On the welfare costs of business cycles: Beyond nondurable goods[†]

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Abstract

This paper constitutes the first effort to measure the welfare cost of economic cycles considering that the consumer derives utility from the consumption of nondurable goods and services, and the stock of durable goods. To accomplish such a task we put forward a novel approach based on a nonseparable utility function. As a result, the complementarities among the consumption categories and the correlations of the shocks in those series are take into account. Our findings for the US economomy suggest that the literature single-good approach tends to underestimate the welfare cost of business cycles.

Keywords: Welfare costs of business cycles, nondurable goods, services, durable goods.
JEL Classifications: C32, E21, E32.

Resumo

Este artigo constitui o primeiro esforço para mensurar o custo de bem-estar dos ciclos econômicos considerando que o consumidor deriva utilidade do consumo de bens não duráveis, serviços e estoque de bens duráveis. Para realizar tal tarefa, apresentamos uma nova abordagem baseada em uma função de utilidade não separável. Como resultado, são consideradas as complementaridades entre as categorias de consumo e as correlações dos choques nessas séries. Nossas descobertas para a economia dos EUA sugerem que a abordagem do bem único da literatura tende a subestimar o custo de bem-estar dos ciclos econômicos.

Palavras-chave: Custo de bem-estar dos ciclos econômicos, bens não duráveis, serviços, bens duráveis.

Área 8 - Microeconomia, Métodos Quantitativos e Finanças

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1 Introduction

Lucas (1987) measured the welfare cost of business cycles by the amount of consumption added to the observed consumption in order to make a risk-averse consumer indifferent between this shifted consumption and the counterfactual risk-free version of the observed consumption. Assuming a representative agent framework with constant relative risk aversion (CRRA) utility and a trend-stationary process to represent the consumption sequence, he concluded that there was little to gain from deepening stabilization policies.

The subsequent literature has been concerned with the simplifying assumptions made by Lucas (1987) and can be split into two branches (Barlevy et al. (2005)): one where idiosyncratic shocks were introduced and calibration-oriented methods were used to calculate the welfare cost of business cycles (Imrohoroğlu, 1989; Krusell and Smith, 1999; De Santis, 2007; Krusell et al., 2009), and another where the computations of the welfare costs of business cycles were performed using alternative econometric models for the consumption process (see, for instance, Obstfeld (1994), Issler et al. (2008), Reis (2009), Guillén et al. (2014)).

These two branches share an important characteristic, namely that the utility function depends on a single good that cannot be stored. In particular, the econometric models for consumption were estimated using different consumption measures: *i)* nondurable goods plus services, or *ii)* total expenditures, including durable goods (Obstfeld (1994), Pallage and Robe (2003), Issler et al. (2008), Reis (2009) and Guillén et al. (2014)). According to Lucas (2003), we can specify the welfare cost of business cycles as compensation in terms of one good or a subset of goods, rather than all of them. Although a clear definition can be established for any measure of consumption, the simple sum of different consumption categories is equivalent to assuming that they are perfect substitutes, which is not the case even for nondurable goods and services. Furthermore, a total consumption measure assumes that consumers derive utility from current expenditures on durable goods, but not from those in the past. However, consumers derive utility from the service flow of the stock of durable goods (Ogaki and Reinhart, 1998a,b).

Hai et al. (2020) calculated the welfare costs of business cycles by assuming that consumer preferences depend on nondurable goods and *memorable goods*. According to the authors, a good is memorable if a consumer can draw current utility from its past consumption experience through memory – e.g., trips, vacations, and entertainment. Hai et al. (2020) classified nondurable and memorable goods, concluding that the welfare costs of business cycles are overestimated when memorable goods are excluded from the analysis. Although this approach is interesting, it is important to take into account the usual categories of consumption – services and nondurable and durable goods – to calculate the welfare cost of business cycles, especially because each category of consumption has its own growth pattern and volatility.[1] Furthermore, durable goods should not be ignored because their volatility may contribute to a mechanism of minimizing welfare losses. Given that consumers derive utility from the stock of durable goods, they could postpone expenditures on durable goods during bad times to smooth expenditures on nondurable goods and services.

