A Welfare Analysis of Economic Fluctuations in South America*

Fábio Augusto Reis Gomes[†] Leandro Gonçalves do Nascimento[‡]

July 8, 2004

Abstract

How large are welfare costs associated with economic aggregate fluctuations is a topic of great concern among economists at least since Robert Lucas' well-known and thought-provoking exercise in the late 1980s. Our analysis assesses the magnitude of such costs for nine countries in South America by means of three alternative trend-cycle decomposition methods. The results suggest South American countries have welfare costs of economic fluctuations notably higher than the U.S. economy.

Key Words: welfare costs of economic fluctuations; consumption; Beveridge-Nelson decomposition.

JEL Classification Numbers: C32, C51, E32, E60.

Área ANPEC: ÁREA 2 - Macroeconomia, Desenvolvimento e Economia do Setor Público

*We would like to thank Guilherme Hamdan, João Victor Issler, Luiz Renato Lima e Pedro Ferreira for many helpful comments. The usual disclaimer applies.

[†]Affiliation: Graduate School of Economics (EPGE), Fundação Getulio Vargas, and Centro de Pesquisa em Economia Internacional (CEPE). E-mail address: freis@fgvmail.br

[‡]Corresponding author. Affiliation: Graduate School of Economics (EPGE), Fundação Getulio Vargas. Address (home): Erotides de Oliveira, 61/1502, Icarai, Niteroi, Rio de Janeiro, 24230230, Brazil. Phone: 55-21-27103752. E-mail address: lgoncalv@fgvmail.br

A Welfare Analysis of Economic Fluctuations in South America

Abstract

How large are welfare costs associated with economic aggregate fluctuations is a topic of great concern among applied economists at least since Robert Lucas' well-known and thought-provoking exercise in the late 1980s. Our analysis assesses the magnitude of such costs for nine countries in South America by means of three alternative trend-cycle decomposition methods. The results suggest South American countries have welfare costs of economic fluctuations notably higher than the U.S. economy.

Key Words: welfare costs of economic fluctuations; consumption; Beveridge-Nelson decomposition.

JEL Classification Numbers: C32, C51, E32, E60.

Resumo

A magnitude dos custos dos ciclos econômicos é uma questão de grande importância entre os economistas desde pelo menos o experimento provocador de Robert Lucas no final dos anos de 1980. Nossa análise avalia a magnitude desses custos para nove países sul-americanos por meio de três métodos alternativos de decomposição tendência-ciclo. Os resultados sugerem que os países da América do Sul incorrem em um custo de bem-estar do ciclo econômico agregado muito maior do que a economia norte-americana.

Palavras-chave: custos de bem-estar das flutuações econômicas; consumo; decomposição de Beveridge-Nelson.

Classificação do JEL: C32, C51, E32, E60.

1 Introduction

In an influential work, Lucas (1987) estimates the welfare costs of business cycles are rather small in the U.S. economy. His original set-up consists of a representative agent with a constant relative risk aversion (CRRA) utility function, where there is no idiosyncratic risk and consumption is trend-stationary. The findings suggested there is little role for marginal counter-cyclical policies, since the upper bound of the welfare gains from such policies could be easily overwhelmed by their costs.

The aim of this paper is to estimate the welfare costs of aggregate economic fluctuations in nine South American countries. First we assume a trend-stationary reduced form for consumption and use two trend-cycle decomposition methods in Lucas (1987), namely linear trend and Hodrik-Prescott filter. Then, we suppose that consumption is first-difference stationary and apply Beveridge-Nelson trend-cycle decomposition, following a method recently used by Issler, Franco and Guillén (2003).

In fact, many authors have modified Lucas' set-up in an attempt to assess if the costs of business cycle are not trivially small. Among others, Obstfeld (1994), Pemberton (1996), Tallarini (2000) and Dolmas (1998) estimate the welfare costs of business cycle using non-expected utility. They consider a class of utility functions introduced by Epstein and Zin (1989) so as to analyze the consumption-based CAPM model and the equity premium puzzle. This class of preferences is recursive and exhibits first-order risk aversion, what could easily lead to large costs from consumption variability, as argued by Pemberton (1996). Obstfeld (1994), Dolmas (1998) and Tallarini (2000) consider a stochastic process for consumption first-difference stationary, beyond the trend-stationary case. Note that, if consumption is an integrate process, the effects of shocks are permanent and this fact certainly increases the variability and the risk of consumption as perceived by agents.

Many authors have also considered an incomplete markets artificial economy inhabited by heterogeneous agents. Imrohoroglu (1989) constructs an environment with many individuals facing idiosyncratic and imperfectly insurable income risk. However, her results are not essentially different from Lucas (1987). Similarly, by means of a model with partially insurable idiosyncratic risk, Krusell and Smith (1999) appraise whether the costs of business cycles are very high for poor or unemployed people. They conclude that such costs are extremely small for almost all consumers, and even negative for some.

Even though recent literature has changed Lucas' set-up in multiple dimensions, especially with respect to the utility function specification and agent heterogeneity, our paper maintains the assumption of a CRRA momentary utility function and a representative agent. Both hypotheses seem to be vindicable on empirical grounds according to Mulligan (2002), who shows that the he equity premium puzzle is associated with a bad proxy to aggregate capital return and not to CRRA utility, and hence a parcimonious model can reliably account for the facts.

