Characterizing the Brazilian Term Structure of Interest Rates in a Cointegrated VAR Model

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Abstract

This article uses a cointegrated VAR model to test the expectation hypothesis of the Brazilian term structure of interest rates. Using monthly data from January 1995 to February 2010, this paper presents evidence that differences between two spreads are stationary. This indicates that the curvature may be more informative indicator of expected future interest rates than the slope. Level and slope are characterized by deriving the common trends using Proietti (1997) methodology. The analysis of this representation allows to link common trends with macroeconomic variables.

Resumo

Este artigo usa o modelo VAR com cointegração para testar a hipótese das expectativas na estrutura a termo das taxas de juros brasileiras. Usando dados mensais de Janeiro de 1995 até Fevereiro de 2010, este artigo apresenta evidências que as diferenças entre dois spreads são estacionárias. Isto indica que a curvatura pode ser um indicador mais informativo das taxas de juros futuras esperadas que a inclinação. O nível e a inclinação são caracterizados pela derivação das tendências comuns usando a metodologia de Proietti (1997) A análise desta representação ajuda a relacionar tendências comuns e variáveis macroeconômicas.

Key words: term structure of interest rates; expectations hypothesis; rational expectations; cointegration; common trends.

Palavras-chave: estrutura a termo das taxas de juros; hipótese das expectativas; expectativas racionais; cointegração; tendências comuns.

Área ANPEC: Área 3 - Macroeconomia, Economia Monetária e Finanças.

JEL classification: E43; G12.

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1. Introduction

The relationship between short- and long-term interest rates, or the process of formation of the term structure of interest rates, is relevant to financial market participants, whose major concern is to make inferences about investment opportunities, and also to monetary authorities, whose major concern is to monitor agents' expectations.

The most widely known theory of the term structure of interest rates, the Expectations Hypothesis (EH), posits that a long-term spot interest rate is the long-term average of the expected future spot short-term rates plus a time-invariant term premium.

According to the Efficient Markets model, the empirical U.S. and European literature tests the EH under Rational Expectations, conventionally called the rational expectations hypothesis (hereinafter referred to as REH). Shiller (1979) rejects the REH by showing that the U.S. long-term rate was relatively more volatile than the one justified by the present value model of short rates for the 1966-77 period. Mankiw and Summers (1984) analyze the behavior of two pairs of maturities in the U.S.bond market: maturities of six months and twenty years, and three months and six months for the 1963-1983 period. They reject the REH and also reject the alternative overreaction hypothesis of the long rate to the current short rate. Mankiw and Miron (1986) use U.S. treasury bonds with maturities between three and six months to test the validity of the REH for the 1890-1979 period. They reject the REH for all subperiods, except for 1890-1914, prior to the founding of the Federal Reserve System (FED). Mankiw (1986), using data on the USA, Canada, the United Kingdom, and Germany, rejects the REH when testing several of its implications. The author reinforces his conclusions by showing that changes in nondiversifiable risk, or changes in asset supply may not satisfactorily explain the large fluctuations in interest rates. Campbell and Shiller (1987) extend the present value model to nonstationary series and, although they reject the REH for U.S. data with maturities of 20 years and one month for the 1959-78 period, they show that the theoretical spread of the REH is closely related to the observed spread. Campbell and Shiller (1991) examine postwar US term structure data and report a behavior that is not consistent with the REH. For any maturity between one month and ten years, they conclude that a high yield spread between long and short rates predicts a long-term increase in the short rate according to the REH, but a short-term decline in the long rate that runs counter to the REH. Hardouvelis (1994) analyzes the behavior of interest rates in G7 countries and, by using instrumental variables, he manages to reverse the negative correlation between the yield spread and the short-run change in the long rate for all countries except the USA, where the yield spread seems to overreact to the expected change in short rates. Quite recently, Longstaff (2000), after working with REPO-agreement series as proxies for U.S. short-term risk-free rates, has not rejected the REH for maturities of up to three months. In brief, the REH is almost always rejected for the USA and often not rejected for other G7 countries.

The frequent rejection of the REH aroused the interest in the construction of a variable term premium. Hardouvelis (1994) suggests that the long rate is measured with noise, Modigliani and Sutch (1966) mention variations in the supply of long bonds encouraged by the public debt management policy, and Engle et al. (1987) build a model with a time-varying risk premium. An alternative to the variable term premium is that the failure of the REH may result from persistent expectational errors. Froot (1989) uses surveys on the expectations of interest rates to show the relevance of systematic expectational errors in long horizons. Campbell and Shiller (1991) suggest an overreaction of the yield spread to the expected future changes in short rates.

Giese (2008) extends a common approach to test the EH of the term structure. If spreads between two yields are nonstationary, the EH fails. However, if differences between two spreads are stationary suggests that the curvature of the yield curve may be a more meaningful indicator of expected future interest rates than the slope.