To properly consider the usual consumption categories, we put forward a novel approach to measure the welfare cost of business cycles by assuming a nonseparable utility function. In our analysis, we avoid the perfect substitution hypothesis and take into account the correlations of the shocks that affect nondurable goods, services, and the stock of durable goods. We keep a simple representation for the time series as in Obstfeld (1994), modeling the consumption series as random walk processes, which allow us to write closed-form solutions to the welfare cost of business cycles. Then, we show that the welfare cost is a function not only of the volatility of the three categories of consumption but also of the correlations of the shocks in the consumption processes, provided that the utility function is nonseparable. Next, we show that there is a map between the welfare cost of business cycles calculated as compensation in all consumption categories and alternative measures of welfare cost in which the compensation occurs only in some of the categories. Moreover, we argue that the compensation should be higher when it is not applied to all three categories of consumption. After all, we conduct an empiri-

[1] These two components, specially the volatility, affects the measures of the welfare costs of business cycles, as detailed in Section 2 by Proposition 1.

cal exercise for the US economy where we find that the literature single-good approach underestimates the welfare cost of business cycles.

The remainder of the paper is structured as follows. Section 2 presents our novel approach to estimate the welfare cost of business cycles. Section 3 explores alternative strategies to compensate the consumer by the risk since the preferences are no longer represented by a single-good utility. Section 4 estimates de welfare costs of business cycles to US economy. Finally, the main conclusions are summarized in Section 5.

2 Welfare cost of business cycles

When the welfare cost is calculated using a single-good setting, implicitly the researchers assume that the utility function is separable in the consumption measure effectively employed and its risk is the only one that matters. However, business cycles are not limited to any specific consumption category and, there is no reason, apart from simplicity, to suppose that preferences are represented by a separable utility function. To avoid such an assumption but maintain the tractability, we adopt the Cobb-Douglas aggregating function:

$$v(C,K,S) = C^{\theta_c} K^{\theta_k} S^{\theta_s} \tag{1}$$

where $\theta_x \in (0,1)$ for $x = \{c,k,s\}$ and $\theta_c + \theta_k + \theta_s = 1$.

We apply the CRRA utility to the aggregating function (1), obtaining:

$$u(C,K,S) = \begin{cases} \frac{1}{1-\gamma}\left(C^{\theta_c} K^{\theta_k} S^{\theta_s}\right)^{1-\gamma} & \text{if } \gamma > 0,\ \gamma \neq 1 \\ \theta_c \ln C + \theta_k \ln K + \theta_s \ln S & \text{if } \gamma = 1 \end{cases} \tag{2}$$

where $\gamma > 0$ is the relative risk aversion coefficient.[2]

[2] Dolmas (1998), Otrok (2001), Tallarini (2000), and Epaulard and Pommeret (2003) assumed different utility functions. As discussed by Lucas (2003) and Reis (2009), although many of these works found larger estimates of the costs of business cycles, this typically came at the cost of assuming that consumers are extremely averse to risk, which seems to be inconsistent with the risk-taking that we observe in their choices. Indeed, Otrok (2001) himself noted that "*it is trivial to make the welfare cost of business cycles as large as one wants by simply choosing an appropriate form for preferences*". Not by chance, Issler et al. (2008) and Guillén et al. (2014) maintain the preferences adopted by Lucas (1987) in order to avoid such critique.

Motivated by the rational expectations–permanent income theory, Obstfeld (1994) assumed a random-walk process for nondurable plus service consumption to evaluate the welfare cost of business cycles. Based on the consumer intertemporal problem, Hall (1978) derived the well-known random-walk hypothesis for nondurable consumption. Mankiw (1982) applied this approach to durable goods, concluding that the stock of durable goods follows a random-walk process as well. Finally, unit root tests generally characterize those series as integrated processes.[3] Thus, we assume random-walk processes and, as a consequence, each consumption category has permanent shocks, as follows:

