We Will Shock You: a coherent Bayesian approach for Stress Test

FILIPI SANGUINO
Universidade de São Paulo
JOÃO VINÍCIUS DE FRANÇA CARVALHO
Universidade de São Paulo

Resumo

A técnica de teste de estresse visa entender as consequências financeiras de cenários improváveis, mas plausíveis. A importância deste exercício é acentuada em ambientes de alta instabilidade e volatilidade, como o Brasil. Neste trabalho desenvolvemos uma metodologia coerente de teste de estresse Bayesiano, preservando as propriedades matemáticas das medidas de risco. Para tanto, utilizamos as Redes Bayesianas Dinâmicas como método e, na plataforma teórica, a *Arbitrage Pricing Theory*. Enquanto o primeiro fornece a topologia de interdependência entre as variáveis, considerando a dinâmica temporal e as possibilidades de propagação do contágio, o segundo capta os efeitos dos choques nos retornos do índice Ibovespa, principal indicador de desempenho das ações brasileiras entre janeiro/1995 e julho/2021. Os resultados indicam que o índice Ibovespa é mais fortemente sensível a fatores de risco vinculados a investidores internacionais (câmbio e S&P500) do que a elementos domésticos (inflação e CDI). Por fim, foram simulados os efeitos de choques extremos no índice Ibovespa, bem como o cálculo de medidas de risco, como *Value-at-Risk* e *Tail Value-at-Risk*. Os resultados sugerem que a combinação S&P500 em estado negativo, câmbio em estado positivo e IPCA em estado neutro constitui uma combinação consistentemente agressiva, com alta capacidade de gerar perdas, com massa de probabilidade significativa.

Palavras-chave: Testes de estresse; crises financeiras; Redes Bayesianas Dinâmicas.

Abstract

The stress test technique aims to understand the financial consequences of unlikely but plausible scenarios. The importance of this exercise is accentuated in high instability and volatility environments, such as Brazil. In this work we develop a coherent Bayesian stress test methodology, preserving the mathematical properties of risk measures. Therefore, we used Dynamic Bayesian Networks as method, and on the theoretical platform, we used Arbitrage Pricing Theory. While the first provides the interdependence topology among the variables, considering temporal dynamics and possibilities of contagion propagation, the second captures the effects of shocks on the returns of Ibovespa index, the main performance indicator for Brazilian equities between January/1995 and July/2021. The results indicates that the Ibovespa index is more strongly sensitive to risk factors linked to international investors (foreign exchange and S&P500) than to domestic elements (inflation and CDI). Finally, the effects of extreme shocks on Ibovespa index were simulated, as well as the computation of risk measures, as Value-at-Risk and Tail Value-at-Risk. The results suggest that the combination S&P500 in negative state, exchange rate in positive state and IPCA in neutral state constitutes a consistently aggressive combination, having a high capacity to generate losses, with a significative mass of probability.

Keywords: *Stress Test*; financial crises; Dynamic Bayesian Networks.

Classificação JEL: G01, C45, C11, C22.
Área 8: Microeconomia, Métodos Quantitativos e Finanças

1. Introduction

"The oldest and strongest emotion of mankind is fear, and the oldest and strongest kind of fear is fear of the unknown", said H.P. Lovecraft, an exponent of horror literature, whose works focused on individual's impotence in the face of the unknown, in a useful analogy to risk.

Risk management is vital for the economic agents, especially in politically or financially unstable environments (Consigli, 2002; Mittnik, 2014). In this regard, Rosengren (2014) cites impacts of capital inadequacy in the real economy, such as reduced liquidity and restricted access to credit. In the face of these impacts, Brazil is a prolific testing environment for quantitative risk management: after all, as narrated by Castro et al. (2019), in sixty years, the country faced up a military dictatorship (1964-1985), a hyperinflation process (1980-1994) and two processes of impeachment of civilian presidents (1992 and 2016).

Political crises and scandals are also frequent in Brazil, with the Operation *Car Wash* as the most notable event. Rensi & Carvalho (2021) point out the consequences of this operation for the insurance market, strongly related with the banking sector, shown by Fonseca & Carvalho (2021). In this context, Brazilian companies suffer intense shocks in the volatility of current prices and, therefore, large fluctuations in equity values (de Oliveira et al., 2018). Thus, it is essential to develop accurate approaches to estimate and mitigate the risks, in addition to constituting sufficient reserves for scenarios in which the extreme events materialize (Obstfeld et al., 2009).

Specifically for banks, aiming to mitigate the effects of the unknown events and unify the standardization of financial risk management, the Basel Committee on Banking Supervision (BCBS) presented a series of ratified agreements by more than one hundred of countries: Basel I (1988), Basel II (2004) and Basel III (2010). In common with all, the need for risk analysis mechanisms is declared, namely: (i) Value at Risk (VaR), and; (ii) Stress Test. After the adoption, this procedures became popular in the global financial market (Bruno, 2008; Righi & Ceretta, 2014).

The VaR is a risk measure that estimate the maximum expected loss, under usual circumstances, with a given confidence interval, in a given time horizon. According to Linsmeier & Pearson (1996), there are different approaches, which can be parametric, based on probabilistic distributions, or non-parametric, based on time series or Monte Carlo simulations. However, despite its popularity and ease of measurement, the VaR is not immune from criticism: (i) it does not report the worst possible losses, and; (ii) it does not respect the important property of sub-additivity, which may make it difficult to diversify assets (Artzner et al., 1999).

The Stress Test, on the other hand, is capable of providing a more complete assessment of the potential shocks from atypical market situations, surpassing purely statistical measures (Rebonato, 2010). This approach is based on generating scenarios and evaluating their impacts on portfolios (Rebonato, 2019). According to Nagpal (2017), scenarios are created based on expert knowledge, shocks in macroeconomic variables, replication of past crises, or even the Reverse Stress Test. In the latter, starting from the lost amount, the stipulated macroeconomic variables are obtained and the plausibility of the simulated scenarios is verified (Kopeliovich et al., 2015).

Given the ability to assess the impact of often unprecedented scenarios, the Stress Test expands the possibilities of the risk management process, especially in cases of catastrophic events. Additionally, episodes of this magnitude are those that require the greatest level of refinement in the methodologies used to measure and control risk. Together, these techniques help to understand the growing relevance of Stress Test exercises. Therefore, researching new techniques or improving existing ones is of paramount importance.

Specifically, obtaining complex conditional probabilities, one of the most arduous tasks in risk management, can be fully achieved through Bayesian Networks. This methodology allows simulating scenarios while maintaining a narrative of causality. Our objective in this work is to propose a coherent approach to the Stress Test using Dynamic Bayesian Networks (Nagarajan et al., 2013), which, in addition to their intrinsic advantages, incorporate temporal dynamics, a feature little explored in the literature in Finance (Fonseca & Carvalho, 2021). It is expected that this technique will provide subsidies for a sophisticated factor model, improving risk management, especially in environments especially susceptible to catastrophic events.