In Brazil, the literature on the topic is quite recent, and so is the formation of a testable term structure. Tabak and Andrade (2001) analyze the REH for the Brazilian term structure using daily data and maturities between two and twelve months from January 1995 and April 2001. By using the lagged yield spread as instrument for the current spread, they find a time dependence of the term premium and conclude for the rejection of the REH. Lima and Issler (2002) test the REH in the context of the present value model developed in Campbell and Shiller (1987) for monthly data and maturities of one month, 180 days and 360 days from January 1995 and December 2001. After they tested the implications of the present value model they concluded that the evidence is only partially favorable to the REH.

This paper examines the Brazilian term structure of interest rates considering the stationarity of the derivatives of the yield curve as suggested in Giese (2008). The differences between spreads allows to test the stationarity of weighted differences between spread in a Vector Error Correction Model. We use monthly data from January 1995 to February 2010 and present evidence that differences between two spreads are stationary. This indicates that the curvature may be a more informative indicator of expected future interest rates than the slope. Level and slope are characterized by deriving the common trends inherent in a cointegrated VAR using Proietti (1997) methodology.

Estes resultados sao importantes para participantes do mercado financeiro e autoridades monetárias porque revelam que apesar das preferencias dos individuos sobre determinada maturidades variarem no tempo, preferencias relativas entre maturidades sao estacionárias

The present paper contains six sections, including the introduction. The second section discusses the theoretical framework. Reduced form and long-run constraints are presented in the third section. The fourth section presents the data analysis and preliminary tests. The fifth section examines the empirical results. Finally, the sixth section concludes.

2. Theoretical Framework

The most widely known theory of the term structure of interest rates, the Expectations Hypothesis (EH), posits that a long-term spot interest rate is the long-term average of the expected future spot short-term rates plus a time-invariant term premium:

$$R_t^{(n)} = \frac{1}{k} \sum_{i=0}^{k-1} E_t R_{t+mi}^{(m)} + \psi_k^n, \tag{2.1}$$

where: $R_t^{(n)}$ is the long-term interest rate with n periods, $R_t^{(m)}$ is the short-term interest rate of m periods, both measured levels, $k = n/m$ is an integer, E_t is the expectation conditioned on the information available at date t, and the constant ψ_k is the term premium. If the EH holds, the term premium is constant and it is easy to understand the relationship between the expected future rates and current rates. Changes in market expectations of future short-term interest rates can result in long rate movements. In other words, except for a constant premium, the long rate is an unbiased predictor of future short rates. On the other hand, if the term premium varies through time, it is difficult to distinguish between changes in the long rate caused by the review of expected future short rates or changes in the long rate caused by a time-varying term premium.

Rearranging equation (2.1) we found a convenient representation in terms of the spread between yields of different maturities which could be tested empirically in this paper. Subtracting $R_t^{(m)}$ from both sides of (2.1) and rearranging it, we have:

$$S_t^{(n,m)} \equiv R_t^{(n)} - R_t^{(m)} = E_t \left[\frac{1}{m} \sum_{i=1}^{m-1} \left[\sum_{j=1}^{i} \left(R_{t+mj}^{(m)} - R_{t+m(j-1)}^{(m)} \right) \right] \right] + \psi_k^n;$$

or

$$S_t^{(n,m)} = E_t S_t^{(n,m)*} + \psi_k^n, \tag{2.2}$$

where,

$$S_t^{(n,m)*} = \frac{1}{m} \sum_{i=1}^{m-1} \left[\sum_{j=1}^{i} \left(R_{t+mj}^{(m)} - R_{t+m(j-1)}^{(m)} \right) \right] = \frac{1}{m} \sum_{i=1}^{m-1} (m-i) \left(R_{t+mi}^{(m)} - R_{t+m(i-1)}^{(m)} \right) \tag{2.3}$$

is a weighted average of future $(k-1)$ short-run changes in the short rate. Actually, if a long-term increase in short-term rates is expected, the current yield of the long bond should be higher than the current yield of the short-term bond, as a way to equalize the return until the maturity of the first bond with the yield of the sequence of k investments in short-term bonds between dates t and $t+n$. The variable $S_t^{(n,m)*}$ is called perfect foresight spread, since, except for the constant ψ_k, it is the spread between long and short bonds if the forecast of future short rates were perfect.

Since bond yields could be well approximated by processes integrated of order one, their differences are integrated of order zero and the first term on the right hand side of equation (2.3) is stationary. If the term premium was stationary, we would expect the spreads (left hand side of equation (2.3)) to be stationary, whilst non-stationary spreads would imply a non-stationary term premium.