Our investigation is relevant at least for two reasons. First, to the best of our knowledge, there is no empirical evidence concerning how large are the costs of business cycles in South America vis-à-vis stable developed economies such as the United States. Previous studies have been mainly concerned with the costs in the U.S. economy, while a few ones analyzed Europe (Duarte, Issler and Salvato (2003)) and even Africa (Pallage and Robe (2003)). Second, the majority of applied studies assumes consumption to be stationary, and the few papers that allow for an integrated process impose a priori restrictions on the cycle of the series (e.g., Obstfeld (1994), Dolmas (1998) and Tallarini(2000)). Here we do not impose any a priori restrictions on the cycle of consumption. Rather, if we find evidence that $\log c_t$ is difference-stationary, then we model $\Delta \log c_t$ as a general stationary ARMA, thereby endogenizing model choice for each country data.

The main results of this paper are that South American countries generally have large welfare losses associated with aggregate economic fluctuations if we consider Lucas' classical framework. Further, if we take into account the approach which allows for $\log c_t \sim I(1)$, such losses are even larger. Therefore, there is a potential positive role for more effective counter-cyclical policies in South American countries, contrary to what is often claimed for the U.S. economy.

We proceed as follows. In section 2, the economic environment required for a welfare analysis of aggregate fluctuations is presented carefully. Data are described in section 3, where we subsequently present the main results regarding welfare costs of business cycles. The conclusion summarizes the findings up to this point.

2 Environment

Agents are supposed to live an infinite number of periods and to derive utility from the stream of consumption (c) throughout their lives according to the following utility function:

$$U(c_0, c_1, \ldots) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where E_0 is the expectation operator given the information set at $t = 0$, $\beta \in (0, 1)$ stands for the intertemporal discount factor, and the momentary utility function is represented by

$$u(c) = \frac{c^{1-\phi} - 1}{1 - \phi}, \tag{2}$$

where $\phi \geq 1$ is the relative risk aversion coefficient.[1] Assume further, as Lucas (1987), that $(c_t)_t$ is log-normal about a deterministic trend, that is:

$$c_t = \alpha_0 \left(1 + \alpha_1\right)^t z_t, \tag{3}$$

where $\log z_t \sim N\left(0, \sigma_z^2\right)$.

In this set-up, a cycle-free consumption stream is given by $c_t^* = E(c_t)$. Thus $c_t^* = \alpha_0 \left(1 + \alpha_1\right)^t \exp\left(\frac{\sigma_z^2}{2}\right)$. Every risk-averse consumer (as the one represented by the concave utility function above) prefers a risk-free stream c_t^* to an uncertain one, c_t, since both series have the same mean. Therefore, the welfare costs associated with aggregate fluctuations in this economy can be represented by the compensating variation in consumption, λ, which solves:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left((1 + \lambda)c_t\right) = \sum_{t=0}^{\infty} \beta^t u(c_t^*). \tag{4}$$

That is, λ is the compensation in all dates and states of nature that makes the representative agent indifferent between these two streams of consumption. Notice that, the higher is λ, the stronger will be an agent's willingness to live in a business cycle-free world instead of a world with aggregate fluctuations.

Solving (4) for λ, given (1) to (3), it is easily checked that:

$$\lambda = \exp\left(\frac{\phi \sigma_z^2}{2}\right) - 1, \tag{5}$$

[1] Notice that, when $\phi \to 1$, $u(c)$ collapses to $\log c$.

for $\phi \geq 1$.[2] This formula for the welfare costs of aggregate fluctuations has two intuitions: (i) the more volatile is consumption time series, in the sense of a higher variance (σ_z^2), the higher are the costs of business cycles; (ii) the welfare costs are also higher for more risk averse agents, that is, λ is increasing in ϕ.

Although Lucas (1987) proposed exactly this analysis, he implemented it in a different way. Instead of estimating σ_z^2 from the residuals associated with the log-linear regression implied by (3), Lucas filtered the logarithm of consumption series using the procedure in Hodrick and Prescott (1997) - HP -, and estimated σ_z^2 from cycle obtained by subtracting the HP-trend from the original series.

In spite of its simplicity, the preceding analysis has one drawback: it does not take into account that $\log c_t$ is frequently considered $I(1)$ in several theoretical and empirical studies - e.g., Hall (1978), Flavin (1981), and Campbell (1987). It is worth noting this error may lead to completely flawed results, despite all the intuition underlying equation (5). In case $\log c_t \sim I(1)$, shocks are permanent and the aggregate risk of the economy would be a function of all z_i, $i = 1, 2, \ldots, t$. On the other hand, λ as described by (5) is merely a function of σ_z^2, not of the entire history of the random variable z_t. Thus, if it were the case of $\log c_t$ being $I(1)$, equation (5) probably would underestimate the costs of aggregate fluctuations, as argued by Obstfeld (1994).

To deal with this fact, we test whether $\log c_t \sim I(1)$. If it is, the trend of the series is also stochastic and, given the evidence of a unit root, unconditional mean and variance of the original series are not well-defined. We then redefine $c_t^* = E_0 c_t$ as in Obstfeld (1994), and then apply the Beveridge and Nelson (1981) decomposition for $\log c_t \sim I(1)$, who have shown that any difference stationary stochastic process can be decomposed into a deterministic term, a random walk trend and a stationary cycle.[3] Then:

$$\log c_t = \log\left[\alpha_0 \left(1+\alpha_1\right)^t\right] + \log X_t + \log Y_t, \tag{6}$$

where $\log\left(1+\alpha_1\right)^t$ is the deterministic trend, $\log X_t = \sum_{i=1}^{t} \varepsilon_i$ stands for the random walk component, and $\log Y_t = \sum_{i=1}^{t} \psi_{t-i} \vartheta_i$ represents the stationary cycle. We assume further that permanent shock (ε_t) and the transitory shock (ϑ_t) have a bivariate normal distribution as

$$\begin{bmatrix} \varepsilon_t \\ \vartheta_t \end{bmatrix} \sim IIDN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right). \tag{7}$$

In this framework, we implicitly define λ as in (4), except for the fact that $c_t^* = E_0 c_t$. It can be shown that, if $\phi > 1$, then:

$$\lambda = \left(\frac{\sum_{t=0}^{\infty} \left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)w_t^2}{2}\right)}{\sum_{t=0}^{\infty} \left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)^2 w_t^2}{2}\right)} \right)^{\frac{1}{1-\phi}} - 1, \tag{8}$$

where $w_t^2 = t\sigma_{11} + 2\sigma_{12} \sum_{j=1}^{t} \psi_j + \sigma_{22} \sum_{j=1}^{t} \psi_j^2$ is the variance of the logarithm of consumption

[2] Appendix A.