$$X_t = \alpha_{0,x}(1+\alpha_{1,x})^t e^{\sum_{i=0}^{t} \varepsilon_{x,i} - 0.5(t+1)\sigma_x^2} \tag{3}$$

where $X_t \in \{C_t, K_t, S_t\}$, $x \in \{c, k, s\}$, and $e^{-0.5(t+1)\sigma_x^2}$ is a usual reparameterization. Furthermore, we allow the shocks to be contemporaneously correlated by assuming that $\boldsymbol{\varepsilon}_t = [\varepsilon_{c,t}\ \varepsilon_{k,t}\ \varepsilon_{s,t}]^\intercal \overset{iid}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$, where

$$\boldsymbol{\Omega} \equiv \begin{bmatrix} \sigma_c^2 & \sigma_{c,k} & \sigma_{c,s} \\ \sigma_{c,k} & \sigma_k^2 & \sigma_{k,s} \\ \sigma_{c,s} & \sigma_{k,s} & \sigma_s^2 \end{bmatrix} \tag{4}$$

Obstfeld (1994) treated the initial consumption, C_0, as known, and, as a result, the welfare cost of business cycles becomes a function of it. However, Lucas (1987) assumed a trend-stationary consumption process, which allowed him to use the unconditional expectation. Consequently, the initial consumption is not known by the consumers, who evaluate their well-being behind a veil of ignorance. To maintain this feature and avoid the dependence of C_0, we assume that the expectation of the lifetime utility is taken before the realization of any shock in the beginning of time, so C_0 does not belong to information set $\mathcal{I}_0$.[4]

[3] Unit root tests suggest that time series C_t, S_t, K_t, and $C_t + S_t$ are integrated of order 1. These results are available upon request.

[4] This explains why the reparameterization $-0.5(t+1)\sigma_x^2$ in equation (3) depends on $t+1$ instead of t. Furthermore, the deterministic term of equation (3) should be $\tilde{\alpha}_{0,x}(1+\alpha_{1,x})^{t+1}$; however, we rewrite it as $\alpha_{0,x}(1+\alpha_{1,x})^t$, in which $\alpha_{0,x} \equiv \tilde{\alpha}_{0,x}(1+\alpha_{1,x})$.

Finally, we define the welfare cost of business cycles as the scalar λ that solves:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u((1+\lambda)C_t,(1+\lambda)K_t,(1+\lambda)S_t)\right]=\sum_{t=0}^{\infty}\beta^t u\left(\mathbb{E}_0[C_t],\mathbb{E}_0[K_t],\mathbb{E}_0[S_t]\right) \quad (5)$$

where $\beta\in(0,1)$ is the intertemporal discount factor. Therefore, the compensation λ is the same for each consumption category and we consider not only the risk of any specific consumption category but of all of them.[5] Proposition 1 present the welfare cost of business cycles λ.

Proposition 1. *Under utility function (2), $\beta\in(0,1)$, and the consumption process (3) with $\boldsymbol{\varepsilon}_t\overset{iid}{\sim}\mathcal{N}(\mathbf{0},\boldsymbol{\Omega})$, solving (5) yields:*

$$\lambda=\begin{cases} e^{0.5\vartheta_1/(1-\beta)}-1, & \text{if } \gamma=1\\ e^{0.5[(\gamma-1)\vartheta_2+\vartheta_1]}\left[\frac{1-\beta(1+\alpha_1)^{1-\gamma}e^{0.5(\gamma-1)[(\gamma-1)\vartheta_2+\vartheta_1]}}{1-\beta(1+\alpha_1)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1, & \text{if } \gamma\neq 1,\ \gamma>0\end{cases}$$

provided that $\beta(1+\alpha_1)^{1-\gamma}<1$ and $\beta(1+\alpha_1)^{1-\gamma}e^{0.5(\gamma-1)[(\gamma-1)\vartheta_2+\vartheta_1]}<1$, where $\alpha_1\equiv(1+\alpha_{1,c})^{\theta_c}(1+\alpha_{1,k})^{\theta_k}(1+\alpha_{1,s})^{\theta_s}-1$, $\vartheta_1\equiv\theta_c\sigma_c^2+\theta_k\sigma_k^2+\theta_s\sigma_s^2$ and $\vartheta_2\equiv\theta_c^2\sigma_c^2+\theta_k^2\sigma_k^2+\theta_s^2\sigma_s^2+2\theta_c\theta_k\sigma_{c,k}+2\theta_c\theta_s\sigma_{c,s}+2\theta_k\theta_s\sigma_{k,s}$.

