

## Debt Financing and Emergent Dynamics of a Financial Fitness Landscape ♦

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*Abstract:* The paper develops an agent-based computational bottom-up model of output production in which the endogenous supply of credit-money by an adaptive banking system and the cash flow of heterogeneous firms are modeled as co-evolutionary phenomena. As it turns out, financial fragility – à la Hyman Minsky – becomes an emergent dynamics in such a complex economy, with the distribution of financing regimes (hedge, speculative and Ponzi, to borrow Minsky's taxonomy) across firms, or the financial fitness landscape of the system, coming out as a spontaneous order.

*Keywords:* complexity approach; financial fragility; emergent dynamics.

*Resumo:* O artigo elabora um modelo de produção, computacional e baseado em agente, no qual a oferta de moeda de crédito por um sistema bancário adaptativo e o fluxo de caixa de firmas heterogêneas são modelados como fenômenos co-evolucionários. Assim, a fragilidade financeira – à Hyman Minsky – torna-se uma dinâmica emergente nessa economia complexa, sendo que distribuição de regimes de financiamento (hedge, especulativo e ponzi, emprestando a taxonomia de Minsky) entre as firmas, ou o grau de robustez financeira do sistema, emerge como uma ordem espontânea.

*Palavras-chave:* abordagem da complexidade; fragilidade financeira; dinâmica emergente.

Classificação JEL: B59; C63; E12

ANPEC: Área 1 – Escolas do Pensamento Econômico, Metodologia e Economia Política.

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♦ We are grateful to Caio Racy, Paulo Scarano, Steve Keen and Mark Setterfield for useful comments. The responsibility for the final content is solely ours, though. Research funding granted by the Brazilian National Council of Scientific and Technological Development (CNPq) is also gratefully acknowledged.

## 1. Introduction

This paper develops an agent-based computational bottom-up model of production in which an endogenous supply of credit-money by an adaptive banking system and the cash flow position of heterogeneous firms are modeled as co-evolutionary phenomena. As it turns out, financial fragility and instability à la Hyman Minsky (1975; 1982) becomes an emergent dynamics in such a complex economy, with the distribution of financing regimes (hedge, speculative and Ponzi, to borrow Minsky's taxonomy) across firms, or the financial fitness landscape of the system, coming out as a spontaneous order.

Following a keynesian monetary approach, the supply of credit-money at any single period is endogenous, demand-driven at the current nominal interest rate. The latter is set by an adaptive banking system through a rule-of-thumb, satisficing procedure as a markup over the base rate, which is determined by the monetary authority. The underlying presumption is that the banking system is price maker and quantity taker in its markets for loans, and price taker and quantity maker in the markets where it raises funding. Firms, in turn, follow a markup-pricing behavior as well, with their individual prices being set as a firm-specific markup over prime costs.

The decision-making of the collection of heterogeneous firms and the banking system that populate the economy, the former with respect to production and pricing while the latter with respect to pricing, is updated periodically through mechanisms of local inference (production and/or pricing by competitors) and/or of intertemporal inference (past performance and expected future one).

Hence, this paper analyzes the dynamics of financial fragility and instability from the perspective of the complexity approach, which has been increasingly adopted in economics after an early fruitful development in mathematics, physics and biology. While there is no agreed-upon definition of such a complex term as "complex system", it can be argued that complexity theory represents an ambitious intellectual effort to analyze the functioning of highly organized but decentralized systems composed of very large numbers of individual components.<sup>1</sup> According to Foley (2003), these systems are seen as having a potential to configure their component parts in an astronomically large number of ways (they are *complex*), constant change in response to environmental stimulus and their own development (they are *adaptive*), a strong tendency to achieve recognizable, stable patterns in their configuration (they are *self-organizing*), and an avoidance of stable, self-reproducing states (they are *non-equilibrium systems*). Furthermore, the methods of complex systems theory are highly empirical and inductive, with the computer playing a critical role in the analysis, as it becomes virtually impossible to say much directly about the dynamics of non-linear systems with a large number of degrees of freedom using classical mathematical analytical methods.

Indeed, Arthur, Durlauf & Lane (1997) describe the (Santa Fe) complexity perspective by first pointing out features of the economy that together present difficulties for the traditional mathematical methods employed in economics. First, *dispersed interaction*, with what happens in the economy being determined by the interaction of many dispersed, possibly

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<sup>1</sup> Major research centers of this intellectual effort have included earlier the Free University in Brussels, Belgium, Stuttgart University in Germany, and later the Santa Fe Institute in New Mexico, USA. Some introductions of to the concepts of complex systems theory are Prigogine & Stengers (1984), Nicolis & Prigogine (1989), Waldrop (1992), Lewin (1992), Albin & Foley (1998), Rosser (1999) and Gribbin (2005).

heterogeneous, agents acting in parallel. Second, *no global controller*, meaning that no global entity controls interactions, as controls are rather provided by mechanisms of competition and coordination among agents. Third, *cross-cutting hierarchical organization*, which means that the economy has many levels of organization and interaction, even though the overall organization is more than hierarchical, with many sorts of tangled interactions across such levels. Fourth, *continual adaptation*, meaning that behaviors, actions, and strategies come to be revised continually as the individual agents accumulate experience. Fifth, *perpetual novelty*, which means that niches are being continually created by new markets, new technologies, new behaviors, new institutions, and the very act of filling a niche may provide new niches. Finally, *out-of-equilibrium dynamics*, which means that with new niches, new potentials, new possibilities, being continually created, the economy operates far from any optimum or global equilibrium, although improvements are always possible and do occur regularly.

It should be mentioned that financial fragility – à la Minsky – as emergent dynamics in a complex system has been also derived in the model developed by Gallegati, Giulioni & Kichiji (2003). In this model, however, the supply of credit-money by the banking system is made to depend on its current equity base, which rises (falls) if past profits were positive (negative), and it is independent from the interest. If a firm goes bankrupt, it is not able to fulfil debt commitments and, therefore, bank's profit and equity base shrink, which will lead to a fall in the supply of credit-money and a rise in interest rate. In the model developed in this paper, in turn, the supply of credit-money at any single period is endogenous, demand-driven at the nominal interest rate, which is set by an adaptive banking system through a rule-of-thumb, satisficing procedure as a markup over the base rate, which is determined by the monetary authority. While this (risk-adjusted) banking markup is given in any single period, it will change over periods according to an indicator of non-performing loans granted to firms. A rise in the default rate in some period will lead to a rise in the expected rate of default for the next period and, therefore, to a rise in the risk-adjusted banking markup for the next period. Firms, therefore, are not quantity-constrained, but price-constrained in the credit-money market. Indeed, unlike the papers mentioned above, the decision-making of firms and the banking system, the former with respect to production and pricing while the latter with respect to pricing, is here updated periodically through mechanisms of local and/or of intertemporal inference. Besides, in this paper firms' expected demand is formed through adaptive rules-of-thumb, while consumers' choices are based on a satisficing procedure of bounded inference, among several other differences.<sup>2</sup>

The remainder of the paper is organized as follows. Section 2 describes the structure of the model economy whose dynamic behavior is intended to be tracked down, whereas Sections 3 and 4 describe and analyze several corresponding simulation results. In turn, the closing section summarizes the main conclusions derived along the way and raises some possibilities for extensions and further research.

