Labor Supply in Pandemic Environments: An Aggregative Games Approach

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Abstract

In this paper we analyze the effects that pandemic processes have on labor supply decisions using an aggregative game framework. The individual payoff depends on her labor supply and on the probability of being infected, which in turn, depends on the aggregate labor supply. We show the effects of social and sanitary public policies on the Nash equilibrium and analyze its expectational stability. The results indicate that compensating policies as well as sanitary policies can attenuate the damaging effect of pandemic and stabilize expectations regarding the aggregate decision of labor supply. We also find a set of parameters where two-period cycles for the expectations revision map may arise, implying the oscillating behavior of the probability of contagion in this class of models.

Keywords: Labor supply, pandemic, aggregative games.

JEL Classification Numbers: C72, J28, J38

ANPEC Classification Area: 8 - Microeconomics, Quantitative Methods and Finance.

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Resumo

Em este trabalho analisamos os efeitos que processos pandémicos têm sobre as decisões de oferta de mão de obra utilizando a estrutura de jogos agregativos. O benefício do indivíduo depende da sua oferta de trabalho e da probabilidade de ser infectado, que por sua vez, depende da oferta agregada de mão de obra. Mostramos os efeitos de políticas públicas sociais e sanitárias sobre o equilíbrio de Nash e analisamos a sua estabilidade expectacional. Os resultados indicam que políticas compensatórias salariais, assim como sanitárias podem atenuar os efeitos prejudiciais da pandemia e estabilizar as expectativas em relação à decisão agregada de oferta de trabalho. Também encontramos um conjunto de parâmetros no qual surgem ciclos de período dois para a aplicação de revisão de expectativas, implicando o comportamento oscilatório da probabilidade de contágio em esta classe de modelos.

Palavras-Chave: Oferta de trabalho, pandemia, jogos agregativos.

Número de Classificação JEL: C72, J28, J38

Área de Classificação da ANPEC: 8 - Microeconomia, Métodos Quantitativos e Finanças.

1 Introduction

The event of Covid-19 constituted an economic shock that affected countries in different ways. Besides the challenges related to quick responses in order to avoid a massive mortality rate, the fact that the disease is mainly spread by social contact called for the design of totally new economic policies. In particular, decision makers tried to balance the goal of avoiding high mortality with the problem related to decrease of economic activity, since policies such as lockdown, tracking and quarantine directly affected the labor market.

Clearly the new environment after Covid-19 largely affected policy makers' decisions, but also firms and workers, who had to balance their decisions based on the trade-off between receiving higher wages and being susceptible of infection. Moreover, the interaction of workers' decisions was evident, as the higher the number of active workers in the market, the higher the probability of contagion. This situation of collective and individual strategic decisions fits well on an aggregative game framework, as we propose in this paper.

Many studies have been done in order to assess how some government responses to the Covid-19 pandemic were effective to contain its spread. Aum et al. (2021) propose a SIR (Susceptible, Infectious, or Recovered) epidemiological model to evaluate the effectiveness of policies that determine lockdowns, testing individuals for their state of infection and the combination of tracking and quarantine. They use data from South Korea and the UK to calibrate and test their model. Birinci et al. (2021) analyze the effects of different labor market policies, namely, unemployment insurance expansions and payroll subsidies. They show that the introduction of payroll subsidies alone is preferred over unemployment insurance expansions and estimate an optimal mix of policies with 20% of the budget directed to payroll subsidies and the remainder to unemployment insurance expansions. Bradley et al. (2021) propose a matching model in which the decision of whether the worker should work from home is a function of the joint surplus of a match. They show that policies that impose lockdowns are costly to young workers and more beneficial to the old, whose rate of mortality decreases.