Proof: *Appendix A.*

For $\gamma=1$, the utility becomes separable and, not coincidentally, λ depends on the volatility of each shock, gathered in ϑ_1, but not on their correlations. As expected, for $\gamma\neq 1$, those correlations, included in ϑ_2, become relevant.

3 Alternative welfare costs measures

While it seems natural to apply the compensation λ to all consumption categories, as done in condition (5), alternative approaches are available. It is instructive to evaluate them. To do so, we define the first alternative measure of the welfare costs of business

[5] In Section 3 we discuss alternative measures of the welfare cost of business cycles in which the compensation does not fall on all consumption categories.

cycles as the scalar λ_c that solves:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u((1+\lambda_c)C_t, K_t, S_t)\right] = \sum_{t=0}^{\infty}\beta^t u\left(\mathbb{E}_0[C_t], \mathbb{E}_0[K_t], \mathbb{E}_0[S_t]\right) \tag{6}$$

In this case, the consumer is compensated by an extra amount of just C by the risk relative to C, K and S. Analogously, we define λ_k and λ_s as compensations just in K and S, respectively. In addition, the welfare cost of business cycles can be measure by compensations over two consumption categories. In this vein, we define another alternative measure as the scalar λ_{ck} that solves:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u((1+\lambda_{ck})C_t, (1+\lambda_{ck})K_t, S_t)\right] = \sum_{t=0}^{\infty}\beta^t u\left(\mathbb{E}_0[C_t], \mathbb{E}_0[K_t], \mathbb{E}_0[S_t]\right) \tag{7}$$

And, we define λ_{cs} and λ_{ks} analogously. Proposition 2 details how each alternative measure of the welfare cost of business cycles is related to the measure λ implied by condition (5).

Proposition 2. *Assume the utility function (2) and $\beta \in (0,1)$. Provided that the scalar λ in (5) is well-defined, the following results hold:*

$$\begin{array}{ll} (i) & 1+\lambda_i = (1+\lambda)^{1/\theta_i}, \\ (ii) & 1+\lambda_{ij} = (1+\lambda)^{1/(\theta_i+\theta_j)} \end{array}$$

where $i, j \in \{c, k, s\}$, with $i \neq j$.
Proof: *Appendix B.*

Therefore, no matter how the welfare cost of business cycle is measured, it is related to the scalar λ. As expected, the compensation needs to be larger when it increases just one consumption category, i.e. $\lambda_i > \lambda$, because $\theta_i < 1$. In the same vein, $\lambda_{ij} > \lambda$ because $\theta_i + \theta_j < 1$.

In the single-good approach of the literature, the scalar λ_x^L measures the welfare-cost of business cycle as follows:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{((1+\lambda_x^L)X_t))^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\beta^t \frac{(\mathbb{E}_0[X_t])^{1-\gamma}}{1-\gamma} \tag{8}$$

where we keep the CRRA utility, and X_t is measure by nondurable goods expenditure, C, or the nondurable goods plus services expendidures, $C+S$, i.e. $X_t \in \{C_t, C_t+S_t\}$. For this reason, λ_x^L is indexed by $x \in \{c, c+s\}$. Assuming that $X_t \in \{C_t, C_t+S_t\}$ follows the consumption process (3), with $\varepsilon_{x,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_x^2)$, the scalar λ_x^L is given by:

$$\lambda_x^L = \begin{cases} e^{0.5\sigma_x^2/(1-\beta)} - 1 \text{ if } \gamma = 1 \\ e^{0.5\gamma\sigma_x^2}\left[\frac{1-\beta(1+\alpha_{1,x})^{1-\gamma}e^{0.5\gamma(\gamma-1)\sigma_x^2}}{1-\beta(1+\alpha_{1,x})^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1 \text{ if } \gamma \neq 1,\ \gamma > 0 \end{cases}, \tag{9}$$

provided that $\beta(1+\alpha_{1,x})^{1-\gamma} < 1$ and $\beta(1+\alpha_{1,x})^{1-\gamma}e^{0.5\gamma(\gamma-1)\sigma_x^2} < 1$, for $x \in \{c, c+s\}$.

