Technological Progress, Income Distribution and Capacity Utilisation

A Computer Simulation-Based Analysis

Fabio Hideki Ono*
José Luís Oreiro†

Resumo

Este artigo apresenta um modelo pós-Keynesiano de crescimento no qual (i) a taxa de mark-up varia no longo-prazo em função da divergência entre a taxa de lucro efetiva e a taxa de lucro 'desejada' pelos capitalistas; e (ii) a relação capital-produto não é necessariamente constante, mas pode variar como consequência do progresso tecnológico, que segundo a classificação de Harrod pode ser neutro, poupador ou dispendioso de capital. Demonstramos que as condições de estabilidade econômica só podem ser atendidas caso o progresso tecnológico seja neutro ou 'dispendioso' de capital e o investimento sensível a variações na margem de lucro. Ao submetermos o modelo a simulações computacionais, observamos ainda a possibilidade de uma transição endógena de um regime de acumulação do tipo wage-led para profit-led. Ademais, verificamos uma tendência à estabilidade das taxas de lucro, sob a condição de que a propensão a poupar dos capitalistas seja elevada.
Palavras-chave: Decisões de investimento, progresso tecnológico, simulações computacionais

Abstract

The paper presents a post-keynesian growth model in which (i) the mark-up rate varies in the long-term due to a misalignment between the actual rate and the 'desired' profit rate; and (ii) the capital-output ratio is not necessarily constant, on the contrary it may shift as a result of the technological progress, which according the Harrod's typology can be neutral, capital saving or capital intensive. We demonstrate that the economic stability is only reached if the technological progress is neutral or capital intensive and the investiment is susceptible to fluctuations in the mark-up rate. After undergoing computer simulations, we noticed that an endogenous transition from a wage-led to a profit-led accumulation regime is feasible. Furthermore, we identified a tendency to the stabilization of the profit rate, conditioned to a high savings out of profits ratio.
Keywords: Investment decisions, technological progress, computer simulations

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*Graduate student in Development Economics Program at UFPR (CNPq scholarship). E-mail: fhono@usa.net. Web-page: http://fhono.conjuntura.com.br

†Phd in Economics (IE/UFRJ), Professor of Economics at UFPR and CNPq researcher. E-mail: joreiro@sociais.ufpr.br . Web-page: http://www.joseluisoreiro.ecn.br

1 Introduction

The long-term stability of capitalist economies has been an exhaustively discussed topic in the *history of economic thought*. Authors such as Marx (1867),Keynes (1936) and Schumpeter (1934) pointed out the inherently unstable nature of capitalist economies. The flourish of the theory of economic growth in XX century, since the seminal articles from Harrod (1939) and Domar (1946), was followed by a recrudescence of scepticism with regard to the supposedly self-controlled characteristic of these economies. In fact, one of the fundamental findings of the Harrod-Domar model is the so-called "Harrod's principle of instability", which states that any slip from the stable growth-path tends to amplify the economic disequilibrium, resulting in explosive growth trajectories or chronic depressions.

In the post-Keynesian tradition, subsequently to original formulations from Kalecki (1954), Robinson (1962) and Rowthorn (1981), the matter of stability remains a quite debated theme. As a result, recent developments in this tradition, for example, Dutt (1994), You (1994) and Lima (1999) focus on the analysis of economic stability assuming alternative hypotheses about the relation between: a) growth and income distribution (wage-led accumulation regimes versus profit-led ones); b) technological progress and market concentration; and c) capital accumulation and productivity expansion. These models have demonstrated that the instability *a la* Harrod is not an essential attribute of modern capitalist economies. Thereby, the existence of non-linearities in these models create a bounded instability, and restrain the likelihood of explosive growth trajectories or massive decrease in production level just as predicted in Harrod's original model.

A common assumption adopted in models that follow a post-Keynesian tradition is the constancy of the capital-output ratio. Thus, these models explicitly suppose a neutral technological progress *a la* Harrod, that is, the type of progress that does not alter the amount of capital that is technically required to produce an additional unit of output. This hypothesis is sustained by two main arguments, a theoretical and an empirical one. From a theoretical standpoint, the assumption of a neutral technological progress la Harrod seems to be the only manner to reconcile the technological progress with the construction of balanced-growth models (Bresser-Pereira, 1988, p.49;Solow, 2000, p.4). From an empirical point of view, the long-term stability of the capital-output ratio was presented by Kaldor (1957) as one of the 'stylized facts' of capitalist economies' growth. So, the assumption of a constant capital-output ratio would be justified not only for a theoretical convenience, but also for its presumed realism.

Nevertheless, recent empirical studies indicate that capital-output ratio is hardly constant in the long-term. Table 1, reproduced from Maddison (1991, p.54), clearly shows an upward trend of the capital-output ratio in a group of six developed countries during the period of 1890-1987.

Table 1: Capital-Output Ratio in Selected Countries (1890-1987)

	1980	**1913**	**1950**	**1973**	**1987**
France	n.a	1.64	1.68	1.75	2.41
Germany	2.29	2.25	2.07	2.39	2.99
Japan	0.91	1.01	1.80	1.73	2.77
Netherlands	n.a.	n.a.	1.75	2.22	2.74
United Kingdom	0.95	1.48	1.68	1.96	2.59
United States	2.09	2.91	2.26	2.07	2.30

Source : Maddison (1991, p.54)

Therefore, the central objective of this paper is to analyse the implications of different assumptions with regard to the behaviour of the capital-output ratio and the dynamics of the investment towards accumulation regimes and the long-term stability of capitalist economies. In particular, we analyse the effects of distinct hypotheses about the nature of technological progress, according Harrod's terminology (neutral, capital saving or capital intensive), over conditions for the long-term economic stability. To that end we develop a post-Keynesian growth model, in the tradition of Robinson (1962), Kalecki (1954) and Rowthorn (1981), in which (i) the mark-up rate varies in the long-term due to a misalignment between the actual profit rate and the "desired"profit rate[1]; and (ii) the capital-output ratio is not necessarily constant, on the contrary, it may shift as a result of the technological progress that the economy faces in the long-term.

In this context, we demonstrate that the conditions for economic stability can only be fulfilled if the technological progress is 'neutral' or 'capital intensive' and the investment is sensitive to fluctuations in the mark-up rate. However, in case of a 'capital-saving' technological progress or if investment is insensitive to variations in profit margins, then the conditions to stability may never be fulfilled, in other words these economies would be essentially unstable. The corollary for these assumptions is the attested behaviour of the capital-output ratio during the last 120 years which complies with the long-term stability of the capitalist economies. Hence, whether these economies are truly unstable, then the reason for this instability must be pursued elsewhere except for the type of technological progress.