Empirically, Miller (2020) estimates the optimal level of wage reduction that minimizes the weighted sum of the average of the cost imposed onto workers who experience the wage reduction and the variance of this cost across workers. Data from a university in the United States is used to verify how the weights a planner puts on minimizing the average pain, compared to equalizing pain across employees affect the optimal policy. Dergiades et al. (2022) analyze the impact of public policies to contain the spread of Covid-19 on the U.S. labor market. They classify policies as non-pharmaceutical interventions (NPIs) or economic support measures (ESM) and show that NPI's quickly cause an increase in the number of unemployment claims, while the reduction of unemployment caused by ESMs does not occur immediately. Famiglietti and Leibovici (2022) analyze the relation between the spread of Covid-19, non-pharmaceutical interventions, and economic activity, showing that the adoption of both policies together contribute to the mitigation of economic negative impacts caused by NPIs. Eichenbaum et al. (2022) have similar results, with a positive interaction between social distancing/mask use and testing/quarantining.

Although many studies have assessed policy effectiveness against the spread of Covid-19 and its economic impact, not much has been made to understand the rationale of workers' decisions in pandemic environments. If the worker is infected, her wage suffers a reduction corresponding to the income compensation she will receive; otherwise, she will receive her total wage. The probability of infection depends on the total labor supply and sanitary policies. Thus, when both kinds of policies are implemented (wage compensation policies and sanitary policies), each worker is confronted with the decision of how much labor she will supply. This paper sheds some light on these aspects of the pandemic by using an aggregative game framework. We show that for quite general functional forms and a large set of parameters there is a Nash equilibrium in which best response functions' second-order conditions are automatically satisfied. Stability properties of the equilibrium are analyzed, with intuitively appealing results. In particular, we show the conditions under which workers increase their individual labor supply whenever they perceive a decreasing in the probability of contagion. As a consequence, the probability of infection increases, with recurrent oscillations.

2 The Model

Consider a labor market in which $N \geq 2$ workers are at risk of infection by a contagious disease. Let us denote $\mathcal{N} = \{1, 2, \cdots, n\}$, the set of workers. A worker $n \in \mathcal{N}$ decides her labor supply $x_n \in [0, 1]$ and receives w_n per unit of work. If she is infected, she is forbidden to work, but eligible to receive a compensating wage $\lambda_n w_n$, where $\lambda_n \in (0,1)$ is a public policy parameter defined by the government to mitigate the economic impact of the pandemic.

Since the contact with other people increases contagion, the total labor supply $X = \sum_{n=1}^{N} x_n$ determines the probability that a worker is infected, defined as $\pi = \pi(X)$. We assume that $\pi : [0, N] \to [0,1]$ is a continuous and strictly increasing function and define the labor supply of all workers but n as $X_{-n} = X - x_n$.

Each worker $n \in \mathcal{N}$ has preferences represented by the Von Neumann-Morgenstern utility function $u_n : \mathbb{R}_+ \to \mathbb{R}$. Given a profile $(x_1, x_2, \cdots, x_n)$ of labor supply, the payoff of worker n is:

$$U_n(x_n, X_{-n}) = \pi(X_{-n} + x_n)\, u_n(\lambda_n w_n x_n) + (1 - \pi(X_{-n} + x_n))\, u_n(w_n x) \tag{1}$$

Individuals decide how much to work before knowing whether they are contaminated. The individual strategy sets S_n and payoffs $U_n(\cdot,\cdot)$ define the aggregative game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, where for all $n, S_n = [0,1]$ and $U_n : [0,1] \times [0,(N-1)] \to \mathbb{R}$ is given by (1).

For each X_{-n}, maximization of $U_n(\cdot, X_{-n})$ generates worker n's best-reply function $x_n^*(X_{-n}) = \arg\max U_n(\cdot, X_{-n})$. Since $\pi(\cdot)$ and $u_n(\cdot)$ are continuous functions, so is $U_n(\cdot,\cdot)$. Therefore, the best reply functions are non-empty and upper hemi-continuous. However, because $U_n(\cdot, X_{-n})$ is not necessarily a quasiconcave function, the best-reply values may not define a convex set. To avoid this problem, we consider isoelastic utility functions and log-linear distributions of probability of contagion.

3 Existence and stability of the equilibrium for some specific functional forms

In this section we analyze the existence and stability of the Nash equilibrium of the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$ given in Section 2 for some particular (although widely used) cases of utility functions and probabilities of contagion. We will state conditions on the fundamentals for obtaining best-response functions of the individuals, existence and stability of the equilibrium.