2. Theoretical Background

2.1. Risk Measures

Due to its standardization and ease of calculation, VaR was widely adopted, enabling the development of a vast literature in Finance. Even in Brazil, VaR became the official measure of risk by Circular No. 3646/2013 of the Central Bank of Brazil. However, Pérignon & Smith (2010) question the quality of VaR reported by commercial banks around the world, concluding that the historical method – the most popular – is ineffective for projecting future volatility. Previously, Krause (2003) had already stated that VaR would not be an adequate risk measure if the assets' correlation matrix had a small number of entries compared to its matrix dimension, in addition to the measure being subject to manipulation by the malicious agents.

About the parametric approach, Abad et al. (2016) evaluated the performance of VaR under asymmetric distributions, which are more realistic than the Gaussian one. They concluded that the asymmetric generalized t distribution had a better fit to global stock indices compared to the traditional t-Student distribution. However, the latter was more conservative, requiring greater capital allocation, being preferred by regulators.

While simple, VaR is not immune from criticism. Artzner et al. (1999) defined axioms to classify a risk measure as coherent: (i) monotonicity; (ii) subadditivity; (iii) positive hegemony, and; (iv) translation invariance. However, they prove that VaR is not coherent, as it does not satisfy the axiom of subadditivity. In other words, in certain cases, VaR does not promote portfolio diversification. In this regard, Daníelsson et al. (2013) found that, by using a sufficiently small number of observations, the historical simulation method is prone to flaws in the subadditivity axiom. Nevertheless, Emmer et al. (2015) consider that the most problematic factor about VaR performance is not its inconsistency, but the impossibility of properly estimating the Tail Risks.

To circumvent these limitations, other risk measures were proposed. Among them, Expected Shortfall (ES), Conditional Value-at-Risk (CVaR) and Tail Value-at-Risk (TVaR). Rockafellar & Uryasev (2002) argue that both have properties superior to VaR in many respects. But the CVaR is special as it informs the expected loss if the value stipulated by the VaR is exceeded, showing the flow behavior of the distribution of results. Aiming at a computationally less costly implementation, Khokhlov (2016) presents CVaR analytical solutions for the main distributions used in market risk management. However, all these techniques are guided by events that have already been witnessed, and their limitation in cases of unprecedented catastrophic occurrences is evident.

2.2. Stress Test

The Stress Test exercise, by its nature, has advantages over traditional risk measures (Gao et al., 2017). After all, its essence is in the analysis of improbable but plausible situations, expanding the simulation possibilities to scenarios probably not observed by conventional measures. Rebonato (2010) highlights the possibility of building a causal narrative, more easily understood by human reasoning compared to purely numerical approaches. This facilitates the assessment of the reasonableness of assumptions and results, in addition to improving communication between risk professionals and the institutions' senior management.

Rebonato (2017) classifies the Stress Tests into three large groups: (i) extreme-tail; (ii) vulnerability-driven, and; (iii) coherent-stress-testing. The first reveals the impact of extreme events that occurred in the past or extrapolates the loss distribution with the help of Extreme Value Theory (EVT). The second is based on the study of the portfolio, with the identification of scenarios that could impact its vulnerabilities. Finally, the third one defines not only the root causes (geopolitical, macro or microeconomic), but mainly identifying the transmission channels that link the causes to the risk factors and, finally, the impact on the portfolio. Still on the last category, the author emphasizes the subjectivity in the attribution of conditional probabilities as a fundamental factor in the construction of scenarios. However, quality and sensitivity tests can be applied, minimizing subjectivity, maintaining the causal narrative.

The Stress Test, due to its high level of complexity and interdependence among market agents, would benefit from the use of network theory (Anand et al., 2015; Battiston & Martinez-Jaramillo, 2018; Gong et al., 2019). The risk of contagion between numerous counterparties, the applicability of financial networks for the study of exogenous shocks, better understanding of systemic risk and tooling flexibility for the study of topics ranging from regulation, macro and micro-prudential, to the impact of climate change in the real economy.

A specific technique in this field is Bayesian Networks (BN), which represent the conditional probabilities through directional acyclic graphs. Empirically, Gao et al. (2017), using the so-called Suppes-Bayes Causal Networks, found gains in precision and reduced computational complexity when compared to Monte Carlo methods. In addition to Rebonato (2010, 2017, 2019), whose methodological focus is given by Expert Static Bayesian Networks, this technique is also used in stress test exercises by Carraro (2018), who uses static Gaussian Bayesian Networks to simulate the resilience of credit institutions to exogenous shocks.

Also, in Carraro (2018) a comparison is made between a network whose topology was determined subjectively, with one obtained directly from the data via the Score-Based Learning method, specifically by the Hill-Climbing algorithm. This enriches risk management decision making by identifying possible biases in causality relationships. Additionally, there is an opportunity to evaluate relational changes if the network topology is obtained continuously.

Still on the BN variants, Nagarajan et al. (2013) argue that temporality is often a determining factor in the study of real-world variables, and for that, Dynamic Bayesian Networks (DBN) should be used. Another advantage of DBN is the possibility of feedbacks between the variables of interest, something ignored by static Bayesian Networks. The DBN configuration is obtained via multivariate time series (MTS), with the aid of vector autoregressive (VAR) or algorithms such as Least Absolute Shrinkage and Selection Operation (LASSO).

Based on the Arbitrage Pricing Theory (APT), Rebonato (2019) uses Bayesian Networks to propagate shocks between the studied variables, building joint probability distributions. With this information, it was possible to obtain several risk measures, considering the multiple states for the variables involved. However, the temporal dimension was not addressed in that work, showing a gap to be explored.

Our proposal is to refine the Coherent-Stress-Testing introduced by Rebonato (2010), using Dynamic Bayesian Networks, a promising technique that encompasses the temporal dimension of the series in the shocks on the variables of interest (Fonseca & Carvalho, 2021; Nagarajan et al., 2013). Thus, we adopt a causal and coherent view, different from the usual risk metrics, something especially relevant in environments with high conditional variance, such as Brazil.

3. Methodology

3.1. Coherent Stress Test

The concept of Coherent-Stress-Testing is explored by Rebonato (2010, 2017) as a simulation modality (microprudential and macroprudential) centered on the causal relationships between the studied variables. According to this approach, the root events as well as the transmission routes between the variables are selected. Due to its causal logic, this analysis becomes more intuitive and consistent with the human perception of cause-effect.

3.2. Bayesian Networks

The tool used to identify, to represent and to operationalize the causal and probabilistic component between the variables will be the Bayesian Network (BN). In the next subsections we present its basic structures.

3.2.1. Graphs, Topology and Parameters

A graph can be described as a relational graphic structure, composed of nodes and arcs. The graph $G = (V, A)$ is constituted by a set of V nodes and A arcs, the arc $a = (u, v)$ represents a relational component between the nodes u and v. Classical graph theory allows the existence of directional $(u \rightarrow v)$ and non-directional $(u - v)$ arcs to denote causality. However, the use of BN supposes directional acyclic graphs (DAG), in which, given the arc of the case $u \rightarrow v$, the node u is called the parent node of v, or even $pa(v)$. The configuration of nodes and arcs is called the BN topology. The values and probability functions that relate the variables are defined as the parameters of the BN.