This framework could be extended to weighted differences between spreads,

$$\begin{aligned}\left(R_t^{(n)} - R_t^{(m)}\right) - c\left(R_t^{(m)} - R_t^{(s)}\right) = &\frac{1}{n}\sum_{i=1}^{n-1}(n-i)\,E_t\left(R_{t+mi}^{(m)} - R_{t+m(i-1)}^{(m)}\right) \\ &- \frac{c}{m}\sum_{i=1}^{m-1}(m-i)\,E_t\left(R_{t+si}^{(s)} - R_{t+s(i-1)}^{(s)}\right) + \psi_k^n - c\psi_k^m\end{aligned} \tag{2.4}$$

Rearranging equation (2.4) we found a convenient representation, as suggested in Giese (2008), that will be tested empirically in this paper,

$$\begin{aligned}\left(R_t^{(n)} - R_t^{(s)}\right) - (1+c)\left(R_t^{(m)} - R_t^{(s)}\right) = &\frac{1}{n}\sum_{i=1}^{n-1}(n-i)\,E_t\left(R_{t+mi}^{(m)} - R_{t+m(i-1)}^{(m)}\right) \\ &- \frac{(1+c)}{m}\sum_{i=1}^{m-1}(m-i)\,E_t\left(R_{t+si}^{(s)} - R_{t+s(i-1)}^{(s)}\right) \\ &+ \psi_k^n - (1+c)\psi_k^m\end{aligned} \tag{2.5}$$

where c is a constant. Equation (2.5) shows that if the spreads are pairwise cointegrated, the weighted differences between the term premia of differing maturities have to be stationary. In the case of cointegrated term premia, a deviation from the usual curvature may reflect changes in the future interest rate expectations and the yield curve could be used as indication of expectations on the future path of interest rates.

3. Reduced Form and Long-Run Constraints

A full discussion of the econometric models employed here can be found in Beveridge and Nelson(1981), Stock and Watson(1988), Engle and Granger(1987), Campbell(1987), Campbell and Deaton(1989), and Proietti(1997). We start by assuming that y_t is a 5×1 vector containing the yields at time t of a zero coupon bonds with n months to maturity. We also assume that all series individually contain a unit-root, and are generated by a p-th order vector autoregression (VAR):

$$y_t = \pi_1 y_{t-1} + \pi_2 y_{t-2} + \cdots + \pi_p y_{t-p} + \varepsilon_t \tag{3.1}$$

or

$$\begin{aligned} y_t - \pi_1 y_{t-1} - \pi_2 y_{t-2} - \cdots - \pi_p y_{t-p} &= \varepsilon_t, \\ \left(I_n - \pi_1 L - \pi_2 L^2 - \cdots - \pi_p L^p\right) y_t &= \varepsilon_t, \\ \Pi\left(L\right) y_t &= \varepsilon_t. \end{aligned}$$

Decomposing $\Pi\left(L\right) = I_n - \pi_1 L - \pi_2 L^2 - \cdots - \pi_p L^p$ as:

$$\Pi\left(L\right) = -\Pi\left(1\right) L^p + \left(1 - L\right) \Gamma\left(L\right),$$

We have the vector error-correction model (VECM):

$$\begin{aligned} \Delta y_t - \Gamma_1 \Delta y_{t-1} - \Gamma_2 \Delta y_{t-2} - \cdots - \Gamma_{p-1} \Delta y_{t-p+1} - \Pi\left(1\right) y_{t-p} &= \varepsilon_t, \text{ or,} \\ \Delta y_t - \Gamma_1 \Delta y_{t-1} - \Gamma_2 \Delta y_{t-2} - \cdots - \Gamma_{p-1} \Delta y_{t-p+1} - \alpha\beta' y_{t-p} &= \varepsilon_t, \end{aligned}$$

where $\Gamma_j = -I_n + \sum_{i=1}^{j} \pi_i$, $j = 1, 2, \cdots, p-1$.

We tested the theoretical framework presented early by determinig the number of stationary cointegrating relations and non stationary commmon trends. Hence, the final reduced form to be estimated, after appropriate testing is:

$$\Delta\, y_t = \Gamma_1\, \Delta\, y_{t-1} + \ \ldots\ + \Gamma_{p-1}\, \Delta\, y_{t-p+1} + \alpha\beta' y_{t-p} + \varepsilon_t \tag{3.2}$$

We turn now to the discussion of how to extract trends and cycles from (3.2). Jumping straight to our results, we found that the system (3.2) is well described by a $VECM(p)$, which can be put in state-space form, as discussed in Proietti(1997):

$$\begin{aligned} \Delta y_{t+1} &= Z f_{t+1} \\ f_{t+1} &= T f_t + Z' \varepsilon_{t+1}, \end{aligned} \tag{3.3}$$

where,

$$f_{t+1} = \begin{bmatrix} \Delta y_{t+1} \\ \Delta y_t \\ \vdots \\ \alpha' y_{t-p} \end{bmatrix}, \; T = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \cdots & \Gamma_{p-1} & -\alpha\beta' & -\alpha \\ I_N & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_N & 0 & 0 \\ 0 & 0 & \cdots & 0 & \beta' & I_r \end{bmatrix}$$

with the associated VECM being,

$$\begin{aligned} \Delta y_t &= \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p+1} + \alpha\beta' y_{t-2} + \varepsilon_t, \text{ and,} \\ Z &= \left[I_N \; \mathbf{0} \; \mathbf{0} \right]. \end{aligned}$$

From the work of Beveridge and Nelson(1981), and Stock and Watson(1988), ignoring initial conditions and deterministic components, the series in y_t can be decomposed into a trend (μ_t) and a cyclical component (ψ_t), as follows:

$$y_t = \mu_t + \psi_t,$$

where,

$$\begin{aligned} \mu_t &= y_t + \lim_{l\to\infty} \sum_{i=1}^{l} E_t \left[\Delta y_{t+i} \right], \text{ and,} \\ \psi_t &= -\lim_{l\to\infty} \sum_{i=1}^{l} E_t \left[\Delta y_{t+i} \right]. \end{aligned}$$

