Forecasting The Yield Curves Using Macroeconomics Expectations and Time-Varying Volatility

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Abstract

This study investigates the predictability of yield curves using the Nelson-Siegel models with macroeconomics expectations and time-varying volatility in yields. The results suggest that introducing macroeconomics expectations from market participants improves out-of-sample forecasting, mainly in short maturities. Also, time-varying volatility improves the economic relevance of these forecasts.

Resumo Este estudo investiga a previsibilidade das curvas de juros usando os modelos de Nelson-Siegel com expectativas macroeconômicas e volatilidade variando no tempo. Os resultados sugerem que a introdução de expectativas macroecoômicas dos participantes do mercado melhora a previsão fora da amostra, principalmente em maturidades curtas. Além disso, a volatilidade variando no tempo melhora a relevância econômica dessas previsões.

Keywords: yield curve forecasting, macroeconomics expectations, dynamic factor models, extended Kalman filter, out-of-sample forecasts

Palavras-chave: previsão da curva de juros, expecativas macroeconômicas, modelos de fatores dinâmicos, filtro de Kalman extendido, previsão fora da amostra

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1. Introduction

It is well known that the yield curves may unveil the market agents' expectations of the path of the interest rates in the short and long-term. The short-term expectation suggests the policy instrument's movement under the central bank's control. In contrast, the long-term rates interest rate indicates the market's averages of expected future short rates. The ability to forecast the behavior of the term structure of interest rates is essential for macroeconomists, financial economists, and fixed income managers

since the pricing of financial assets and their derivatives and risk management rely heavily on these forecasts.

The literature on yield curves proposes non-arbitrage, equilibrium, and factor models to capture information not directly observable in the market. Researchers, notably Nelson & Siegel (1987) (henceforth NS), Svensson (1994) and Diebold & Li (2006), investigated and developed parametric interpolation models to fitting and forecasting the yield curve. The reinterpretation of Diebold & Li (2006) of the Nelson & Siegel (1987) model explores the model's performance dynamically as a tool for forecasting the yield curve in such a way that the factors of the NS model were interpreted as level (long-term of interest rate), slope (short-term), and curvature (mid-term).

These models have gradually broadened in the past decades by introducing time-varying loading parameters and volatility, once considered constant over time. The first extension allow the loading terms vary over time as function of a single parameter λ, which became a latent factor, see Laurini & Hotta (2010), Hevia *et al.* (2015), while Koopman *et al.* (2010), Caldeira *et al.* (2010), Laurini & Caldeira (2016), innovated with both parameters time-varying. The available evidence suggests that time-varying parameters outperform the DNS model with fixed or estimated λ and variance. Also, Diebold *et al.* (2006), for instance, included macroeconomic variables in the DNS model and analyzed its relationship with the yield curve's behavior. Fernandes & Vieira (2019) employ a factor-augmented dynamic NS (DNS-Macro) model to predict the yield curve in the US that relies on an extensive data set of mostly forward-looking macroeconomic variables. The DNS-Macro with forward-looking macro factors significantly improves interest rate forecasts in out-of-sample analysis relative to many extant models in the literature. Vieira *et al.* (2017) suggest the same improvements in forecasting the Brazilian yield curve using forward-looking variables.

Challenging research in this domain is whether introducing time-varying volatility in yields with macroeconomics expectations in the DNS model may result in better out-of-sample results. In other words, will the DNS model with time-varying volatility outperform the forecasts against the benchmark model (without extensions)? And if so, will adding forward-looking macroeconomic factors improve the results against backward-looking macroeconomic indicators? In all two preliminary questions, we think so. Given the improvement of DNS models, their interpretations, and macro-finance literature, our research aims to fill gaps in this field. Our first goal is to evaluate the DNS-Macro with observed macroeconomic factors such as interest rate, economic activity, and inflation rate, then estimate the DNS-GARCH-Macro model in the same manner. Our second goal is to run the initial framework, including forward-looking macroeconomic factors, then examine out-of-sample results against the DNS benchmark model.

We provide a comprehensive evaluation of the forecasting performance in terms of statistical accuracy and economic relevance. First, we carry out a traditional evaluation based on statistical measures of forecasting performance: RMSFE, CSFE, MCS, and GW [1]. Second, we use different forecasts to construct mean-variance portfolios; then, we compare their Certainty equivalent. The results suggest

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[1]Root Mean Squared Forecast Error, Cumulative Squared Forecast Error, Model Confidence Set, and Giacomini-White tests, respectively

that introducing macroeconomics expectations from market participants improves out-of-sample forecasting, mainly in short maturities. Also, time-varying volatility improves the economic relevance of these forecasts.

2. Dynamic Factor Models for the Yield Curve

Dynamic factor models play a major role in econometrics since allowing the explanation of a large set of time series in terms of a small number of unobserved common factors (e.g Jungbacker *et al.* , 2014, and the literature therein). Many specifications for the yield curve can be viewed as dynamic factor models with a set of restrictions imposed on factor loadings (see, for example, Joslin *et al.* , 2013). In this section, we discuss the fourteen individual yield curve models beginning with the three-factor DNS model.

2.1. Dynamic Nelson-Siegel model

Diebold & Li (2006) proposed to study and estimate the term structure of interest rates using the model proposed by Nelson & Siegel (1987), assuming that the parameters vary over time. The following equation would describe the dynamics of the term structure

$$y_{i,t}(\tau_i) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i}\right), \tag{1}$$

where y_{it} denotes the yield at time t of a security with maturity τ_i, for $t = 1, \ldots, T$ and $i = 1, \ldots, N$, and λ is a decay parameter that can capture a variety of shapes of the yield curve through time, such as upward and downward sloping, and inversely humped. The $\beta_{1t}, \beta_{2t},$ and β_{3t} are time-varying parameters, or the state variables, that can be interpreted as the level, slope, and curvature latent components of the yield curve.

The DNS model is our starting point to model and forecast the yield curve. The dynamic movements or evolution of the yield curve factors, $\beta_{1t}, \beta_{2t},$ and β_{3t}, are assumed to follow a vector autoregressive process of first order, which allows for casting the yield curve latent factor model in state-space.

2.2. DNS in State-Space Representation

Diebold *et al.* (2006) note that DNS framework can be represented as a state space model by treating $\boldsymbol{\beta_t} = \beta_{j,t}$, for $j = 1, \ldots, 3$, as a latent vector. For these purpose, the general specification of the dynamic factor model is given by:

$$y_t = \Lambda(\lambda_t)\boldsymbol{\beta_t} + \boldsymbol{\varepsilon_t}, \qquad \boldsymbol{\varepsilon_t} \sim \mathcal{N}\left(0, \Sigma_\varepsilon\right), \tag{2}$$

where Λ is a $N \times K$ matrix of factor loadings, $\boldsymbol{\beta_t}$ is a K-dimensional stochastic process, and $\boldsymbol{\varepsilon_t}$ is the $N \times 1$ vector of measurement errors, whose covariance matrix given by Σ_ε. For any given, strictly

positive λ_1, the $N \times K$ factor loading matrix $\Lambda(\lambda_t)$ is given by:

$$\Lambda_{ij}(\lambda_k) = \begin{cases} 1, & j = 1 \\ \psi_{i2} = \dfrac{1 - z_{1i}}{\lambda_1 \tau_i}, & j = 2 \\ \psi_{i3} = \dfrac{1 - z_{1i}}{\lambda_1 \tau_i} - z_{1i}, & j = 3, \end{cases}$$

where $z_{1,i} = \exp(-\lambda_1 \tau_i)$.