It is interesting to compare λ_c and λ_c^L. While the former measures the extra amount of C to compensate the consumer by the risk related to C, K and S, the latter adjusts C taking into account only its own risk. Thus, at first, we expect to observe $\lambda_c > \lambda_c^L$, especially for $\gamma \neq 1$ because ϑ_2 gathered the shocks correlations (see Proposition 1). Despite that, it is worth remember that consumers derive utility from the stock of durable goods and, as a result, they could postpone expenditures on durable goods during bad times to fund expenditures on nondurable goods and services. Thus, take into account K gives consumers flexibility to deal with negative shocks.

Another interesting comparison is between λ_{cs} and λ_{c+s}^L. The latter, not only neglects K, but the summation of C and S could imply an additional smoothing. In this vein, we expect to observe $\lambda_{cs} > \lambda_{c+s}^L$.

4 Application for the U.S. economy

We extracted annual data from the National Income and Product Accounts for 1929-2020 for personal consumption expenditures on nondurable goods and services and the current-cost net stock of consumer durable goods. On the one hand, it is usual to exclude the war period from the econometric analyses and, for this reason, our analysis encompasses a Sub-Sample for the years between 1950 and 2020. However, on the other hand, we investigate precisely the welfare costs of economic fluctuations and eliminating the most turbulent periods certainly underestimates these costs. Therefore, we also examine the full sample period in the same spirit as Guillén et al. (2014).

We use the implicit price deflator of each consumption category and midperiod population to calculate the real per capita series for C, S, and K. Thus, we take the log and first-difference of equation (3) and apply the ordinary least squares (OLS) estimator to the system for C, K, and S considering the full period and the post-war period. To apply the literature single-good approach, we also calculate real per capita series for $C + S$, using the divisia index and the midperiod population. Then, for C and $C + S$ we apply the single-equation OLS estimation based on the first difference of the log of the equation (3).

Table 1 reports the system estimations. As can be seen, K has the largest growth rate and variance in both samples, and the consumption category shocks show positive correlations. Table 1 also displays the Cobb–Douglas parameters. Using the nominal values of C, K and S, these parameters were calibrated by the average value of $X_t/(C_t + K_t + S_t)$ for $X_t \in \{C_t, K_t, S_t\}$. Note that S and K have the largest participation in consumers' budgets in both samples. Those characteristics of the series of S and K reinforce the importance in modeling preferences on those categories.[6]

Table 1: Estimation of the systems based on consumption processes (3)

	Full sample (1930–2020)					Sub-sample (1950–2020)				
Variable	θ_x	$1 + \hat{\alpha}_{1,x}$	$1000 \times \hat{\Omega}$			θ_x	$1 + \hat{\alpha}_{1,x}$	$1000 \times \hat{\Omega}$		
C	0.241	1.014	0.649	0.347	0.213	0.216	1.014	0.223	0.090	0.175
S	0.391	1.020	0.347	0.524	0.097	0.414	1.021	0.090	0.295	0.082
K	0.367	1.035	0.213	0.097	1.170	0.370	1.041	0.175	0.082	0.664

Table 2 displays the single-equation estimations. It is worth mentioning that the volatility of $C + S$ is lower than the one of C and S in both periods. In this vein, the sum of expenditures on non-durable goods and services results in additional smoothing.

Table 3 presents the welfare costs of business cycles, considering $\beta = 0.95$ and $\gamma = \{1,\ 2.5,\ 5.0,\ 7.5\}$. In addition to λ, we report the alternative and the literature measures of the welfare costs of business cycles. Each measure is larger in the full sample than

[6] As we mention in the introduction, the literature has already incorporated the analysis of services, but as a substitute for nondurables. We extend those studies by adding durables and relaxing the separability assumption.