It is straightforward to show that μ_t is a multivariate random-walk. Using the state-space representation (3.3), we can compute the limits above. The cyclical and trend components will be, respectively:

$$\begin{aligned} \psi_t &= -Z \left[I_m - T \right]^{-1} T f_t, \\ \mu_t &= y_t - \psi_t. \end{aligned} \tag{3.4}$$

or, using formulas (6) and (7) in Proietti(1997),

$$\psi_t = -K\Gamma^* (L) \Delta y_t + P y_t, \tag{3.5}$$

and

$$\mu_t = K \sum_{i=1}^{t} \varepsilon_i, \tag{3.6}$$

where K and P are projection matrices (see the Appendix).

We can also use (3.3) to forecast trend and cyclical components at any horizon into the future. The forecast of ψ_{t+s}, given information up to t, is:

$$\widehat{\psi}_{t+s|t} = E_t\left[\psi_{t+s}\right] = -K\Gamma^*\left(L\right)ZTf_{t+s-1} + Py_t + PZ\left(\sum_{i=1}^{s} T^i\right)f_t,$$

and the forecast of μ_{t+s}, given information up to t, is:

$$\widehat{\mu}_{t+s|t} = \mu_t,$$

since the best forecast of a random walk $t+s$ periods ahead is simply its value today.

To fully characterize the elements in (3.2), we need to compute the variance and the covariance of forecasts of the trend and cyclical components. Recall that the conditional expectation of a log-Normal random variable is just a function of the mean and variance of the normal distribution associated with it. Hence, to compute the variances of these forecasts, we have just to apply standard results of state-space representations. It is straightforward to show that:

$$E_t\left[\left(\mu_{t+s} - \widehat{\mu}_{t+s|t}\right)\left(\mu_{t+s} - \widehat{\mu}_{t+s|t}\right)'\right] = s \cdot KQK',$$

where $E_t\left[\varepsilon_{t+i}\varepsilon'_{t+i}\right] = Q$. We also have,

$$E_t\left[\left(\psi_{t+s} - \widehat{\psi}_{t+s|t}\right)\left(\psi_{t+s} - \widehat{\psi}_{t+s|t}\right)'\right] = VQV' + P\left(\sum_{i=1}^{s-1} W(i)QW(i)'\right)P'$$

and

$$E_t\left[\left(\mu_{t+s} - \widehat{\mu}_{t+s|t}\right)\left(\psi_{t+s} - \widehat{\psi}_{t+s|t}\right)'\right] = KQV' + K\left[\sum_{i=1}^{s-1} QW(i)'\right]P',$$

where $V = \left[P - K\Gamma^*\left(1\right)\right]$, as computed in the Appendix.

Equation (3.6) shows that the trend innovation is a rank k linear combination of ε_i, then the common trends are $\mu_t^* = \alpha'_{\perp}\Gamma(L)y_t$.

4. Data Analysis and Preliminary Tests

The Brazilian term structure of interest rates was observed at a monthly frequency from September 1996 to February 2010. Figure 1 shows the constant maturity continuously compounded spot rate per year. Three major aspects may be seen: the shift in the Brazilian exchange rate system in January 1999, the energy crisis and the 2002 Brazilian election.