The state-space framework is achieved by assuming that the dynamic movements or evolution of the yield curve factors $\boldsymbol{\beta_t}$ are modeled by the following first-order vector-autoregressive process:

$$\boldsymbol{\beta_{t+1}} = \boldsymbol{\mu} + \Phi\left(\boldsymbol{\beta_t} - \boldsymbol{\mu}\right) + \eta_t, \qquad \eta_t \sim \mathcal{N}\left(0, \Sigma_\eta\right), \tag{3}$$

where μ is a $K \times 1$ vector of constants, Φ is a $K \times K$ coefficient matrix, and Σ_η is the covariance matrix of the disturbance vector η_t, which is independent of the vector of residuals $\boldsymbol{\varepsilon_t}$ for all t.

The matrix of variance-covariance of the innovations to the measurement system Σ_ε is assumed to be diagonal. This assumption implies that deviations of the observed yields of various maturities from those implied by the fitted yield curve are uncorrelated. While the matrix of variance-covariance of the innovations to the transition system Σ_η is unrestricted so that shocks to the three yield-curve factors are correlated.

2.3. The Factor-Augmented Nelson-Siegel Model

The first extension of the DNS model is the inclusion of macro-finance indicators in the model. In addition to the yield data, we have p factors available representing macroeconomic information available at the monthly frequency, covering the same period as the yield data. We include these factors in the state vector such that it becomes $(\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, f_{1,t}, \ldots, f_{p,t})' = \boldsymbol{\beta}_t^{FA}$. With the extension of the state factor, the size of the coefficient matrix Φ^{FA} in the state equation increases from 3×3 to $(3+p) \times (3+p)$. The resulting state-space form is then given by

$$\mathbf{y}_t = \underbrace{\begin{bmatrix} \boldsymbol{\Lambda}(\lambda) & \mathbf{0}_{N \times p} \end{bmatrix}}_{\boldsymbol{\Lambda}(\lambda)^{FA}} \underbrace{\begin{bmatrix} \boldsymbol{\beta}_t \\ f_t \end{bmatrix}}_{\boldsymbol{\beta}_t^{FA}} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}_\varepsilon), \tag{4}$$

$$\underbrace{\begin{bmatrix} \boldsymbol{\beta}_{t+1} \\ f_{t+1} \end{bmatrix}}_{\boldsymbol{\beta}_{t+1}^{FA}} = (I_{3+p} - \Phi^{FA}) \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{0}_{p \times 1} \end{bmatrix} + \Phi^{FA} \underbrace{\begin{bmatrix} \boldsymbol{\beta}_t \\ f_t \end{bmatrix}}_{\boldsymbol{\beta}_t^{FA}} + \boldsymbol{\eta}_t^{FA}, \quad \boldsymbol{\eta}_t^{FA} \sim \mathcal{N}(\mathbf{0}_{3+p}, \boldsymbol{\Sigma}_\eta^{FA}) \tag{5}$$

for t = 1, ..., T , where the dimensions of $\boldsymbol{\Phi}$, $\boldsymbol{\eta}_{t+1}$, and $\boldsymbol{\Sigma}_\eta$ are increased as appropriate. The coefficient matrix structure implies that the macro-factors affect the individual yields through the Nelson-Siegel factors and feedback from the yields to the macro-factors. Therefore, we estimated the DNS-Macro in that framework. In the following section, we explain the algorithm used in the estimation procedure.

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2.4. Estimation Procedure

The estimation of the loading parameters λ in the measurement matrix in Eq. (2) is the key to estimating the state-space model. Keeping λ's fixed over the whole sample period, the equations (2) and (3) characterize a linear and Gaussian state-space model; thus, the Kalman filter can be used to obtain the likelihood function via the prediction error decomposition. On the other hand, treating the decay parameters λ as a stochastically time-varying latent factor and time-varying volatility in the yield processes introduce nonlinearities in the models handle by the extended Kalman filter. The estimation procedures are as discussed below.

2.4.1. Estimation of Linear State Space Models Based on the Kalman Filter

Assuming that the decay parameters are constant, the measurement equation becomes linear. In this case, the DNS model is treated as linear Gaussian state-space models. Given the state-space formulation of the dynamic factor model presented in (2) and (3), the Kalman filter can be used to obtain the likelihood function via the prediction error decomposition. An optimization algorithm is used to maximize the likelihood function estimated by the Kalman filter, an iterative process of estimating and updating the measurement and transition equations until an optimal point is obtained. In short, the filter computes the optimal yields forecasts and the corresponding forecasting errors, after which the Gaussian likelihood function is evaluated using the prediction-error decomposition of the likelihood function for the forecasts and the states. It sequentially updates the measurement and transition equations until an optimal yield forecast is achieved.

Consider the general state-space representation in (2) and (3). This state-space model is estimated by applying a Kalman filter, a recursive formula running forwards through time to estimate latent factors from past observations. The Kalman filter evaluates the conditional means and variances of the latent factors $\boldsymbol{\beta}_{t+1}$ conditional on the information available up to and including time t, denoted as $\hat{b}_{t+1|t}$ and $P_{t+1|t}$ respectively. Using the transition equation in (3), the optimal predicted estimates is then given by

$$\hat{b}_{t+1|t} = \boldsymbol{\mu} + \varPhi \left(b_{t|t} - \boldsymbol{\mu} \right), \tag{6}$$

$$P_{t+1|t} = \varPhi P_t \varPhi' + \Sigma_\eta, \tag{7}$$

where $P_{t+1|t}$ is mean square error (MSE), or covariance, matrix. Hence, the optimal filtered estimates $\hat{b}_{t+1}$ and P_{t+1} is given by

$$\hat{b}_{t+1} = \hat{b}_{t+1|t} + P_{t+1|t} \varLambda' F_{t+1|t}^{-1} v_{t+1}, \tag{8}$$

$$P_{t+1} = P_{t+1|t} - P_{t+1|t} \varLambda' F_{t+1|t}^{-1} P_{t+1|t}, \tag{9}$$

where $v_{t+1} = y_{t+1} - \varLambda \hat{b}_{t+1|t}$ is the prediction error, $F_{t+1|t} = \varLambda P_{t+1|t} \varLambda' + \Sigma_\varepsilon$ is the measurement prediction variance, and $P_{t+1|t} \varLambda' F_{t+1|t}^{-1}$ is called the Kalman gain.