Table 2: Estimation of the single-equations based on consumption processes (3)

	Full sample (1930–2020)		Sub-sample (1950–2020)	
Variable	$1+\hat{\alpha}_{1,x}$	$1000 \times \hat{\sigma}_x^2$	$1+\hat{\alpha}_{1,x}$	$1000 \times \hat{\sigma}_x^2$
C	1.014	0.649	1.014	0.223
$C+S$	1.017	0.500	1.018	0.213

in the subsample, and any measure increases with γ. In particular, in the full sample period λ is 0.795% for $\gamma = 1$ and 0.953% for $\gamma = 7.5$. For the subsample period these percentages become 0.417% and 0.470%, respectively. In both sample, fixing γ, $\lambda_c > \lambda_c^L$ and $\lambda_{ck} > \lambda_{c+k}^L$. In this vein, the literature single-good approach underestimate the welfare costs of business cycles. The lower percentages for λ_c^L and λ_{c+s}^L are not surprising, but our approach leads to substantially larger welfare costs. For instance, considering the subsample period and $\gamma = 2.5$, λ_c is five times bigger than λ_c^L and λ_{ck} is twice λ_{c+k}^L.

Table 3: Percentage welfare cost of business cycles

	Full sample (1930-2020)				Sub-sample (1950-2020)			
γ	1	2.5	5	7.5	1	2.5	5	7.5
λ	0.795	0.851	0.908	0.953	0.417	0.432	0.451	0.470
λ_c	3.340	3.579	3.820	4.014	1.944	2.015	2.107	2.192
λ_k	2.175	2.330	2.485	2.611	1.131	1.172	1.225	1.274
λ_s	2.046	2.191	2.337	2.456	1.010	1.046	1.094	1.138
λ_{ck}	1.309	1.401	1.495	1.570	0.712	0.738	0.772	0.803
λ_{cs}	1.261	1.350	1.440	1.512	0.662	0.686	0.717	0.746
λ_{ks}	1.049	1.123	1.197	1.258	0.532	0.551	0.576	0.599
λ_c^L	0.651	1.182	1.662	1.965	0.223	0.402	0.557	0.647
λ_{c+s}^L	0.501	0.856	1.146	1.317	0.213	0.356	0.466	0.528

Note: We set $\beta = 0.95$. For λ, see Proposition 1. For λ_i and λ_{ij}, with $i, j \in \{c, k, s\}$ and $i \neq j$, see Proposition 2. For λ_x^L, $x \in \{c, c+s\}$, see condition (9).

Considering $\gamma = 2.5$, Table 4 presents the welfare costs of business cycles measures for different values of the intertemporal discount factor, β. The costs increase with β. In particular, λ_c reaches 6.815% for $\beta = 0.99$. Furthermore, $\lambda_c > \lambda_c^L$ and $\lambda_{ck} > \lambda_{c+s}^L$ for any of the β considered, which reinforces the conclusion that the literature single-good approach underestimate the welfare costs of business cycles.

Table 4: Percentage welfare cost of business cycles

	Full sample (1930-2020)				Sub-sample (1950-2020)			
β	.93	.95	.97	.99	.93	.95	.97	.99
λ	0.690	0.851	1.111	1.602	0.353	0.432	0.555	0.777
λ_c	2.892	3.579	4.693	6.815	1.647	2.015	2.597	3.650
λ_k	1.885	2.330	3.049	4.412	0.958	1.172	1.508	2.115
λ_s	1.773	2.191	2.867	4.147	0.856	1.046	1.346	1.888
λ_{ck}	1.135	1.401	1.832	2.643	0.604	0.738	0.949	1.330
λ_{cs}	1.093	1.350	1.764	2.546	0.562	0.686	0.883	1.237
λ_{ks}	0.909	1.123	1.467	2.115	0.451	0.551	0.709	0.992
λ_c^L	0.919	1.182	1.654	2.756	0.314	0.402	0.561	0.926
λ_{c+s}^L	0.676	0.856	1.169	1.841	0.283	0.356	0.482	0.743

Note: We set $\gamma = 2.5$. For λ, see Proposition 1. For λ_i and λ_{ij}, with $i, j \in \{c, k, s\}$ and $i \neq j$, see Proposition 2. For λ_x^L, $x \in \{c, c+s\}$, see condition (9).