Figure 1 Graph of monthly of the spot interest rates

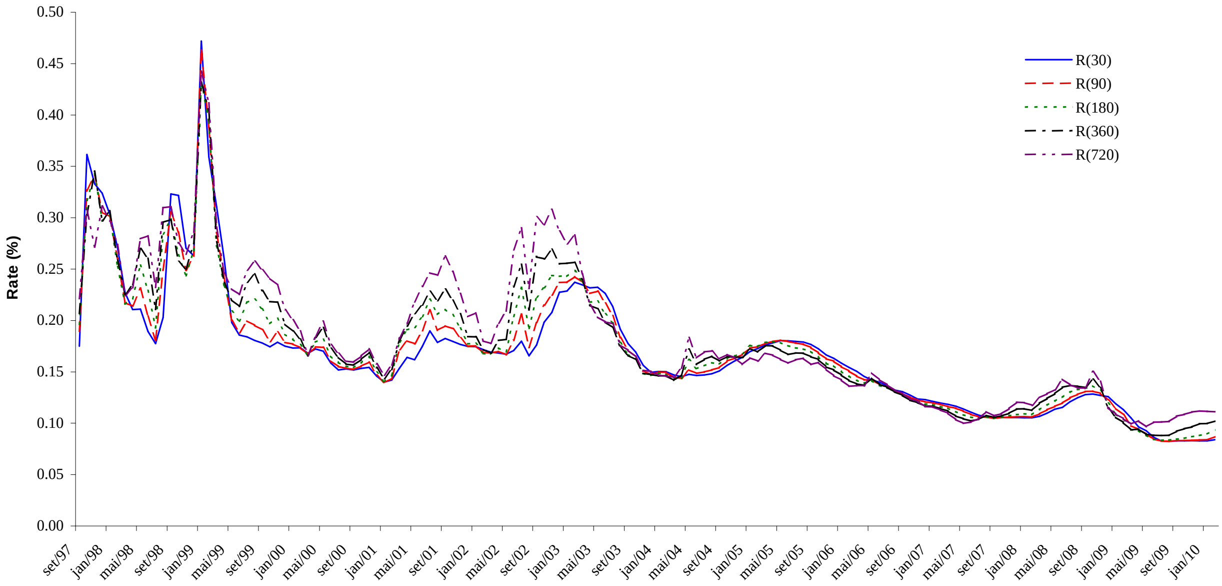


Table 1 shows some descriptive statistics of the levels and first differences of monthly spot interes rate. Similarly to international evidence, the Brazilian term structure of interest rates was positively sloped, with higher volatility on shorter and longer maturities. As often occurs with interest rate series, a high autocorrelation indicates that the available information in the sample is actually smaller than its size could indicate (150 observations). The nonstationarity of the series was assessed by the Dickey-Fuller Test with GLS Detrending (DF-GLS) of Elliott, Rothenberg and Stock (1996), whose null hypothesis is nonstationarity. The results provide evidence of nonstationarity in levels and stationarity in differences.

Table 1
Descriptive statistics of Brazilian annual spot rates from September 1997 to February 2010

	Rates in the level					Monthly difference in the rates				
Statistics	$R^{(30)}$	$R^{(90)}$	$R^{(180)}$	$R^{(360)}$	$R^{(720)}$	$R^{(30)}$	$R^{(90)}$	$R^{(180)}$	$R^{(360)}$	$R^{(720)}$
Mean	0.1679	0.1693	0.1720	0.1768	0.1836	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007
Median	0.1636	0.1658	0.1653	0.1656	0.1644	-0.0015	-0.0015	-0.0010	-0.0013	-0.0023
Maximum	0.4720	0.4631	0.4322	0.4315	0.4425	0.2091	0.2004	0.1642	0.1567	0.1565
Minimum	0.0824	0.0825	0.0834	0.0880	0.0965	-0.1127	-0.1028	-0.1213	-0.1277	-0.1208
Std. Dev.	0.0615	0.0605	0.0606	0.0631	0.0669	0.0293	0.0264	0.0249	0.0248	0.0256
Skewness	1.6899	1.4909	1.1805	1.0543	0.9728	4.1355	3.4439	2.1491	1.4099	1.3465
Kurtosis	7.4664	6.9981	5.4238	4.6053	3.8699	32.8367	30.4729	22.3112	18.9364	15.7427
Jarque-Bera	196.0704	155.4763	71.5585	43.8921	28.3861	5991.4900	5013.7570	2446.2350	1637.0040	1053.1100
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_1	0.8800	0.8980	0.9090	0.9170	0.9220	-0.0910	0.0800	0.1100	0.0930	0.0070
ρ_2	0.7760	0.7760	0.7940	0.8160	0.8380	-0.1260	-0.3080	-0.3540	-0.3120	-0.2140
ρ_3	0.7000	0.7110	0.7380	0.7600	0.7870	-0.0080	-0.0570	-0.0830	-0.0990	-0.1020
ρ_4	0.6260	0.6580	0.6960	0.7210	0.7480	0.1140	0.0800	0.0110	-0.0430	-0.0030
ρ_5	0.5270	0.5890	0.6520	0.6880	0.7100	-0.1190	0.0200	0.0880	0.1000	0.0850
DF-GLS	-0.11	-1.94	0.09	-0.22	-1.08	-3.14	-1.60	-2.03	-2.31	-2.01

Notes:
(i) Size of the samples: 150 observations in levels and 149 in differences;
(ii) DF-GLS tests H_0: nonstationary series. Daily level includes intercept with a window of 13 lags. Daily difference uses 12 lags;
(iii) DF-GLS critical values: -2.58 (1%), -1,94 (5%) and -1.62 (10%);
(v) ρ_i indicates autocorrelation of order i.

Although the nonstationarity in interest rates seems questionable in light of the economic theory, several studies assume the nonstationarity of the interest rate levels and focus on modeling differences of interest rates or of some difference between maturities, such as the yield spread or the holding return. The cointegration of longer- and shorter-term with unit coefficientis is a necessary condition for the REH to be valid. The condition is not sufficient because the cointegration requires stationarity on expectations errors and term premium. In other words, the cointegration is consistent with a time-varying term premium.

5. Empirical Results

The theoretical relationships presented in sections 2 and 3 are investigated by determining the number of cointegrating relations and nonstationary common trends, and testing explicit hypotheses about the parameters of these relations.

The vector equilibrium correction model (VECM(p)), equation (3.2), consists of monthly end-of-period yields for Brazilian zero-coupon bonds of five maturities, one month ($R^{(30)}$), three months ($R^{(90)}$), six-months ($R^{(180)}$), one-year ($R^{(360)}$) and two-years ($R^{(720)}$) bonds. The choice reflects the structure of the yield curve with short-, medium- and long-term maturities.