The Kalman filter iterative process is initialized by using the unconditional mean and variance of $\boldsymbol{\beta}_t$. For this purpose, we carry out the 2-step procedure as described in Diebold & Li (2006). Specifically,

the unconditional mean and covariance matrix of the state vector is started as follows

$$b_{1|0} = \mathbb{E}\left[\boldsymbol{\beta}_t\right] = \boldsymbol{\mu} \quad \text{and} \quad P_{1|0} = \mathbb{E}\left[\boldsymbol{\beta}_t\boldsymbol{\beta}_t'\right] = \Sigma_\beta,$$

where the unconditional covariance matrix of the state vector is the solution of $\Sigma_\beta - \Phi\Sigma_\beta\Phi = \Sigma_\eta$, which we can solve using the properties of the vectorization operator *vec* (see Christensen & van der Wel, 2019, for details).

Let the vector θ collects all unknown coefficients in the in the VAR parameter matrix Φ, variance matrices Σ_ε and Σ_η, and Λ and $\boldsymbol{\mu}$. To estimate the parameters vector θ, the likelihood function is constructed from the update step by assuming that the forecasting errors v_t are Gaussian. The Gaussian log-likelihood function is computed as

$$\ell\left(\theta\right) = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T}v_t'F_t^{-1}v_t. \tag{10}$$

As a result, $\ell\left(\theta\right)$ can be evaluated by Kalman filter for a given value of θ. By maximizing this log-likelihood function with respect to the parameters (collective represented as a vector θ) using a quasi-Newton optimization method results in maximum likelihood estimates of the parameters. The algorithm BFGS is used to maximize the log-likelihood function specified in (10) to obtain the estimates of the parameters θ.

Under this framework, we estimated the linear models. In the next sections, we extend the DNS model by considering conditional heteroskedasticity in the yield processes and treating the loading parameters as a stochastically time-varying latent factor.

2.4.2. Time-Varying Volatility

In the DNS model, we assume volatility is constant over time, which may be a restrictive assumption since yield curves are related to trading in the financial markets, then changes in volatility may occur. In general, heteroscedasticity is a constant problem in economics, especially in finance. The Kalman filter can not handle this problem, that is, the filter works under the hypothesis that the variance matrices are constant, or at least known. Assuming the GARCH structure, the matrix Σ_ε is time-varying.

To allow for conditional heteroscedasticity in the yield processes, we modify the DNS model by following Koopman *et al.* (2010), who propose capturing yield curve volatility allowing for a common variance component jointly affecting all individual yields. The common variance component is modeled as a generalized autoregressive conditional heteroscedasticity (GARCH) process. Harvey *et al.* (1992) already provides an extensive framework for incorporating this GARCH$(1,1)$ model into unobserved component time series models and how to deal with corresponding implications for estimation procedures. This factor can be interpreted as the volatility of an underlying bond market portfolio according to Engle & Ng (1993). The error in the measurement equation (2) are decomposed as

$$\boldsymbol{\varepsilon}_t = \Gamma_\varepsilon\varepsilon_t^* + \boldsymbol{\varepsilon}_t^\dagger, \quad t = 1, \ldots, T, \tag{11}$$

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where Z_ε and $\varepsilon_t^\dagger$ are $N \times 1$ vectors of loadings and noise component respectively, and ε_t^* is a scalar representing the common disturbance term. The error components are mutually independent of each other and are distributed as follows

$$\varepsilon_t^* \sim \mathrm{NID}(0, h_t), \quad \text{and} \quad \varepsilon_t^\dagger \sim \mathrm{NID}(\mathbf{0}, \Sigma_\varepsilon^\dagger), \quad t = 1, \dots T, \tag{12}$$

where $\Sigma_\varepsilon^\dagger$ is a diagonal matrix and h_t is the variance specified as a GARCH process, according to Bollerslev (1986). In this case, we have

$$h_{t+1} = \gamma_0 + \gamma_1 \varepsilon_t^{*2} + \gamma_2 h_t, \quad t = 1, \dots, T, \tag{13}$$

and the estimated parameters have the constraints $\gamma_0 > 0$, $0 < \gamma_1 < 0$, $0 < \gamma_2 < 0$, $h_1 = \gamma_0(1 - \gamma_1 - \gamma_2)^{-1}$, and $(\gamma_1 + \gamma_2) < 1$. The weights vector Γ_ε can be normalized to avoid identification problems, such that $\Gamma_\varepsilon' \Gamma_\varepsilon = 1$, but we follow Koopman *et al.* (2010) and fixed γ_0 at 1×10^{-4}. The resulting time-varying variance matrix for ε_t is given by

$$\Sigma_\varepsilon(h_t) = h_t \Gamma_\varepsilon \Gamma_\varepsilon' + \Sigma_\varepsilon^\dagger, \tag{14}$$

where $\Sigma_\varepsilon(h_t)$ depends on a single factor described by the GARCH process in (13). The (unconditional) time-varying variance matrix of y_t is $\Lambda(\lambda)\Sigma_\beta\Lambda(\lambda)' + \Sigma_\varepsilon(h_t)$, where Σ_β is the solution of $\Sigma_\beta - \Phi\Sigma_\beta\Phi' = \Sigma_\eta$. The GARCH factor ε_t^* is incorporated in the measure equation (2), which is treated as a latent factor. Hence, we include ε_t^* in the state vector alongside the DNS factors.

The resulting observation and state equations of the DNS models with time-varying volatility can be rewritten into the state-space formulation as

$$y_t = \begin{bmatrix} \Lambda(\lambda) & \Gamma_\varepsilon \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_t \\ \varepsilon_t^* \end{bmatrix} + \varepsilon_t^\dagger, \quad \varepsilon_t^\dagger \sim \mathcal{N}(0, \Sigma_\varepsilon^\dagger), \tag{15}$$

$$\begin{bmatrix} \boldsymbol{\beta}_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix} = \begin{bmatrix} (I_j - \Phi_j)\boldsymbol{\mu} \\ 0 \end{bmatrix} \begin{bmatrix} \Phi_j & \mathbf{0}_{j \times 1} \\ \mathbf{0}_{1 \times j} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_t \\ \varepsilon_t^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_t \\ \varepsilon_{t+1}^* \end{bmatrix}, \qquad \begin{bmatrix} \boldsymbol{\eta}_t \\ \varepsilon_{t+1}^* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \Sigma_\eta & \mathbf{0}_{j \times 1} \\ \mathbf{0}_{1 \times j} & h_{t+1} \end{bmatrix} \right), \tag{16}$$

for $t = 1, \dots, T$. The addition of GARCH disturbances and extra parameters requires applying certain adjustments to the estimation procedure.