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<sup>2</sup> Nonetheless, see Delli Gatti, Gallegati, Giulioni & Palestrini (2003) for an interesting complex formal analysis of the interactions among changes in financial fragility, industrial dynamics and macroeconomic performance and Russo, Delli Gatti & Gallegati (2005) for a promising scaling approach to business cycle, in which the shifting behavior of the firms size power law distribution over the business cycle is explained by a mechanism based on the interplay among R&D investment, technological innovation, wage dynamics and – along the lines of that model developed in Gallegati, Giulioni & Kichiji (2003) -- financial fragility.

## 2. Structure of the model

The economy is populated by a large number of heterogeneous firms, each one of them producing a single homogeneous and non-perishable good. They share a single fixed-coefficient technology, with homogeneous labor being the only physical input used:

$$X_{it} = L_{it} / a \quad (1)$$

where  $X_i$  and  $L_i$  are respectively the levels of output and employment of the  $i$ -th firm in a given period  $t$ , while  $a$  is the shared labor-output ratio. The latter is assumed to remain unchanged throughout, as we abstract from technological change.<sup>3</sup>

Firms produce – and hire labor – in a given period  $t$  according to expected demand, as we abstract from strategic accumulation – and scrapping – of inventories. Now, since effective demand faced by firms may happen to be lower than expected, a firm may hold unplanned inventories. Individual production flow in a given period, if any, will therefore be determined by the excess of expected demand over accumulated inventories:

$$\begin{aligned} X_{it} &= X_{it}^D - X_{it}^S & \text{if } X_{it}^D > X_{it}^S \\ X_{it} &= 0 & \text{if } X_{it}^D \leq X_{it}^S \end{aligned} \quad (2)$$

where  $X_{it}^D$  and  $X_{it}^S$  are respectively expected demand and inventories of the  $i$ -th firm.

Firms will use two sources of funding to cover labor costs, which are actually the only production costs involved – for simplicity, the holding of inventories is assumed to be costless. Labor is always in excess supply, as we assume that labor supply in a given period is infinitely elastic at a wage rate,  $V$ , that remains unchanged over periods. From eqs. (1) and (2), the wage bill,  $W_i$ , which is due in the beginning of the production period, will then be proportional to the amount of expected demand that cannot be met by accumulated inventories:

$$W_{it} = VaX_{it} \quad (3)$$

Firms will use external funding in the form of credit-money granted by the banking system to cover whatever portion of the wage bill that cannot be covered by the stock of accumulated profits,  $R_{it}^S$ , which represents their internal funding. Formally, the amount of external funding employed to finance production is given by:

$$\begin{aligned} D_{it} &= W_{it} - R_{it}^S & \text{if } X_{it}^D > X_{it}^S \text{ and } R_{it}^S < W_{it} \\ D_{it} &= 0 & \text{if } X_{it}^D > X_{it}^S \text{ and } R_{it}^S \geq W_{it} \\ & & \text{or } X_{it}^D \leq X_{it}^S \end{aligned} \quad (4)$$

<sup>3</sup> An aggregate dynamic model of capital accumulation, growth and distribution in which labor-saving technological innovation plays a pivotal role is developed in Lima (2000). The innovation rate is made to depend non-linearly on market concentration, thus incorporating a possibility raised in the neo-schumpeterian literature on the double-sided relationship between industrial dynamics and technological change. An alternative specification of endogenous technological change can be found in Lima (2004), where labor-saving innovations by firms depend non-linearly on distribution itself, the idea being that functional distribution determines both the incentives to innovate and the availability of funding to carry it out. In both models, the non-linearity involved gives rise to multiple equilibria and endogenous, self-sustaining fluctuations.

Therefore,  $D_{it}$  represents the demand for credit-money by an individual firm to finance that portion of its flow of production that cannot be covered out of accumulated profits. Following a keynesian monetary approach, the supply of credit-money at any single period is in turn endogenous, demand-driven at the current interest rate. The latter, in turn, is set by an adaptive banking system through a rule-of-thumb, satisficing pricing procedure as a markup over the base rate:

$$i_t = (1 + h_t) i^* \quad (5)$$

where  $h_t > 0$  is the (risk-adjusted) banking markup and  $i^*$  is the base rate, which is exogenously set by the monetary authority and is assumed to remain unchanged over periods.

Firms also follow a markup-pricing behavior according to which their individual prices are set as a firm-specific markup over prime costs:

$$P_{it} = (1 + z_{it})(D_{it}^S + D_{it})(1 + i_t) / X_{it}^D \quad (6)$$

where  $z_{it}$  is the markup,  $D_{it}^S$  is the stock of accumulated debt,  $D_{it}$  is the demand for credit-money given by eq. (4) and  $X_{it}^D$  is the level of expected demand. Hence, individual price in a given period is set so as to generate a level of expected revenue,  $P_{it} X_{it}^D$ , which is greater than the actual total debt service at the end of that period,  $(D_{it}^S + D_{it})(1 + i_t)$ . In case an individual firm does not hold any stock of accumulated debt and will not demand any credit-money in the current period, we assume that it will follow a satisficing pricing procedure of charging in the current period the same price it charged in the previous one.

Regarding expected demand for a given period, two possibilities will be considered. In the first, the procedure to form expectations is not revised along the way, and we will refer to this case as *rigid expectation formation* (REF). In this case, the collection of firms will follow throughout a set of adaptive rules-of-thumb that differ from each other according to how many past realizations of effective demand are to be taken into account and whether they are more or less optimistic. To put it precisely, one such an optimistic rule will make for an expected demand growth – with respect to the previous period – that is equal to the highest effective demand growth in the two past periods. An even more optimistic rule will make for an expected demand growth that is equal to the highest demand growth in the three past periods. Analogously, two pessimistic rules will make for an expected demand growth that is equal to the lowest demand growth in the two and three past periods, respectively. In turn, two other rules prescribe that expected demand growth should be a moving arithmetic average of the two or three past periods, respectively. Analogously, two other rules prescribe that expected demand growth should be a moving geometric average of the two or three past periods, respectively. Another rule will make for an expected demand growth that is equal to the median of the three past periods. Finally, one of these rules prescribes that the expected demand growth should be equal to the actual one observed in the previous period.