Initially, we need to determine the lag length of the model. The choice of lag length reflects the persistence of short-term effects. We use the usual selection criteria and chose thirteen lags in levels and twelve lags in differences, which means that twelve matrices need to be estimated in the VECM (3.2).

Then we estimate the cointegration rank using the trace statistics and the maximum eigenvalue statistic. Table 2 reports the results of Johansen (1988) cointegration test applied to the set of interest rates used in this exercise. As can be seen from this table, we accept the hypothesis of three cointegrating vectors using the two previously cited statistics. Cointegrating rank equal to three implies the existence of two common trends which in turn imply four spreads that can not be stationary, and the expectations hypothesis is not valid.

Table 2

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Rank Test (Trace) Trace Statistic	Critical Value (5%)	Prob.**	Rank Test (Maximum Eigenvalue) Max-Eigen Statistic	Critical Value (5%)	Prob.**
None *	0.5638	188.70	69.82	0.00	113.66	33.88	0.00
At most 1 *	0.2349	75.04	47.86	0.00	36.68	27.58	0.00
At most 2 *	0.1678	38.37	29.80	0.00	25.16	21.13	0.01
At most 3	0.0866	13.20	15.49	0.11	12.42	14.26	0.10
At most 4	0.0057	0.78	3.84	0.38	0.78	3.84	0.38

Notes:

(i) Trace test indicates 3 cointegrating eqn(s) at the 0.05 level;

(ii) * denotes rejection of the hypothesis at the 0.05 level;

(iii) **MacKinnon-Haug-Michelis (1999) p-values.

Table 3 displays the normalized coefficients of the three cointegrating vectors chosen by the Johansen (1988) methodology. We note that one-month bonds belongs only to the first cointegration relation, three-months bonds belongs only to the second cointegrating relation and six-months bonds belong only to the third cointegrating relation, while one-year and two-years bonds belong to all three relations. The first cointegrating vector determines a long-run relationship between yields of short-, medium- and long-term maturities, while the other two vectors determine long-run relationships between yields of medium- and long-term maturities.

Table 3

Cointegrating vectors

Normalized cointegrating coefficients				
$R^{(30)}$	$R^{(90)}$	$R^{(180)}$	$R^{(360)}$	$R^{(720)}$
1	0	0	-2.02	1.10
			(0.22)	(0.20)
0	1	0	-1.89	0.93
			(0.16)	(0.14)
0	0	1	-1.62	0.63
			(0.07)	(0.07)

Notes:
(i) standard error in parentheses;
(ii) $R^{(n)}$ denotes the $\log(1+R^{(n)}/100)$

The estimated coefficients of the adjustment matrix are shown in Table 4 below. We can note by the significance of the estimated coefficients that the importance of cointegration relations in the adjustment matrix fall with maturity.

Table 4

	Adjustment coefficients		
$D(R^{(30)})$	2.29	-7.57	8.84
	(0.96)	(2.37)	(2.39)
$D(R^{(90)})$	2.90	-8.69	9.48
	(1.13)	(2.78)	(2.81)
$D(R^{(180)})$	2.57	-7.68	8.62
	(1.20)	(2.95)	(2.97)
$D(R^{(360)})$	2.60	-7.77	9.13
	(1.39)	(3.42)	(3.46)
$D(R^{(720)})$	1.91	-6.44	8.12
	(1.69)	(4.16)	(4.20)

Notes:
(i) standard error in parentheses;
(ii) $D(R^{(n)})$ denotes the difference of $\log(1+R^{(n)}/100)$

Table 5 shows some descriptive statistics of the levels of yield spreads The nonstationarity of the series was assessed by the Dickey-Fuller Test with GLS Detrending (DF-GLS) of Elliott, Rothenberg and Stock (1996), whose null hypothesis is nonstationarity The result provides evidence of nonstationarity in levels of yields spreads, except the yield spread for three months and a month.

Table 5
Descriptive statistics of levels of yield spreads from September 1997 to February 2010

	Yield spreads						
Statistics	$R^{(90)}$ - $R^{(30)}$	$R^{(180)}$ - $R^{(30)}$	$R^{(360)}$ - $R^{(30)}$	$R^{(720)}$ - $R^{(30)}$	$R^{(360)}$ - $R^{(90)}$	$R^{(360)}$ - $R^{(180)}$	$R^{(720)}$ - $R^{(360)}$
Mean	0.0014	0.0041	0.0089	0.0157	0.0075	.0047	0.0068
Median	0.0004	0.0025	0.0054	0.0092	0.0043	.0035	0.0050
Maximum	0.0466	0.0816	0.0927	0.1248	0.0640	.0391	0.0389
Minimum	-0.0356	-0.0578	-0.0642	-0.0615	-0.0316	-.0146	-0.0738
Std. Dev.	0.0098	0.0179	0.0248	0.0328	0.0165	.0085	0.0123
Skewness	0.1976	0.4694	0.6864	0.9396	0.9450	1.1309	-1.2660
Kurtosis	8.0254	6.0255	4.4861	4.0161	4.1949	4.9225	14.6606
Jarque-Bera	158.8158	62.7163	25.5821	28.5219	31.2516	55.0707	889.8798
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_1	0.3000	0.4610	0.5980	0.7500	0.7310	.7920	0.5710
ρ_2	0.2710	0.4110	0.5260	0.6090	0.6170	.6670	0.5700
ρ_3	0.1290	0.2720	0.3610	0.4840	0.4660	.5330	0.4430
ρ_4	0.0400	0.0910	0.1640	0.3060	0.2540	.3780	0.4320
ρ_5	-0.0210	-0.0620	0.0330	0.1870	0.1090	.2700	0.3880
DF-GLS	-1.89	-2.26	-1.30	-1.27	-1.53	-0.64	-2.34