Since h_{t+1} in (13) is a function of its past values and unobserved values of ε_t^*, it is not possible to calculate the values required for h_{t+1} at time t. Specifically, Harvey *et al.* (1992) explains that, although the models are not conditionally Gaussian because knowledge of past observations does not imply knowledge of past GARCH errors, we may treat the models as though they are conditionally Gaussian. Because of that, in the presence of GARCH errors, the Kalman filter can be regarded as a quasi-optimal filter instead of optimal. Harvey *et al.* (1992) propose to take the expectation of the latent term in the volatility specification such that we obtain an estimate for h_{t+1}, given by

$$\hat{h}_{t+1|t} = \gamma_0 + \gamma_1 \mathbb{E}\left[\varepsilon_t^{*2} | \mathfrak{I}_t\right] + \gamma_2 \hat{h}_{t|t-1}, \quad t = 1, \dots, T, \tag{17}$$

where $\mathfrak{I}_t$ denotes all information available up to and including time t. To calculate the expectation term we note that

$$\varepsilon_t^* = \mathbb{E}\left[\varepsilon_{t-1}|\mathfrak{I}_t\right] + \left(\varepsilon_t^* - \mathbb{E}\left[\varepsilon_t^*|\mathfrak{I}_t\right]\right).$$

By squaring and taking conditional expectations we can shown that

$$
\begin{aligned}
\mathbb{E}\left[\varepsilon_t^{*\,2}|\mathfrak{I}_t\right] &= \mathbb{E}\left[\varepsilon_t^*|\mathfrak{I}_t\right]^2 + \mathbb{E}\left[\left(\varepsilon_t^* - \mathbb{E}\left[\varepsilon_t^*|\mathfrak{I}_t\right]\right)^2\right], \\
&= \hat{\varepsilon}_{t|t}^{*\,2} + P_{t|t}^{\varepsilon},
\end{aligned}
\tag{18}
$$

where $\hat{\varepsilon}_{t|t}^*$ is the last element of the filtered state $b_{t|t}$ and $P_{t|t}^{\varepsilon^*}$ is the last diagonal element of the $P_{t|t}$, the filtered variance of $\hat{\varepsilon}_{t|t}^*$. Then, we substitute the expression $\mathbb{E}\left[\varepsilon_t^{*\,2}|\mathfrak{I}_t\right]$ into (17) to obtain a prediction for the volatility component h_{t+1}. Lastly, we insert the predicted value h_{t+1} in the position (j,j) of the variance matrix Σ_η, corresponding to the location of ε_t^* in the state vector. With this framework we estimated the DNS-GARCH and extensions

3. Out-of-Sample Analysis

In order to forecasts h-months ahead, the steps below follow after the filtering step and estimation of the optimal set of parameters $\boldsymbol{\theta}$ throughout the sample, see Durbin & Koopman (2012) for more details. Therefore, we have the following

$$
\begin{aligned}
\mathbf{y}_{t+1} &= \boldsymbol{\Lambda}(\lambda)\mathbb{E}(\boldsymbol{\beta}_{t+1}|Y_t), \\
\bar{\mathbf{y}}_{t+1} &= \boldsymbol{\Lambda}(\lambda)\bar{\mathbf{b}}_{t+1},
\end{aligned}
\tag{19}
$$

where $\bar{\mathbf{b}}_{t+1}$ is the state vector and $\bar{\mathbf{B}}_{t+1}$ the variance matrix of the states calculated by the Kalman filter in (6) and (7). For other forecasts, the filter can be rewritten to $h = 2, \ldots, H$, as follows

$$
\begin{aligned}
\bar{\mathbf{b}}_{t+h} &= \boldsymbol{\mu} + \boldsymbol{\Phi}(\bar{\mathbf{b}}_{t+1} - \boldsymbol{\mu}), \tag{20}\\
\bar{\mathbf{B}}_{t+h} &= \boldsymbol{\Phi}\bar{\mathbf{B}}_{t+1}\hat{\boldsymbol{\Phi}}' + \boldsymbol{\Sigma}_\eta, \tag{21}\\
\bar{\mathbf{y}}_{t+h} &= \boldsymbol{\Lambda}(\lambda)\bar{\mathbf{b}}_{t+h}, \tag{22}
\end{aligned}
$$

where the states vector and the variance matrix of the previous estimation states are used to calculate the predictions in the step $h+1$. With the time-varying loading parameter, the difference for predictions lies in the loading matrix that multiplies the state vector, that is, $\mathbf{Z}_t(\mathbf{a}_{t|t-1})$ instead of $\boldsymbol{\Lambda}(\lambda)$ in (19) and (22).

The out-of-sample predictions are assessed by the relative sizes of the root mean square error (RMSPE) of all considered models relative to those from the DNS baseline model. The RMSPE is calculated as follows:

$$R(h,\tau) = \sqrt{\frac{1}{n}\sum_t [\hat{y}_{t+h|t}(\tau) - y_{t+h}(\tau)]^2}, \tag{23}$$

where n is the number of forecasts previously defined in 312. The drawback of using RMSPE is that this is a single statistic summarizing individual forecasting errors over an entire sample. Although often used, they do not give any insight as to where in the sample a particular model makes its largest and smallest forecast errors. Therefore, we also graphically analyze the cumulative squared forecast errors (CSFE) proposed by Welch & Goyal (2008). These cumulative prediction errors series clearly depicts when a model outperforms or underperforms a given benchmark and could motivate the use of adaptive forecast combination schemes. The CSFE is given by:

$$\text{CSFE}_m(h,\tau) = \sum_t \left[\left(\hat{y}_{t+h|t,\text{bench}}(\tau) - y_{t+h}(\tau) \right)^2 - \left(\hat{y}_{t+h|t,m}(\tau) - y_{t+h}(\tau) \right)^2 \right]. \tag{24}$$

In the case a model outperforms the benchmark, the $\text{CSFE}_m(h,\tau)$ will be an increasing series. If the benchmark produces more accurate forecasts, then $\text{CSFE}_m(h,\tau)$ will tend to be decreasing.

We use the Giacomini & White (2006) test to assess whether the forecasts of two competing models are statistically different. The Giacomini-White (GW) test is a test of conditional forecasting ability and is constructed under the assumption that forecasts are generated using a moving data window. This is a test of equal forecasting accuracy and as such can handle forecasts based on both nested and non-nested models, regardless from the estimation procedures used in the derivation of the forecasts.

Lastly, we implement the Model Confidence Set (MCS), approach developed by Hansen *et al.* (2011), which consist on a sequence of tests which permits to construct a set of 'superior' models, where the null hypothesis of Equal Predictive Ability (EPA) is not rejected at a certain confidence level. The EPA statistic tests is calculated for an arbitrary loss function, in our case we test squared errors of DNS model against competing models.

3.1. The Economic Value of the Yield Curve Predictability

Although our analysis is focused on statistical measures of predictive accuracy, it is important to evaluate the extent to which the apparent gains in predictive accuracy can be used in real time to improve investors' economic utility, that is, translate into better investment performance. Given that statistical significance does not necessarily imply economic significance (Thornton & Valente, 2012; Sarno *et al.* , 2016; Caldeira *et al.* , 2016; Gargano *et al.* , 2019), we assess the economic value of the predictive power of interest rates by investigating the utility gains accrued to investors who exploit the predictability of yield curve relative to a no-predictability alternative associated with the random-walk model.