In the other case, to which we will refer as *flexible expectations formation* (FEF), a reinforcing mechanism is introduced, with a firm becoming more or less optimistic depending on the evolution of its financial fitness. The collection of firms will follow a set of nineteen rules-of-thumb that differ from each other according to how optimistic or pessimistic they are, with each firm being randomly distributed an initial rule to be followed. While the most

optimistic rule makes for an expected demand growth – with respect to the level of effective demand observed in the previous period – that is equal to the highest effective demand growth in the last ten periods, the most pessimistic rule makes for an expected demand growth that is equal to the lowest effective demand growth in the last ten periods. Accordingly, the second most optimistic (pessimistic) rule makes for an expected demand growth that is equal to the highest (lowest) effective demand growth in the last nine periods, while the third most optimistic (pessimistic) rules makes for an expected demand growth that is equal to the highest (lowest) effective demand growth in the last eight periods, and so on. In addition, the least optimistic rule makes for an expected demand growth that is equal to the effective demand growth in the last period, which implies that the least pessimistic rule makes for an expected demand growth which is the lowest in the last two periods.

In case a firm finds itself in a hedge (ponzi) position at the end of a given period, it will become more optimistic (pessimistic) in the very next period. In case the would-move individual firm is already adopting the most optimistic (pessimistic) rule, no update will take place. Similarly, in a case a firm finds itself in a speculative position, it will not become either more optimistic or pessimistic in the very next period.

Effective aggregate demand,  $X_t^E$ , in turn, is assumed to follow a first-order autoregressive process, AR(1), given by:

$$X_t^E = \phi X_{t-1}^E + \varepsilon_t \quad (7)$$

where  $\varepsilon$  is white noise. The realization of this shock will be given by a component,  $\tilde{X}_t^E$ , which will be a control variable in the simulation results reported in the next section, multiplied by a value  $\mu_t$ , drawn from a zero mean normal distribution, with  $\mu_t \in (-3,3)$ . In any given period, the corresponding realization of effective aggregate demand will be distributed uniformly among a large number of consumers. Each one of these consumers will then make a certain number of phone calls to firms so as to learn about the price they would quote her. Having made all these phone calls, whose actual number will be a control variable in the simulation results presented in the next section, consumers will call back to the firm whose price is the lowest to make a purchase. In case the lowest-price firm is unable to satisfy all of the effective demand submitted by this individual consumer, she will then call back the second lowest-price firm – and so on, if necessary – to complete her desired purchase. Once any purchase is made, in turn, the corresponding individual stock of the good in the firm is immediately reduced accordingly. Besides, consumers do not imitate each other and rely solely on their own price-quote to decide from whose firm(s) they are willing to purchase.

Even though firm-specific markups are given in any period, they will change over periods according to the following mechanism of local inference. In the very end of the period, a certain fraction of the firms that ranked highest with respect to accumulated debt will then make phone calls to 4 competitors – pretending to be consumers, let us say – so as to learn about the price they had charged. Each one of these firms, whose actual number will be a control variable in the simulation results presented in the next section, will compare the average current price of these competitors with its own. In case its own price is higher (lower) than such an average price, the firm will lower (raise) its markup for the next period by 10%.

Likewise, the (risk-adjusted) banking markup is given in any period but will change over periods according to an indicator of non-performing loans, while the base rate will remain unchanged.<sup>4</sup> In the end of the period, the banking system will compute a default rate given by:

$$d_t = (D_t^T - D_t^P) / D_t^T \quad (8)$$

where  $D_t^T = (D_t^S + D_t)(1+i_t)$  is aggregate total debt and  $D_t^P$  is the aggregate debt actually paid back. Since the default rate so computed will be the expected one for the next period, the risk-adjusted banking markup in a given period is given by:

$$h_t = \frac{1+h_{t-1}}{1-d_{t-1}} - 1 \quad (9)$$

Hence, a rise in the default rate in some period will lead to a rise in the expected rate of default for the next period and, therefore, to a rise in the risk-adjusted banking markup for the next period.<sup>5</sup> Some empirical evidence for the above specification is the following. Saunders & Schumacher (2000) studied the determinants of bank interest margins in six selected European countries and the United States during the period 1988-95, having found that interest margins are positively affected by credit risk. Angbazo (1997), using interest data for a sample of North-American banks during the period 1989-93, found that the ratio of non-performing loans to total loans is a significant explanatory variable for bank spreads. In turn, Brock & Rojas-Suarez (2000) studied the determinants of bank spreads in six Latin American countries during the mid-1990s, having found evidence that high levels of non-performing loans raise spreads. Besides, there is empirical evidence on the countercyclical behavior of bank markups, especially with respect to output growth (Angelini & Cetorelli 2003, Olivero 2004, Mandelman 2005).<sup>6</sup>

Finally, the end-of-period cash flow position of the collection of heterogeneous firms that inhabit the economy can be interestingly described and analyzed by employing a version of Hyman Minsky's (1975; 1982) taxonomy of financing regimes – hedge, speculative and

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<sup>4</sup> The assumed constancy of the base rate implies that changes in the interest rate are driven solely by changes in the banking markup. Nevertheless, it will be seen that the dynamic behavior of this simplified model is complex enough and gives rise to several possibilities. Since our purpose is to focus on the flexibility of the banking markup, we do not incorporate a policy rule regarding the base rate. However, a natural candidate would be a Taylor-type monetary policy rule (Taylor, 1993) according to which the base rate responds to the difference between current and target inflation rate and to the output gap. Indeed, this quite an interesting extension has already joined our research-in-progress – for which we invite the reader to stay tuned.

<sup>5</sup> Arguably, banking markups are influenced by a variety of determinants – bank characteristics, macroeconomic conditions, explicit and implicit taxation, deposit insurance regulation, overall financial structure, and underlying legal and institutional framework. For tractability, however, in this paper we focus solely on the influence exerted by the rate of non-performing loans.

<sup>6</sup> A macrodynamic model that adopts a similar specification for the monetary side and, by having much simpler microfoundations, can be solved analytically, is developed in Lima & Meirelles (2003). Over time, banking markup falls with firms' profit rate and rises with the rate of inflation. The dynamic stability properties of the system are studied analytically for both cases regarding capacity utilization, full and less-than-full, which makes for the possibility of multiple equilibria. Unlike the present complex, computational model, however, firms' finance regime – and thus the financial fragility of the economy – is not modeled. In turn, a dynamic model in which firms' debt position is modeled, but which has much simpler microfoundations as well and can be solved analytically, is developed in Lima & Meirelles (2007). Regarding dynamics, it is shown the possibility of relating the stability properties of a system having the interest rate and the debt position by firms as state variables to the type of minskyan finance regime that prevails across firms.

Ponzi. Indeed, simulation results to be reported in the next two sections will show that financial fragility à la Minsky becomes an emergent dynamics in such a complex economy, with the distribution of financing regimes across firms, or the financial fitness landscape of the system, coming out as a spontaneous order.

According to Minsky's broad characterization, hedge financing units are those that can fulfill all of their contractual payment obligations by their cash flows, while speculative units are those that can meet their interest payment commitments on outstanding debts, even as they are unable to repay the principle out of income cash flows. For Ponzi units, in turn, the cash flow from operations are not enough to fulfill either the repayment of principle or the interest due on outstanding debts.

