THE EFFECT OF A CARBON PERMIT MARKET ON THE SIZE OF STABLE COALITIONS OF AN INTERNATIONAL ENVIRONMENTAL AGREEMENT ON CLIMATE CHANGE – A DIFFERENTIAL GAME APPROACH

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Abstract: In this study an International Environmental Agreement with the presence of a market for pollution permits is analyzed. A two-stage differential game is formulated, where in the first stage agents decide whether to joint or not the agreement and in the second stage agents negotiate over the level of emissions. The aim thus was to introduce the permit market system to investigate if the existence of such market creates any further incentive of agents to form a large coalition. Two such markets were introduced: the regulated market, where signatory of the agreements have access to and the voluntary market, which non-signatories have access to. Our results were positive in the sense that the presence of permit market, a larger number of signatories is achieved when compared with the baseline case without the permit market system. However, it is shown that if the cap set on signatories is too stringent then the incentive diminishes. It is also shown by calculating the steady-state of the stock of pollution that cooperation among agents is fundamental given that the larger the number of signatories the smaller is the steady-state of the stock of pollution.

Keywords: International Environmental Agreements. Climate Change. Carbon Markets. Differential Games.
Classificação JEL: C02, C61, C73, Q54

Resumo: Neste estudo analisa-se um Acordo Ambiental Internacional com a presença de um mercado de licenças de poluição. Formula-se um jogo diferencial em dois estágios onde no primeiro estágio os agentes decidem se vão ou não aderir ao acordo e no segundo estágio os agentes negociam o nível de emissões. O objetivo, portanto, foi introduzir o sistema de mercado de licenças ambientais para investigar se a existência de tal mercado cria algum incentivo adicional de agentes para formar uma grande coalizão. Introduz-se dois mercados: o mercado regulado, ao qual os signatários dos acordos têm acesso e o mercado voluntário, ao qual os não signatários têm acesso. Os resultados foram positivos no sentido de que com a presença do mercado de licenças, um maior número de signatários é alcançado quando comparado com o caso base, sem o sistema de mercado de licenças. No entanto, mostra-se que, se o limite estabelecido para os signatários for muito rigoroso, o incentivo diminui. Mostra-se, calculando o estado estacionário do estoque de poluição, que a cooperação entre os agentes é fundamental, visto que quanto maior o número de signatários, menor é o estado estacionário do estoque de poluição.

Palavras-chave: Acordos Ambientais Internacionais. Mudanças Climáticas. Mercados de Carbono. Jogos Diferenciais.
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1 Introduction

International environmental cooperation is of surmounting importance, and increasingly. Climate change caused by global warming is a real threat on the incoming decades. The emissions of greenhouse gases is considered the main source of anthropogenic influence on the natural environment. The main greenhouse gases are: carbon dioxide, methane, nitrous oxide and flourinated gases. As a result of industrial activity, energy generation and agricultural activity, these gases are emitted and stay in the atmosphere thereby "trapping" solar radiation emitted from the sun and raising temperatures. At the same time that emission of greenhouse gases is what sustain the modern lifestyle.

The IPCC (Intergovernmental Panel for Climate Change) in the 2014 synthesis report is clear by stating: "Human influence on the climate system is clear, and recent anthropogenic emissions of greenhouse gases are the highest in history. Recent climate changes have had widespread impacts on human and natural systems." (IPCC, 2014, p. 2). Moreover: "Warming of the climate system is unequivocal, and since the 1950s, many of the observed changes are unprecedented over decades to millennia. The atmosphere and ocean have warmed, the amounts of snow and ice have diminished, and sea level has risen.*"* (IPCC, 2014, p. 2).

In a special report, the IPCC has raised the objective of keeping the average surface temperature of the planet below 1,5º C from the pre-industrial average temperature level in order to avoid the most serious effects of climate change. The costs of mitigation, however, have to drastic. According to the Special Report, the IPCC claims that emission would have to be reduced about 45% from the 2010 levels and reach net 0% emission by 2050 (IPCC, 2018, p. 12). This is a goal that is highly unlikely to be met without cooperation. This is why international environmental agreements on climate change are important. To stop climate change coordination among countries is vital if the problem of climate change and its impact are to be faced adequately.