[3] Notice this redefinition is without loss of generality, since $E_0 c_t$ as defined here and $E c_t$ as in Lucas' approach are risk-free, and the same intuition applies in both cases.

conditional in the information at $t = 0$.[4] When $\phi = 1$, equation (8) amounts to:

$$\lambda = \exp\left((1-\beta)\sum_{t=0}^{\infty}\beta^t \frac{w_t^2}{2}\right) - 1. \tag{9}$$

Notice that, as before, the more volatile is the consumption series, the higher are the costs associated with business cycles. Furthermore, even though those costs now depend upon the growth rate of consumption, they are not functions of the initial level, α_0: linear shifts on the logarithm of consumption series do not affect λ. Therefore, richer societies do not necessarily have lower costs of cyclical fluctuations.

The following proposition gives sufficient conditions for the convergence of (8) and (9) and will be useful in empirically computing values for the compensating variation in consumption.

Proposition 1 *Assume $\psi_t \to 0$ as $t \to \infty$ (which is often the case in our set-up: see (A14) below). Then **(i)** (9) always converges, and **(ii)** (8) converges whenever $\alpha_1 \geq 0$ and $\beta(1+\alpha_1)^{1-\phi}\exp\left(\frac{(1-\phi)^2\sigma_{11}}{2}\right) < 1$.*

Proof. *Appendix B.* ■

Whereas the above proposition gives sufficient conditions in order to guarantee the convergence of λ, it is by no means necessary. In every case those conditions did not hold in our empirical implementation, the computed value of λ was significantly large so as to allow us to infer that the costs of business cycles diverged. Fortunately, in most relevant cases (i.e., $\phi \in \{1, 2, 5\}$) those conditions were frequently satisfied.

3 Empirical Results

The directions to empirically implement the computations of the welfare costs of cyclical fluctuations are as follows. With respect to Lucas' framework, given the specification in (3), we first run a linear least squares regression of the logarithm of consumption in a time trend and a constant, and then store the estimated standard deviation of residuals using it in computing (5). An analogous exercise accounts for the calculations using the HP filter: σ_z is computed using the deviation of the original series from the HP trend.

When consumption is difference-stationary after testing, we model the "best" stationary ARMA process for $\Delta \log c_t$[5], and then follow the procedure described in Appendix A on how to identify each component of equations (8) and (9) to evaluate λ.

Our data set consists of constant price annual per capita consumption in nine South American countries and U.S. dating from 1951 to 1999, extracted from Penn World Table - Summers, Heston, and Aten (2002). South American countries are: Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Peru, Paraguay and Uruguay. Although we lack specific information for non-durable consumption as it would be preferable, our data are widely used in econometric studies and very reliable for direct comparisons among countries.

[4] All calculations are presented in Appendix A, jointly with a discussion on how to identify relevant parameters.
[5] See section 3.2 for the meaning of "best".

3.1 Lucas' classical set-up

Equation (3) was separately estimated via ordinary least squares. All country estimates were significative at 5%.[6] Our benchmark is $\beta = 0.95$ and $\phi = 2$, and the results in this case are presented in table 1. As it was already expected, the costs associated with business in the American economy are quite small, both with linear trend specification and HP filter. The costs in South American countries are, however, very large vis-à-vis U.S.: they typically average 10 times the corresponding estimate for U.S. with HP filter, and 17 times in the linear trend case.

Table 1 - Welfare Costs of Business Cycles

$\lambda(\%)$ when $\beta = 0.95$ and $\phi = 2$			
Country	Beveridge-Nelson	Hodrick-Prescott	Linear Trend
USA	0.48	0.04	0.10
Argentina	3.68	0.25	0.76
Brazil	4.15	0.24	2.34
Bolivia	–	0.20	0.64
Chile	24.39	0.88	2.59
Colombia	0.19	0.07	0.29
Ecuador	0.26	0.08	0.97
Paraguay	1.97	0.35	0.70
Peru	2.73	0.37	5.94
Uruguay	2.53	0.36	1.47

3.2 Modified framework: Beveridge-Nelson decomposition

The first step consisted in testing for unit root. Only for Bolivia we have found evidence that $\log c_t \sim I(0)$ using Augmented Dickey-Fuller test (see table 2). In the second step, the estimation of the "best" stationary ARMA(p, q) for $\Delta \log c_t$ when $\log c_t \sim I(1)$, lag length was selected so as to minimize Schwarz information criterion conditional on passing in diagnostic testing. In particular, we first checked if the Ljung-Box Q-statistics associated with partial and autocorrelegram of first difference of consumption were not significative. If so, we merely modeled the demeaned series as an innovation. Differently, whenever the Q-statistic was significative for any lag, we have chosen the best ARMA(p, q) in order to minimize information criterion.