In this section, we explore the empirical evidence linking statistical forecasting evaluation with economic utility. To this purpose, we consider a mean-variance investor with quadratic utility and relative risk aversion γ who allocates her portfolio on a risky bond with τ periods to maturity versus a one-month T-bill that pays the risky free rate (Rapach & Zhou, 2013). At the end of t, the investor allocates the following share of her portfolio to bond with maturity τ_i during $t+1$:

$$w_{i,t} = \left(\frac{1}{\gamma} \right) \left(\frac{\hat{r}_{t+h}^{(\tau_i)}}{\hat{\sigma}_{t+h}^{2,(\tau_i)}} \right) \tag{25}$$

where $\hat{r}_{t+h}^{(\tau_i)} = \tau_i y_t^{\tau_i} - (\tau_i - h)\hat{y}_{t+h}^{\tau_i - h}$ is a return forecast for the bond with maturity τ_i in time t and $\hat{\sigma}_i^{2\,2}$ is a forecast of the variance of bond returns. Over the forecast evaluation period, the investor realizes the average utility,

$$\hat{\nu}_i = \hat{\mu}_i - 0.5\gamma\hat{\sigma}_i^2, \tag{26}$$

where $\hat{\mu}_i$ ($\hat{\sigma}_i^2$) is the sample mean (variance) of the portfolio formed on the basis of $\hat{r}_{t+h}^{(\tau_i)}$ and $\hat{\sigma}_i^2$ over the forecast evaluation period. The resulting sequences of allocation weights are next used to calculate realized utilities. For each model m, the realized utility are converted into equivalent returns CER, i.e., the difference between utility (26) with model m and the DNS represents the utility gain accruing to using the competitors models forecast of the bond yields in place of the DNS benchmark forecast in the asset allocation decision. This utility gain (certainty equivalent return) can be interpreted as the portfolio management fee that an investor would be willing to pay to have access to the information in the model forecast relative to the information in the benchmark DNS model.

4. Data and Results

4.1. Data

This paper's data set consists of monthly closing prices observed for yields of future DI contracts. Based on the observed rates for the available maturities, the data were converted to fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 36, 42, 48, and 60 months, through interpolations using cubic splines. The database contains the maturities with the highest liquidity for January 2003 through December 2019 (T = 204 observations) and represents the most liquid DI contracts negotiated during the analyzed period. We assess the model's performance by splitting the sample into two parts: the first one includes 132 observations used to estimate all models' parameters. The second part is used to analyze the performance out-of-sample of bond portfolios obtained from the model, with 72 observations.

Table 1 displays the descriptive statistics for the Brazilian interest rate curve. For each of the 14 time series, we report average, standard deviation, minimum, maximum, and the last three columns contain sample autocorrelations at displacements of 1, 6, and 12 months. Descriptive statistics presented in Table 1 seem to confirm key stylized facts about yield curves: the sample average curve is upward sloping and concave, volatility is decreasing with maturity, autocorrelations are very high and increasing with maturity. Also, there is a high persistence in the yields: the first-order autocorrelation for all maturities is above 0.87 for each maturity.

Figure **??** presents a three-dimensional plot of the data set and illustrates how yield levels and spreads vary substantially throughout the sample. Although the yield series change heavily over time for each of the maturities, a strong common pattern in the 14 series over time is apparent. The sample contains 204 monthly observations with maturities of τ = 3, 6, 9, 12, 15, 18, 21 24, 27, 30, 36, 39, 48, and 60 months.

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[2]We follow the strategy of Rapach & Zhou (2013) and estimate the variance of bond returns using the sample variance computed from a one-year (252-obs) rolling window of historical returns.

Table 1: Descriptive statistics for the term structure of interest rates

Maturity	Mean	Std.dev.	Min	Max	Skewness	Kurtosis	$\hat{\rho}_1$	$\hat{\rho}_6$	$\hat{\rho}_{12}$
M3	12.19	4.52	4.30	27.50	0.90	4.11	0.96	0.70	0.48
M6	12.20	4.47	4.30	28.30	0.90	4.24	0.96	0.69	0.48
M9	12.24	4.42	4.40	29.00	0.92	4.56	0.95	0.68	0.48
M12	12.30	4.39	4.50	29.60	0.98	4.93	0.94	0.67	0.47
M15	12.38	4.34	4.50	30.40	1.06	5.39	0.94	0.65	0.46
M18	12.48	4.31	4.60	31.30	1.16	5.95	0.93	0.64	0.45
M21	12.57	4.28	4.70	32.30	1.29	6.64	0.93	0.62	0.44
M24	12.65	4.27	4.90	33.40	1.43	7.41	0.92	0.61	0.43
M27	12.74	4.25	5.00	34.30	1.57	8.15	0.92	0.60	0.42
M30	12.80	4.26	5.10	35.10	1.69	8.86	0.91	0.59	0.41
M36	12.92	4.28	5.30	36.70	1.95	10.35	0.90	0.57	0.40
M42	13.02	4.34	5.60	38.40	2.20	11.93	0.90	0.55	0.38
M48	13.10	4.38	5.70	39.40	2.37	12.96	0.89	0.54	0.38
M60 (Level)	13.18	4.35	6.00	39.40	2.44	13.12	0.89	0.55	0.38
Slope	0.99	2.10	-3.90	11.90	0.75	6.77	0.84	0.35	0.01
Curvature	-0.24	1.17	-3.00	3.10	0.18	2.86	0.87	0.40	0.08

NOTE: The table reports summary statistics for Brazil yield curve over the period 2003-2019. We examine monthly data, constructed using the spline method. For each maturity we show mean, standard deviation, skewness, raw kurtosis, minimum, maximum, and three auto-correlations coefficients, $\hat{\rho}_1$, $\hat{\rho}_6$, $\hat{\rho}_{12}$. Also the table reports proxy estimates for level, slope, and curvature of the yield curve. The proxies are defined as follows: for level, the highest maturity bond (60 months); for slope, the difference between the bond of 60 months and the bond of 3 months; and for curvature, two times the bond of 18 months minus the sum of bond of 3 months and bond of 60 months.

We use three macroeconomic factors: the Selic interest rate is the monetary policy interest rate, i.e., the key tool used by the Central Bank of Brazil (BCB) to implement the monetary policy. The Selic rate, or 'over Selic', is the Brazilian federal funds rate. Precisely, Selic rate is the weighted average interest rate of the overnight interbank operations – collateralized by federal government securities – carried out at the Special System for Settlement and Custody (Selic).

Under the inflation-targeting regime, the BCB's Monetary Policy Committee (Copom) regularly sets the target for the Selic rate. Within the relevant horizon for the monetary policy, the Copom aims to keep the Extended National Consumer Price Index (IPCA inflation rate) around the target and anchor inflation expectations. Accordingly, the BCB performs daily open market operations to keep the effective Selic rate at the target set by Copom.

Brazil has several price indexes that differ significantly in scope, depending on their particular purposes. A price index can be designed to reflect the cost of living for a specific group of households, but each household will have its price index based on its consumer basket. In this sense, there can be different inflation perceptions between what the citizen notices as inflation and the variation of several price indexes. The inflation index adopted here is the IPCA which is the reference for the Brazilian inflation-targeting system. The BCB ensures that the IPCA's annual inflation is centered at the inflation target set by the National Monetary Council (CMN).