While identifying into which category any firm falls gives a measure of its individual financial fragility, classifying all the collection of firms in an economy according to these categories provides an analytically useful way of assessing the level of economywide financial fragility at any point in time. Indeed, the higher the proportion of speculative and Ponzi firms, the greater the financial fragility of the economy. For Minsky, capitalist financial structures have an inherent tendency to move from states of robustness to fragility over time, evolving in a way that we believe can be better understood through an evolutionary, complex dynamics approach.

Since in the model developed here individual prices for a given period are set so as to generate expected revenues that are greater than the total debt service at the end of that period, we employ the following version of Minsky's taxonomy of financing postures:

$$\text{Hedge: } R_{it}^S(1+i_t) + P_{it}x_{it}^E \geq (D_{it}^S + D_{it})(1+i_t) \quad (10)$$

$$\text{Speculative: } D_{it}^S + D_{it} \leq R_{it}^S(1+i_t) + P_{it}x_{it}^E < (D_{it}^S + D_{it})(1+i_t) \quad (11)$$

$$\text{Ponzi: } R_{it}^S(1+i_t) + P_{it}x_{it}^E < D_{it}^S + D_{it} \quad (12)$$

where  $x_{it}^E$  is the individual amount of effective demand corresponding to the proportion of the realization of effective aggregate demand, eq. (7), that turns out to be satisfied by the  $i$ -th firm.

### 3. Simulation results (I)

We first simulate the dynamic behavior of a setting with one bank, 100 firms and 5,000 consumers over 500 periods. The base rate,  $i^*$ , a parameter, is set at 1%. As regards initial values, the one for the realization of effective aggregate demand is 200,000, with a trend given by  $\phi=1.002$ , and for the banking markup is 10% – with an implied rate of non-performing loans for the last period equal to 0. Three values are considered for the shock component of effective aggregate demand,  $\tilde{X}_t^E$ , namely 5,000, 10,000 and 20,000. Individual firms' characteristics such as initial markup, ranging from 1% to 30%, and the rule to be followed throughout to form demand expectations – since we first consider the case in which expectations formation is rigid (REF) – are then randomly distributed among the collection of firms. In order to allow firms to form demand expectations regarding the first period, we start by randomly generating a record of four past realizations of effective demand.

As mentioned in the preceding section, two control variables are the number of phone calls consumers will make to firms to quote prices,  $c$ , a measure of price transparency, and the number of firms that are highest-ranked in accumulated debt as of the very end of the



period that will make phone calls to 4 competitors so as to learn about their price and eventually change their markup for the next period,  $f$ , a measure of markup rigidity. Besides, another control variable is the shock term of effective aggregate demand,  $\tilde{X}_t^E$ , which gives the realization of the perturbation component in the AR(1) process for effective aggregate demand when multiplied by a value,  $\mu_t$ , drawn from a zero mean normal distribution, with  $\mu_t \in (-3, 3)$ .

Figures 1-3 show simulation results for a situation in which the number of phone calls made to firms by consumers,  $c$ , is relatively low and remains constant, while the number of firms that revise their markups,  $f$ , assumes only the extreme – none and all of them – values and the mean one. The shock component of effective aggregate demand is equal to 5,000. It can be seen that the system remains quite financially robust, with the proportion of ponzi firms remaining equal to zero throughout. Hence, financial fragility is quite low no matter how rigid are individual markups, an emergent dynamics that is probably related to the corresponding behaviour of effective aggregate demand, as shown by Figure 4, which is common to those three values for the number of phone calls made to firms by consumers,  $c$ .

Figure 1: Distribution of financing regimes ( $c = 5; f = 0$ )

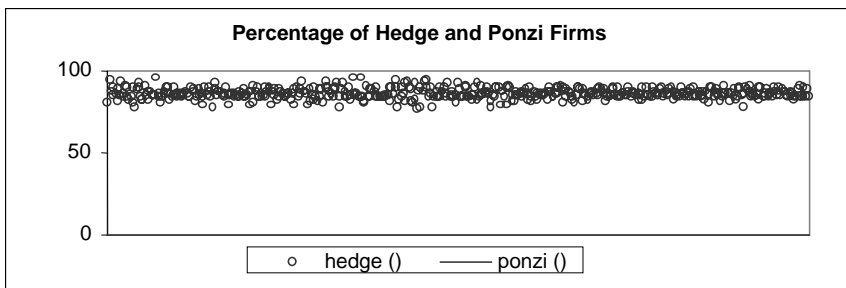


Figure 2: Distribution of financing regimes ( $c = 5; f = 50$ )

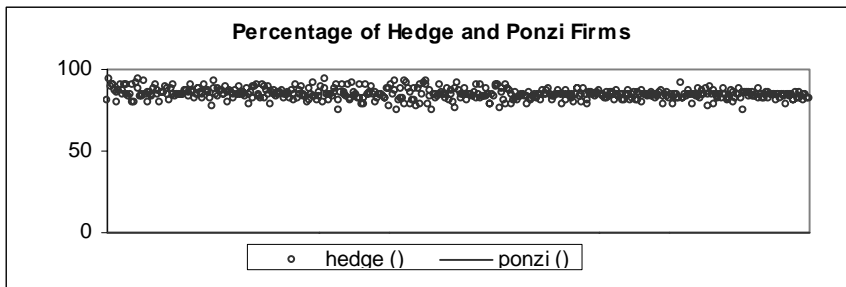


Figure 3: Distribution of financing regimes ( $c = 5; f = 100$ )

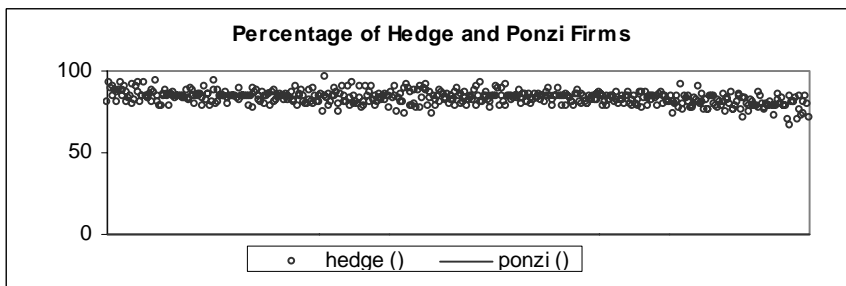
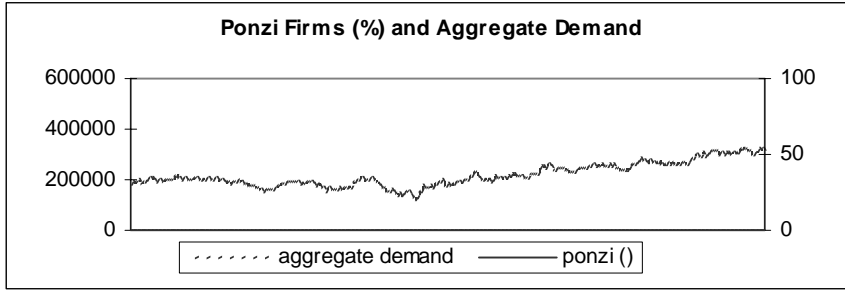


Figure 4: Evolution of effective aggregate demand



Figures 5-7, in turn, show that higher values for the number of phone calls made to firms by consumers, when combined with no markup rigidity,  $f = 100$ , although with the same shock component of effective aggregate demand as before, will make for the emergence of financial fragility in a system that remains quite financially robust for a considerable number of periods.