Results of the benchmark case are reported in table 1. If we do not consider the extremely large costs in Chile, South American countries have costs associated with cyclical fluctuations 5 times the corresponding estimate for the U.S. economy. Moreover, the welfare costs of aggregate fluctuations using the Beveridge-Nelson approach average 2 times the costs in the linear trend case, and 8 times the costs with the HP filter. As expected, to impose that consumption is $I(0)$ when it is $I(1)$ leads to underestimating λ.

[6] Given evidence of serial correlation in residuals, we have used Newey-West covariance estimator. Specific estimation results not reported in this paper are available upon request from the authors.

Table 2

Unit Root Test (ADF) - Logarithm of Consumption (H_0 : series has unit root)		
Country	Level (p-value)	1st difference (p-value)
USA	0.29	0.00
Argentina	0.35	0.00
Brazil	1.00	0.00
Bolivia	0.02	-
Chile	0.49	0.00
Colombia	0.99	0.04
Ecuador	0.89	0.00
Paraguay	0.06	0.00
Peru	0.14	0.00
Uruguay	0.90	0.00

When we plot benchmark values of λ against the initial level of output, we corroborate the claim (see section 2) that richer societies do not necessarily have lower costs of aggregate fluctuations. In this case, the initial level of consumption (in log) poorly explains the costs of business cycles (figure 4), for the associated measure of fit (R^2) is nearly negligible in every case.

Given the results (see Appendix C for a detailed report) outlined above, we classify South American countries and the U.S. in three groups according to the magnitude of their welfare costs. The first group is labeled "small costs" and consists of the U.S. economy, Bolivia, Colombia, and Ecuador (see figure 1). Even though those countries are not alike with respect to economic performance, their consumption series is indeed quite smooth. This supports the view that welfare costs of aggregate fluctuations are not necessarily correlated with good economic outcomes (income, equity, etc.). Brazil is included in the second group, which we name "medium costs" and comprises five additional countries (namely, Argentina, Paraguay, Peru, and Uruguay). The third group consists of Chile, whose consumption series behavior is exceedingly volatile (see figure 3).[7]

With respect to the U.S. economy, our empirical findings may be compared with Obstfeld (1994), who performs an estimation using total consumption data in PWT. In this case, our estimates are slightly larger, which might be explained by the fact that, when one endogenizes the stochastic process driving the business cycles, underestimates hardly arise. But, at the same time, our estimates of the costs in U.S. allow us to infer they are small relatively to South American countries.

4 Concluding Remarks

This study was concerned with the welfare costs of economic fluctuations in developing South American countries. We considered a non-trivial extension of Lucas' (1987) set-up based on a representative agent framework and CRRA utility. That is, we allowed for a more general consumption process instead of imposing it is trend-stationary. In fact, to estimate the consumption integration order and its cyclical component as in this paper represents a crucial change, since a trend-stationary series and

[7] Chile underwent a process of structural reforms recently. Notice that, starting at 1985, consumption is quite smooth in this country relatively to the 1951-84 period. However, our estimates for the welfare costs of aggregate fluctuations for this country are still very large, since we have not analyzed the post-reforms period separately. As a topic of future research, pari passu with the disclose of new post-reforms data, we intend to perform a more careful evaluation of changes of regime in South American countries like Chile.

a first-difference one have different characteristic. Particularly, the former has mean reversion while the latter has permanent shocks. Thus, if consumption is an integrated process, the effects of shocks are permanent and this certainly increases the variability and the risk of consumption as perceived by agents.

One important contribution of this work was to shed some light on possibly misleading results arising from misespeficication of the consumption series, since unit root test indicated that all countries have a consumption series first-difference stationary, except Bolivia. In this sense, it was shown that, when we endogenize the reduced-form of consumption, the costs of the aggregate fluctuations are no longer substantially small as the underestimates (of at least one order of magnitude) suggest when we impose the logarithm of consumption is trend-stationary.

The results reported in the text also suggest that, as opposed to the U.S. economy, many countries in South America have substantial welfare costs associated with aggregate fluctuations. At first glance, this finding is quite intuitive and could imply that a sizable counter-cyclical policy is desirable. Nonetheless, even if the costs associated with aggregate fluctuations were large, it is by no means obvious that government intervention is welfare-improving. For example, if a large fraction of aggregate fluctuation were associated with real shocks such as changes in household's preferences and technology, then governmental counter-cyclical policies would be seldom effective.

References

BEVERIDGE, Stephen; NELSON, Charles R. (1981). "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'." *Journal of Monetary Economics*, v. 7, p.151-74.

CAMPBELL, John. (1987). "Does Saving Antecipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis." *Econometrica*, v.55, p.1249-73.

DOLMAS, Jim. (1998). "Risk Preferences and the Welfare Costs of Business Cycles." *Review of Economic Dynamics*, v.1, p.646-76.

DUARTE, Angelo M.; ISSLER, João Victor; SALVATO, Márcio A. (2003). "Are Business Cycles All Alike in Europe?" *Mimeo*.

EPSTEIN, Larry G.; ZIN, Stanley E. (1989). "Substituition, risk aversion and the temporal behavior of consumption growth and asset returns I: a theoretical framework." *Econometrica*, v.57, p.937-69.

FLAVIN, Majorie A. (1981). "The Adjustments of Consumption to Changing Expectations About Future Income." *Journal of Political Economy*, v.89, p.974-1009.

FRANCO, Afonso A. de M.; GUILLÉN, Osmani T. de C.; ISSLER, João Victor. (2003). "On the Welfare Costs of Business Cycles in the 20th Century." *Mimeo*.

GOMES, Fábio Augusto Reis; PAZ, Lourenço Senne. (2004). "Especificações para a Função Consumo: Testes para Países da América do Sul." *Pesquisa e Planejamento Econômico*, v.34, p.42-59.