Before stating the results of this section, let us recall the following concepts of replacement functions and Perceived-to-Actual maps, already used by Cornes et al. (2021) to define and analyze the expectational stability of the Nash equilibrium in aggregative games. For the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$ we define the replacement function of individual n as the function $r_n : [0, N] \to [0, 1]$ satisfying:

$$U_n(r_n(X), X - r_n(X)) \geq U_n(z, X - z);\ \forall z \in S_n.$$

Thus, $r_n(X)$ is the individual optimal participation of n in the total contribution X of the game. That replacement function was largely used in aggregative games (Okuguchi (1993) and Cornes and Hartley (2007)). To define the expectational stability of the Nash equilibrium in that kind of games, Cornes et al. (2021) considered the perceived-to-actual map as $\rho(X) = \sum_{n=1}^{N} r_n(X)$, and a Nash equilibrium of the game X^* is called expectational stable (unstable) if $|\rho'(X^*)| < 1$ ($|\rho'(X^*)| > 1$).

3.1 Isoelastic utility functions and log-linear probabilities of contagion

There is large evidence that the probability of contagion is increasing on social contact. For instance, using data for Atlanta, Boston, Chicago, New York (NYC), and Philadelphia, Glaeser et al. (2022) estimate that total Covid-19 cases per capita decrease on average by approximately 20% percent for every ten percentage point fall in mobility between February and May 2020. According to their findings, we assume that $\pi(X)$ follows a distribution that can be estimated according to the following specification:

$$\ln \pi = \beta_0 + \beta_1 \ln X$$

where $\beta_0 = -\sigma \ln N$ and $\beta_1 = \sigma$. Note that the latter is affected by policies aimed to decrease the contagion rate. Large values of σ are associated with more effective policies. Therefore, expected utility functions and probability of contagion follow the specification below:

$$u(z) = \frac{z^{1-\gamma} - 1}{1 - \gamma};\quad \pi(X) = \left(\frac{X}{N}\right)^{\sigma} \tag{2}$$

where $0 < \gamma < 1$ and $\sigma > 1$. It is important to say that most of the results in this work are valid even with individuals having different risk aversion coefficients γ; however, to keep the analysis simple we are supposing that all individuals have the same value for that parameter. Thus, according to this specification, we have:

$$U_n(x_n, X_{-n}) = \left(\frac{X_{-n} + x_n}{N}\right)^{\sigma}\left(\frac{(\lambda_n w_n x_n)^{1-\gamma} - 1}{1-\gamma}\right) + \left(1 - \left(\frac{X_{-n} + x_n}{N}\right)^{\sigma}\right)\left(\frac{(w_n x_n)^{1-\gamma} - 1}{1-\gamma}\right)$$

$$= w_n\left[\left(\frac{X_{-n} + x_n}{N}\right)^{\sigma}\left(\frac{(\lambda_n x_n)^{1-\gamma}}{1-\gamma}\right) + \left(1 - \left(\frac{X_{-n} + x_n}{N}\right)^{\sigma}\right)\left(\frac{{x_n}^{1-\gamma}}{1-\gamma}\right)\right] - \frac{1}{1-\gamma}$$

Maximization of the payoff function above does not depend on w_n and the best-response function is the solution of:

$$\max_x \left[\left(\frac{X_{-n} + x}{N}\right)^{\sigma}\left(\frac{(\lambda_n x)^{1-\gamma}}{1-\gamma}\right) + \left(1 - \left(\frac{X_{-n} + x}{N}\right)^{\sigma}\right)\left(\frac{{x_n}^{1-\gamma}}{1-\gamma}\right)\right] \tag{3}$$

The following proposition states that the objective function of problem (3) is strictly concave for an open set of parameters:

Proposition 1. *Suppose that the payoff functions U_n given in (1) of the game Γ have the functional specifications given in (2), and the parameters belong to the set*

$$\mathcal{C} = \left\{\left(\gamma, \sigma, (\lambda_n)_{n=1}^N\right) \in (0,1) \times (1, +\infty) \times (0,1)^N ; (\sigma - \gamma)\left(1 - \lambda_n^{1-\gamma}\right) < \gamma(\sigma - 1)\right\}, \tag{4}$$

then, the objective function of the maximization problem (3) is strictly concave for all $n \in \mathcal{N}$.