3.2.2. Topological Properties

The absence of arcs between nodes indicates conditional independence. In addition, the d-separation principle guarantees the node's independence from all nodes other than its parents or descendants. A Markov Blanket is defined as the subset of nodes that have all the useful information to infer about a node u. If the subset is minimal, making it impossible to remove any node without loss of relevant information, the structure is called Markov Boundary. This minimal structure includes u, $pa(u)$, the children of u, as well as the parents of the children of u.

Extending the Law of Total Probability, Nagarajan et al. (2013) define the Markov Property, representing the joint probability distribution of a random vector X as the product of the conditional probability distributions, where Π_i is the set $pa(X_i)$:

$$f_X(X) = \prod_{i=1}^{p} f_{X_i}(X_i | \Pi_i) \tag{1}$$

3.2.3. Learning

Carvalho & Chiann (2013) argue that network learning presents a Bayesian behavior. As it is a sequential and adaptive process to new information, it is possible to estimate the new probabilistic distributions based on updates in the prior distributions. Therefore,

$$p(\theta | d) = \frac{p(d|\theta) \times p(\theta)}{p(d)}, \theta \in \Theta \tag{2}$$

with $p(\theta)$ representing the *a priori* probability, d is the random sample of the probability distribution $p(x|\theta)$. The likelihood function of d is represented by $p(d|\theta)$, Θ is the parametric space in which the parameters θ are inserted. Finally, $p(\theta|d)$ is the *a posteriori* distribution.

3.2.4. Temporal Dynamics: Vectors Autoregression

By including the temporal dimension of a BN, Dynamic Bayesian Networks (DBN) are obtained, whose differential lies in the ability to express the conditional relationships, chronologically, between the temporal series of random variables (Nagarajan et al., 2013). To implement this technique, vectors autoregression with memory p, $VAR(p)$, are used, so that for $t \geq p$, the vector of observed variables $X(t)$ is

$$X(t) = A_1 X(t-1) + \cdots + A_i X(t-1) + \cdots + A_p X(t-p) + B + \varepsilon(t) \tag{3}$$

where $A_i, i = 1, \ldots, p$ are matrices of coefficients of dimensions $k \times k$, with k representing the number of variables, B is a vector of constants of the same size, $\varepsilon(t)$ is defined as the white noise vector with expected value equal to zero and positive and invariant covariance matrix. By verifying significant entries in the matrices A_i, the conditional arcs are established.

3.2.5. Network Significance

Given the arcs obtained by estimating the $VAR(p)$, it is necessary to assess the quality of the network through tests. Although the p-value is widely used, it is not a measure of probability (Amrhein et al., 2019; Greenland et al., 2016) and fails in multiple testing problems (Benjamini & Hochberg, 1995). For this reason, the q-value will be used (Storey, 2002), a measure of significance that explains the expected percentage of false positives among the tests. However, this measure must be adjusted by the Positive False Discovery Rate (pFDR), defined by Storey (2002) as the proportion of false positives among all rejected hypotheses. The formal definition of the q-value is:

$$q(t) = inf_{\{\Gamma : t \in \Gamma\}}\{pFDR(\Gamma)\} \tag{4}$$

with $\{\Gamma\}$ representing the set of reject regions.

3.3. Arbitrage Pricing Theory (APT)

Aiming at the development of a theory that extended the factor logic of the Capital Asset Pricing Model (CAPM), Ross (1976) modeled the expected return of an asset as a function of a vector of macroeconomic and systemic factors. As argued by Rebonato (2019), APT can be described in the form of multiple states, potentiating adverse events. For that, and without loss of generality, suppose the existence of two macroeconomic variables Ω, Γ, with two states each $\{\omega_0, \omega_1\}, \{\gamma_0, \gamma_1\}$. Thus:

$$\begin{gathered} p(\omega_0, \gamma_0) \Leftrightarrow p^{00} = 1 \\ p(\omega_0, \gamma_1) \Leftrightarrow p^{01} = 1 \\ p(\omega_1, \gamma_0) \Leftrightarrow p^{10} = 1 \\ p(\omega_1, \gamma_1) \Leftrightarrow p^{11} = 1 \end{gathered} \tag{8}$$

$$r_i(t) = r_f + \beta_{i\Omega}[\lambda_\Omega + f_\Omega^0(t)] + \beta_{i\Gamma}[\lambda_\Gamma + f_\Gamma^0(t)] + \varepsilon_i(t), \tag{9}$$

where p^{xy} is a binary indicator, which is numerically equal to 1 (one) if the state of the macroeconomic variables is (ω_x, γ_y) and 0 (zero) otherwise. The risk-free interest rate is denoted r_f. The sensitivity of assets as a function of a certain macroeconomic variable is measured by β. The risk premium of the macroeconomic variable is represented by λ, while $f_\Omega^0(t)$ is the portfolio mimicking factor, associated with variable Ω and state 0. The measurement of β of asset i in relation to factor Π is given by Equation 10, while the risk premium λ is described in Equation 11.

$$\beta_{i\Pi} = \frac{\sigma_i}{\sigma_\Pi} \rho_{i\Pi} \tag{10}$$

$$\lambda_i = \hat{r}_i - r_f \tag{11}$$

Still on Equation 10, the Pearson correlation coefficient ($\rho_{i\Pi}$) will be replaced by the partial correlation coefficient obtained via DBN. This change mitigates the risk of improper results or spurious associations. In Equation 11, the risk-free rate considered will be set at 5% p.y., as it is close to the SELIC rate (the basic rate of return of the Brazilian economy) in effect at the time of writing this text. “0” is the normal state, in which the APT converges to the risk-free rate plus white noise ($E[f_\Omega^0(t)] = 0, E[f_\Omega^1(t)] = \omega_1$). Therefore, the return expectation can be obtained by decomposing the probabilities by their states.

$$r_i = r_f + [p^{00}(\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}\lambda_\Gamma)] + \{p^{01}[\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1)]\} + \{p^{10}[\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}\lambda_\Gamma]\} + \{p^{11}[\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1)]\} \tag{12}$$