The paper is divided into four sections, including this introduction. In section 2 we present the model's framework. In section 3 we examine the long-term dynamics of this model assuming different hypothesis concerning the types of technological progress and the intensity of the accelerator and profitability effects on investment decisions. In section 4 we conduct computer simulations on the model. Finally, in section 5 we provide a summary of our findings.

[1]This assumption is inspired on Bresser-Pereira (1988). He states that: "the hypothesis, therefore, is that firms, specially in oligopolistic sectors, would establish some sort of 'desired' profit-rate, that would be historically determined, according to what managers and share-holders consider to be a reasonable profit rate. This rate would probably be estimated around 10% to 15% of the firm's total capital."(1988, p. 125)

2 The model's structure

We suppose an economy where firms produce a homogenous output and they have market power, that is to say, these firms set prices by adding a mark-up rate on the unit costs, which is constant in the short-term. Then, mark-up price equation is written as:

$$p = (1 + z)wq \tag{1}$$

where, p is the price level, z is the mark-up rate ($z > 0$), w is the money wage rate and q is inverse of labour productivity, a technical coefficient that represents the amount labour needed per unit of final output[2]

Let R be the profit rate generated by the capital stock, assumed given in the short-term, u is the degree of capacity utilisation (defined as $u = \frac{X}{\overline{X}}$, where X is the actual output/income and $\overline{X}$ the full-capacity potential output/income), m is the share of profits in income, which is equal to $\frac{z}{1+z}$ and σ is the output-capital ratio (defined as $\sigma = \frac{\overline{X}}{K}$, where K is the value of the stock of capital in the economy), which is the reciprocal of the capital-output ratio (Bresser-Pereira, 1988, p.196). It is straightforward to show that:

$$R = um\sigma \tag{2}$$

By following the post-Keynesian tradition of Kaldor (1956), Robinson (1962) and Pasinetti (1962), we assume the separation between two social classes: capitalists and workers. The capitalists save a constant fraction (s_c) of their income which is only compounded of profits while workers 'spend what they earn', in other words, they consume all their wages ($s_w = 0$). Thus, this assumptions permit us to show that the aggregate saving per capital unit is given as:

$$\frac{S}{K} = s_c R \tag{3}$$

The desired growth rate of the capital stock in this economy is denoted by:

$$\frac{I}{K} = \alpha_0 + \alpha_1 m + \alpha_2 u\sigma \quad where : \alpha_0 > 0, \alpha_1 \geq 0, \alpha_2 \geq 0 \tag{4}$$

Equation (4) is nothing less than an investment function. The specification of this function is inspired in Bhaduri and Marglin (1990) for whom the desired growth rate of the capital stock is a separable function of m and u. The only difference with respect this previous work consists in including the output-capital ratio as an independent argument in the investment function.

This formulation is justified by the fact that the accelerator effect of the output growth in investment decisions depends upon the rate of capacity utilisation and 'capital productivity' as well. In fact, given the rate of capacity utilisation, the higher is σ the higher would be the output level associated with the current capital stock in the economy, and so, the higher

[2]In the following analysis, we will suppose that labour productivity is growing at an exogenous rate α, so that q is decreasing at a rate $\frac{1}{\alpha}$. Equation 1 can be rewritten as: $\frac{w}{p} = \frac{1}{1+\bar{z}}\frac{1}{q}$. Since the mark-up ($\bar{z}$) is constant in the short-term, then any change in labour productivity (q) is entirely incorporated into real wages ($\frac{w}{p}$). This effect implies that unions and workers are well succeeded in bargaining with capitalists, which also suggests the prevalence of the second stage of capitalism as stated by Kaldor (1957)

would be the total level of sales. Thus, an increase in the output-capital ratio, *ceteris paribus*, would induce a higher investment level due to the accelerator effect.

Finally, if we suppose a closed economy without government, then the goods market equilibrium requires an equivalence between the saving per unit of capital and the desired growth rate of the capital stock:

$$\frac{S}{K} = \frac{I}{K} \tag{5}$$

Substituting (2) into (3) and the resulting equation into (5) we have:

$$\frac{I}{K} = s_c m u \sigma \tag{6}$$

Substituting (4) into (6) and solving the resulting equation for u, we have:

$$u^* = \frac{\alpha_0 + \alpha_1 m}{(s_c m - \alpha_2)\,\sigma} \tag{7}$$

Equation (7) presents the short-term equilibrium of the degree of capacity utilisation, that is to say, the degree of capacity utilisation that equalises the planned investment to savings out of profits. To ensure that $u^* > 0$, it is necessary that $s_c m - \alpha_2 > 0$, which implies that the share of profits in income must exceed a critical value m^* denoted by $\frac{\alpha_2}{s_c}$.

The determination of u^* is pictured in figure 1 below:

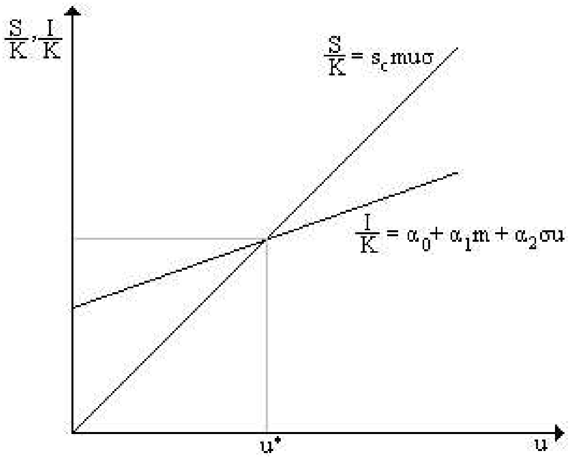

Figure 1:

The effect of a change in the share of profits in income over the short-term equilibrium of the degree of capacity utilisation is given by the derivative below:

$$\frac{\partial u^*}{\partial m} = -\frac{\alpha_1 \alpha_2 + \alpha_0 s_c}{\sigma\,(s_c m - \alpha_2)^2} \tag{8}$$

We observe from equation (8) that an increase in the share of profits in income will result in a decrease of the degree of capacity utilisation. This is a surprising outcome since, in principle, there are two forces exerting pressure against the degree of capacity utilisation. On the one hand, an increase in the share of profits in income gives rise to an increase in aggregate saving because capitalists save a higher share of their income than workers.

This effect tends to reduce the volume of effective demand and thus, the degree of capacity utilisation.

On the other hand, investment depends on the share of profits in income which is a proxy for 'profitability'. So, investment would raise as a consequence of an income redistribution towards profits, and this would engenders a higher volume of effective demand and a higher degree of capacity utilisation. However, according to our specification of investment function, the first effect is stronger than the second one, so that the degree of capacity utilisation will reduce in response to an increase in the degree of capacity utilisation. Hence, we conclude that a wage-led accumulation regime prevails on the economy under consideration.