There have been, so far, two major attempts to curb climate change caused by global warming: The Kyoto protocol and more recently, the Paris Agreement. Both of them recognize that climate change cannot be adequately dealt with if countries unilaterally take actions, efforts must be coordinated. The Kyoto protocol, although has been somewhat effective in reducing carbon emissions (Grunewald and Martinez-Zarzoso, 2015). However, as of now, the amount of carbon dioxide in the atmosphere has reach record high levels.

The problem of the lack of supranational government and the enforcement of hard laws is an important aspect of our approach. In fact, if the rules of treaties could be enforced somehow, there wouldn't even be a problem to begin with, countries would simply implement environmental rules and the problem would be solved. The Kyoto protocol couldn't reach its objective because despite massive number of signatories, the countries were sovereign and didn't abide to the rules.

The Kyoto protocol created flexible mechanisms to aid countries to meet their obligations to cut emissions. One of these mechanism, described in the article 17 of the agreement, was the establishment of a market for tradable pollution permits. The so called Annex B countries could buy carbon permits if they needed to exceed their quota of emissions. This is a market based solution which allowed countries to better manage their plans to meet their targets accorded in the framework convention. By placing a price on a ton of carbon, for example, the real cost of pollution can be partly dealt with, since the country that emits pollutants must compensate for the damage imposed on other countries.

Being a cap-and-trade mechanism, the cap is usually set according to the targeted level for emissions reduction. If an agreement has stricter rules than the amount of pollution permits issued should naturally be smaller, if the agreement is weaker, in the sense of allowing higher emission levels, than the supply of permits should be higher. As Ibikunle and Gregoriou (2018) argue, while regulators cannot decide the price of permits, they can influence its supply and thus influence the price of permits.

In our model, alongside the mechanisms created by the Kyoto protocol, there is a voluntary carbon market. The voluntary carbon market creates carbon offset through projects approved by independent organizations who certify that submitted projects qualify standards of emission reduction and thus creates a certificate that can be sold or bought in the voluntary market. (Bayon et al, 2009).

In this research, our approach will be that of game theory. Game theoretical models of environmental agreements are a major tool to analyze these issues. How does game theory maps to what we have been saying so far? First, an IEA game is a model of negotiation, that is, agents negotiate over emission levels. That presupposes an international institution and a legal framework on which such negotiation take place. Since international law has limited power over actions of sovereign countries, the players in our game cannot be forced into signing the agreement, that is, they sign the agreement if it is in their best interest to do so, in game theory language, they behave noncooperatively. Since the game is noncooperative, the size of the coalition is determined in self-enforcing way. And finally, once the agents decide to enter the agreement, their emission level is binding and they fulfill their obligations. We don't consider the possibility of cheating.

In line with the noncooperative, self-enforcing approach, the works of Carraro and Sinicalco (1993) and Barrett (1994), the general conclusion is that only small stable coalition are possible. Barrett (1994) also states that large coalitions are possible only if the pay-off of the cooperative agreement doesn't deviate too much from the noncooperative scenario. Given that the baseline models in the noncooperative case have not given a good prospect for full cooperation, in this research we try to enlarge this class of models by introducing a mechanism that could alter the trade-off that agents face when deciding if they will join an international environmental agreement.

We consider marked-based mechanism. Particularly carbon markets, carbon credits, or pollution permits. With this type of mechanism an agent which exceed his quota of emissions can buy permits in a market to offset his excessive amount of emissions. Such mechanism were present in the Kyoto protocol and is also present in the recently accorded Paris Agreement. Additionally, we posit that there exists two such markets, one defined by the agreement and a voluntary market. In fact those voluntary markets do exist and are growing steadily since 2005, and have reached 63 million credits worth of dollars issued in 2017, (Fearnehough et al,2020). Therefore we can analyze the role that decentralized solutions impact on the size of stable coalitions and the impact on the total level of emissions as well.

Also, it is important to add that most of the literature in IEA games utilizes static or repeated games. The approach that we will follow is different, we are going to model our game as a differential game. The literature with this particular approach is still incipient, therefore, we think it is important to extend the literature with this particular approach. The main advantage of the differential game structure in comparison with the static or repeated approach is that we can take into account the effect that carbon dioxide accumulates in the atmosphere as a stock which slowly decays in time. This important effect is not present in the static game literature, when only flows of emission cause damage. In the differential game approach, the stock of greenhouse gases is the main driver of damages. We believe thus, that differential games add more realism to the phenomena being described. Since it is the stock of pollutants on the atmosphere that matter and cause damage, not the current flow of emissions. Differential games, as will be described latter puts the issue into a system dynamics perspective, which in our view is fundamental in understanding the coupling of human-climate interactions.