4. Results Analysis

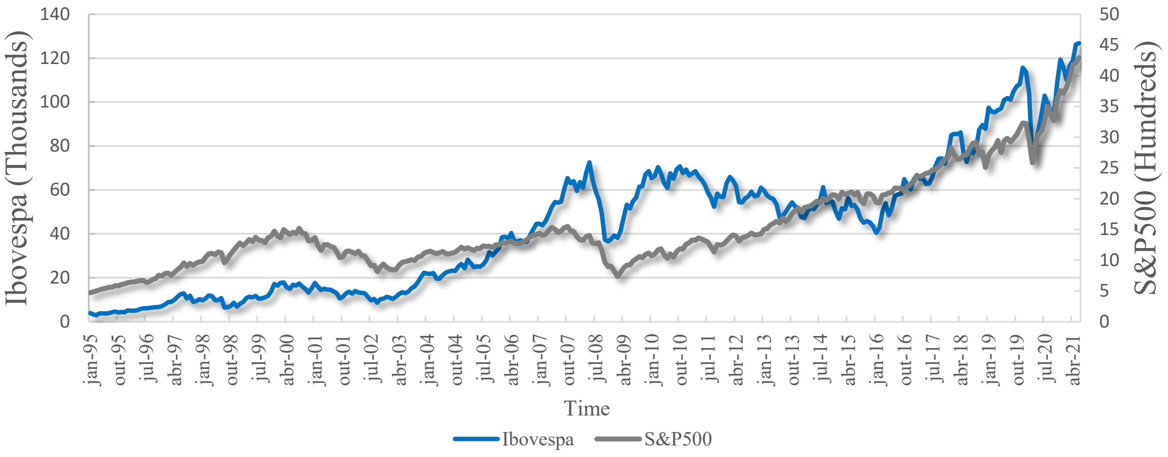
4.1. Data and Descriptive Statistics

Our objective in this work is to evaluate the impact of extreme exogenous shocks of incident risk factors on the Ibovespa, the main index of the Brazilian stock exchange. To make the study feasible, historical series of monthly closings of the Ibovespa and S&P500 indices were used, both obtained from the Yahoo!Finance website. The choice of the S&P500 index over another important indicator (Dow Jones) is explained by its composition methodology. While the Dow Jones is driven by the price of the main shares, the dynamics of the S&P500 is governed by the companies’ market value, in a similar way to the Ibovespa.

The monthly periodicity was defined so that all series were on the same basis since the official local inflation index has monthly release. The monthly historical series of the fixed income returns *Certificado de Depósito Interbancário* (CDI), the inflation rate *Índice Nacional de Preços ao Consumidor Amplo* (IPCA) and the US Dollar Exchange Rate (BRL/USD) were also analyzed. All were obtained from the Central Bank of Brazil Time Series Management System (series 4391, 433 and 3698, respectively). The CDI was prioritized over the SELIC due to its daily fluctuation. For all series, the period is between January/1995 and July/2021.

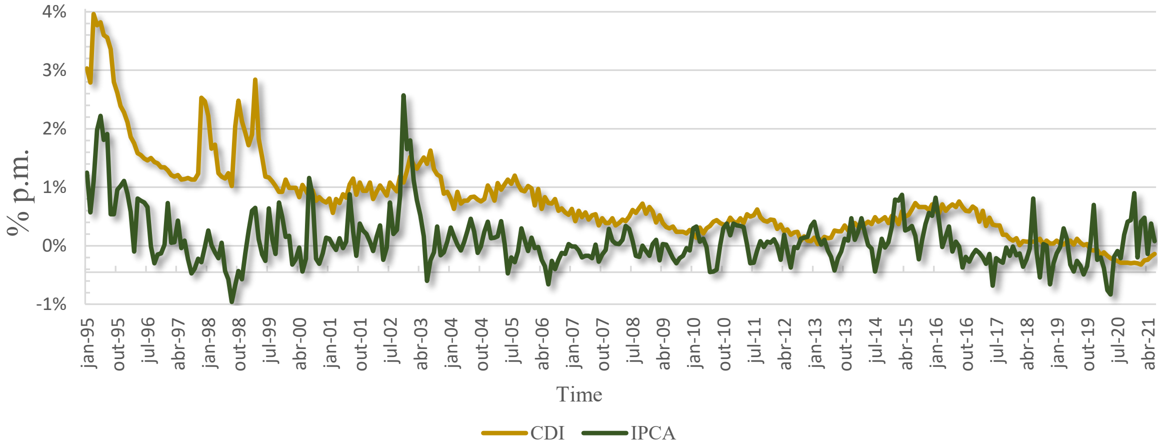
Figure 1 shows the performance of the Ibovespa and S&P500 indices, while Figure 2 shows the evolution of the CDI and IPCA over the years.

Figure 1 – Ibovespa and S&P500 Historical Series



Source: own elaboration

Figure 2 – CDI and IPCA Historical Series



Source: own elaboration

Once the series is obtained, it is necessary to evaluate the stationarity premise of each one. For that, we used the classical Augmented Dickey-Fuller (ADF) test, which evaluates the null hypothesis of the presence of a unit root. This procedure is important because the unit root incidence interferes in the econometric modeling of the series by violating its stationarity. For all series, the results of the respective ADF tests are shown in Table 1. When the null hypothesis was not rejected, the transformation into log-returns was performed. Such transformation was not necessary for the CDI and IPCA series as they are already in the form of rates.

Table 1 - ADF Test results for the historical series

Series	Original Series p-Value	Log-Return p-Value
Ibovespa	0.59	0.01
CDI	0.01	-
IPCA	0.01	-
S&P500	0.99	0.01
BRL/USD	0.63	0.01

Source: own elaboration

4.2. Estimating the Network

Having the series in log-returns, the DBN is estimated. From it, the partial correlations between the time series are extracted. Table 2 shows the partial correlations and respective q-values of all the edges of the graph. Rebonato (2019) states that a small number of factors is sufficient to satisfactorily describe the variation in asset prices. Therefore, as a cutoff criterion, we defined as the financial results from edges which partial correlation modulus is greater than 0.05. The algorithm used to estimate the DBN performs measurements only for VAR(1) models. Therefore, it is necessary to evaluate the suitability of a VAR(1) to the dynamical system. Table 3 shows the highest estimated eigenvalues of the VAR(1), VAR(2) and VAR(3) models, as well as the result of the Breusch-Godfrey (BGT) test for the Ibovespa's VAR as a function of the other variables. This test aims to assess the stationarity of each of the estimated models through the inexistence of autocorrelation between the residuals.

As the largest eigenvalue of the VAR(1) model is less than one, and the null hypothesis of the BGT was not rejected, we can say that it is a stationary model. From the complete VAR(1) model, the significant edges of the estimated DBN were selected. Figure 3 shows the dimension of the remaining edges after the significance criterion (partial correlation modulus greater than 0.05, as suggested by Efron, 2010).

Table 2 - Partial Correlations and q-value of Network Edges

Edge	PCOR	q-Value
S&P500~Ibovespa	0.523	<0.001
IPCA~CDI	0.387	<0.001
USD~Ibovespa	-0.159	<0.001
USD~S&P500	-0.147	<0.001
USD~CDI	0.080	<0.001
IPCA~Ibovespa	0.067	<0.001
CDI~S&P500	0.043	<0.001
CDI~Ibovespa	0.042	<0.001
IPCA~S&P500	-0.029	<0.001
USD~IPCA	-0.025	<0.001