Notes:
(i) Size of the samples: 150 observations in levels and 149 in differences;
(ii) DF-GLS tests H_0: nonstationary series. Daily level includes intercept with a window of 13 lags. Daily difference uses 12 lags;
(iii) DF-GLS critical values: -2.58 (1%), -1,94 (5%) and -1.62 (10%);
(v) ρ_i indicates autocorrelation of order i.

We identify the adjustment matrix and cointegration matrix together. Identifying cointegrating vectors, we can use these stationary relations and the weighted differences between spreads as shown in equation (2.5). Table 6 shows the normalized long-run relations. The first equation indicates that the short, medium and long-end of the yield curve can be characterized by a approximately stationary curvature, while the remaining two equations indicate that the medium and long-end can also be characterized by stationary curvatures.

Table 6
Normalized long-run relations

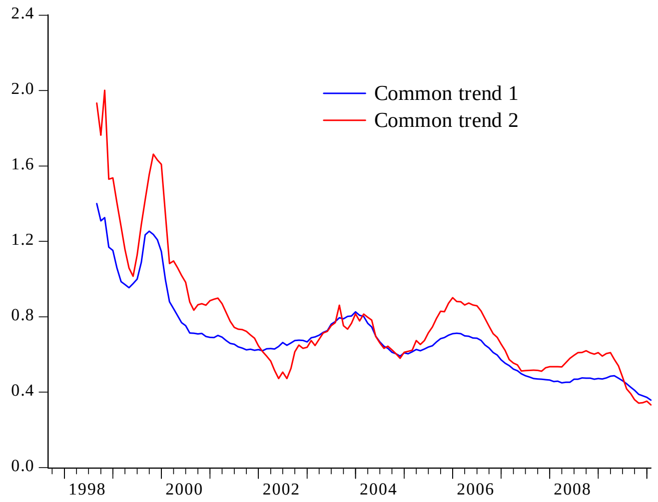
Equation 1		Equation 2		Equation 3	
$\beta_1 * y_t$		$\beta_2 * y_t$		$\beta_3 * y_t$	
$R^{(360)}$ - $R^{(30)}$	-0.57	$R^{(360)}$ - $R^{(90)}$	-0.59	$R^{(360)}$ - $R^{(180)}$	-0.59
	(0.08)		(0.07)		(0.07)
$R^{(720)}$ - $R^{(360)}$	0.60	$R^{(720)}$ - $R^{(360)}$	0.52	$R^{(720)}$ - $R^{(360)}$	0.35
	(0.16)		(0.11)		(0.06)
C	0.00	C	0.00	C	0.00
	(0.00)		(0.00)		(0.00)

Notes:
(i) standard error in parentheses;
(ii) $R^{(n)}$ - $R^{(m)}$ is the spread between yields of different maturities.

We found that spreads are not stationary, while linear combinations of these spreads are stationary. This finding is important for understanding how expectations on interest rates are formed and consequently for monetary policy. This result suggest that deviations from spreads are persistent, while equilibrium mean reversion is fast for differences.

We used the methodology discussed in Proietti (1997) to decompose each maturity series into a trend and cycle component and thus we could identify the two common trends. Figure 2 illustrates the shape of the common trends.

Figure 2 - Common trends



We found that common trends and inflation are positively correlated, indicating that shocks to the long rates that shift the yield curve appear to be related to inflationary shocks.

6. Conclusions

This paper examines the Brazilian term structure of interest rates considering the stationarity of the derivatives of the yield curve as suggested in Giese (2008). The differences between spreads allows to test the stationarity of weighted differences between spread in a Vector Error Correction Model.

We accept the hypothesis of cointegration rank equal to three, which implies the existence of two common trends, which in turn imply four spreads that can not be stationary, and the expectations hypothesis is not valid. The first cointegrating vector determines a long-run relationship between yields of short-, medium- and long-term maturities, while the other two vectors determine long-run relationships between yields of medium- and long-term maturities.

We used the cointegrating relations and the weighted differences between spreads to identify normalized long-run relations, one equation indicates that the short, medium and long-end of the yield curve can be characterized by a approximately stationary curvature, while the remaining two equations indicate that the medium and long-end can also be characterized by stationary curvatures.This finding is important for understanding how expectations on interest rates are formed and consequently for monetary policy. This result suggest that deviations from spreads are persistent, while equilibrium mean reversion is fast for differences.