The effect of a change in the output-capital ratio on the short-term equilibrium of the degree of capacity utilisation is denoted by the partial derivative below:

$$\frac{\partial u^*}{\partial \sigma} = -\frac{u^*}{\sigma} < 0 \tag{9}$$

We observe from equation (9) that an increase in the output-capital ratio (in other words, a decrease in the capital-output ratio) will result in a fall of the short-term equilibrium of the degree of capacity utilisation.

From (8) and (9) it is easy to rewrite u^* as an implicit function of m and s:

$$u^* = u^*(m, \sigma) \quad ; u_m < 0,\ u_\sigma < 0 \tag{10}$$

3 Long-term dynamics and stability

In the long-term, the share of profits in income and the output-capital ratio cannot be regarded constant anymore. With reference to the income distribution between capitalists and workers, the devisal of a 'desired' profit rate by the capitalists – which is a "social convention"that prevails at a particular time (cf. Bresser-Pereira (1988, p.125)) – makes the mark-up rate, and also the share of profits in income, an *endogenous* variable in the long-term. Therefore, if the actual profit rate is *lower* than the 'desired' profit rate, then the capitalists would increase the mark-up rate as a device to raise the actual profit rate[3]. If the actual profit rate if *higher* than the 'desired' rate, then capitalists would reduce the mark-up rate in order to reduce the actual profit rate to the level given by this 'desired' rate[4]. Thus, we have the following differential equation:

$$\dot{m} = -\theta\left(R - \bar{R}\right) \qquad \theta > 0 \tag{11}$$

[3]We must emphasize that the actual profit rate will only raise, in response to an increase in the mark-up rate, if the degree of capacity utilisation has a small sensitivity to mark-up rate variations. The reason for that is the prevalence of a wage-led accumulation regime in this economy. Under these circumstances, an increase in the mark-up rate will raise the share of profits in income that would ultimately result in a lower degree of capacity utilisation. If the decline in the degree of capacity utilisation is very steep, then the actual profit rate may reduce by virtue of an increase in the mark-up rate.

[4]Why should the capitalists deliberately undertake measures to reduce the profit rate? A possible answer for this question is given by the theory of "barriers to new competitors"from Bain (1956) and Labini (1984). According to this theory, the desired profit rate can be considered the profit rate that is compatible with the long-term stability of an industry, that is to say, the maximum profit rate level that would not induce the entrance of new competitors in the industry. Hence, if the actual profit rate is above its 'desired' level, then this would invite new competitors to enter and consequently, in the long-term, it would cause a decrease in profits and reduce the market power of established firms. A post-Keynesian macroeconomic model consistent with the theory of barriers to new competitors is presented in Oreiro (2004).

where, $\bar{R}$ is the 'desired' profit rate and θ determines the speed of adjustment of profit rate to the 'desired' level[5].

3.1 The dynamics of different technological progress modalities

The type of technological progress that is occurring int the economy determines the dynamics of the output-capital ratio. The output-capital ratio can only be regarded constant if the technological progress is neutral in Harrod's sense. In this configuration we have:

$$\frac{\dot{\sigma}}{\sigma} = 0 \tag{12}$$

If the technological progress is 'capital saving', then the output-capital ratio will increase over time,indicating that the production of one unit of output requires less and less capital. In this case, we suppose that the output-capital ratio increases at a constant and exogenous rate $h > 0$, in such a manner that:

$$\frac{\dot{\sigma}}{\sigma} = h > 0 \tag{13}$$

Eventually, if the technological progress is 'capital intensive', then the output-capital ratio will diminish over time, indicating that more and more capital is necessary to produce a unit of output. Again, we suppose that the output-capital ratio decreases at a constant and exogenous rate $h < 0$, i.e.

$$\frac{\dot{\sigma}}{\sigma} = h < 0 \tag{14}$$

3.1.1 Long-term dynamics when technological progress is neutral

When the technological progress is neutral *a la* Harrod, the output-capital ratio is constant and the long-term dynamics of this economy is completely outlined by equation (11).

Substituting (10) into (2) and the resulting equation into (11) we obtain the following differential equation that presents the dynamic behaviour of the share of profits in income:

$$\dot{m} = -\theta \left[m\sigma u^* (m, \sigma) - \bar{R} \right] \tag{15}$$

Differentiating (15) with respect to $\dot{m}$ and m, we have:

$$\frac{\partial \dot{m}}{\partial m} = -\theta\sigma u^* (1 - \varepsilon_{u,m}) \tag{16}$$

where, $\varepsilon_{u,m} \equiv -\left(\frac{m}{u}\right) u_m$ denotes the elasticity of the degree of capacity utilisation with respect to the share of profits in income.

[5]Lavoie (2002) proposes another adjustment mechanism for profit rate misalignments. For him, the actual profit rate results from a conflict of interests between workers - who desire a high real wage level and thus, a low profit rate - and capitalists - who desire a high profit rate. In this framework, however, capitalists - for some unspecified reasons - are not capable to induce the adjustment of actual level of profit rate towards its 'desired' level so, capitalists can only adjust their 'desired' profit rate, according to the following equation: $\dot{\bar{R}} = -\theta(\bar{R} - R)$.
In equation 11, on the other hand, we assume that capitalists change their mark-up rate in order to adjust the profit rate to an exogenously determined 'desired' level, and then they implicitly change the share of profits in income. In Lavoie's specification, the 'desired' profit rate is endogenous in the long-term, and in our terms, he would have $\theta < 0$.

Let m^* be a fixed value from (15), that is, the value of m that holds the share of profits in income constant over time. This value is a *stable equilibrium* if and only if $\frac{\partial \dot{m}}{\partial m} < 0$ (cf. Takayama (1993, p.336)). But that requires the fulfilment of one condition, namely:

$$\varepsilon_{u,m} < 1 \tag{17}$$

In words: the long-term equilibrium will be *stable* if and only if the elasticity of the degree of capacity utilisation with respect to the share of profits in income is *less* than an unity.

In order to better understand the economic logic beneath this result let us suppose an initial situation where the actual profit rate is below the 'desired' profit rate. In these circumstances, the capitalists would increase the mark-up rate with the aim to raise the actual profit rate up to the 'desired' level. However, the increase in mark-up rate will result in an income redistribution towards profits, which will reduce the volume of effective demand and consequently, the degree of capacity utilisation. If the fall in the degree of capacity utilisation is very steep, then it will outweigh the effects of an increase in the mark-up rate on the profit rate. Thus, the actual profit rate will decrease instead of increase, which would persuade the capitalists to raise the mark-up rate even further and so evidently the profit rate dynamics will be non-convergent. In order to avoid this instability, the degree of capacity utilisation must present a lower sensitivity to changes in the share of profits in income, in other words, the condition (17) must be satisfied.