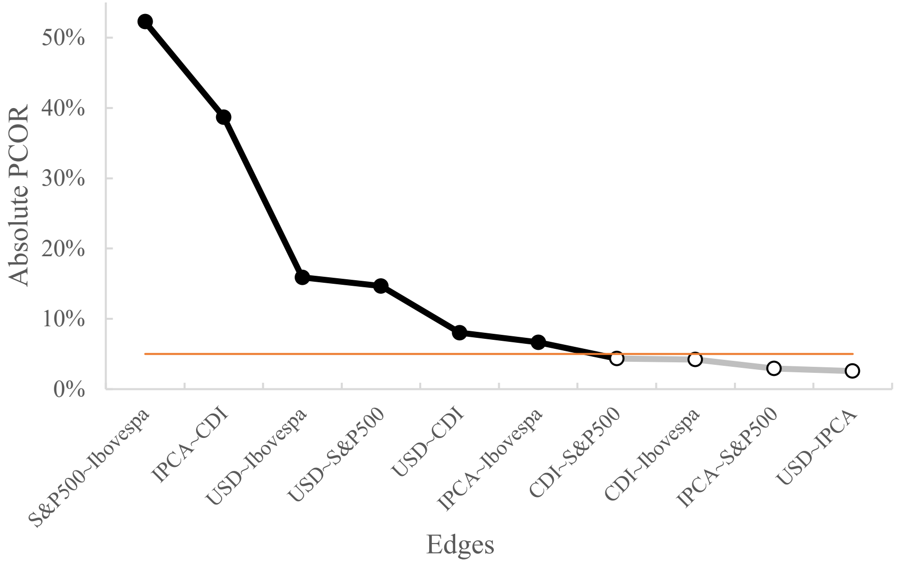
Source: own elaboration

Table 3 – Breusch-Godfrey Stationarity Test and Higher Eigenvalue of Multivariate Systems

Order (p)	BG Test		Higher Eigenvalue
	Statistic	p-Value	
1	0.324	0.569	0.982
2	1.193	0.551	0.979
3	1.281	0.734	0.978

Source: own elaboration

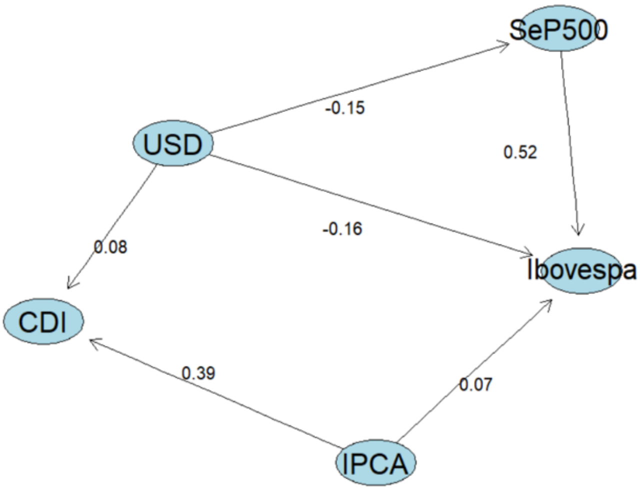
Figure 3 – Criteria for selecting relevant edges for modeling



Source: own elaboration

Once the significant edges are selected by the partial correlation criterion greater than 0.05, Figure 4 presents the final graph of the DBN and the magnitudes of the respective significant arcs. It is important to emphasize that the direction of the arc does not necessarily (but often) denote a causal relationship, but a factorization of the distribution of the joint probability density function as a function of conditional probabilistic dependence. The values represent the partial correlation between the historical series. Given the network's topology and parameters, it is possible to carry out a concept test, evaluating whether the graph relations have, in a greater or lesser degree, qualitative significance.

Figure 4 – DBN representative graph with significant edges



Source: own elaboration

Figure 4 shows a strong correlation between the S&P500 and Ibovespa indexes. Thus, the performance of the Brazilian index is conditionally dependent on the performance of the S&P500, signaling that the external environment (S&P500 and exchange rate) is more relevant than domestic risk factors, materialized in the inflation rate (IPCA) and fixed-income rate (CDI), which follows the economy's basic interest rate (SELIC). Corroborating this hypothesis, the exchange rate is negatively correlated, simultaneously with the Ibovespa and the S&P500. This is due to the use of the US dollar as a store of value, strengthening the currency in times of pessimism. In the domestic scenario, the correlation between the IPCA and the CDI is remarkable, a fact consistent with the dynamics of Brazilian monetary policy. In Brazil, the Central Bank uses SELIC as a formal instrument to control inflation. Finally, it should be noted that the edge linking the IPCA to the Ibovespa, despite having a less relevant parameter, is consistent with the stylized fact of the correction of the shares' fair value for inflation.

4.3. Financial Modeling

Once the DBN that lists all the variables was estimated, allowing the identification of the propagating risk sources and the adverse shock receptors, the Ibovespa and the CDI were identified as shock receptors from diverse sources. To meet the objective of measuring Ibovespa returns (as a function of the exchange rate, IPCA and S&P500), it is necessary to write the mathematical dependence function of each one of them following the procedure of (Rebonato, 2019). Therefore, Table 4 presents the symbology used to define the risk factors.

Table 4 – Definition of risk factors on the Ibovespa

Symbol	Subtitle
Ω	Factor S&P500
ω_x	Return of S&P500 in x state
Γ	Factor USD
γ_x	Return of Exchange Rate in x state
Θ	Factor IPCA
θ_x	Return of IPCA in x state

Source: own elaboration

For each factor presented in Table 4, three states are designed: (i) negative; (ii) normal, and; (iii) positive, represented by indices -1, 0 and 1, respectively. As presented in section 3, the expected return of the incident mimicked risk factor in the normal state is equal to zero. However, for the positive state, the expectation of the return of the mimicking risk factor will suffer a shock of magnitude equal to two standard deviations above the mean, while in the negative state two standard deviations below the mean. Equation 13 presents the mathematical relation of evaluation of effects, considering all possible impact scenarios of shocks from risk sources on Ibovespa returns, estimated from the DBN.

$$r_{ibov} = r_f + [\boldsymbol{p}^{\mathbf{000}}(\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{001}}[\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{00-1}}([\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{010}}(\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{011}}[\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{01-1}}([\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{0-10}}(\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{0-11}}[\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{0-1-1}}([\beta_{i\Omega}\lambda_\Omega + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{100}}(\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{101}}[\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{10-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{110}}(\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{111}}[\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{11-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{1-10}}(\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{1-11}}[\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{1-1-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_1) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{-100}}(\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{-101}}[\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{-10-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}\lambda_\Gamma + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{-110}}(\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{-111}}[\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{-11-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_1) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} + [\boldsymbol{p}^{\mathbf{-1-10}}(\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}\lambda_\Theta)] + \{\boldsymbol{p}^{\mathbf{-1-11}}[\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_1)]\} + \{\boldsymbol{p}^{\mathbf{-1-1-1}}([\beta_{i\Omega}(\lambda_\Omega + \omega_{-1}) + \beta_{i\Gamma}(\lambda_\Gamma + \gamma_{-1}) + \beta_{i\Theta}(\lambda_\Theta + \theta_{-1})])\} \tag{13}$$

4.4. Obtaining Parameters

After the mathematical structure development, the necessary parameters for the simulation are obtained. Table 5 shows the values obtained for β and λ listed in Equation 13.