Figure 5: Distribution of financing regimes ( $c = 20; f = 100$ )

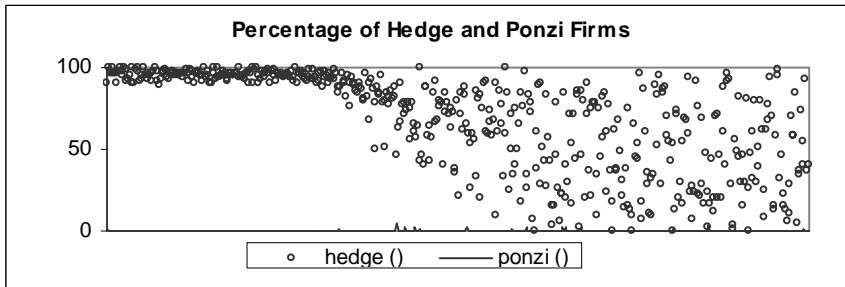


Figure 6: Distribution of financing regimes ( $c = 50; f = 100$ )

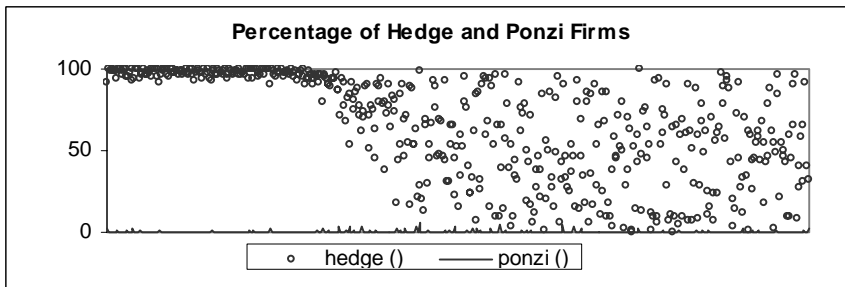
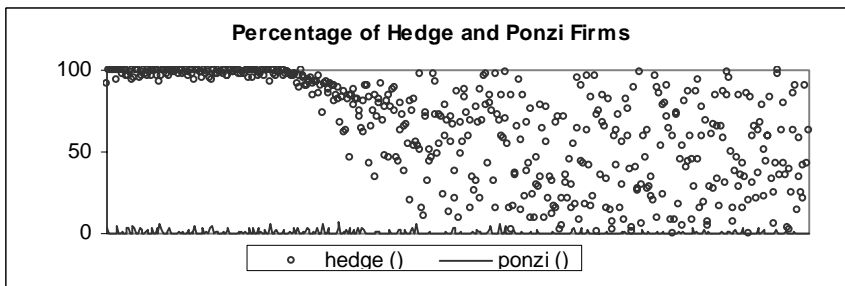


Figure 7: Distribution of financing regimes ( $c = 100; f = 100$ )



However, it is worth stressing that the financial fragility that emerged in all of these three previous cases is not actually associated with a rise in the proportion of ponzi firms, but with a considerable rise in the proportion of speculative firms. Once again, such an emergent financial fragility dynamics is probably related to the behaviour of effective aggregate demand, as shown in Figure 4.

Figures 8-10, in turn, describe simulation results for a situation in which the number of phone calls made to firms by consumers is relatively low and remains constant, while the number of firms that revise their markups assumes only the extreme – none and all of them – values and the mean one. Hence, this situation is exactly the same as the one pictured in Figures 1-3 as far as price transparency and markup rigidity are both concerned, while the shock component of effective aggregate demand has been doubled to reach 10,000. Interestingly, the system actually becomes much more financially robust over time when either none or half of the firms revise their markups in light of realized profitability, as shown in Figures 8 and 9. However, emergent dynamics changes considerably when all of the firms revise their markups, and that increase in financial robustness to the limit no longer emerges.

Figure 8: Distribution of financing regimes ( $c = 5; f = 0$ )

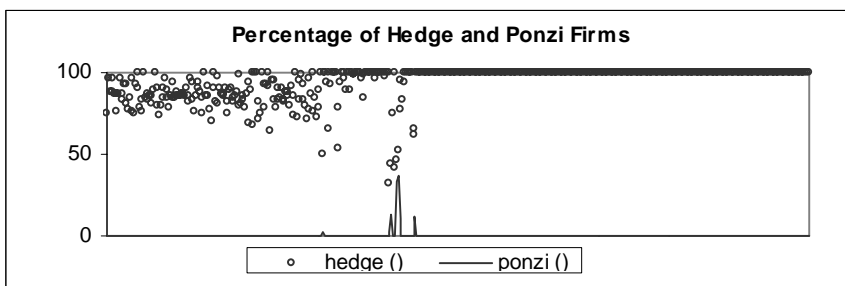


Figure 9: Distribution of financing regimes ( $c = 5; f = 50$ )

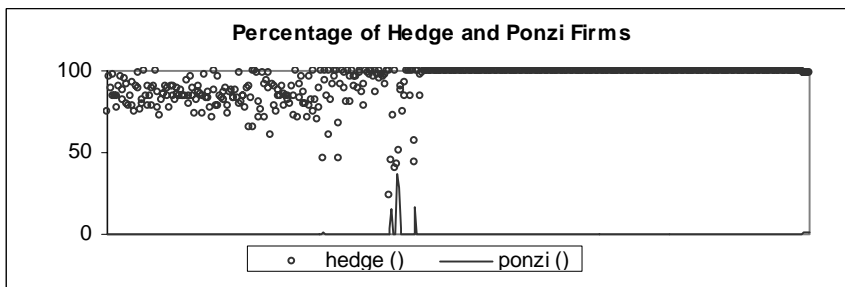
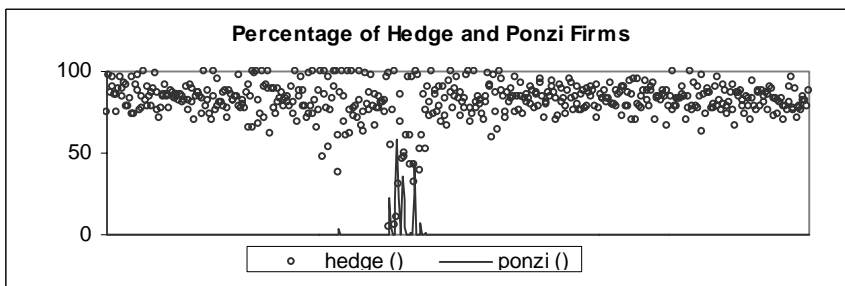


Figure 10: Distribution of financing regimes ( $c = 5; f = 100$ )



A similar non-linear response of the emergent financial fragility to a fall in the markup rigidity is obtained when the shock component of effective demand is doubled again, now to reach 20,000. As in the two previous situations, the number of phone calls made to firms by consumers is low and remains constant, while the number of firms that revise their markups increases. The system becomes more financially robust over time when either 10 or 30 or 50 firms revise their markups, but financial fragility dynamics changes considerably when all of the firms revise markups, as pictured in Figure 14, with such a large increase in financial robustness no longer emerging when markups are fully flexible,  $f = 100$ .