3.1.2 Long-term dynamics when technological progress is 'capital saving'

When the technological progress is 'capital saving', the dynamic behaviour of the economy is characterized by a system of differential equations in σ (13) and m (15).

Linearising the system around its steady state and expressing the resulting equations in the matrix form we have:

$$\begin{bmatrix} \dot{m} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} -\theta\sigma u^* \left(1 - \varepsilon_{u,m}\right) & -m\theta\left(\sigma u^* + \sigma u_\sigma\right) \\ 0 & h \end{bmatrix} \begin{bmatrix} m - m^* \\ \sigma - \sigma^* \end{bmatrix} \tag{18}$$

From (9) we have that $-m\theta\left(u^* + u_\sigma \sigma\right) = -m\theta\left(u^* - \frac{u^*}{\sigma}\sigma\right) = 0$.

The system of equations represented in (18) will be stable only if the *determinant* and the *trace* of the jacobian matrix are respectively positive and negative (cf. Takayama (1993, pp.407-408)). From these relations follow the conditions for system's stability:

$$DET = -h\theta\sigma u^* \left(1 - \varepsilon_{u,m}\right) \tag{19}$$

$$TR = -\theta\sigma u^* \left(1 - \varepsilon_{u,m}\right) + h \tag{20}$$

From equation (19) we may notice that the determinant of the jacobian matrix will be positive if and only if $\varepsilon_{u,m} > 1$, namely, if the degree of capacity utilisation is very sensitive to changes in the share of profits in income. Notwithstanding, if this condition holds then the trace of the jacobian matrix will be necessarily positive, making the system unstable. On the other hand, if the previously referred elasticity is less than one, then the trace of the jacobian matrix can be negative but its determinant will be also negative and the system is also inevitably unstable. Therefore, we conclude that in case of a 'capital saving' technological progress the conditions for stability will never be met and the system is intrinsically unstable.

3.1.3 Long-term dynamics when technological progress is 'capital intensive'

When the technological progress is 'capital intensive', the dynamic behaviour of the economy is characterized by a system of differential equations analogous to that represented in (18), with a difference that in the new system we have $h < 0$. Thus, the determinant and the trace of the jacobian matrix remain expressed by (19) and (20).

Given that $h < 0$, the determinant of the jacobian matrix will be positive if the elasticity of the degree of capacity utilisation with respect to the share of profits in income is less than unity. In these conditions, the trace of the jacobian matrix is necessarily negative and the system is stable. As a consequence of this, we conclude that in case of a 'capital intensive' technological progress the economic system is stable only if the degree of capacity utilisation presents low sensitivity to variations in the share of profits in income.

3.2 The acceleration and profitability effects over investment

The investment function specified in (4) complies with a post-Keynesian tradition[6] and is influenced by an interaction between the profitability and acceleration effects[7], respectively represented by the terms $\alpha_1 m$ and $\alpha_2 \sigma u$. The investment sensitivity to both effects is straightly bound to the values of the parameters α_1 and α_2. Consequently, these parameters define the long-term dynamics of the model and the particular conditions for the steady-state equilibrium. In the next subsections we identify the attributes required for stability and assess the mentioned effects separately and simultaneously.

3.2.1 Acceleration effect

Assuming that $\alpha_1 = 0$, we isolate the acceleration effect on investment. In this situation, the short-term equilibrium of the degree of capacity utilisation is given as

$$u^* = \frac{\alpha_0}{\sigma\left(s_c m - \alpha_2\right)} \tag{7a}$$

This will be a positive value if and only if $s_c > \frac{\alpha_2}{m} = s_c^*$, in other words, the savings out of profits ratio must exceed a critical value s_c^*, which inversely depends upon the share of profits in income.

With regard to the elasticity of the degree of capacity utilisation with respect to the share of profits in income we conclude, after algebric transformations, that $\varepsilon_{u,m} \equiv -\left(\frac{m}{u^*}\right) u_m = \frac{s_c m}{s_c m - \alpha_2} > 1$, given that $\alpha 2 > 0$. Therefore, from (16) we have $\frac{\partial \dot{m}}{\partial m} = -\theta\sigma u^*\left(1 - \varepsilon_{u,m}\right) > 0$, so we deduce that an increase in the share of profits in income would lead the capitalists to raise the mark-up rate even further. The explosive trajectory of the share of profits in income suggests the occurrence of a "worker's euthanasia"in the long-term.

[6]c.f. Robinson (1962, cap.2), Rowthorn (1981) and Bhaduri and Marglin (1990)

[7]The acceleration effect occurs when investment, which is a decision to enlarge the productive capacity, reacts to current or foreseen changes in aggregate demand (that in our analysis is represented by through the degree of capacity utilisation). When the profit rates increase capitalists are impelled to expand the productive capacity, that is, to invest and this is called profitability effect.

3.2.2 Profitability effect

Supposing that $\alpha_2 = 0$, we separately consider the profitability effect over investment. In this case, the equilibrium level of the degree of capacity utilisation is given by:

$$u^* = \frac{\alpha_0 + \alpha_1 m}{s_c m \sigma} \tag{7b}$$

and the dynamics of m is given by equation

$$\dot{m} = -\theta \left(\frac{\alpha_0 + \alpha_1 m}{s_c m} - \bar{R} \right) \tag{11b}$$

The steady-state value share of profits in income is:

$$m^* = \frac{s_c \bar{R} - \alpha_0}{\alpha_1} \tag{21}$$

Two conditions result from this outcome: firstly, m^* is a positive value provided that $s_c > \frac{\alpha_0}{\bar{R}}$; secondly, it is economically inconceivable that $m^* > 1$. Therefore, to ensure that $1 > m^* > 0$ a coherent combination of parameters is needed, so that $\frac{\alpha_0}{\bar{R}} > s_c > \frac{\alpha_0 + \alpha_1}{\bar{R}}$ (22).