Table 5 – Betas and Lambdas

Betas		Risk Premia	
Factor	**Value**	**Factor**	**Value**
$\beta_{i\Omega}$	1,02718	λ_Ω	0,00291
$\beta_{i\Gamma}$	-0,33439	λ_Γ	0,00154
$\beta_{i\Theta}$	1,24718	λ_Θ	0,00137

Source: own elaboration

To obtain the probabilities of combinations between states, each observation in the time series was classified into one of three groups: (i) observations smaller than the mean minus two standard deviations; (ii) observations greater than the mean plus two standard deviations, and; (iii) intermediate observations between these other two groups. After classification, it is possible to count the number of occurrences in each of the possible combinations of states. Considering that the dataframe has 317 observations, some combinations were not observed. Aiming at exercising stress, it is desirable that all combinations have some probability of occurrence, even if small. Therefore, the probability of 0.316% (corresponding to 1 observation in a sample of 317 occurrences) was added to each of the 26 least frequent combinations. It is important to note that for each probability value added to some combination, the same amount must be subtracted from another combination, usually the most frequent one. Furthermore, the nomenclature of the state will follow an informative order, denoting the states of the S&P500, exchange rate (USD) and IPCA, respectively.

Table 6 – Observed and Defined Probabilities

State	Short name	p(Observed)	P(Final)	State	Short name	p(Observed)	P(Final)
p000	p1	86.8%	78.5%	p11-1	p15	0.0%	0.3%
p001	p2	3.8%	4.1%	p1-10	p16	0.0%	0.3%
p00-1	p3	0.3%	0.6%	p1-11	p17	0.0%	0.3%
p010	p4	2.2%	2.5%	p1-1-1	p18	0.0%	0.3%
p011	p5	0.0%	0.3%	p-100	p19	3.5%	3.8%
p01-1	p6	0.0%	0.3%	p-101	p20	0.0%	0.3%
p0-10	p7	1.3%	1.6%	p-10-1	p21	0.3%	0.6%
p0-11	p8	0.0%	0.3%	p-110	p22	0.9%	1.3%
p0-1-1	p9	0.0%	0.3%	p-111	p23	0.0%	0.3%
p100	p10	0.6%	0.9%	p-11-1	p24	0.0%	0.3%
p101	p11	0.0%	0.3%	p-1-10	p25	0.0%	0.3%
p10-1	p12	0.0%	0.3%	p-1-11	p26	0.0%	0.3%
p110	p13	0.3%	0.6%	p-1-1-1	p27	0.0%	0.3%
p111	p14	0.0%	0.3%				

Source: own elaboration

Finally, to obtain the mimicked factors, probability distributions were parameterized in order to represent the behavior of risk factors. For that, numerical methods were used that minimized the Bayesian Information Criterion (BIC), obtaining the most adherent distribution among an universe of 106 different possible distributions. After obtaining the distributions, described in Table 7, 100,000 simulations of each random variable were generated and, later, the results were classified with the same three criteria of the combination of states.

Table 7 – Observed and Defined Probabilities

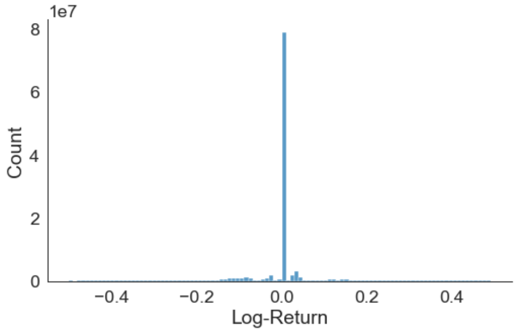
Factor	Distribution	BIC	Parameters
Ibovespa	Logistic	-545.3	loc≅0.0149, scale≅0.0450
CDI	Exponentially Modified Gaussian	1,362.6	x≅2.1501, loc≅0.0058, scale≅0.0029
IPCA	Johnson's SU	-1,487.9	a≅-0.7780, b≅1.2174, loc≅0.0024, scale≅0.0031
S&P500	Laplace	34.6	loc≅0.0121, scale≅0.0321
USD	Double Gamma	305.4	a≅1.1425, loc≅0.0053, scale≅0.0247

Source: own elaboration

4.5. Simulation

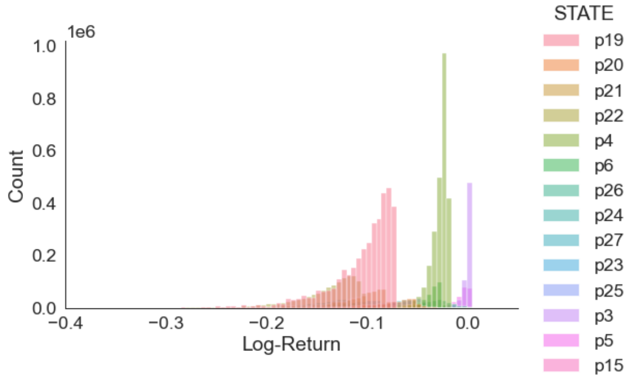
As a last step, a Monte Carlo simulation was carried out, repeating the following procedure one hundred million times: (i) a combination of states is drawn weighted by the probability of occurrence described in Table 6; (ii) the binary indicator of this combination of states is set to 1, while all other binary indicators are set to 0; (iii) Equation 13 is calculated, if the state combination is not neutral, i.e., it has a mimicry factor different from zero, one of the random variables generated and classified above is drawn; (iv) this portion is added to the risk-free rate, and; (v) the return value obtained is stored. At the end of the simulation procedure, we obtained the histograms of the estimated log-return results presented in Figures 5 and 6.

Figure 5 – Histogram of Log-Returns



Source: own elaboration

Figure 6 – Left Tail Log-Returns



Source: own elaboration

While Figure 5 presents the complete histogram of the simulation, Figure 6 shows exclusively the left tail, that is, the values below the risk-free rate, coloring the incidence of each combination of states brought in Table 6. The purpose of this separation is to show how different combinations of states are concentrated in different regions of the histogram. Complementarily, Table 8 describes the absolute frequencies of combinations of states in the worst-case results (1%, 0.1%, 0.01% and 0.001% worst-case scenarios).

Table 8 – Absolute Frequencies in Worst Cases

	Worst-Case Scenarios			
State	**1%**	**0.10%**	**0.01%**	**0.001%**
p-110	363,524	37,115	3,895	644
p-100	356,654	34,452	3,330	0
p-11-1	116,375	11,411	1,338	231
p-10-1	73,703	7,260	710	59
p-111	41,685	4,128	421	65
p-1-1-1	15,140	1,867	102	1
p-101	14,323	1,887	101	0
p-1-10	12,205	1,340	103	0
p-1-11	6,391	540	0	0

Source: own elaboration

As explained in Table 8, we can see that the situations which presented the greatest losses were all linked to the *"-1"* status of the S&P, i.e., precisely the main risk factor incident on the Ibovespa. Also significant is the presence of state *"1"* for the exchange rate, corroborating the mathematical formulation of the simulation (Equation 13). Given the state combinations, the one that can imply the maximum loss is the *"p-11-1"* combination, as it places the factors as harmful as possible among the possibilities, so that it was this combination that generated the greatest simulated loss (Table 9).