Being a more peripheral methodology, not much has been accomplished so far. There are only a small number of papers dealing with IEA games. We can mention the works of Rubio and Casino (2005), Rubio and Ulph (2007), De Zeeuw (2008) and Breton et al (2010). All of these works have analyzed an IEA game in a dynamic setting with different approaches upon which the coalition is formed (the first stage of the game). Rubio and Casino (2005) consider that agents take a once-and-for-all decision to join the agreement. Rubio (2007) seeks to provide a dynamic coalition formation, while De Zeeuw (2008) considers far-sighted stability. Breton et al (2010) considers an evolutionary approach to coalition formations, which agents switch strategies depending on the most successful strategy. We have looked into the literature and could not find more recent papers, we would like to make this explicit. The once-and-for-all approach to coalition formation has yielded small coalitions, namely two, while the approach by Rubio (2007) being dynamic, has sustained a larger number of signatories, it starts with 15 out of 100 but it decreases to 7 as time passes. The evolutionary approach by Breton et al (2010) has been able to sustain partial and full-cooperation, but it is dependent on some range of parameter values, with no cooperation also being a possible outcome.

We use the approach by Rubio and Casino (2005) for the formation of the coalition, that is we take the once-and-for-all approach to the stage one of our game, but we add a market for carbon emission permits and analyze how does the addition of the market mechanism alters the size of self-enforcing coalitions. It is important to realize that the addition of a permit market to the class of dynamic IEA games is not a mere breach in the literature that we hope to fill. Our results could be important for policy making. A well designed structure of incentives is paramount in any policy-making. Therefore part of our objectives is to show, also, under which conditions the permit market creates incentives for countries to join and international environmental agreement.

2 The Model

Now we effectively present our model. We consider N countries indexed by $i \in \{1,2,\ldots,N\} \subset \mathbb{N}$ each country has amount of production $Y_i = Q(E_{it})$ which is a function of its levels of emissions at any given time and it's given by its technology $Q(.)$ which is shared by all countries and each country has a fixed amount of endowments. The technology is assumed to be strictly concave and satisfy $Q(0)=0$. Given that countries derive benefit from the consumption of Y , we assume the follow benefit function:

$$B(t) = aE_i(t) - \frac{1}{2}E_i(t)^2 \tag{1}$$

Where $a>0$ is a parameter that measure the marginal benefit from a unit of emissions. Also $B'(t)>0$ and $B''(t)<0$. This particular form for the benefit function is widely adopted in the literature due to its nice properties when applied to optimal control problems, virtually every work in this area which derive analytical solution adopts this particular form, see Dockner and Long (1993), Rubio and Casino (2005), Bretton et al (2010) and Li (2013). This form for the benefit function also has the property of diminish marginal returns to the emission level, which is realistic and desirable.

We also assume that countries suffer from environmental damage from a global stock of pollution P . Therefore, the damage function is:

$$D(t) = \gamma P(t) \tag{2}$$

Where $\gamma > 0$ and $D'(t)>0$, that is damages are increasing in the level of pollution stock. A damage function that is linear in the stock of pollution is also adopted by Bretton et al (2010) and Li (2013), but for a quadratic damage function the game is also solvable, but the calculations are more involved and according to Breton et al (2010) without significant difference in results, so we adopt a linear damage function.

The stock of pollution $P(t)$ evolves according to the following law of motion:

$$\begin{gathered} \dot{P} = \sum_{i=1}^{N} E_i - kP \\ P(0) = P_0 \, given \end{gathered} \tag{3}$$

Where:

$\dot{P} = dP/dt$ = derivative of P with respect to time;

$\sum_{i=1}^{N} E_i(t)$ = the sum of emissions from all countries;

k = the rate of decay of pollution;

P_0 = initial stock of pollution;

Equation (3) is a linear differential equation, it provides the link of the emission levels of all countries with the stock of pollution. It is a way of coupling the economic activity of countries with the environmental degradation, that is, as countries emit more pollution, that pollution accumulates in the atmosphere according to (3). The particular form for the evolution of the stock of pollution is an extremely simple one but also popular in the literature. As Bretton et al (2010) points out, it cannot grasp the occurrence of catastrophic events, but for tractability of the model is a popular way of modeling.