3.2.3 Simultaneous operation of acceleration and profitability effects

In this situation we assume that $\alpha_1 > 0$ and $\alpha_2 > 0$. Substituting (7) into (15), the dynamics of the share of profits in income is written as the equation below:

$$\dot{m} = -\theta \left[\frac{m(\alpha_0 + \alpha_1 m)}{s_c m - \alpha_2} - \bar{R} \right] \tag{11c}$$

Deriving (11c) with respect to m we obtain $\frac{\partial \dot{m}}{\partial m} = \theta \frac{[\alpha_0 \alpha_2 - \alpha_1 m (s_c m - \alpha_2)]}{(s_c m - \alpha_2)^2}$. As we have verified in section 3.1.1, the system's stability depends upon that the condition $\frac{\partial \dot{m}}{\partial m} < 0$ holds, that is to say, an increase in the share of profits in income must not induce capitalists to raise the the mark-up rate more and more. Thus, the savings out of profits ratio must be sufficiently high to ascertain that:

$$s_c > \frac{\alpha_2 (\alpha_0 + 2\alpha_1 m)}{\alpha_1 m^2} \tag{23}$$

This condition must be held along the entire trajectory of m, since its initial condition until its steady-state value. Additionally, from (7), in order that the degree of capacity utilisation is a positive number, another condition must be satisfied:

$$s_c > \frac{\alpha_2}{m} \tag{24}$$

Nevertheless, once the condition (23) holds, so does the condition (24) since $\frac{\alpha_2(\alpha_0 + 2\alpha_1 m)}{\alpha_1 m^2} > \frac{\alpha_2}{m}$.

3.3 Configurations for the model's stability

In table 2 below we compile the previous analysis for different combinations among technological progress modalities and specifications of investment function, and we present the long-term behaviour of the output-capital ratio and the share of profits in income.

Table 2: Stability conditions

Technological Progress / Investment	Neutral $h = 0$	Capital Saving $h > 0$	Capital Intensive $h < 0$
Acceleration Effect $\alpha_1 = 0$	m: Explosive σ: Constant	m: Explosive σ: Explosive	m: Explosive $\sigma \rightarrow 0$
Profitability Effect $\alpha_2 = 0$	m: Stable in the long-term σ: Constant	m: Stable in the long-term σ: Explosive	m: Stable in the long-term $\sigma \rightarrow 0$
Combined Effects $\alpha_1 > 0$ and $\alpha_2 > 0$	m: Stable in the long-term σ: Constant	m: Stable in the long-term σ: Explosive	m: Stable in the long-term $\sigma \rightarrow 0$

We verify that the steady-state equilibrium is a feasible result in the long-term only if the technological progress is neutral or capital intensive and besides, if the investment is susceptible to variations in the share of profits in income.

4 Computer simulations

To evaluate the dynamics and interactions between the variables of the model in three different types of technological progress we carry out a computer simulation[8], by adopting a top-down methodology[9]. So, we assigned economically plausible[10] values for the model's parameters, according to table 3 below:

[8]To proceed the numerical simulations we used Maple 7.0 from Waterloo Maple Inc.

[9]This procedures involves the adjustment of a preexistent economic model to a numerical simulation environment. An opposite methodology would involve the creation of a model designed to simulations. The first method entails some difficulties in setting up real and/or plausible values for the variables and parameters, however it enriches the assessment of interactions among the selected variables, which could hardly be accomplished with an ordinary quantitative analysis.

[10]For example, the savings out of profits ratio was assigned with the purpose of fulfilling conditions (3.2.2) and (23). It is not a "manna that falls from heaven"once a savings out of profits ratio of 42% seems to be a reasonable value and is also coherent to Samuelson's "principle of correspondence"that corroborates the reasoning that values might be chosen in order that a model's dynamics would prove to be minimally realistic.

Table 3: Parameters

Parameter	Value	Description
θ	0.5	Adjustment factor of the profit rate.
α_0	0.02	Autonomous term of the investment function which represents the *animal spirits* of capitalists.
α_1	0.02	Coefficient that expresses the investment's sensibility to profit fluctuations
α_2	0.02	Coefficient that measures the accelerator effect, namely the influence of changes in the degree of capacity utilisation and the output-capital ratio on investment.
$\bar{R}$	0.09	'desired' profit rate
s_c	0.42	Savings out of profits.

We used as an initial values for the economy under consideration, a share of profits in income of 30% and a output capital ratio of 0.5, which is tantamount to a capital-output ratio of 2. When the technological progress is capital saving or capital intensive we suppose respectively a variation rate of the output-capital ratio equals to 0.01 and -0.01, that is to say, a variation rate of 1% in each period. We arbitrarily stipulate 100 periods as range for the analysis.

In the simulations we examine the dynamics of the share of profits in income (m), the degree of capacity utilisation (u_*), the output-capital ratio (s) and the profit rate over time and under different assumptions with regard to technological progress (neutral, capital saving and intensive) and to the investment's sensibility to changes in the profit margins and the degree of capacity utilisation. With reference to the later we focus our analysis on the trajectory over time of the variables under consideration in three distinct scenarios, which are: $\alpha_1 = 0$ (investment is not sensitive to changes in profit margins); $\alpha_2 = 0$ (investment is not sensitive to changes in the degree of capacity utilisation) and; $\alpha_1 > 0, \ \alpha_2 > 0$(the investment is sensitive to both changes in m and u).

4.1 Case 1: Accelerator effect only ($\alpha_1 = 0$)

As we have demonstrated in section 3.1.1, the dynamics of the share of profits in income implies that $\frac{\partial \dot{m}}{\partial m} > 0$, which causes the system to be unstable[11]. The figure 2 below represents the economic dynamics when the technological progress is neutral:

[11] The elasticity of the degree of capacity utilisation with respect to the share of profits in income is $\varepsilon_{u,m} > 1$ so that $\frac{\partial \dot{m}}{\partial m} = -\theta \sigma u^* \left(1 - \varepsilon_{u,m}\right) > 0$. Consequently, given the parameters in table 2 and independently if $h < 0$ or $h > 0$, the trace of the jacobian matrix is positive thereby indicating a systemic instability.

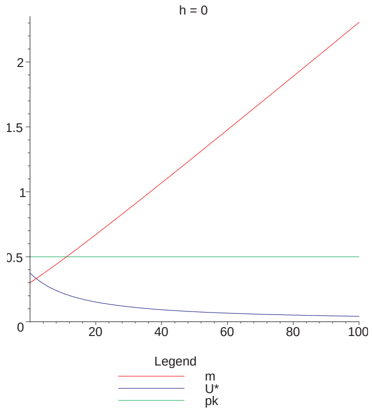

Figure 2: The curve pk represents σ, namely the dynamics of the output-capital ratio.

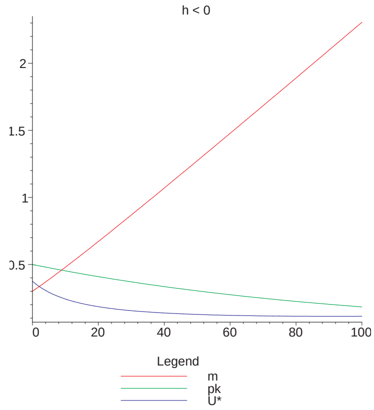

Figure 3:

As we may notice, in both cases the share of profits in income clearly presents an explosive trajectory over time.