Figure 11: Distribution of financing regimes ( $c = 5; f = 10$ )

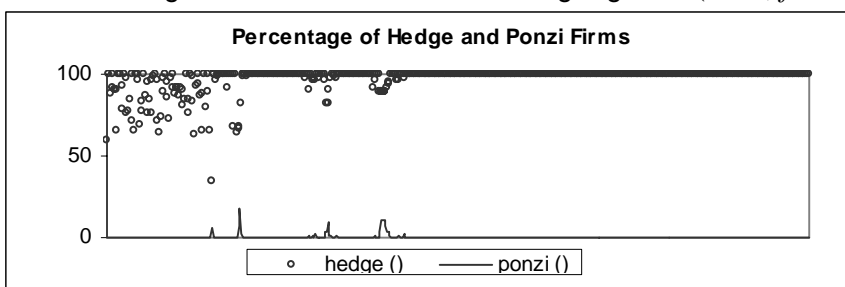


Figure 12: Distribution of financing regimes ( $c = 5; f = 30$ )

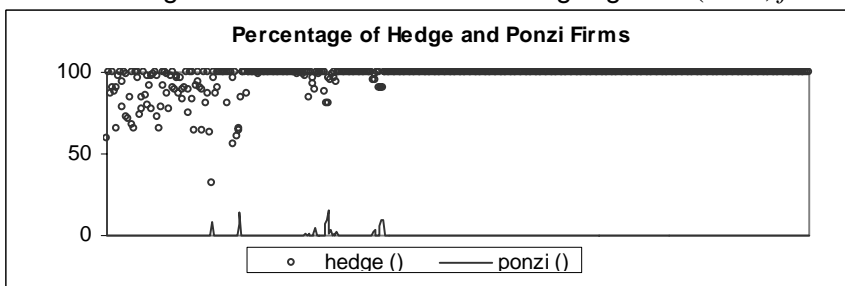


Figure 13: Distribution of financing regimes ( $c = 5; f = 50$ )

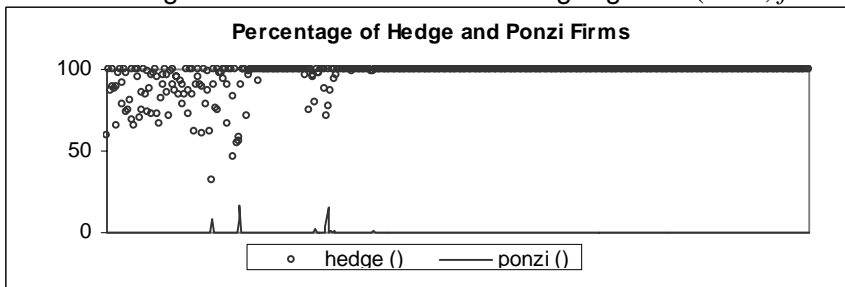
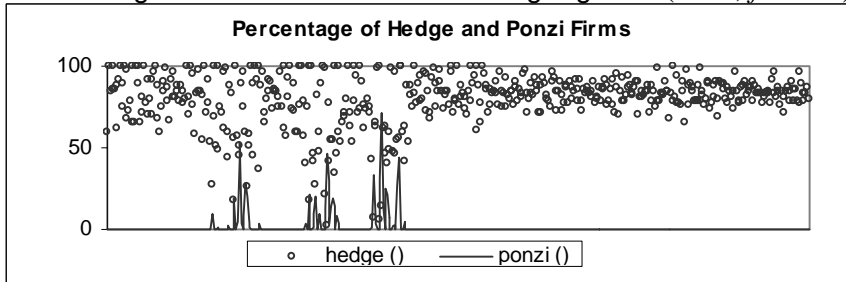


Figure 14: Distribution of financing regimes ( $c = 5; f = 100$ )



#### 4. Simulation results (II)

We now simulate the dynamic behavior of a setting with one bank, 100 firms and 5,000 consumers over 1,000 periods. The base rate is again set at 1%, while the banking markup in the first period is again equal to 10%. As in the previous simulation, firms' characteristics such as the initial markup, ranging from 1% to 30%, and the rule-of-thumb to be followed in the first period to form demand expectations, are randomly distributed among the collection of firms. Effective demand is now assumed to be given by the following autoregressive process:

$$X_t^E = (1 + g + s_t)X_{t-1}^E \quad (13)$$

where  $g$  is the (exogenous) growth rate of effective demand and  $s_t$  is a noise drawn from a normal distribution with zero mean and standard deviation equal to 0.05. Hence, this process replaces the process described in eq. (7). In order to allow firms to form demand expectations regarding the first period, we start by generating a record of ten past realizations of effective demand according to the autoregressive process given by eq. (13), with 200,000 being the level of effective demand observed in the tenth past realization. Besides, the exogenous growth of effective demand is assumed to be equal to 0.8%.

Since we now assume that expectations formation is flexible (FEF), the rule-of-thumb followed to form demand expectations will change over time according to a reinforcing mechanism, with a firm becoming more or less optimistic with respect to expected demand depending on the evolution of its financial fitness. In case a firm finds itself in a hedge position at the end of a given period, it will move to the immediately higher level of optimism in the next period. Similarly, in case a firm finds itself in a ponzi position at the end of a given period, it will move to the immediately higher level of pessimism in the next period. In case the would-move firm is already following the most optimistic or pessimistic rule, no update will take place. Finally, in a case a firm finds itself in a speculative position, it will not become either more optimistic or pessimistic in the next period.

Finally, it is now assumed that the only control variable in the simulation is the number of firms that are highest-ranked in accumulated debt that will make a phone call to four competitors to learn about their price and eventually change their markup for the next period,  $f$ , a measure of markup rigidity.

Figure 15 shows simulation results for a situation in which no single firm revises its markup. It can be seen that the system remains financially robust, with the proportion of hedge firms remaining high throughout. Hence, financial fragility is low despite individual markups are rigid, an emergent dynamics that is probably somehow related to the corresponding behaviour of effective aggregate demand, which is shown in Figure 16.