HALL, Robert E. (1978). "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy*, v.86, p.971-87.

HODRICK, R.J.; PRESCOTT, Edward C. (1997). “Postwar U.S. Business Cycles: An Empirical Investigation.” *Journal of Money, Credit and Banking*, v.29, p.1-16.

IMROHOROGLU, Ayse. (1989). “The cost of business cycles with indivisibilities and liquidity constraints.” *Journal of Political Economy*, v.97, p.1364-83.

KRUSSEL, Per; SMITH, Antony A., Jr. (1999). “On the welfare effects of eliminating business cycles.” *Review of Economic Dynamics*, v.2, p.245-72.

KYDLAND, Finn E.; PRESCOTT, Edward C. (1982). “Time to Build and Aggregate Fluctuations.” *Econometrica*, v.50, p.1345-70.

LUCAS, Robert, Jr. (1987). Models of Business Cycles. Oxford: Blackwell.

LUCAS, Robert, Jr. (2003). “Macroeconomic Priorities.” *American Economic Review*, v.93, p.1-14.

MULLIGAN, Casey. (2002). “Capital, Interest, and Aggregate Intertemporal Substitution.” *NBER working paper #9373*.

OBSTFELD, Maurice. (1994). “Evaluating Risky Consumption Paths: the Role of Intertemporal Substitutability.” *European Economic Review*, v.38, p.1471-86.

PALLAGE, Stéphane; ROBE, Michel A. (2003). “On the Welfare Cost of Economic Fluctuations in Developing Countries.” *International Economic Review*, v.44, p.677-98.

PEMBERTON, James. (1996). “Growth trends, cyclical fluctuations, and welfare with non-expected utility preferences.” *Economics Letters*, v.50, p. 387-92.

SUMMERS, Robert; HESTON, Alan; ATEN, Bettina. (2002). Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October (http://pwt.econ.upenn.edu/).

TALLARINI, Thomas D., Jr. (2000). “Risk-sensitive real business cycles.” *Journal of Monetary Economics*, v.45, p.507-32.

A Appendix

A.1 Lucas' λ

From (4),

$$E\left(E_0\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right)\right) = E\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right) = \sum_{t=0}^{\infty}\beta^t u(c_t^*). \tag{A1}$$

Given the functional assumption (2) and the stochastic process for consumption (3), (A1) yields:

$$(1+\lambda)^{1-\phi}\sum_{t=0}^{\infty}\beta^t E c_t^{1-\phi} = \alpha_0^{1-\phi}\exp\left(\frac{(1-\phi)\sigma_z^2}{2}\right)\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t. \tag{A2}$$

Notice the left-hand side of (A2) can be simplified by using log-Normal's usual properties:

$$E c_t^{1-\phi} = \left(\alpha_0(1+\alpha_1)^t\right)^{1-\phi}\exp\left(\frac{(1-\phi)^2\sigma_z^2}{2}\right). \tag{A3}$$

Then, (5) follows from (A2) and (A3) after some straightforward algebra.

A.2 Beveridge-Nelson Approach

Initially, we demonstrate (8). Our task is to find λ such that, given (1), (2), (6) and (7), it solves:

$$E_0\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right) = \sum_{t=0}^{\infty}\beta^t u(E_0 c_t). \tag{A4}$$

It is easily verified that (A4) simplifies to:

$$(1+\lambda)^{1-\phi}\sum_{t=0}^{\infty}\beta^t E_0 c_t^{1-\phi} = \alpha_0^{1-\phi}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t\left(E_0 X_t Y_t\right)^{1-\phi}, \tag{A5}$$

where $E_0 X_t Y_t = E_0\exp\left(\sum_{i=1}^{t}\varepsilon_i + \sum_{i=1}^{t}\psi_{t-i}\vartheta_i\right)$. Let $\zeta_t = \sum_{i=1}^{t}\varepsilon_i + \sum_{i=1}^{t}\psi_i\vartheta_{t-i}$. Then its conditional distribution is $\zeta_t \sim N\left(0, w_t^2\right)$, and $w_t^2 = t\sigma_{11} + 2\sigma_{12}\sum_{j=1}^{t}\psi_j + \sigma_{22}\sum_{j=1}^{t}\psi_j^2$. Hence, the right-hand side of (A5) is:

$$RHS = \alpha_0^{1-\phi}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t\exp\left(\frac{(1-\phi)\,w_t^2}{2}\right). \tag{A6}$$

Since $\zeta_t \sim N\left(0, w_t^2\right)$, the left-hand side of (A5) can be further simplified:

$$LHS = (1+\lambda)^{1-\phi}\alpha_0^{1-\phi}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t\exp\left(\frac{(1-\phi)^2 w_t^2}{2}\right). \tag{A7}$$

Therefore, after some algebra, (A6) and (A7) imply (8).

If $\phi = 1$, λ must solve:

$$\sum_{t=0}^{\infty} \beta^t E_0 \log\left((1+\lambda)c_t\right) = \sum_{t=0}^{\infty} \beta^t \log E_0 c_t. \tag{A8}$$

Notice that $E_0 \log c_t = \log \alpha_0 \left(1+\alpha_1\right)^t$ and $E_0 c_t = \alpha_0 \left(1+\alpha_1\right)^t \exp\left(\frac{w_t^2}{2}\right)$. Then (A8) yields:

$$\frac{1}{1-\beta}\log\left(1+\lambda\right) + \sum_{t=0}^{\infty} \beta^t \log \alpha_0 \left(1+\alpha_1\right)^t = \sum_{t=0}^{\infty} \beta^t \log \alpha_0 \left(1+\alpha_1\right)^t + \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t^2}{2}\right)$$

$$\Longrightarrow \lambda = \exp\left((1-\beta)\sum_{t=0}^{\infty} \beta^t \left(\frac{w_t^2}{2}\right)\right) - 1, \tag{A9}$$

exactly as in (9).