5 Conclusions

We investigate the welfare cost of business cycles by considering expenditures on non-durable goods and services and the stock of durable goods. To do so, we propose a novel approach based on a nonseparable utility function and a per-period compensation λ over these consumption categories. We also show how this measure is related to alternative measures that emerges from other compensation schemes. This is particularly interesting as it allows a more direct comparison of our results with those based on the literature single-good approach.

By applying our approach to US economy, we found larger welfare costs of business cycles, concluding that the literature single-good approach underestimate such costs. Although this result are not surprising, because our approach take into account three sources of risk instead of just one, it is worth remembering that consumer could postpone expenditures on durable goods during bad times to smooth expenditures on nondurable goods and services. Therefore, calculate the welfare cost of business cycles considering all these consumption categories is a valuable contribution.

Finally, it is worth remembering that to implement our approach we assume a Cobb–Douglas aggregating function, and other assumptions present in the literature such as the CRRA utility and the random-walk consumption process. These hypotheses keep the tractability of the problem, but can be relaxed in future works.

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A Lemma 1 and Proposition 1

Lemma 1 establishes the results necessary to solve (5) for λ, which is stated in Proposition 1.

Lemma 1. *Under consumption process (3) with $\boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$, the following results hold:*

$$
\begin{aligned}
&(i) \quad \mathbb{E}_0[X_t] = \alpha_{0,x}(1+\alpha_{1,x})^t, \\
&(ii) \quad \mathbb{E}_0[\ln X_t] = \ln \alpha_{0,x} + t \ln(1+\alpha_{1,x}) - 0.5(t+1)\sigma_x^2, \\
&(iii) \quad \mathbb{E}_0\left[\left(C_t^{\theta_c} K_t^{\theta_k} S_t^{\theta_s}\right)^{1-\gamma}\right] = \alpha_0^{1-\gamma}(1+\alpha_1)^{(1-\gamma)t} e^{0.5(\gamma-1)[(\gamma-1)\vartheta_2+\vartheta_1](t+1)}
\end{aligned}
$$

where $\alpha_0 \equiv \alpha_{0,c}^{\theta_c}\alpha_{0,k}^{\theta_k}\alpha_{0,s}^{\theta_s}$, $\alpha_1 \equiv (1+\alpha_{1,c})^{\theta_c}(1+\alpha_{1,k})^{\theta_k}(1+\alpha_{1,s})^{\theta_s} - 1$, $\vartheta_1 \equiv \theta_c\sigma_c^2 + \theta_k\sigma_k^2 + \theta_s\sigma_s^2$ and $\vartheta_2 \equiv \theta_c^2\sigma_c^2 + \theta_k^2\sigma_k^2 + \theta_s^2\sigma_s^2 + 2\theta_c\theta_k\sigma_{c,k} + 2\theta_c\theta_s\sigma_{c,s} + 2\theta_k\theta_s\sigma_{k,s}$.

Proof of Lemma 1. Given that $\boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$, $e^{\sum_{i=0}^t \varepsilon_{x,i}}$ is log-normal, and

$$
\mathbb{E}_0\left[e^{\sum_{i=0}^t \varepsilon_{x,i}}\right] = e^{0.5\mathbb{V}_0\left[\sum_{i=0}^t \varepsilon_{x,i}\right]} = e^{0.5(t+1)\sigma_x^2},
$$

which proves (i). Given that $\mathbb{E}_0\left[\sum_{i=1}^t \varepsilon_{x,t}\right] = 0$, taking log of (3) and the conditional expectation lead to (ii). To prove (iii), substitute (3) in $\mathbb{E}_t\left[C_t^{\theta_c} K_t^{\theta_k} S_t^{\theta_s}\right]$. Collect the intercepts in α_0, the growth rates in $1+\alpha_1$ and the reparameterization terms in ϑ_1. Finally, log-normality implies that