However, when analyzing Table 6, we notice that the states *"p-100"* and *"p-110"* have the highest observed frequencies and, consequently, the highest occurrences in the simulation. Although the state *"p-100"* has a higher probability of occurrence, this combination does not corroborate the extreme losses, as one can see when evaluating the last column of Table 8. The state *"p-110"* (S&P in negative state, exchange rate in positive state and IPCA in neutral state) has proven to be the main generator of extreme losses, having a non-negligible probability mass.

Comparing with traditional risk measures (i.e., VaR), we performed a quantile analysis of the simulation. For this purpose, TVaR was obtained at different percentiles of the left tail

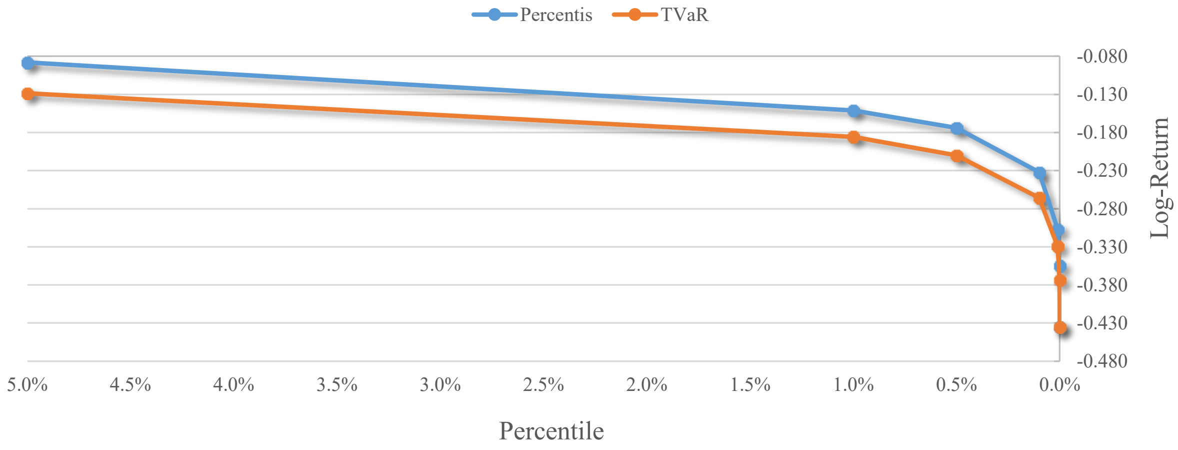
(Table 9) and, finally, graphically compared (Figure 7). The tail's heavy weight is noticeable, having a kurtosis of 9.79 points higher than the normal distribution.

Table 9 – Percentile Analysis and TVaR

Percentile	Log-Return	TVaR
5.000%	-0.088	-0.129
1.000%	-0.151	-0.186
0.500%	-0.174	-0.210
0.100%	-0.232	-0.266
0.010%	-0.307	-0.329
0.001%	-0.355	-0.374
Worst Case	-0.436	-0.436

Source: own elaboration

Figure 7 – Evolution of Percentiles and TVaR



Source: own elaboration

In view of the results, it is necessary to emphasize the importance of the differentiation of states shown in Table 6. Because, ultimately, it is through this segregation in multiple situations that the probabilities can be distributed in states of interest, according to the need for evaluation. This tactic can be used by risk or investment managers, unifying the theoretical tools of different areas of financial institutions under the aegis of the Bayesian perspective.

5. Final remarks

Our objective was to develop a coherent stress test approach, based on Bayesian logic and compatible with Brazilian peculiarities and instabilities. For that, two large tool blocks were used. The first is the methodological pillar: through Dynamic Bayesian Networks, we capture the structures of temporal interdependencies and possibilities of contagion between relevant factors in the financial and capital markets. The second is related to the theoretical platform: we used the Arbitrage Pricing Theory to assess the effects of shocks on the returns of the Ibovespa index with inputs estimated via DBN.

The use of Dynamic Bayesian Networks, still little explored in the literature of Finance, has great potential in identifying factors, guiding the modeling by showing which factors are in fact more relevant, as well as the way in which the factors are connected. In the context of stress testing, approaches involving networks are important for the study of systemic risk, denoting a future gap to be explored. This must be a latent concern in an increasingly interconnected and complex environment.

The results were consistent with the proposed formulation, as well as demonstrating the practicality of the method. The time series together form a stationary system, showing that the adjusted Dynamic Bayesian Network was adequate to represent the joint behavior of economic factors. Subsequently, the estimated graph provided the structure to be modeled by the APT, as well as the magnitudes of dependence for calculating the sensitivities and risk premiums. At this point, the Ibovespa index's characteristic is to reflect more strongly the risk appetite of the international investor than internal factors.

The states' probability matrix was also estimated, making it possible to unify, on the same tooling framework, the management of assets and risk. Finally, the simulation of the effects of extreme shocks on the Ibovespa index was carried out, as well as it was possible to quickly compute the measures of interest, such as VaR or TVaR. Finally, despite the mathematical complexity of some steps in the process, the procedure is easy to implement, allowing for a simplified but powerful approach to the stress test.

For future research, one can mention the use of a greater number of factors, as well as replicating the process with another network topology. Transition zones between states can also be studied (as seen in Figure 6), or a formulation that extends this approach to the process of composition of investment portfolios. Other possible sophistications of this process can be robust methodologies for measuring the probability of states, as well as the flexibility of sensitivity values and risk premiums. Furthermore, it is worth questioning how the model would behave when obtaining daily periodicity variables.

References

Abad, P., Benito, S., López Martín, C., & Ángel Sánchez-Granero, M. (2016). Evaluating the performance of the skewed distributions to forecast value-at-risk in the global financial crisis. *The Journal of Risk*, *18*(5), 1–28. https://doi.org/10.21314/J0R.2016.332

Amrhein, V., Trafimow, D., & Greenland, S. (2019). Inferential Statistics as Descriptive Statistics: There Is No Replication Crisis if We Don't Expect Replication. *The American Statistician*, *73*(sup1), 262–270. https://doi.org/10.1080/00031305.2018.1543137

Anand, K., Craig, B., & von Peter, G. (2015). Filling in the blanks: network structure and interbank contagion. *Quantitative Finance*, *15*(4), 625–636. https://doi.org/10.1080/14697688.2014.968195

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, *9*(3), 203–228. https://doi.org/10.1111/1467-9965.00068

Battiston, S., & Martinez-Jaramillo, S. (2018). Financial networks and stress testing: Challenges and new research avenues for systemic risk analysis and financial stability implications. *Journal of Financial Stability*, *35*, 6–16. https://doi.org/10.1016/j.jfs.2018.03.010

Benjamini, Y., & Hochberg, Y. (1995). Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing. *Journal of the Royal Statistical Society: Series B (Methodological)*, *57*(1), 289–300. https://doi.org/10.1111/j.2517-6161.1995.tb02031.x