Proposition 1 states sufficient conditions for the existence of a unique solution of the workers' maximization problem. Note that if $\left(\bar{\gamma}, \bar{\sigma}, (\bar{\lambda}_n)_{n=1}^N\right) \in \mathcal{C}$ then $\left(\bar{\gamma}, \sigma, (\bar{\lambda}_n)_{n=1}^N\right) \in \mathcal{C}$ for all $\sigma > \bar{\sigma}$. Therefore, if a police against dissemination of the disease is associated with strictly concave payoff functions, so are all other more effective policies. Similar relations are valid for the parameters $\bar{\lambda}_n$ and $\bar{\gamma}$ in the set $\mathcal{C}$, since for any $\lambda_n > \bar{\lambda}_n$ and for any $\gamma > \bar{\gamma}$, $\left(\bar{\gamma}, \bar{\sigma}, (\lambda_n)_{n=1}^N\right) \in \mathcal{C}$ and $\left(\gamma, \bar{\sigma}, (\bar{\lambda}_n)_{n=1}^N\right) \in \mathcal{C}$. Therefore, increases in the compensating wage rates or risk aversion preserve the uniqueness property of problem (3).

Now, we will present the results of existence of equilibrium for the game with parameters in $\mathcal{C}$. To do this, it will be useful to define the following parameters. For $\lambda = (\lambda_1, \cdots, \lambda_n)$ and $\gamma \in (0,1)$, define:

$$K(\gamma, \lambda) = \sum_{n=1}^{N} \left[\frac{1-\gamma}{1-\lambda_n^{1-\gamma}} \right] \tag{5}$$

and for $\gamma = 1$, $K(1, \lambda) = -\sum_{n=1}^{N} [\ln(\lambda_n)]^{-1}$. Notice that $K(\gamma, \lambda) > 0$ for all $\gamma \in (0,1]$, since for all $n \in \mathcal{N}$, $\lambda_n \in (0,1)$. We can also observe that $K(\gamma, \lambda)$ is strictly increasing in any λ_n.

Proposition 2. *Consider the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions given in (1) and the specification (2). If the parameters of the game belong to the set $\mathcal{C}$ given in (4), then the replacement functions are:*

$$r_n(X) = \left(\frac{1-\gamma}{\sigma} \right) \left(\left(1-\lambda_n^{1-\gamma}\right)^{-1} \left(\frac{X}{N} \right)^{-\sigma} - 1 \right) X \tag{6}$$

and the Perceived-To-Actual (P-T-A) map is:

$$\rho(X) = \left(\frac{K(\gamma, \lambda) N^{\sigma}}{\sigma} \right) X^{1-\sigma} - \frac{(1-\gamma)}{\sigma} N X. \tag{7}$$

Furthermore, the aggregate labor supply Nash equilibrium is:

$$X^* = \left(\frac{K(\gamma, \lambda)}{\sigma + (1-\gamma) N} \right)^{1/\sigma} N. \tag{8}$$

From Proposition 2, the replacement and P-T-A are increasing functions on the parameters λ, as can be seen in the equations (6) and (7). Therefore, individual and collective responses to the perception of the total labor supply are larger if the workers receive a larger compensation rate in case of infection. Moreover, as the P-T-A function is decreasing in the perception of the aggregate labor supply, from Proposition 2 and definition (5), we can verify that the aggregate labor supply Nash equilibrium X^* is strictly increasing in the compensation policy, as we can see in the equation (8).

An interesting impact of the wage compensation policy is its cross effect among individuals. Consider a worker n whose compensation λ_n increases. Because X^* strictly increases, there must be a reduction in the labor supply of all other workers whose compensation does not change. Therefore, individual n is the only worker who bears the increase in the total labor supply. This effect should be taken into account in the policy design of those compensations.

From the equation (8), the aggregate labor supply Nash equilibrium X^* is also strictly increasing in σ. Therefore, labor supply is positively affected by higher efforts of fighting the spread of the disease.

Finally, we can observe the effect of the population size N and the sanitary policy parameter σ on the probability of contagion in equilibrium $\pi(X^*) = K(\gamma, \lambda)/(\sigma + (1-\gamma)N)$. The greater those parameters are, the lower the probability of contagion results.