When technological progress is capital intensive, as in figure 3, the output-capital ratio exhibits a descendent path and, as stated by equation (7a), this attenuates the decline of the degree of capacity utilisation.

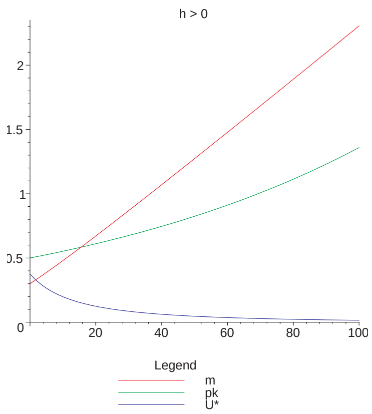

Figure 4:

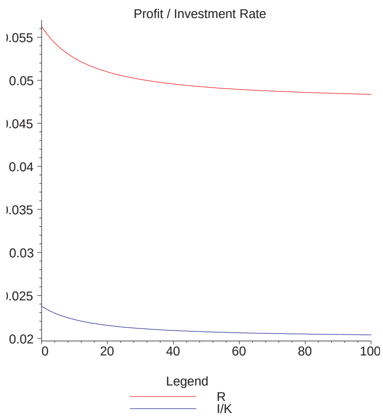

Figure 5:

When technological progress is capital saving (Figure 4), both m and σ increase indefinitely, thus the degree of capacity utilisation falls even faster to a close to zero level.

We confirm that the share of profits out of income truly presents an explosive trend: $\lim_{t\to\infty} m = \infty$. As a result, the degree of capacity utilisation tends to fall down to zero: $\lim_{t\to\infty} u^* = 0$. Since these variables follow opposite directions, they sustain the profit rate at a level of 4.37%. So, the actual profit rate (R) will never reach the 'desired' profit rate level ($\bar{R}$). The growth rate of the capital stock will remain relatively stable over time, moving from a initial level of 2.375% to stabilise itself at 2%, which is equal to α_0. This outcome reveals that capitalist will keep investing even in a total stagnation scenario ($u^* \to 0$). In this situation, the investment depends solely on the entrepreneurs' *animal spirits*.

Notably, the type of technological progress have no effects on the dynamics of share of profits in income and the degree of capacity utilisation.

4.2 Case 2: Profitability effect only ($\alpha_2 = 0$)

When we disregard the acceleration effect on investment and consider the profitability effect only, the share of profits in income become stable over time, despite the kind of technological progress. This phenomenon is explained by equation (11b) which shows that m's dynamics is not affected by σ.

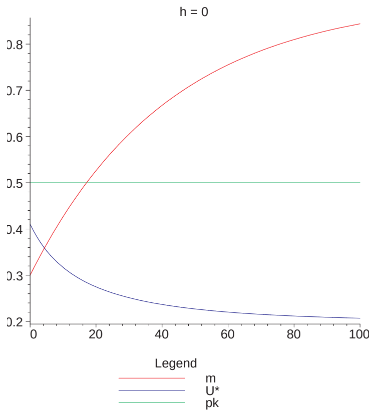

Figure 6:

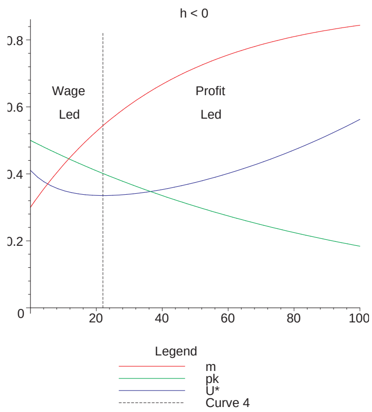

Figure 7:

In figure 6 above, we may observe the asymptotic behaviour of the share of profits in income, which according to equation (21) presents a steady-state equilibrium around 90%. The short-term equilibrium of the degree of capacity utilisation goes from an initial value around 41% to stabilise itself at 20%. Therefore, in case of a neutral technological progress all variables converge toward their long-term equilibrium value.

When technological progress is capital intensive (Figure 7), we observe that the share of profits in income and the output-capital ratio both converge toward their own steady-state equilibrium and σ falls down to zero in the long-term.

At the beginning of the analytical period, m presents a sharp increase which causes a decrease of the degree of capacity utilisation (see equation 7b). However, as σ declines (tending to zero) and the m increases slightly, then the degree of capacity utilisation reverts its downward trend and displays increasing variation rates. In this situation, the output grows more slowly than the capital stock growth rate (because technological progress is capital intensive: $\sigma \to 0$), hence it imposes an additional effort over the installed productive capacity with intention to maintain the equilibrium level of the share of profits in income. It is interesting to notice that in the initial stage when an increase in the share of profits in income induces a reduction of u^*, indicating a wage-led accumulation regime. However, the behaviour of u^* endogenously changes[12] over time and the model exhibits a profit-led accumulation regime, in other words, as m increases it raises the degree of capacity utilisation.

[12]Bhaduri and Marglin (1990) identified the requisites for the profit-led and wage-led accumulation regimes. Given that their model involved comparative statics analyses, the transition from one accumulation regime to another does not take place over time but it would depend on the degree of capacity utilisation level. By using our specification for variables and parameters we have, in accordance with Bhaduri and Marglin (1990), that: a) a wage-led accumulation regime occurs if $u^* > \frac{\alpha_1}{s_c}$; b) a profit-led accumulation regime occurs if $u^* < \frac{\alpha_1}{s_c}$. This relation is not supported by figure 7, because we observe situations where distinct accumulation regimes happens for identical values of u^*.

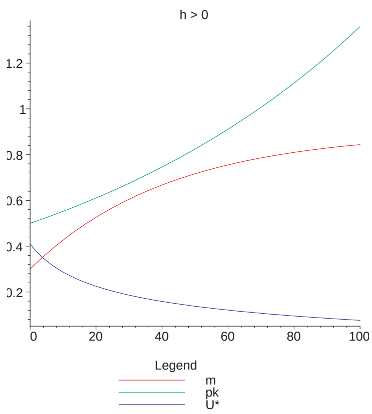

Figure 8:

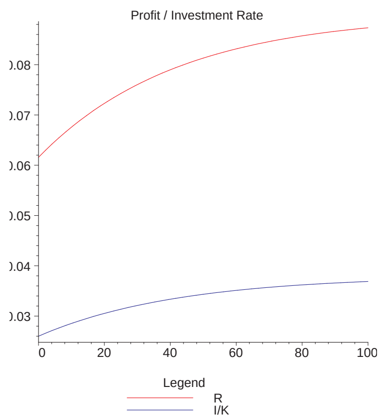

Figure 9:

When technological progress is capital saving (figure 8), the output-capital ratio presents an explosive behaviour but it does not alters the stability of the degree of capacity utilisation: $\lim_{t\to\infty} u^* = 0$. A higher capital productivity ($\uparrow \sigma$) and the increase of m up to its equilibrium value implies a reduction in the short-term equilibrium level of the degree of capacity utilisation, which begins the process around 41%. These trajectories denote a wage-led accumulation regime.