A.3 Identification

Let $\Delta \log c_t \sim I(1)$. According to Beveridge and Nelson (1981), $\log c_t$ can be decomposed as:

$$\log c_t = \log \alpha_0 + t\log(1+\alpha_1) + \log X_t + \log Y_t,$$

where $\log X_t = \sum_{i=1}^{t} \varepsilon_i$, $\log Y_t = \sum_{i=1}^{t} \psi_i \vartheta_{t-i}$ and $(\varepsilon_t, \vartheta_t)'$ has a bivariate normal distribution as in (7). Using the Wold decomposition, Beveridge and Nelson have demonstrated that

$$\Delta \log c_t = \log(1+\alpha_1) + \mu_t + \nu_1\mu_{t-1} + \nu_2\mu_{t-2} + \nu_3\mu_{t-3} + \ldots$$

implies

$$\Delta \log X_t = \left(\sum_{i=0}^{\infty} \nu_i\right)\mu_t \tag{A10}$$

and

$$\log Y_t = -\left(\left(\sum_{i=1}^{\infty} \nu_i\right)\mu_t + \left(\sum_{i=2}^{\infty} \nu_i\right)\mu_{t-1} + \left(\sum_{i=3}^{\infty} \nu_i\right)\mu_{t-2} + \ldots\right). \tag{A11}$$

Therefore, the definition of $\log X_t$ combined with (A10) yields:

$$\varepsilon_t = \left(\sum_{i=0}^{\infty} \nu_i\right)\mu_t. \tag{A12}$$

Moreover, it is easily checked that the definition of $\log Y_t$ and (A11) imply:

$$\vartheta_t = \mu_t \tag{A13}$$

$$\begin{aligned} \psi_0 &= -\sum_{i=1}^{\infty} \nu_i \\ \psi_1 &= -\sum_{i=2}^{\infty} \nu_i \\ &\vdots \\ \psi_{t-1} &= -\sum_{i=t}^{\infty} \nu_i \\ &\vdots \end{aligned} \tag{A14}$$

Thus, (7) and (A12) to (A14) imply:

$$\sigma_{11} = \left(\sum_{i=0}^{\infty} \nu_i\right)^2 \text{var}(\mu_t), \tag{A15}$$

$$\sigma_{12} = \left(\sum_{i=0}^{\infty} \nu_i\right) \text{var}(\mu_t), \tag{A16}$$

$$\sigma_{22} = \text{var}(\mu_t). \tag{A17}$$

We must, then, obtain var(μ_t) and $(\nu_i)_{i=1}^{\infty}$ as a means to identify the relevant parameters in our model, (A15)-(A17) and α_1. This is indeed a straightforward task inasmuch as, by estimating an ARMA(p, q) for $\Delta \log c_t$, $(\mu_t)_t$ is consistently estimated by the residuals and, inverting the AR polynomial, we also find $(\nu_i)_{i=1}^{\infty}$. Lastly, α_1 is a function of the constant in the ARMA(p, q) and the coefficients of the AR polynomial.

B Appendix

Proof of Proposition 1. Firstly, we check (i). Let $\zeta_t = \beta^t \frac{w_t^2}{2}$. By applying D'Alembert's convergence test for infinite series, it suffices to assure that $\lim_{t\to\infty} \frac{\zeta_{t+1}}{\zeta_t} = c < 1$. Toward this end, initially notice that

$$\frac{1}{\beta}\frac{\zeta_{t+1}}{\zeta_t} = \frac{\left(\frac{t+1}{t}\right)\sigma_{11} + 2\sigma_{12}\frac{\sum_{j=1}^{t+1}\psi_j}{t} + \sigma_{22}\frac{\sum_{j=1}^{t+1}\psi_j^2}{t}}{\sigma_{11} + 2\sigma_{12}\frac{\sum_{j=1}^{t}\psi_j}{t} + \sigma_{22}\frac{\sum_{j=1}^{t}\psi_j^2}{t}}. \tag{B1}$$

For any sequence $(x_t)_t$ such that $\lim_{t\to\infty}(x_{t+1} - x_t) = 0$, it is true that $\frac{x_t}{t} \to 0$ as $t \to \infty$. Therefore, if we set $x_t = \sum_{j=1}^{t}\psi_j$ or $\sum_{j=1}^{t}\psi_j^2$, then $x_{t+1} - x_t = \psi_t$ or ψ_t^2 and the result applies. An analogous argument also holds for $\sum_{j=1}^{t+1}\psi_j$ and $\sum_{j=1}^{t+1}\psi_j^2$. Thus, given σ_{11}, σ_{12} and $\sigma_{22} < \infty$, the right-hand side of (B1) converges to 1, implying that $\lim_{t\to\infty}\frac{\zeta_{t+1}}{\zeta_t} = \beta < 1$.