To end this subsection, we establish some results of expectational stability for the specification (2). In addition to classify the cases of stability and instability, we can find the existence of expectational cycles around the Nash equilibrium.

Proposition 3. *Consider the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions given in (1) and the specification (2). Assume that its parameters belong to the set $\mathcal{C}$ given in (4). Then, the Nash equilibrium X^* given in Proposition 2 is:*

(i) Expectational Stable if $\sigma + (1-\gamma) N < 2$;

(ii) Expectational Instable if $\sigma + (1-\gamma) N > 2$;

(iii) If $\sigma_0 + (1-\gamma_0) N_0 = 2$, there exists a neighborhood of those parameters where there are cycles of period 2 around the expectational instable X^*.

According to Part (i) of Proposition 3, in the case of a large population, if the relative risk aversion is sufficiently close to one and the curvature of the contagion probability is sufficiently low, then the equilibrium is expectational stable. In the proof provided in the Appendix, we show that $\rho'(X^*) < 0$, thus the stability is oscillating. Therefore, the convergence to X^* is through greater and lower values of the aggregate labor supply, and as a consequence, oscillating in the contagion probability. Although this oscillation is empirically sound, the more realistic case that combines a strong policy against the pandemic with not so low relative risk aversion is unstable, as shown in part (ii) of the proposition. In other words, when workers are confident on the sanitary policies, the aggregate labor supply exceeds the corresponding equilibrium.

Part (iii) of the proposition is the most interesting result, as it shows the existence of two-period cycles, which is a consequence of the period-doubling bifurcation theorem (Devaney (1989)). This result is illustrated by recurrent oscillations that can be observed in pandemic processes. Workers increase their individual labor supply whenever they perceive a decreasing in the probability of contagion. As a result, the probability of infection increases, restarting the cycle.

3.2 Logarithmic utility functions and general contagion probability functions

Since the assumption of $\gamma \neq 1$ is crucial in most of the results given in the previous subsection, we will extend the analysis for the case of logarithmic utilities of the individuals. As we will see, this case allows us the analysis of more general forms of the probability of contagion and clearer conclusions on individual and aggregate labor supply.

Therefore, we will also study the following specification:

$$u(z) = \ln z; \quad \pi'(X) > 0, \ \pi''(X) > 0 \tag{9}$$

Thus, for that case we can state the following proposition:

Proposition 4. *Suppose that the payoff functions U_n given in (1) of the game Γ have the functional specifications given in (9), then the objective function of the maximization problem (3) is strictly concave for all $n \in \mathcal{N}$.*

It is worth noting how the assumption of logarithmic utilities let the functional form of the contagion probability without any restriction, except that of strict monotonicity and strict convexity, in order to obtain a unique solution for the individual problem.

Now, we can state the characterizations of the replacement functions and perceived-to-actual maps as well as the Nash equilibrium for the case of logarithmic utility functions of individuals.

Proposition 5. *Suppose that the game $\Gamma = \{S_n, U_n\}_{n=1}^{N}$, with payoffs functions given in (1) assumes specification (9). Then, the replacement function and the P-T-A map are given* *by:*

$$r_n(X) = \frac{1}{\pi'(X)\ln(\lambda_n^{-1})}; \quad \rho(X) = \frac{K(1.\lambda)}{\pi'(X)} \tag{10}$$

If, in addition, $\pi'(N) \geq \frac{K(1,\lambda)}{N}$*, then there exists a unique equilibrium that satisfies:*

$$X^* = \frac{K(1.\lambda)}{\pi'(X^*)} \tag{11}$$

Similarly to Proposition 2, increases in the compensation parameter λ_n are associated with a larger total labor supply X^*. As a consequence, the individual labor supply of workers whose compensation does not change decreases.

Figure 1 shows the typical shape of the Perceived-To-Actual map for this case of logarithmic utility. It is quite interesting the decreasing behavior of this map indicating the countercyclical response of the actual labor supply to the perceived labor supply. The intersection with the diagonal corresponds to the Nash equilibrium aggregate labor supply and it is easy to argue the possibility of cycles around it, just as stated in Proposition 3 for the case of isoelastic utility functions and probability of contagion with constant elasticity with respect to the total labor supply (log-linear probability of contagion with respect to the total labor supply).