Bruno, S. V. D. B. (2008). Otimização de Portfólio de Ativos Reais Utilizando uma Medida de Risco Coerente. In *VIII Encontro Brasileiro de Finanças* (Issue February). http://www.impa.br/opencms/pt/ensino/downloads/dissertacoes_de_mestrado/dissertacoe s_2008/Sergio_Bruno.pdf

Carraro, A. (2018). *Bayesian Networks and Financial Stress Testing* [Ca' Foscari University of Venice]. http://dspace.unive.it/bitstream/handle/10579/14116/848303-1224377.pdf?sequence=2

Carvalho, J. V. de F., & Chiann, C. (2013). Redes Bayesianas: um método para avaliação de interdependência e contágio em séries temporais multivariadas. *Revista Brasileira de Economia*, *67*(2), 227–243. https://doi.org/10.1590/S0034-71402013000200003

Castro, F. H., Eid Junior, W., Santana, V. F., & Yoshinaga, C. E. (2019). Fifty-Year History of the Ibovespa. *Brazilian Review of Finance*, *17*(3), 47. https://doi.org/10.12660/rbfin.v17n3.2019.80028

Consigli, G. (2002). Tail estimation and mean–VaR portfolio selection in markets subject to financial instability. *Journal of Banking & Finance*, *26*(7), 1355–1382. https://doi.org/10.1016/S0378-4266(02)00267-4

Daníelsson, J., Jorgensen, B. N., Samorodnitsky, G., Sarma, M., & De Vries, C. G. (2013). Fat tails, VaR and subadditivity. *Journal of Econometrics*, *172*(2), 283–291. https://doi.org/10.1016/j.jeconom.2012.08.011

de Oliveira, F. A., Maia, S. F., de Jesus, D. P., & Besarria, C. da N. (2018). Which information matters to market risk spreading in Brazil? Volatility transmission modelling using MGARCH-BEKK, DCC, t-Copulas. *The North American Journal of Economics and Finance*, *45*(January), 83–100. https://doi.org/10.1016/j.najef.2018.02.003

Efron, B. (2010). Local False Discovery Rates. In *Large-Scale Inference* (Vol. 0, pp. 70–88). Cambridge University Press. https://doi.org/10.1017/CBO9780511761362.006

Emmer, S., Kratz, M., & Tasche, D. (2015). What is the best risk measure in practice? A comparison of standard measures. *The Journal of Risk*, *18*(2), 31–60. https://doi.org/10.21314/JOR.2015.318

Fonseca, N. C., & Carvalho, J. V. F. (2021). Analysis of Financial Contagion among Economic Sectors through Dynamic Bayesian Networks. *Anais Do XLV Encontro Da ANPAD*, 1–16.

Gao, G., Mishra, B., & Ramazzotti, D. (2017). Efficient Simulation of Financial Stress Testing Scenarios with Suppes-Bayes Causal Networks. *Procedia Computer Science*, *108*, 272–284. https://doi.org/10.1016/j.procs.2017.05.167

Gong, X.-L., Liu, X.-H., Xiong, X., & Zhang, W. (2019). Financial systemic risk measurement based on causal network connectedness analysis. *International Review of Economics & Finance*, *64*(July), 290–307. https://doi.org/10.1016/j.iref.2019.07.004

Greenland, S., Senn, S. J., Rothman, K. J., Carlin, J. B., Poole, C., Goodman, S. N., & Altman, D. G. (2016). Statistical tests, P values, confidence intervals, and power: a guide to misinterpretations. *European Journal of Epidemiology*, *31*(4), 337–350. https://doi.org/10.1007/s10654-016-0149-3

Khokhlov, V. (2016). *Conditional Value-at-Risk for Elliptical Distributions*.

Kopeliovich, Y., Novosyolov, A., Satchkov, D., & Schachter, B. (2015). Robust Risk Estimation and Hedging: A Reverse Stress Testing Approach. *The Journal of Derivatives*, *May*, 150514062927006. https://doi.org/10.3905/jod.2015.2015.1.043

Krause, A. (2003). Exploring the Limitations of Value at Risk: How Good Is It in Practice? *The Journal of Risk Finance*, *4*(2), 19–28. https://doi.org/10.1108/eb022958

Linsmeier, T. J., & Pearson, N. D. (1996). *Risk Measurement: An Introduction to Value at Risk*. *6*(July). https://doi.org/http://dx.doi.org/10.22004/ag.econ.14796

Mittnik, S. (2014). VaR-implied tail-correlation matrices. *Economics Letters*, *122*(1), 69–73. https://doi.org/10.1016/j.econlet.2013.10.025

Nagarajan, R., Scutari, M., & Lèbre, S. (2013). Bayesian Networks in R. In *Bayesian Networks in R*. Springer New York. https://doi.org/10.1007/978-1-4614-6446-4

Nagpal, K. M. (2017). Designing stress scenarios for portfolios. *Risk Management*, *19*(4), 323–349. https://doi.org/10.1057/s41283-017-0024-x

Obstfeld, M., Shambaugh, J. C., & Taylor, A. M. (2009). Financial Instability, Reserves, and Central Bank Swap Lines in the Panic of 2008. *American Economic Review*, *99*(2), 480–486. https://doi.org/10.1257/aer.99.2.480

Pérignon, C., & Smith, D. R. (2010). The level and quality of Value-at-Risk disclosure by commercial banks. *Journal of Banking & Finance*, *34*(2), 362–377. https://doi.org/10.1016/j.jbankfin.2009.08.009

Rebonato, R. (2010). *Coherent Stress Testing: A Bayesian Approach to the Analysis of Financial Stress*. Wiley.

Rebonato, R. (2017). Stress Testing with Bayesian Nets and Related Techniques: Meeting the Engineering Challenges. In *The Palgrave Handbook of Unconventional Risk Transfer* (Vol. 36, Issue 2, pp. 537–575). Springer International Publishing. https://doi.org/10.1007/978-3-319-59297-8_17

Rebonato, R. (2019). A financially justifiable and practically implementable approach to coherent stress testing. *Quantitative Finance*, *19*(5), 827–842. https://doi.org/10.1080/14697688.2018.1532103

Rensi, R. T., & Carvalho, J. V. F. (2021). Operation Car Wash: Impacts on the Executives' Liability Insurance Market. *Revista de Administração Contemporânea*, *25*(2), 1–19. https://doi.org/10.1590/1982-7849rac2021190386.en

Righi, M. B., & Ceretta, P. S. (2014). Teoria de Medidas de Risco: uma revisão abrangente. *Brazilian Review of Finance*, *12*(3), 411. https://doi.org/10.12660/rbfin.v12n3.2014.42537

Rockafellar, R. T., & Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, *26*(7), 1443–1471. https://doi.org/10.1016/S0378-4266(02)00271-6

Rosengren, E. S. (2014). Our Financial Structures-Are They Prepared for Financial Instability? *Journal of Money, Credit and Banking*, *46*(s1), 143–156. https://doi.org/10.1111/jmcb.12084

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, *13*(3), 341–360. https://doi.org/10.1016/0022-0531(76)90046-6

Storey, J. D. (2002). A direct approach to false discovery rates. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *64*(3), 479–498. https://doi.org/10.1111/1467-9868.00346