Figure 15: Distribution of financing regimes ( $f = 0$ )

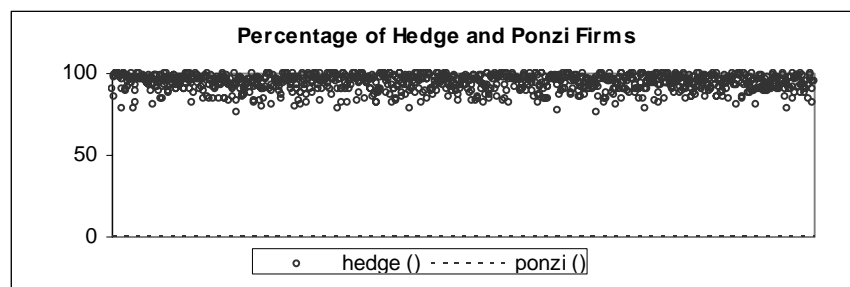


Figure 16: Evolution of aggregate demand and aggregate debt ( $f = 0$ )

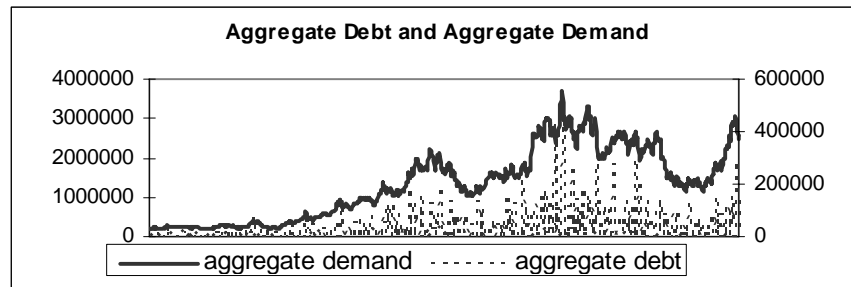


Figure 17 shows simulation results for a case in which only a small fraction, 10%, of the collection of firms revise markups. As in the situation with complete markup rigidity, it can be seen that the system remains financially robust, with the proportion of hedge firms remaining relatively high throughout. Figure 18 shows that in this case average total debt seems to be higher than before, though.

Figure 17: Distribution of financing regimes ( $f = 10$ )

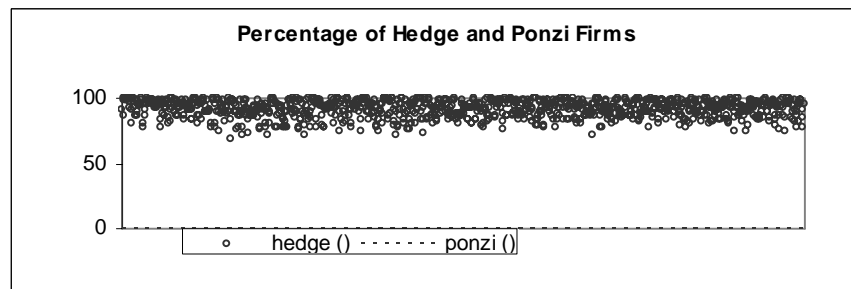


Figure 18: Evolution of aggregate demand and aggregate debt ( $f = 10$ )

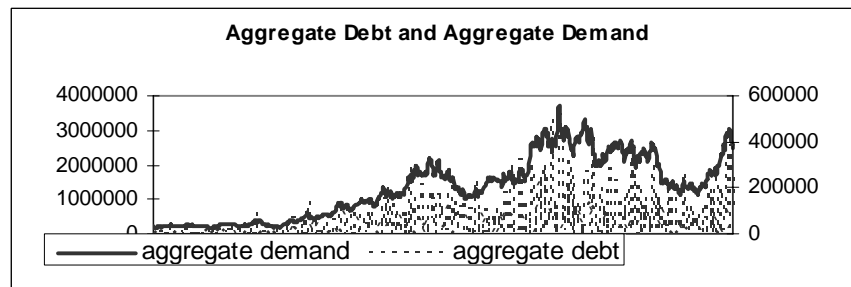


Figure 19 shows that an even lower degree of markup rigidity, ( $f = 40$ ), makes for a lower average financial robustness than before, although average aggregate debt seems to be lower than in the case with ( $f = 10$ ), as shown in Figure 20. Also, the lower average financial robustness that emerges by the middle of the simulation time period comes to be reversed back later on, with the system resuming a level of financial fitness closer to the initial one.

Figure 19: Distribution of financing regimes ( $f = 40$ )

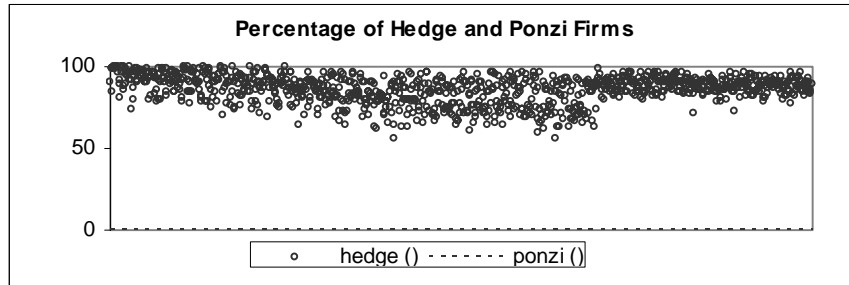


Figure 20: Evolution of aggregate demand and aggregate debt ( $f = 40$ )

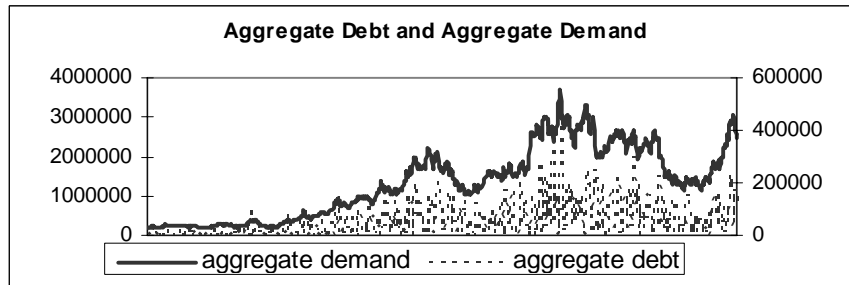


Figure 21 shows that an even higher proportion of firms that revise their markups, ( $f = 60$ ), makes for a pattern of emergence of financial fragility which is similar to that observed in the previous situation with ( $f = 40$ ), a difference being that average financial robustness seems to be lower than before. As shown in Figure 22, average aggregate debt seems to be similar in both cases, though.