In order to verify the second claim, (ii), define $\tilde{\zeta}_t = \left[\beta(1+\alpha_1)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)w_t^2}{2}\right)$. It is easily checked that

$$\frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t} = \beta(1+\alpha_1)^{1-\phi} \exp\left(\frac{(1-\phi)\left(\sigma_{11} + 2\sigma_{12}\psi_{t+1} + \sigma_{22}\psi_{t+1}^2\right)}{2}\right). \tag{B2}$$

Using the fact that $\psi_t \to 0$ as $t \to \infty$, (B2) gives: $\lim_{t\to\infty} \frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t} = \beta(1+\alpha_1)^{1-\phi} \exp\left(\frac{(1-\phi)\sigma_{11}}{2}\right)$. Hence, given $\phi > 1$ and $\alpha_1 \geq 0$, the numerator in (8) is clearly convergent. A similar line of reasoning implies that the denominator in the same equation is convergent if $\beta(1+\alpha_1)^{1-\phi} \exp\left(\frac{(1-\phi)^2\sigma_{11}}{2}\right) < 1$. ■

C Figures and Tables

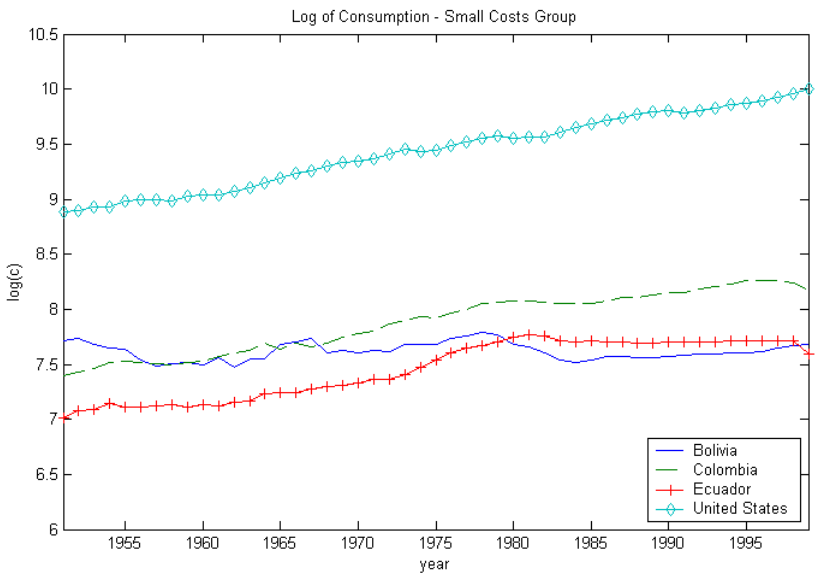

Figure 1

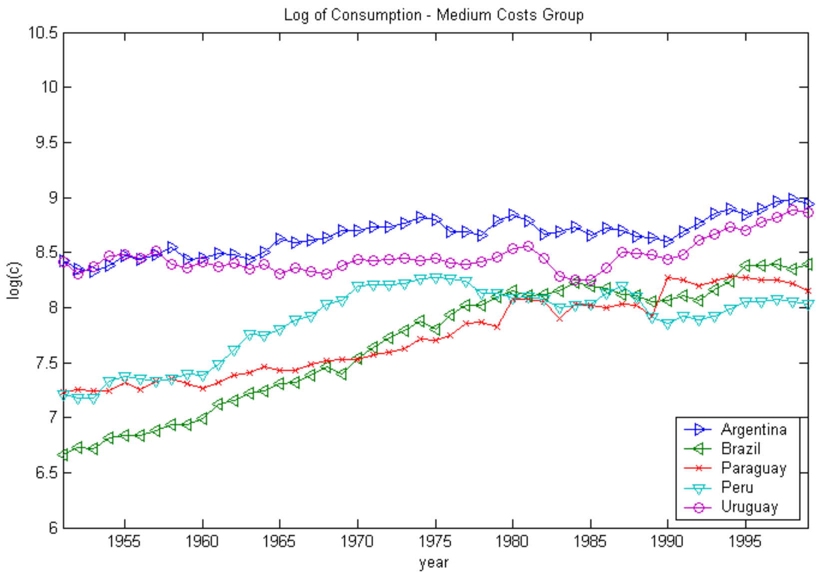

Figure 2

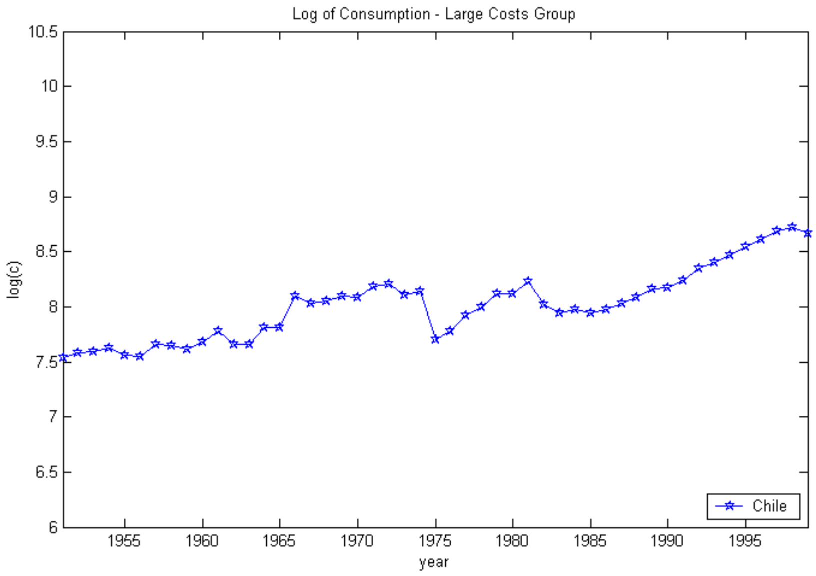

Figure 3

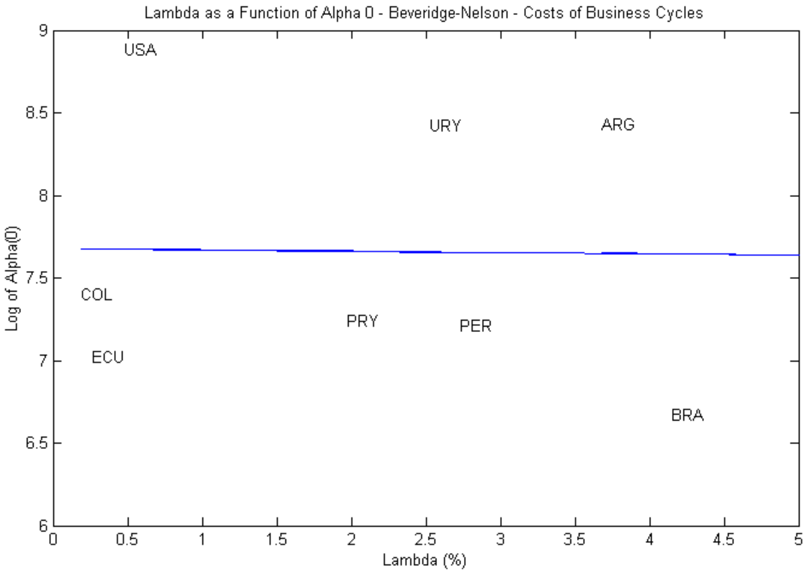

Figure 4

Table C1

Welfare Costs of Business Cycles (λ%) - Argentina							
	Beveridge-Nelson					Linear	HP
$\phi \setminus \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.54	0.85	1.82	3.24	6.45	0.38	0.13
2	1.08	1.72	3.68	6.58	13.34	0.76	0.25
5	2.82	4.63	10.90	23.41	$\infty^{(*)}$	1.92	0.64
10	7.47	16.19	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	3.88	1.28
20	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	7.90	2.57

$^{(*)}$ Convergence conditions in propostion 1 do not apply.

Table C2

Welfare Costs of Business Cycles (λ%) - Bolivia		
ϕ	Linear (for all β)	HP (for all β)
1	0.32	0.10
2	0.64	0.20
5	1.60	0.51
10	3.23	1.02
20	6.56	2.05

Table C3

Welfare Costs of Business Cycles (λ%) - Brazil							
	Beveridge-Nelson					Linear	HP
$\phi \setminus \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	1.14	1.71	3.45	6.03	11.98	1.16	0.12
2	1.90	2.61	4.15	5.51	7.06	2.34	0.24
5	3.37	4.08	5.17	5.82	6.36	5.95	0.60
10	5.07	5.74	6.62	7.08	7.42	12.26	1.20
20	9.23	10.45	12.18	13.17	13.97	26.03	2.42

Table C4

Welfare Costs of Business Cycles (λ%) - Chile							
	Beveridge-Nelson					Linear	HP
$\phi \setminus \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	3.69	5.59	11.49	20.62	43.68	1.29	0.44