The IBC-Br is an indicator of the monthly periodicity, which incorporates the pathway of the variables considered proxies to economic sectors such as Agriculture and livestock, Industry, and Services. The well-known adherence of the trajectory of the IBC-Br to the GDP behavior confirms the importance of monitoring the indicator to understand better and anticipate the activity analysis.

We use market expectations factors from the BCB's Market Expectations System, which monitors market expectations regarding the main macroeconomic variables, providing important inputs for the monetary policy decision-making process.

The BCB carries out the "Focus Survey", compiling forecasts of about 140 banks, asset managers, and other institutions (real sector companies, brokers, consultancies, etc.). The Survey daily monitors the market expectations for several inflation indices, the GDP and industrial production growth, the exchange rate, the Selic rate, fiscal indicators, and external sector variables. Based on this Survey, the BCB compiles daily – and releases weekly – the Focus Market Readout, which brings the summary of the statistics calculated over the information collected. We use the IPCA inflation accumulated median percent change, Over-Selic Target median percent p.y., and total GDP median percent change over the next 12 months (see Figure 1).

5. Results

We use a rolling estimation window of 72 monthly observations (i.e., six years) for computing our results. We produce forecasts for 1-month, 3-month, 6-month, and 12-month-ahead. To compare the performance of out-of-sample forecasts, we compute the root mean square forecast error (RMSFE). Moreover, the Giacomini & White (2006) test (GW-test) is used to assess whether each model outperforms the DNS. Table 2 report statistical measures of the out-of-sample forecasting performance at various

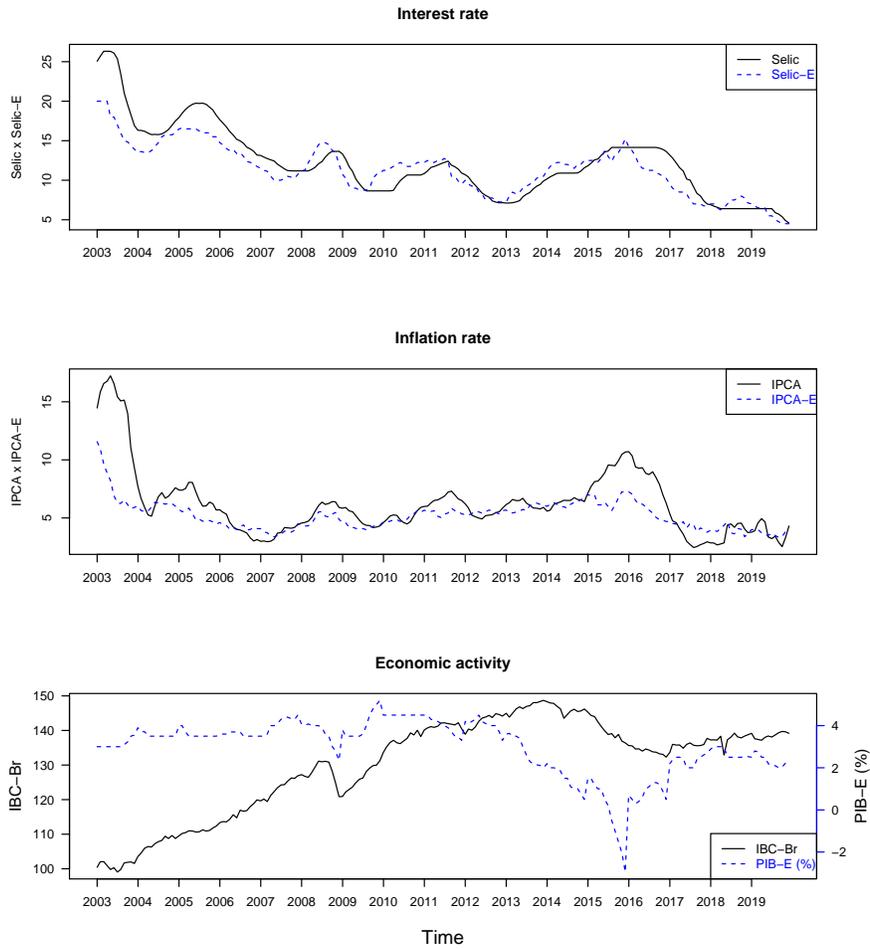Figure 1: This figure shows the SELIC interest rate, the Extended National Consumer Price Index (IPCA), and Index of Economic Activity of the Central Bank (IBC-Br) in solid lines and his expectations (GDP for IBC-Br) of one year in dashed lines, respectively. The sample contains 194 monthly observations from January 2003 through December 2019 for realized series and from 2002 through 2018 for expectations.

horizons. The first row of entries in each panel of the tables report the value of RMSFE (expressed in basis points) for the DNS model, while all other rows report statistic relative to the DNS.

Table 2 reports out-of-sample forecast performance measured in terms of RMSPE. This table is divided in four panels A through D, each corresponding to a different forecast horizon $(1, 3, 6$ and 12 steps ahead).The first row in each panel contains the RMSPE of the DNS baseline forecasts, whereas the remaining rows report RMSPE of a given model relative to those of the benchmark. Therefore any number below one indicates outperformance relative to the benchmark, whereas any number larger than one indicates underperformance. Asterisks to the right of entries indicate that, at the 10% level of significance, the null hypothesis of the GW test is not rejected. Bold type indicates that the model belongs to $\widehat{M}^*_{0.75}$, the set of superior models containing the best models with probability no less than 75%.

Table 2: Relative Root Mean Squared Forecast Errors

NOTE: The Table reports the relative root mean squared forecast errors (RMSFE) relative to the Dynamic Nelson Siegel benchmark (DNS) model for the 1-month, 3-months, 6-months, and 12-months forecast horizons. The evaluation sample is 2014:1 to 2019:12 (73 out-of-sample forecasts). The first line in each panel of the table reports the value of RMSFE (expressed in basis points) DNS model, while all other lines reports statistics relative to the DNS. The following model abbreviations are used in the table: DNS-Macro model for realized macroeconomic factors , DNS-MacroE1 model for market expectations of macroeconomic factors, and the last three models have time-varying volatility (-GARCH). Numbers smaller than one (shown inbold) indicate that models outperform the DNS, whereas numbers larger than one indicate underperformance. The stars on the right of the cell entries indicate the level at which the Giacomini & White (2006) test rejects the null of equal forecasting accuracy ($\star$ means rejection at least 10% level). The colors of the cell entries indicate the model belong to Model Confidence Set (MCS) Hansen *et al.* (2011).

Panel A: 1-month ahead forecasts.