Figure 21: Distribution of financing regimes ( $f = 60$ )

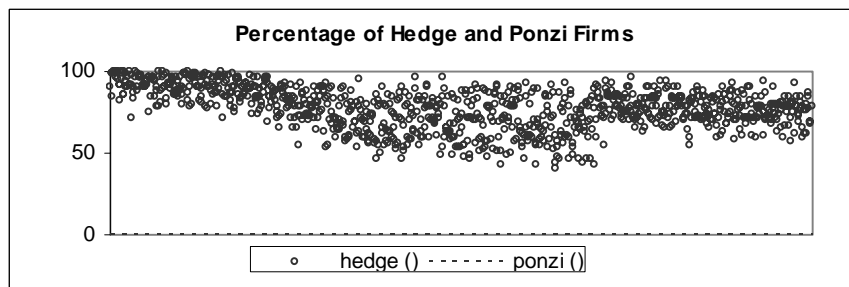
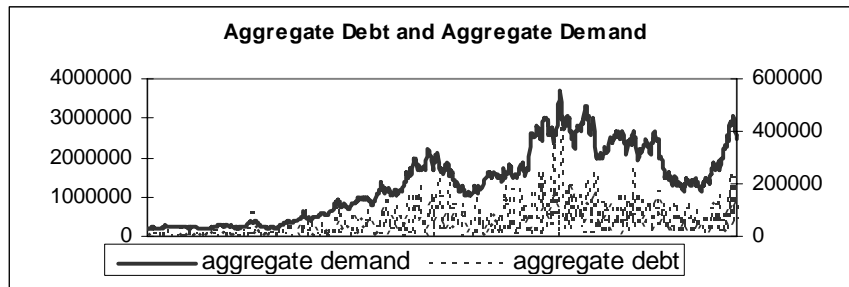


Figure 22: Evolution of aggregate demand and aggregate debt ( $f = 60$ )



Finally, Figure 23 describes a situation in which all of the firms revise their markup, which makes for the emergence of a higher degree of financial fragility than before. Also, a considerable drop in the average financial robustness emerges relatively early in the simulation time period, with the system never resuming its initial financial fitness. Interestingly, as shown in Figure 24, average aggregate debt in this case with markup flexibility does not seem to be much different from the level observed in the previous cases. Besides, it is worth stressing that the financial fragility that emerged in all of the previous cases is not actually associated with an increase in the proportion of ponzi firms, but with a considerable rise in the proportion of speculative firms.

Figure 23: Distribution of financing regimes ( $f = 100$ )

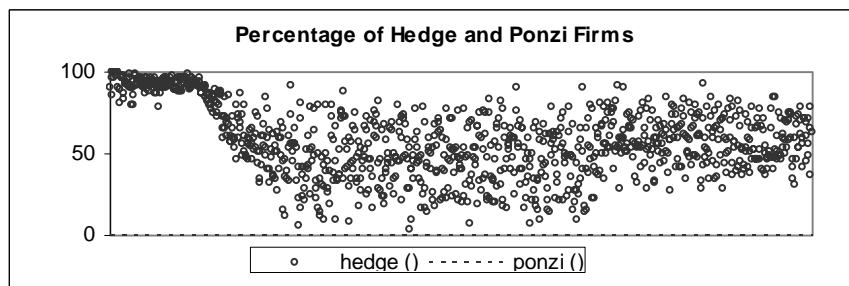
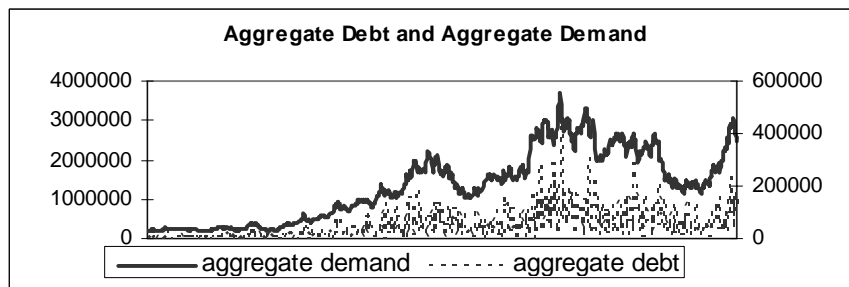


Figure 24: Evolution of aggregate demand and aggregate debt ( $f = 100$ )





## 5. Conclusion

This paper developed an agent-based computational model of production in which the endogenous supply of credit-money by an adaptive banking system and the cash flow position of heterogeneous firms are modeled as co-evolutionary phenomena. Both the collection of heterogeneous firms and the banking system follow a satisficing markup pricing procedure that is updated periodically through mechanisms of local inference (production and/or pricing by competitors) and/or intertemporal inference (past performance and expected future one). As it turns out, the financial fitness landscape of the system becomes an emergent property, as shown through simulation. For Minsky, capitalist financial structures have an inherent tendency to move from states of robustness to fragility over time, evolving in a manner that we do believe can be better described and analyzed through an evolutionary, complex dynamics approach.

Admittedly, the simplified complex model developed in this paper could be extended and improved in several directions, and our research-in-progress – for which we invite the reader to stay tuned – has been pursuing all of them. First, it would be interesting to have an endogenous determination of aggregate effective demand, which would allow the incorporation of several other mechanisms of interaction between financial fragility and production, and in both directions. Second, and somehow related to the former one, an explicit introduction of labor market dynamics would allow a discussion of issues related to employment, wage dynamics and consumers' debt. Third, the introduction of capital as a production factor would allow the discussion of relevant issues related to investment and capital accumulation.

Fourth, in the model of this paper the banking markup changes over periods according to a mechanism of intertemporal inference, while the base rate remains unchanged. As it turns out, a rise in the default rate in some period will lead to a rise in the expected rate of default for the next period and, therefore, to a rise in the risk-adjusted banking markup for the next period. Therefore, the supply of credit-money is not subject to any quantity rationing, but only to an intertemporal price-rationing. Now, in case a firm remains ponzi for a certain number of consecutive periods, with such a number eventually being a control variable in the simulation, some quantity-rationing could well come to apply. One possibility would be for the banking system to simply refuse to grant any further credit-money in the very next period to a firm that had remained ponzi during that defined number of periods, but then start granting it fully again as soon as such firm would cease to be ponzi. Another possibility would be for the banking system to simply state that a firm that remained ponzi during that defined number of periods would not be granted any further credit-money in the very next period and would go bankrupt in case it remained ponzi in such a very next period. Still, the recurrent ponzi could be told by the banking system that it would be granted credit-money in the very next period but it would go bankrupt in case it came to remain ponzi for another consecutive period.

In any case, the introduction of bankruptcy as an exit mechanism would require the incorporation of an entry mechanism as well, and a simple specification would be to have a one-to-one replacement mechanism, with the entrant firm having the characteristics of the 'mean' firm. Also, the banking system would take over whatever remaining assets of the bankrupted firm, which would be unsold goods only. In this case, another modification would be to have the banking system selling those assets to existing firms through a public auction, which would then allow the incorporation of some interesting features drawn from the literature on complex mechanism design.

Finally, it would be interesting to model the co-evolution of the satisficing procedures of production and pricing by firms and banks through a classifier system and a genetic algorithm. For instance, those rules that better fit the purpose of reaching desired results would have a higher probability of being – or keep being – followed, a mechanism of reinforcement, positive feedback similar to those underlying Minsky’s insightful analysis of the propensity of a market economy with a complex financial system to experience endogenous, self-sustaining fluctuations in economic activity.

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