The trajectories of the profit rate and investment rate are pictured in figure 9 and they are not subject to different types of technological progress. This phenomenon happens because both variables depends only on m, which, after presenting an upward trend, becomes stable at its steady-state equilibrium. Simplifying and rewriting (2) and (4) we have:

$$R = u^* m\sigma = \frac{\alpha_0 + \alpha_1 m}{s_c} \tag{2a}$$

$$\frac{I}{K} = \alpha_0 + \alpha_1 m + \frac{\alpha_2\alpha_0}{s_c m} + \frac{\alpha_2\alpha_1}{s_c} \tag{4a}$$

As figure 9 illustrates, the profit rate starts its trajectory at 6.15% and then converges in the long-run to the 'desired' growth rate: $\lim_{t\to\infty} R = \bar{R} = 0.9$. The desired growth rate of the capital stock also raises from 2.6% up to its equilibrium value of 3.8%.

4.3 Case 3: Profitability and accelerator effects on investment($\alpha_1 > 0$ e $\alpha_2 > 0$)

Now, we will analyse the dynamics resulting from both profitability and accelerator effects on investment decisions. Just as in case 2, the share of profits in income tends to become stable in the long-term. This is a consequence of the fulfilment of the condition expressed in (23) during the whole dynamics of m, since its initial value $m_0 = 30\%$ until its steady-state equilibrium $m^* = 78{,}5\%$. It is important to notice that this value for m is very near the actual profit share for the Brazilian economy.

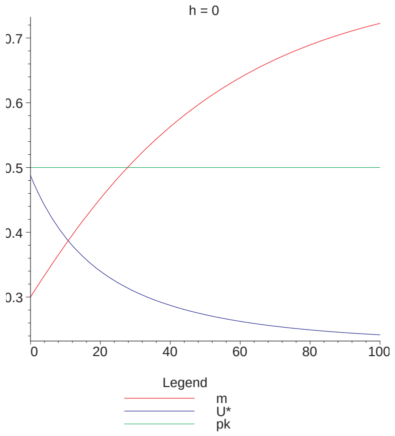

Figure 10:

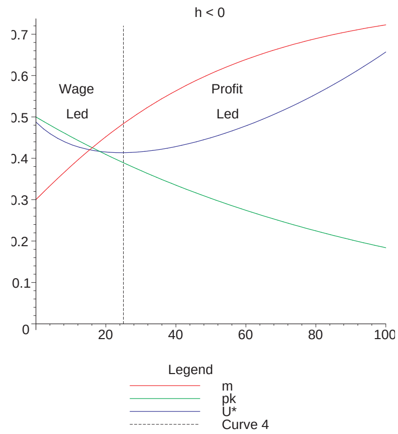

Figure 11:

When technological progress is neutral, as figure 10 illustrates, all variables converge to their own equilibrium values in the long-run. As the share of profits in income increases we observe that the degree of capacity utilisation decreases, and this indicates a wage-led accumulation regime.

When technological progress is capital intensive (Figure 11), a phenomenon similar to case 2 occurs: σ tends to zero and the share of profits in income converge to its long-term equilibrium. It is noteworthy to observe again an endogenous transition from a wage-led to a profit-led accumulation regime.

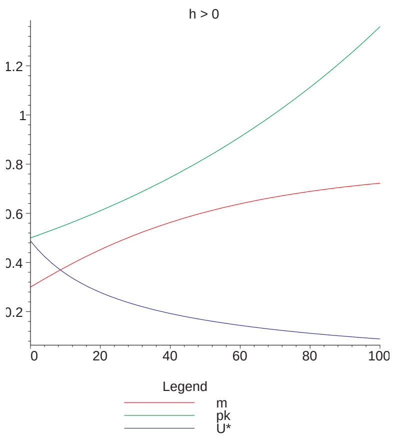

Figure 12:

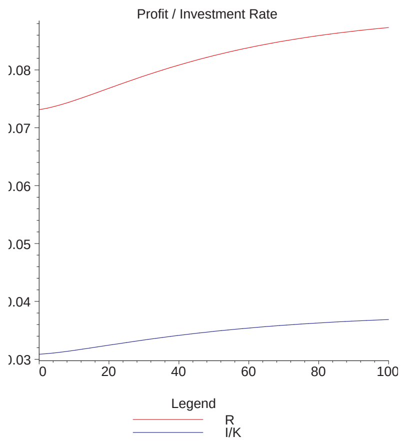

Figure 13:

When technological progress is capital saving, according to the analysis done in section 3, even though m and u^* reach stable values, the system is not entirely stable since the output-capital ratio presents an explosive trajectory.

In figure 13 we notice that both profit and investment rate become stable in the long-term. They converge respectively to 9% (the desired profit rate value) and 3.8%.

When we observe the figures above we verify that the dynamics of variables in case 3 are similar to case 2 where the profitability effect on investment was considered separately. Remarkably, when we take the sensibility of investment to changes in the output-capital ratio (through the accelerator effect) into account, the transition of all other variables to their long-term equilibrium happens more slowly. Furthermore, the steady-state equilibrium values also change, except for the profit rate which still converge to the 'desired' level. For instance, when technological progress is neutral, the degree of capacity utilisation will stabilise itself around 22.9% while in case 2 the equilibrium value is 20%.

Two interesting outcomes derive from the computer simulations above:

1. In all cases, m raised over time which implied a higher income concentration on profits (possessed by capitalists) whereas the degree of capacity utilisation fell down in almost every cases, except for $\alpha_2 = 0$ and $h < 0$ and for $\alpha_1 > 0$, $\alpha_2 > 0$ and $h < 0$. We then deduce that the investment's sensibility to changes in $u\sigma$ and m are only relevant for the dynamics of the degree of capacity utilisation when technological progress is capital intensive.

2. The profit and investment rate remained relatively stable in all situations indicating that capitalists are well succeeded in sustaining these rates, even though in different values from the 'desired' profit rate level (in case of $\alpha_0 = 0$). We also verified that both profit rate and investment rate are not determined by the type of technological progress and hence, by changes in output-capital ratio.