$$
\mathbb{E}_0\left[e^{(1-\gamma)\sum_{i=0}^t(\theta_c\varepsilon_{c,i}+\theta_k\varepsilon_{k,i}+\theta_s\varepsilon_{s,i})}\right] = e^{0.5\mathbb{V}_0\left[(1-\gamma)\sum_{i=0}^t(\theta_c\varepsilon_{c,i}+\theta_k\varepsilon_{k,i}+\theta_s\varepsilon_{s,i})\right]} = e^{0.5(1-\gamma)^2(t+1)\vartheta_2},
$$

which proves (iii).

Proof of Proposition 1. For $\gamma = 1$: Lemma 1 (i) and (ii) are used to calculate the RHS and the LHS of (5) respectively, which are finite provided that $\beta < 1$. For $\gamma \neq 1$: Lemma 1 (iii) is used to calculate the LHS of (5), which is finite provided that $\beta(1+\alpha_1)^{1-\gamma}e^{0.5(\gamma-1)[(\gamma-1)\vartheta_2+\vartheta_1]} < 1$; Lemma 1 (i) is used to calculate the RHS of (5), which is finite provided that $\beta(1+\alpha_1)^{1-\gamma} < 1$.

B Proposition 2

Proof of Proposition 2. For $\gamma = 1$, equation (2) implies that

$$u\left((1+\lambda)C, (1+\lambda)K, (1+\lambda)S\right) = \ln(1+\lambda) + \theta_c \ln C + \theta_k \ln K + \theta_s \ln S$$

$$u\left((1+\lambda_c)C, K, S\right) = \theta_c \ln(1+\lambda_c) + \theta_c \ln C + \theta_k \ln K + \theta_s \ln S$$

$$u\left((1+\lambda_{ck})C, (1+\lambda_{ck})K, S\right) = (\theta_c + \theta_k) \ln(1+\lambda_{ck}) + \theta_c \ln C + \theta_k \ln K + \theta_s \ln S$$

Therefore, provided that λ is well-defined, $1+\lambda_c = (1+\lambda)^{1/\theta_c}$ and $1+\lambda_{ck} = (1+\lambda)^{1/(\theta_c+\theta_k)}$. Analogously, $1+\lambda_i = (1+\lambda)^{1/\theta_i}$ and $1+\lambda_{ij} = (1+\lambda)^{1/(\theta_i+\theta_j)}$ for $i, j \in \{c, k, s\}$, with $i \neq j$.

For $\gamma \neq 1$, equation (2) implies that

$$u\left((1+\lambda)C, (1+\lambda)K, (1+\lambda)S\right) = (1+\lambda)^{1-\gamma} \frac{1}{1-\gamma} \left(C^{\theta_c} K^{\theta_k} S^{\theta_s}\right)^{1-\gamma}$$

$$u\left((1+\lambda_c)C, K, S\right) = (1+\lambda_c)^{\theta_c(1-\gamma)} \frac{1}{1-\gamma} \left(C^{\theta_c} K^{\theta_k} S^{\theta_s}\right)^{1-\gamma}$$

$$u\left((1+\lambda_{ck})C, (1+\lambda_{ck})K, S\right) = (1+\lambda_c)^{(\theta_c+\theta_k)(1-\gamma)} \frac{1}{1-\gamma} \left(C^{\theta_c} K^{\theta_k} S^{\theta_s}\right)^{1-\gamma}$$

Once again, provided that λ is well-defined, $1+\lambda_c = (1+\lambda)^{1/\theta_c}$ and $1+\lambda_{ck} = (1+\lambda)^{1/(\theta_c+\theta_k)}$. As before, $1+\lambda_i = (1+\lambda)^{1/\theta_i}$ and $1+\lambda_{ij} = (1+\lambda)^{1/(\theta_i+\theta_j)}$ for $i, j \in \{c, k, s\}$, with $i \neq j$.