2	7.52	11.50	24.39	46.12	113.52	2.59	0.88
5	26.72	64.43	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	6.61	2.22
10	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	13.66	4.49
20	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	29.17	9.18

Table C5

Welfare Costs of Business Cycles (λ%) - Colombia							
	Beveridge-Nelson					Linear	HP
$\phi \setminus \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.03	0.06	0.12	0.22	0.44	0.15	0.04
2	0.06	0.09	0.19	0.29	0.43	0.29	0.07
5	0.11	0.16	0.26	0.33	0.41	0.73	0.18
10	0.15	0.20	0.28	0.33	0.37	1.46	0.35
20	0.17	0.21	0.26	0.29	0.31	2.95	0.70

Table C6

Welfare Costs of Business Cycles (λ%) - Ecuador							
	Beveridge-Nelson					Linear	HP
$\phi \setminus \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.11	0.12	0.13	0.13	0.13	0.49	0.04
2	0.23	0.24	0.26	0.26	0.27	0.97	0.08
5	0.57	0.61	0.65	0.66	0.67	2.45	0.20
10	1.15	1.22	1.30	1.33	1.35	4.97	0.40
20	2.34	2.48	2.62	2.69	2.73	10.18	0.81

Table C7

Welfare Costs of Business Cycles (λ%) - Paraguay							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.25	0.43	0.98	1.78	3.59	0.35	0.18
2	0.50	0.86	1.97	3.60	7.32	0.70	0.35
5	1.29	2.26	5.40	10.70	27.89	1.76	0.89
10	3.03	5.82	25.04	$\infty^{(*)}$	$\infty^{(*)}$	3.56	1.78
20	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	7.24	3.60

Table C8

Welfare Costs of Business Cycles (λ%) - Peru							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.41	0.65	1.36	2.40	4.74	2.93	0.18
2	0.82	1.30	2.73	4.85	9.72	5.94	0.37
5	2.12	3.42	7.69	15.19	46.79	15.52	0.93
10	5.11	9.49	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	33.44	1.86
20	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	78.06	3.75

Table C9

Welfare Costs of Business Cycles (λ%) - Uruguay							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.41	0.63	1.26	2.17	4.22	0.73	0.18
2	0.83	1.26	2.53	4.38	8.61	1.47	0.36
5	2.14	3.29	6.98	13.28	35.58	3.70	0.90
10	4.96	8.51	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	7.54	1.81
20	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	$\infty^{(*)}$	15.66	3.65

Table C10

Welfare Costs of Business Cycles (λ%) - United States							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.11	0.17	0.34	0.59	1.15	0.05	0.02
2	0.20	0.29	0.48	0.67	0.92	0.10	0.04
5	0.38	0.48	0.65	0.75	0.85	0.25	0.09
10	0.56	0.65	0.77	0.83	0.88	0.50	0.18
20	0.79	0.86	0.93	0.97	0.99	1.01	0.36