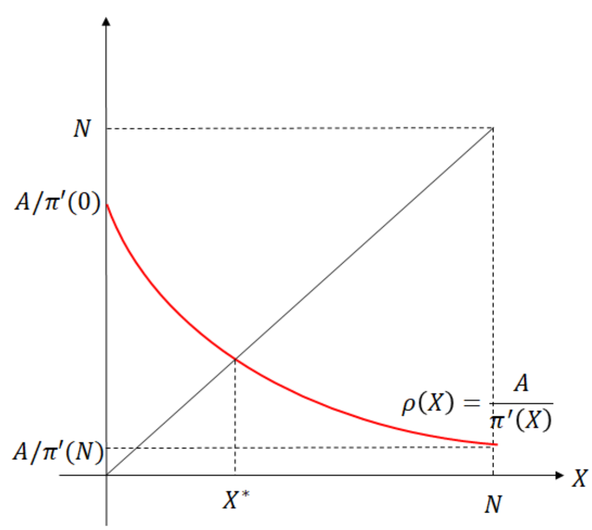

Figure 1: The Perceived-To-Actual map for logarithmic utility and general form of the probability of contagion.

Finally, the following proposition establishes the conditions to obtain expectational stability of the Nash equilibrium for the case we are considering here.

Proposition 6. *Suppose that the game* $\Gamma = \{S_n, U_n\}_{n=1}^{N}$*, with payoffs functions given in (1) has the specification given in (9). Then, the Nash equilibrium given in (11) is expectational stable if and only if:*

$$\frac{X^* \pi'' (X^*)}{\pi' (X^*)} < 1$$

Analysis of the existence of bifurcations from an expectational stable to an expectational unstable Nash equilibrium and the corresponding existence of attracting cycles around it would require more information about the function π.

4 Final Remarks

The pandemic triggered by Covid-19 brought challenges to the scientific community, not restricted to fields directly related to Health Sciences. The massive impacts of the pandemic on economic growth and workers' income demanded a quick response from policy makers. As it seems that pandemics will be globally recurrent, policymakers must consider sanitary and economic policies to face this new economic environment.

From the individual point of view, the pandemic affects workers' decisions of labor supply. At the aggregate level, individuals' choices impact the probability of contagion, which is taken as a focal point for a worker's decision that depends on her risk aversion. Since that probability of contagion is clearly affected by the aggregate decision of labor supply, aggregative games constitute a simple, although embracing framework to model the dynamics of those decisions.

The model presented in this work assumes that the capacity to work of infected individuals is reduced, implying a decrease or even elimination of her wage. The government must define a social policy that may provide a compensation wage for infected workers. At the same time, the aggregate decision of labor supply affects the probability of contagion, which can also be influenced by sanitary policy decisions. All of these aspects are considered in the model to analyze individual and aggregate labor supply decisions.

The first result shows the existence of a large set of parameters of the proposed functional forms for which the second-order conditions of best-response functions are automatically satisfied. This feature of the model is not always overcome in aggregative games. Following the result of existence, we explicitly calculate the individual and collective responses to the perception of the aggregate labor supply, the so-called replacement functions, and the perceived-to-actual map of the game. This exercise is important for the Nash equilibrium solution of the game, since it is defined as the fixed point of its perceived-to-actual map. The explicit forms of those functions allow us to perform some static comparative analysis, where the Nash equilibrium presents very intuitive responses to parameter variations. Lastly, we study the expectational stability of the equilibrium. We show that the equilibrium is unstable for a large set of parameter values related to sanitary policy , population size, and risk aversion. As expected, contagion follows a rapid increasing path. We also prove the existence of two-period cycles around the Nash equilibrium, which is compatible with the persistent oscillations around some levels of stationary aggregate labor supply.

These results shed light on the best social and sanitary policies design. From the theoretical point of view, the model provides a clear analysis of how the Nash equilibrium is affected by workers's decisions and public policies. Empirically, it suggests particular functional forms that may be consistent with the observed data available since Covid-19 became a pandemic.