4.4 The robustness of results

In order to prove the robustness of simulation's results, we submit the parameters to a stress test. We consider investment function to be depedent on both accelerator and profitability effects ($\alpha_1 > 0$ e $\alpha_2 > 0$) and then we change each one of the parameters on table 3, *ceteris paribus*, under two types of technological progress: neutral and capital intensive.

Table 4: Robustness test - Neutral technological progress

Parameter	**Value**	Long-term values			
		m	u^*	$\frac{I}{K}$	R
Original Values	See Table 3	0.774	0.233	0.0378	0.09
α_0	0.025	0.431	0.417	0.0378	0.09
	0.015	1.055	0.171	0.0377	0.09
α_1	0.040	0.290	0.620	0.0378	0.09
	0.015	1.080	0.167	0.0379	0.09
α_2	0.030	0.696	0.259	0.0378	0.09
	0.010	0.836	0.215	0.0378	0.09
$\bar{R}$	0.100	1.000	0.200	0.0420	0.10
	0.080	0.529	0.303	0.0336	0.08
s_c	0.500	1.170	0.154	0.0449	0.09
	0.370	0.476	0.378	0.0333	0.09
θ	0.750	0.774	0.233	0.0378	0.09
	0.720	0.774	0.233	0.0378	0.09

Table 4 exhibits the long-term steady-state values of the share of profits in income, the degree of capacity utilization, the investment and profit rates, when technological progress is neutral. Conditions denoted by equations 23 and 24 impose some restrictions on parameters' values that fulfill some notional limits on variables (for example, $0 < m < 1$). As we can see, when we change parameters such as α_0 by some amount, for example 25%, some conditions may not hold and the reason some variables 'unleash' ($m > 1$) is because the primary values, presented in table 3 are on the threshold of these conditions. The tests disclosed the sensibility of the share of profits in income to the 'desired' profit rate. Changes in θ had no effects over the long-term values of the variables under considerations, it changed the speed of convergence.

Table 5: Robustness test - Capital intensive technological progress

Parameter	**Value**	Long-term values			
		m	u^*	$\frac{I}{K}$	R
Original Values	See Table 3	0.774	∞	0.0378	0.09
α_0	0.025	0.431	∞	0.0378	0.09
	0.015	1.055	∞	0.0377	0.09
α_1	0.040	0.290	∞	0.0378	0.09
	0.015	1.080	∞	0.0379	0.09
α_2	0.030	0.696	∞	0.0378	0.09
	0.010	0.836	∞	0.0378	0.09
$\bar{R}$	0.100	1.000	∞	0.0420	0.10
	0.080	0.529	∞	0.0336	0.08
s_c	0.500	1.170	∞	0.0449	0.09
	0.370	0.476	∞	0.0333	0.09
θ	0.750	0.774	∞	0.0378	0.09
	0.250	0.774	∞	0.0378	0.09
h	-0.030	0.774	∞	0.0378	0.09
	-0.005	0.774	∞	0.0378	0.09

When we considered a capital intensive technological progress, we firstly noted that the parameters had no effects over the long-term value of capacity utilization, which increased indefinitely. When we changed h, there was no effect over long-term values. Just as θ, this parameters only set the speed of convergence. Once more, we noticed that in some cases, for example when we reduced α_1 by 25%, the share of profits in income became inconsistent ($m > 1$), but then again this happened because the primary values were set almost on the limit of conditions 23 and 24.

5 Concluding remarks

In previous sections we analysed the dynamics of some economic variables: the share of profits in income, the output-capital ratio, the profit and investment rate under different assumptions about the type of technological progress and the specification of investment function. They let us enquire the conditions for the long-term stability of capitalist economies.

We verified, by means of computer simulations, that the type of technological progress is essential to determine the requirements for economic stability. Thus, if the technological progress is capital saving then the conditions for stability will never be met, which implies that such capitalist economy is necessarily unstable. Stability is a probable result only if the technological progress is neutral or capital intensive. Under these situations, it is a necessary condition to the stability, that the elasticity of the degree of capacity utilisation with respect to the share of profits in income is *less* than an unity, that is to say, changes in the functional income distribution must have a slight influence on effective demand, and consequently, on the degree of capacity utilisation. Another determinant of a long-term stability is the prevalence of a high propensity to save out of profits. If this ratio is not comparatively low, it could not ensure the equilibrium (at a positive value) of the share of profits in income, in other words, in face of an increase in the share of profits in income, the entrepreneurs would be impelled to increase the profit margin even further and it would cause the profit's trajectory to be explosive. Besides, if investment is quite dependent on the accelerator effect, which means that profit changes does not affect investment, then the share of profits in income will not achieve its long-term equilibrium value.

Computer simulations have revealed that, despite the modalities of technological progress and the specifications of investment function, the profit rate have kept at stable levels during its long-term trajectory. The variation range of the profit rate, from its initial level until equilibrium, was relatively small. This outcome denies Marxist predictions that the profit rate would inexorably decline over time.

Moreover, when technological progress is capital intensive and the investment is sensitive to changes in the share of profits in income, the surprising phenomenon of an endogenous transition from a wage-led to a profit-led accumulation regime arise. The reason for that is the simultaneous effect of the decrease in the output-capital ratio, and the increase in the share of profits in income up to an equilibrium level. In all other situations, as the income distribution becomes more concentrated on profits we may observe a decrease in the degree of capacity utilisation (in some situations it falls down to zero), and this indicates a wage-led accumulation regime during the whole economy's trajectory.

Recent empirical studies about the long-term dynamics of capitalist economies have shown an apparent upward trend in the capital-output ratio in the last 120 years. Hence, we conclude that the technological progress have been, up to present, 'capital intensive' *a la* Harrod. Unless there are strong motives to believe that the growth rate of the capital stock is independent to the share of profits in income, the requirements for the long-term stability of capitalist economies would have been satified. In other words, the technological progress, at least in its present state, is not the source of economic instability.

Nevertheless, instability is actually a distinctive attribute of capitalist economies. According to Maddison (1991), the average growth rate of developed capitalist countries have been varying substantially over time. Long periods of rapid growth - such as the 'golden age' of capitalism during the period of 1950-1973 - alternates with periods of semi-stagnation or moderate growth.

The post-Keynesian literature has pointed out two alternative sources for economic instability. The first, with a Marxist inspiration, would involve a class struggle between capitalists and workers. Goodwin (1967) have demonstrated that a class conflict may breed a limit cycle dynamics for the share of wages (and profits) in income and for the unemployment rate. The second, inspired in Minsky's (1975; 1982) approach, is based on the interaction between

Harrod's principle of acceleration and the endogenous money supply assumptions. This approach is formally developed by González-I-Calvet (1999), among other authors. It follows from these analyses that the causes of economic instability might be investigated taking into account these alternative sources except for the technological progress.

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