Model	Maturity													
	3	6	9	12	15	18	21	24	27	30	36	42	48	60
DNS	38.49	39.37	42.97	47.56	50.15	52.93	54.84	55.62	56.47	58.19	59.61	60.92	62.70	64.63
DNS-Macro	1.008	1.019	**0.990**	**0.981**	**0.988**	**0.999**	1.007	1.013	1.019	1.021	1.026	1.026	1.026	1.025
DNS-MacroE1	**0.945**	**0.899***	**0.979**	1.014	1.028	1.039	1.048	1.055	1.062	1.063	1.069	1.071	1.080	1.094
DNS-GARCH	1.034	**0.987**	1.041	1.061	1.075	1.075	1.082	1.088	1.091	1.087	1.081	1.081	1.078	1.077
DNS-GARCH-Macro	1.008	1.019	**0.990**	**0.981**	**0.988**	**0.999**	1.007	1.013	1.019	1.021	1.026	1.026	1.026	1.025
DNS-GARCH-MacroE1	**0.945**	**0.900***	**0.979**	1.015	1.029	1.039	1.048	1.055	1.062	1.064	1.069	1.071	1.080	1.094

Panel B: 3-months ahead forecasts

Model	3	6	9	12	15	18	21	24	27	30	36	42	48	60
DNS	74.35	80.63	88.28	95.65	98.95	102.2	103.9	104.1	104.7	105.9	106.7	107.3	108.9	110.0
DNS-Macro	**0.964**	**0.983**	**0.977**	**0.977**	**0.984**	**0.995**	1.004	1.012	1.020	1.025	1.038	1.045	1.050	1.058
DNS-MacroE1	**0.811***	**0.897**	**0.939**	**0.969**	**0.989**	1.004	1.022	1.033	1.041	1.046	1.054	1.063	1.069	1.085
DNS-GARCH	**0.992**	1.042	1.046	1.039	1.049	1.056	1.064	1.077	1.086	1.088	1.096	1.107	1.109	1.118
DNS-GARCH-Macro	**0.964**	**0.983**	**0.977**	**0.977**	**0.984**	**0.995**	1.004	1.012	1.020	1.025	1.038	1.045	1.050	1.058
DNS-GARCH-MacroE1	**0.811***	**0.897**	**0.939**	**0.969**	**0.989**	1.005	1.022	1.033	1.041	1.046	1.054	1.063	1.069	1.085

Panel C: 6-months ahead forecasts

Model	3	6	9	12	15	18	21	24	27	30	36	42	48	60
DNS	127.7	139.1	149.3	156.7	159.3	160.4	160.2	158.7	157.5	156.5	153.5	152.4	151.0	149.2
DNS-Macro	**0.950**	**0.980**	**0.993**	1.003	1.011	1.022	1.030	1.038	1.046	1.053	1.066	1.076	1.083	1.093
DNS-MacroE1	**0.880**	**0.936**	**0.962**	**0.982**	**0.996**	1.010	1.024	1.033	1.045	1.053	1.062	1.071	1.076	1.089
DNS-GARCH	1.117	1.097	1.064	1.048	1.048	1.051	1.056	1.067	1.073	1.079	1.093	1.108	1.115	1.132
DNS-GARCH-Macro	**0.950**	**0.980**	**0.993**	1.003	1.011	1.022	1.030	1.038	1.046	1.053	1.066	1.076	1.083	1.093
DNS-GARCH-MacroE1	**0.880**	**0.937**	**0.962**	**0.982**	**0.997**	1.011	1.024	1.033	1.045	1.053	1.062	1.071	1.076	1.089

Panel D: 12-months ahead forecasts

| Model	3	6	9	12	15	18	21	24	27	30	36	42	48	60
DNS	244.0	257.6	265.8	269.5	268.0	264.4	259.3	253.6	248.0	243.5	234.3	226.7	221.2	213.2
DNS-Macro	1.008	1.055	1.085	1.102	1.114	1.123	1.129	1.134	1.139	1.141	1.147	1.150	1.152	1.154
DNS-MacroE1	1.056	1.079	1.090	1.096	1.098	1.098	1.099	1.098	1.097	1.096	1.093	1.089	1.087	1.081
DNS-GARCH	1.086	1.055	1.040	1.030	1.026	1.029	1.027	1.031	1.033	1.036	1.045	1.056	1.065	1.082
DNS-GARCH-Macro	1.008	1.055	1.085	1.102	1.114	1.123	1.129	1.134	1.139	1.141	1.147	1.150	1.152	1.154
DNS-GARCH-MacroE1	1.056	1.079	1.090	1.096	1.098	1.098	1.099	1.099	1.097	1.096	1.093	1.089	1.087	1.082

Figure 2: **Cumulative squared forecast errors (Nelson-Siegel Extensions)**

Note: Figures show the cumulative squared forecast errors (CSFE) of Nelson-Siegel Extensions relative to the DNS baseline model. Figure shows CSFEs for a 1- and 6-month forecast horizon. The evaluation sample is from January 2014 through December 2019 (73 out-of-sample forecasts).

(a) One-month ahead
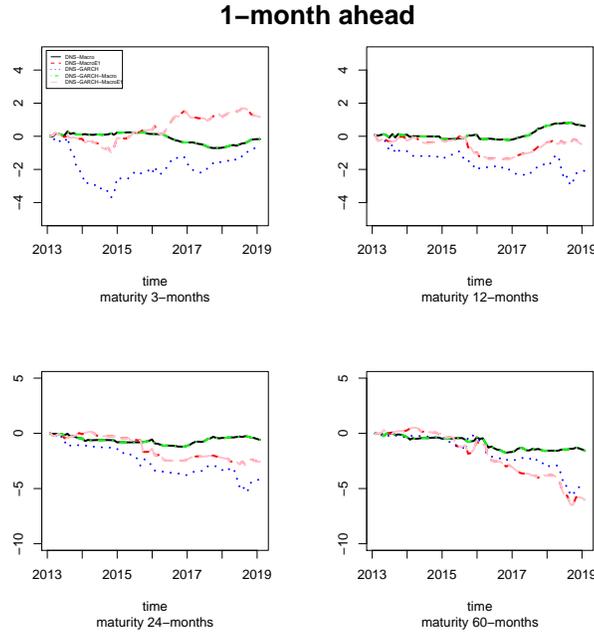
(b) Three-month ahead
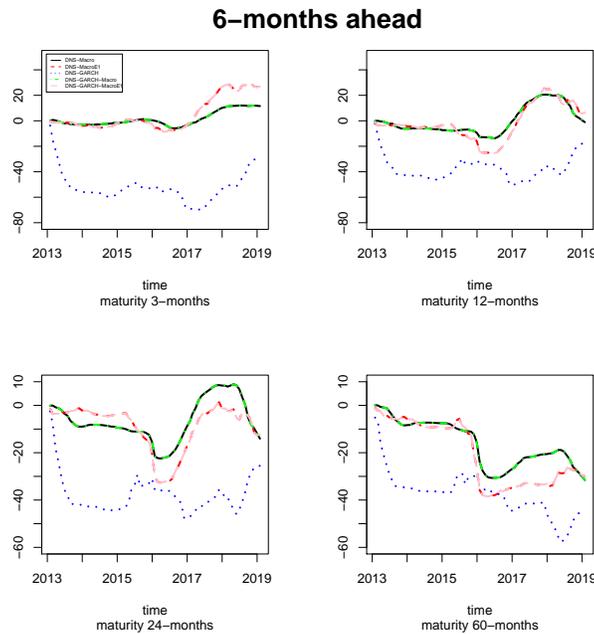
5.1. Economic Evaluation Results

Table 3 reports certainty equivalent (average utility gains in annualized percent return) for a mean-variance investor with $\gamma = \{0.1, 0.5, 1, 5\}$ who allocates among 1 to 5 years bonds and risk-free rate using forecasts based on competitors models in place of DNS forecasts.