Appendix

Proof of Proposition 1 To prove the proposition it is enough to show that the objective function of problem (3) is strictly concave. Its first and second derivatives are, respectively:

$$f'(x) = (1-\gamma)\, x^{-\gamma} \left\{ \frac{\sigma u(\lambda_n)}{N^\sigma} x X^{\sigma-1} + (1-\gamma)\, u(\lambda_n) \frac{X^\sigma}{N^\sigma} + 1 \right\}$$

and:

$$\frac{f''(x)}{1-\gamma} = \frac{\sigma(\sigma-1)\, u(\lambda_n)}{N^\sigma} X^{\sigma-2} x^2 - \frac{2(1-\gamma)\,\sigma u(\lambda_n)}{N^\sigma} X^{\sigma-1} x - \gamma \left\{ (1-\gamma)\, u(\lambda_n) \frac{X^\sigma}{N^\sigma} + 1 \right\};$$

recall that $u(\lambda_n) = (1-\gamma)^{-1}(\lambda_n^{1-\gamma} - 1)$. Since the coefficient of x^2 is strictly negative (because $\lambda_n < 1$ and $\sigma > 1$), $f''(x) < 0$ if and only if the discriminant of the right-hand side is negative, namely:

$$\left[\frac{(1-\gamma)\,\sigma u(\lambda_n)}{N^\sigma} X^{\sigma-1}\right]^2 + \left(\frac{\sigma(\sigma-1)\,u(\lambda_n)}{N^\sigma} X^{\sigma-2}\right)\gamma\left((1-\gamma)\,u(\lambda_n)\frac{X^\sigma}{N^\sigma} + 1\right) < 0$$

Simplifying $\sigma, N^\sigma, u(\lambda_n)$ and $X^{\sigma-2}$, we have:

$$\begin{aligned} & (1-\gamma)^2 \sigma u(\lambda_n) \tfrac{X^\sigma}{N^\sigma} + \gamma(\sigma-1)\left((1-\gamma)\,u(\lambda_n)\tfrac{X^\sigma}{N^\sigma} + 1\right) > 0 \\ \Leftrightarrow\quad & X < \left[\tfrac{\gamma(\sigma-1)}{(\sigma-\gamma)\left(1-\lambda_n^{1-\gamma}\right)}\right]^{1/\sigma} N, \end{aligned}$$

Note that because $u(\lambda_n)$ is negative, the inequality sign is reversed. It follows that $f''(x) < 0$ as long as the parameters $\left(\gamma, \sigma, (\lambda_n)_{n=1}^N\right) \in \mathcal{C}$, it results that the payoff function is strictly concave.□

Proof of Proposition 4 The payoff function in specification (9) is:

$$\begin{aligned} & f(x) = \pi(X_{-n} + x)\ln(\lambda_n w_n x) + (1 - \pi(X_{-n} + x))\ln(w_n x) \\ = \quad & \pi(X_{-n} + x)\ln(\lambda_n) + \ln(x_n) + \ln(w_n) \end{aligned}$$

Its second derivative is, $f''(x) = \pi''(X)\ln(\lambda_n) - \frac{1}{x^2} < 0$, which implies that f is a concave function. □

Proof of Proposition 2 Proposition 1 guarantees that the first order condition characterizes the solution of problem (3). Therefore, the replacement function $x = r_n(X)$ satisfies:

$$\begin{aligned} \frac{\sigma u(\lambda_n)}{N^\sigma} x X^{\sigma-1} + (1-\gamma)\,u(\lambda_n)\frac{X^\sigma}{N^\sigma} + 1 = 0 \\ \Rightarrow x^* = r_n(X) = \left(\frac{1-\gamma}{\sigma}\right)\left(\left(1-\lambda_n^{1-\gamma}\right)^{-1}\left(\frac{X}{N}\right)^{-\sigma} - 1\right) X \end{aligned}$$

Summing up the last equation above, we obtain the P-T-A function:

$$\rho(X) = \left(\frac{K(\gamma,\lambda)\,N^\sigma}{\sigma}\right) X^{1-\sigma} - \frac{(1-\gamma)}{\sigma} N X$$

