for reserves is higher so that the bank accomplishes the required mean for the period and thus avoid the deficit cost of the mean reserve requirement.

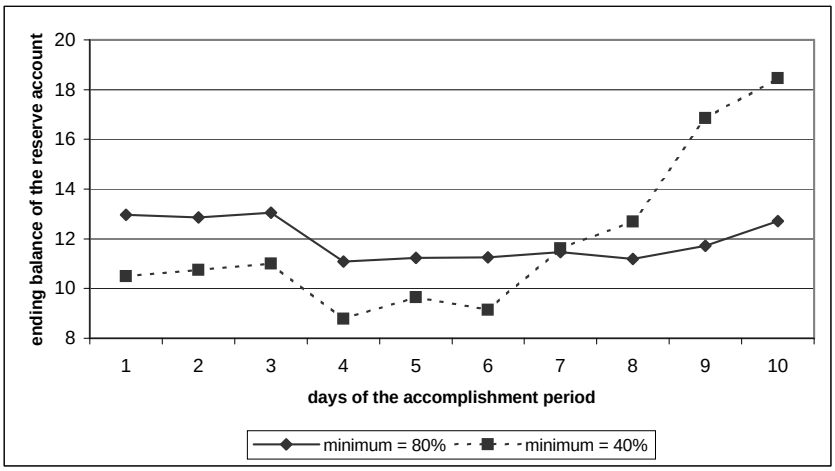

Fig. 6. Comparison between the demand for reserves in the case in which the minimum limit for daily deficit ranges between 80% and 40%

5 Conclusion

In this study, we have modeled the demand for reserves in the Brazilian market by using stochastic dynamic programming.

The results reproduce, to some extent, empirical evidence for Brazil, available from Queiroz (2004). It should be noted that by using the same model proposed by Clouse and Dow (2002) adapted to the Brazilian case, we managed to obtain empirical adherence to the theoretical model.

We showed that, in the Brazilian case, the demand for reserves is higher on the first three days due to the uncertainty about liability caused by the overlap of the accounting and accomplishment periods. The model indicates that if this uncertainty were removed, the variance in the demand for reserves during the accomplishment period would be smaller, which is preferable in terms of monetary policy, given the fact that the Central Bank

uses the interbank interest rate as a major tool.

The model proved consistent with the expectations in several exercises of sensitivity analysis shown during the study. We verified that banks manage the end-of-day balances of their reserve account depending on the expected rate to be defined by the Copom. Banks increase (decrease) the demand for reserves on the days that precede the decision and, consequently, decrease (increase) the demand for reserves on the other days during the accomplishment period, an increase (decrease) in the Selic interest rate is expected.

Other exercises that verify the sensitivity of the model to the hypotheses are interesting. For instance, how much would the demand for reserves change if the interest rate were not constant over time, and followed a certain stochastic process? Such exercises should be the concern of future studies.

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Appendix

In this appendix, we will explain the methodology used to solve the dynamic programming problem in this study.

We solved the intraperiod problem by backward induction. We started out with the last day of the accomplishment period and took an initial guess for the shape of the value function ($V(C_{t+1})$). After this guess, we found R^*_{10t} as a function of C_t and A_{10t} thus solved the minimization problem. If we have $R^*_{10t} = f(C_t, A_{10t})$ we can find the shape of function $W_{10}(C_t, A_{10t})$ by substituting $R^*_{10t} = f_{10}(C_t, A_{10t})$ into $W_{10}(C_t, A_{10t}, R^*_{10t})$:

$$\begin{aligned} W_{10}(C_t, A_{10t}) = E[&i_{co}(f_{10}(C_t, A_{10t}) + z_{10t}) \\ &+ o(f_{10}(C_t, A_{10t}) + z_{10t}) \\ &+ d(C_t, f_{10}(C_t, A_{10t}) + z_{10t}, A_{10t}) + \beta V(C_{t+1})] \end{aligned}$$

If we have the shape of $W_{10}(C_t, A_{10t})$ we can return to day nine and solve the minimization problem of that day. After solving this problem, we found $R^*_{9t} = f_9(C_t, A_{9t})$. As before, we substituted $R^*_{9t} = f_9(C_t, A_{9t})$ into $W_9(C_t, A_{9t}, R^*_{9t})$ and we found the shape of the value function for day nine. We proceeded with this iterative process up to the first day of the accomplishment period. On this day, we solved the optimal problem in R^*_{1t}, we found $R^*_{1t} = g(C_t)$, we substituted this function into $W_1(C_t, R^*_{1t})$ and we found function $W_1(C_t) = V(C_t)$. This function became our new guess for the value function of the intraperiod dynamic programming problem. With this new function, we restarted the process of backward induction until we found the next guess for the value function. This process continued until the shape of the value function did not change between the last two iterations. The theory of dynamic programming guarantees that for any

initial guess that turns out to be a well-behaved function, there will be a solution to the problem and this solution will be exactly the value function to which the problem converges [16] .

We considered that the bank has a VSRR equal to 40 monetary units. Thus, the mean daily balances necessary to accomplish the compulsory reserve corresponds to 18 monetary units. As banks can accomplish up to one third of this cash value, we consider that the mean to be accomplished in each period should be of 12 monetary units. A Selic rate of 16% per year was used. The intertemporal discount rate β amounts to 0.994, which represents the opportunity cost due to the adopted interest rate.

Computationally, we used discrete intervals for variables andR^*_{it}, z_{it}, A_{it}, q_{it} and C_t so as to solve the problem $R^*_{it} \in [6 : 0,5 : 24]$, $z_{it} \in [-3 : 1 : 3]$, $A_{it} \in [3 : 0,5 : 27]$, $q_t \in [-6 : 1 : 6]$ and $C_t \in [0 : 0,05 : 0.5]$.

We chose the variance and magnitude for shock q greater than the variance of shock z, since in Brazil, uncertainty about the flow of payments is relatively small, due to the large volume of reserve requirements on demand deposits. In the United States, for instance, the turnover ratio amounts to 133 compared to around 15 in Brazil [17] . In this study we used $\frac{\sigma_q}{\sigma_z} = 4$.

To find the mean values of daily ending balance during the accomplishment period, we simulated a model with 30,000 periods, taking the shocks from a normal distribution.

[16] For further details, see Sargent and Ljungqvist (2000).
[17] The U.S. turnover rate is that observed in Clouse and Dow (2002). For the Brazilian case, we used the daily mean for June 2004. The daily mean of payments amounted to R$ 289.7 billion, whereas the daily mean for the ending balance of the reserve accounts was R$ 19.3 billion. The data are available from *www.bcb.gov.br*.