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A. State-Space Representation for Error-Correction Models and the Beveridge and Nelson Decomposition (Proietti(1997))

Proietti(1997) discussed in some length how the Beveridge and Nelson(1981) decomposition can be put in state-space form. Here we adapt some of this discussion. If the series y_t are generated by a vector autoregression (VAR):

$$\begin{aligned} y_t - \pi_1 y_{t-1} - \pi_2 y_{t-2} - \cdots - \pi_p y_{t-p} &= \varepsilon_t, \\ \left(I_n - \pi_1 L - \pi_2 L^2 - \cdots - \pi_p L^p\right) y_t &= \varepsilon_t, \text{ or,} \\ \Pi\left(L\right) y_t &= \varepsilon_t. \end{aligned}$$

and we decompose $\Pi\left(L\right) = I_n - \pi_1 L - \pi_2 L^2 - \cdots - \pi_p L^p$ as:

$$\Pi\left(L\right) = -\Pi\left(1\right) L^p + \left(1 - L\right) \Gamma\left(L\right),$$

leading to the vector error-correction model (VECM):

$$\begin{aligned} \Delta y_t - \Gamma_1 \Delta y_{t-1} - \Gamma_2 \Delta y_{t-2} - \cdots - \Gamma_{p-1} \Delta y_{t-p+1} - \Pi\left(1\right) y_{t-p} &= \varepsilon_t, \text{ or,} \\ \Delta y_t - \Gamma_1 \Delta y_{t-1} - \Gamma_2 \Delta y_{t-2} - \cdots - \Gamma_{p-1} \Delta y_{t-p+1} - \alpha\beta' y_{t-p} &= \varepsilon_t, \end{aligned}$$

where $\Gamma_j = -I_n + \sum_{i=1}^{j} \pi_i$, $j = 1, 2, \cdots, p-1$, it is straightforward to put the latter into state space form. To save space, and jumping straight to the series modelled here, we start by assuming that Δy_t is a 5×1 vector containing the instantaneous yield at time t of a zero-coupon bond with maturity m, which can be modelled as VECM($p-1$) (or a VAR(p)), where $\beta' y_t$ is the error-correction vector and α is the adjustment coefficient vector. The state-space form of the VECM($p-1$) is as follows:

$$\begin{aligned} \Delta y_{t+1} &= Z f_{t+1} \\ f_{t+1} &= T f_t + Z' \varepsilon_{t+1}, \end{aligned} \tag{A.1}$$

where,

$$f_{t+1} = \begin{bmatrix} \Delta y_{t+1} \\ \Delta y_t \\ \vdots \\ \alpha' y_{t-p} \end{bmatrix}, \; T = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \cdots & \Gamma_{p-1} & -\alpha\beta' & -\alpha \\ I_N & 0 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_N & 0 & 0 \\ 0 & 0 & \cdots & 0 & \beta' & I_r \end{bmatrix}$$

with the associated VECM being,

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p+1} + \alpha\beta' y_{t-2} + \varepsilon_t, \text{ and,}$$
$$Z = [I_N \ \mathbf{0} \ \mathbf{0}] \, .$$

A.1. Trends and Cycles and State-Space Representation

The basic idea in Beveridge and Nelson(1981) is that, for unit-root processes with zero drift, the random-walk trend in the series and its long-run forecast will both be the same. Hence, for series in y_t can be decomposed into a trend (μ_t) and a cyclical component (ψ_t), as follows:

$$y_t = \mu_t + \psi_t,$$

where,

$$\mu_t = y_t + \lim_{l \to \infty} \sum_{i=1}^{l} E_t \left[\Delta y_{t+i}\right], \text{ and,}$$
$$\psi_t = -\lim_{l \to \infty} \sum_{i=1}^{l} E_t \left[\Delta y_{t+i}\right].$$

Using (**??**), we can compute the limits above. The cyclical component will be:

$$\psi_t = -Z \left[I_m - T\right]^{-1} T f_t. \tag{A.2}$$

The trend in y_t can be simply computed as:

$$\mu_t = y_t - \psi_t.$$

A.2. Computing Mean Squared Errors

From Proposition 2 in Proietti(1997),

$$\psi_{t+1} = -(I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\Gamma^*(L)\Delta y_{t+1} + P y_{t+1}, \tag{A.3}$$

and,

$$\mu_{t+1} = (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\Gamma(L) y_{t+1}$$

or,

$$\Delta\mu_{t+1} = (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\varepsilon_{t+1}, \tag{A.4}$$

where $P = (\Gamma(1)+\gamma\alpha')^{-1}\gamma\,[\alpha'(\Gamma(1)+\gamma\alpha')^{-1}\gamma]\,\alpha'$, and $\Gamma(L) = I_2 - \Gamma_1 L$, which is decomposed as:

$$\begin{aligned}\Gamma(L) &= \Gamma(1) + (1-L)\,\Gamma^*(L)\,, \text{ where,}\\ \Gamma^*(L) &= \Gamma_1,\end{aligned}$$

in the present context.