Table 3: **Out-of-sample economic evaluation of the yield curve forecasting from 2011:01 to 2017:04**

Note: This table reports average utility gain (δ) the portfolio management fee (in annualized percent return) that an investor with mean-variance preferences and risk aversion coefficient of 0.1 to 5 would be willing to pay to have access to the forecasting method relative to the DNS benchmark forecast. The following model abbreviations are used in the table: DNS-Macro model for realized macroeconomic factors, DNS-MacroE1 model for market expectations of macroeconomic factors, and the last three models have time-varying volatility (-GARCH). The sample starts on January 2000 and the evaluation period is January 2011 to June 2017.

Model	$\gamma = 0.1$					$\gamma = 0.5$					$\gamma = 1$					$\gamma = 5$				
	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$
horizon = 1-month ahead																				
DNS − Macro	−118.33	8.325	3.410	1.268	0.367	−23.67	1.665	0.682	0.254	0.073	−11.83	0.832	0.341	0.127	0.037	−2.367	0.166	0.068	0.025	0.007
DNS − MacroE1	−198.17	7.018	4.611	2.723	1.759	−39.63	1.404	0.922	0.545	0.352	−19.82	0.702	0.461	0.272	0.176	−3.963	0.140	0.092	0.054	0.035
DNS − GARCH	665.06	47.87	20.69	11.70	7.621	133.01	9.574	4.137	2.34	1.524	66.51	4.787	2.068	1.170	0.762	13.30	0.957	0.414	0.234	0.152
DNS − GARCH − Macro	665.17	47.94	20.74	11.71	7.585	133.03	9.588	4.148	2.342	1.517	66.52	4.794	2.074	1.171	0.758	13.30	0.959	0.415	0.234	0.152
DNS − GARCH − MacroE1	665.03	47.87	20.69	11.71	7.619	133.01	9.574	4.138	2.341	1.524	66.50	4.787	2.069	1.171	0.762	13.30	0.957	0.414	0.234	0.152
horizon = 3-month ahead																				
DNS − Macro	−34.57	5.770	2.158	0.882	0.362	−6.917	1.154	0.432	0.176	0.072	−3.459	0.577	0.216	0.088	0.036	−0.692	0.115	0.043	0.018	0.007
DNS − MacroE1	−60.29	3.836	2.779	2.162	1.732	−12.06	0.767	0.556	0.432	0.346	−6.029	0.384	0.278	0.216	0.173	−1.206	0.077	0.056	0.043	0.035
DNS − GARCH	210.45	25.16	11.29	7.253	5.131	42.09	5.029	2.258	1.451	1.026	21.05	2.515	1.129	0.725	0.513	4.209	0.503	0.226	0.145	0.103
DNS − GARCH − Macro	210.65	25.26	11.36	7.214	4.948	42.13	5.052	2.272	1.443	0.99	21.07	2.526	1.136	0.721	0.495	4.213	0.505	0.227	0.144	0.099
DNS − GARCH − MacroE1	210.39	25.19	11.35	7.298	5.115	42.08	5.038	2.271	1.460	1.023	21.04	2.519	1.136	0.730	0.512	4.208	0.504	0.227	0.146	0.102
horizon = 6-month ahead																				
DNS − Macro	4.031	3.649	1.845	1.481	1.354	0.806	0.730	0.369	0.296	0.271	0.403	0.365	0.185	0.148	0.135	0.081	0.073	0.037	0.03	0.027
DNS − MacroE1	−0.267	4.840	3.623	3.3	2.993	−0.053	0.968	0.725	0.66	0.599	−0.027	0.484	0.362	0.33	0.299	−0.005	0.097	0.072	0.066	0.060
DNS − GARCH	88.66	16.16	9.802	7.331	5.336	17.73	3.232	1.96	1.466	1.067	8.866	1.616	0.980	0.733	0.534	1.773	0.323	0.196	0.147	0.107
DNS − GARCH − Macro	88.89	16.19	9.666	6.87	4.38	17.78	3.238	1.933	1.374	0.876	8.889	1.619	0.967	0.687	0.438	1.778	0.324	0.193	0.137	0.088
DNS − GARCH − MacroE1	88.61	16.27	9.814	7.113	4.835	17.72	3.253	1.963	1.423	0.967	8.861	1.626	0.981	0.711	0.483	1.772	0.325	0.196	0.142	0.097
horizon = 12-month ahead																				
DNS − Macro	2.452	−21.12	−9.624	−4.667	−2.205	0.490	−4.224	−1.925	−0.933	−0.441	0.245	−2.112	−0.962	−0.467	−0.22	0.049	−0.422	−0.192	−0.093	−0.044
DNS − MacroE1	2.601	−9.637	−1.619	0.926	1.791	0.520	−1.927	−0.324	0.185	0.358	0.260	−0.964	−0.162	0.093	0.179	0.0520	−0.193	−0.032	0.019	0.036
DNS − GARCH	119.38	26.33	11.35	8.786	5.27	23.876	5.266	2.270	1.757	1.054	11.94	2.633	1.135	0.879	0.527	2.388	0.527	0.227	0.176	0.105
DNS − GARCH − Macro	119.78	25.83	8.993	3.498	−3.588	23.96	5.165	1.799	0.700	−0.718	11.98	2.582	0.899	0.350	−0.359	2.396	0.516	0.180	0.070	−0.072
DNS − GARCH − MacroE1	119.44	26.57	10.42	5.839	−0.441	23.89	5.313	2.085	1.168	−0.088	11.94	2.657	1.042	0.584	−0.044	2.389	0.531	0.208	0.117	−0.009

The performance of economic evaluation is evaluated in terms of average utility gain (δ) excess return relative to the risk-free rate [3].

6. Concluding remarks

This research aims to explore forward-looking macroeconomic data in the factor-augmented DNS model to evaluate forecastability performance. Also, we include time-varying volatility to evaluate the out-of-sample performance. We assess the forecastability of the FADNS with time-varying volatility and forward-looking macroeconomic factors. Our first goal was to run the FADNS with observed macroeconomic factors such as interest rate, economic activity, and inflation rate. Also, evaluate the FADNS-GARCH model in the same manner. Our second goal was to run the initial framework, including forward-looking macroeconomic factors, then examine out-of-sample results against the DNS benchmark model.

The results in statistical analysis suggest the DNS with macroeconomics expectations outperforms the benchmark model in short maturities and the first three horizons of forecasts(1 month, 3 months, and 6 months). The DNS with macroeconomics expectations and time-varying volatility also had similar results. The economic evaluations s, on the other hand, indicate time-varying volatility improved the results. In other words, to estimate and forecast the volatility aggregates value to an investor, even in high-risk aversion.

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