Finally, after solving the equation $\rho(X^*) = X^*$ we find the aggregate supply Nash equilibrium:

$$X^* = \left(\frac{K(\gamma,\lambda)}{\sigma + (1-\gamma)N}\right)^{1/\sigma} N. \square$$

Proof of Proposition 3

Part (i): From Cornes et al. (2021), the expectational stability of the equilibrium occurs if an only if $|\rho\prime(X^*)| < 1$. Then,

$$\rho'(X^*) = \left(\frac{(1-\sigma)K(\gamma,\lambda)N^\sigma}{\sigma}\right)(X^*)^{-\sigma} - \frac{(1-\gamma)}{\sigma}N$$

$$= \left(\frac{(1-\sigma)K(\gamma,\lambda)N^\sigma}{\sigma}\right)\left(\frac{K(\gamma,\lambda)}{\sigma+(1-\gamma)N}\right)^{-1} N^{-\sigma} - \frac{(1-\gamma)}{\sigma}N$$

$$\Rightarrow \rho'(X^*) = 1 - \sigma - (1-\gamma)N$$

Since $\sigma > 1$ and $\gamma > 0$, the above expression is negative. Therefore, a necessary and sufficient condition for the expectational stability of the equilibrium is $\sigma + (1-\gamma)N - 1 < 1 \Leftrightarrow \sigma + (1-\gamma)N < 2$.

Part (ii): The proof of this part follows the same steps as the proof of Part (i).

Part (iii): For this part, we must show that: $\left.\frac{\partial\rho^{(2)}}{\partial\gamma}\right|_{X=X^*} \neq 0$ and the same for the other parameters (σ and N), where $\rho^{(2)}(X) = \rho(\rho(X))$, which can be done after some tedious calculations. The existence of attracting two-cycles is a consequence of the fact that the Schwarzian derivative of ρ is negative, since $\rho''' < 0$ (see Devaney (1989) for details). $\square$

Proof of Proposition 5 The payoff function of individual n (up to a constant) is:

$$f(x) = \pi(X)\ln(\lambda_n) + \ln(x)$$

Since $\lambda_n \in (0,1)$, the second-order derivative satisfies:

$$f''(x) = \pi''(X)\ln(\lambda_n) - 1/x^2 < 0$$

Therefore, the first order condition defines the replacement function:

$$f'(x) = \pi'(X)\ln(\lambda_n) + \frac{1}{x} = 0 \Rightarrow x_n \equiv r_n(X) = \frac{1}{\pi'(X)\ln(\lambda_n^{-1})}$$

and the P-T-A map is:

$$\rho(X) = \sum_{n=1}^{N} r_n(X) = \frac{K(1.\lambda)}{\pi'(X)}$$

To prove the existence and uniqueness of X^* in (11), consider the function $\phi : [0, N] \to \mathbb{R}$ defined by:

$$\phi(y) = y\pi'(y) - K(1, \lambda).$$

The function $\phi(\cdot)$ is strictly increasing. Since $\phi(0) = -A < 0$ and $\phi(N) = N\pi'(N) - A = N\left(\pi'(N) - \frac{A}{N}\right)$, $\pi'(N) \geq \frac{A}{N}$ implies $\phi(N) \geq 0$. Therefore, there exists $X^* \in (0, N]$ such that $\phi(X^*) = 0$, and

$$X^* = \frac{A}{\pi'(X^*)}$$

is an equilibrium of Γ. Strict monotonicity of ρ guarantees uniqueness. □

Proof of Proposition 6 The expectational stability is characterized by $|\rho'(X^*)| < 1$. The derivative of ρ evaluated in the Nash equilibrium is:

$$\rho'(X^*) = -\frac{K(1,\lambda)\pi''(X^*)}{(\pi'(X^*))^2} = -\frac{K(1,\lambda)}{\pi'(X^*)}\left(\frac{\pi''(X^*)}{\pi'(X^*)}\right) = -\frac{X^*\pi''(X^*)}{\pi'(X^*)}$$

where the last equality results from equation (11). Expectational stability is verified, since $\pi' > 0$, $\pi'' > 0$, and $\frac{X^*\pi''(X^*)}{\pi'(X^*)} < 1$. □

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