We can now define the net welfare function for a country i, as:

$$W_i(E_i(t), P(t)) = aE_i(t) - \frac{1}{2}E_i(t)^2 - \gamma P(t) \qquad (4)$$
$$\forall i \in \{1, 2, \ldots, N\}$$

2.1 The Self-enforcing game

The self-enforcing game, as we have already stated, is a two stage game. Where in the first stage each country decide if it will join the agreement or not, and in the second stage it decides upon its emission levels. We solve the game by backwards-induction, that is, we solve first for the second stage and the for the first stage.

Just for ease of exposition, we now drop the time argument from each function, but be aware that both emissions and the stock of pollution are functions with respect to time.

2.2 Emission Game

The second stage is the emission game, and proceed as follows. We suppose that as a result of the first stage a coalition of *n* signatory countries has been formed. Then the signatories play a non-cooperatively against the signatories. Both the signatories and the non-signatories maximize their welfare according to the infinite-horizon optimal control problem:

$$\sum_{i=1}^{n} W_S = \sum_{i=1}^{n} \int_0^{\infty} e^{-rt}\left(aE_i - \frac{1}{2}E_i^2 - \gamma P\right) dt \qquad (5)$$

$$W_{NS} = \int_0^{\infty} e^{-rt}\left(aE_j - \frac{1}{2}E_j^2 - \gamma P\right) dt \qquad (6)$$

Where:

W_S is the total discounted welfare of a signatory;

W_{NS} is the total discounted welfare of a non-signatory;

r is the discount rate;

And the differential equations in (3), becomes

$$\dot{P} = \sum_{i=1}^{N} E_i + \sum_{j=1}^{N-n} E_j - kP \qquad (7)$$

Notice that the signatories maximize their joint welfare. That is, they act cooperatively among themselves to reduce emissions while playing non-cooperatively against the non-signatories.

Now, consider that the signatories by signing the agreement are imposed an emission target from which if they exceed that target they must buy pollution permits in a regulated market. Analogously, the non-signatories countries have access to the voluntary carbon market, but since they do not have to comply with any emission target, we assume that they impose on themselves a cap, which can be determined by their environmental awareness. Also we can suppose alternatively that each non-signatory country have their own environmental laws and an internal permit market. Anyways, we assume the existence of two permit markets, one for each type of agent, signatory and non-signatory. We also assume that the price of permits is given exogenously, which is a strong assumption to make, but we can argue for it by assuming that no country can significantly alter the market price, or each government posts its price for the permits, so the permits could work as a sort of carbon tax. In our model if a given country stays above its quota it could be interpreted as a tax to be paid, if it stays below, a tax rebate or subsidy. The assumption of an exogenous price has been used by Li (2013) and Chang et al (2018).Adding the permit markets, our problem becomes:

$$\sum_{i=1}^{n} W_S=\sum_{i=1}^{n}\int_0^{\infty} e^{-rt}\left(aE_i-\frac{1}{2}E_i^2-\gamma P-\rho_1(E_i-E_S)\right)dt \quad (8)$$

$$W_{NS}=\int_0^{\infty} e^{-rt}\left(aE_j-\frac{1}{2}E_j^2-\gamma P-\rho_2(E_j-E_{NS})\right)dt \quad (9)$$

The maximization in (8) and (9) are subject to (7).

Where:

ρ_1 is the price of a ton of carbon dioxide in the regulated market;

ρ_2 is the price of a ton of carbon dioxide in the voluntary market;

E_{S0} is the initial quota distributed by the signatories of the agreement;

E_{NS0} is a self-imposed cap on emissions or a limit set by the domestic environmental law of a non-signatory.

Notice that:

If $E_i-E_{S0}>0$, then the signatories have exceed their quota and then must buy permits.

If $E_i-E_{S0}<0$, then the signatories have not exceed their quota and then can sell the excess permits

If $E_i-E_{S0}=0$, then the signatories have matched their quotas and neither buy or sell permits.

The same is valid for the non-signatories.

After rearranging (8) and (9) we obtain:

$$\begin{gathered}\max_{E_1,E_2,\dots E_n}\sum_{i=1}^{n} W_S=\int_0^{\infty} e^{-rt}\left[(a-\rho_1)\sum_{i=1}^{n} E_i-\frac{1}{2}\sum_{i=1}^{n} E_i^2+n\rho_1 E_{S0}-n\gamma P\right]dt\\ s.t\ \dot{P}=\sum_{i=1}^{N} E_i+\sum_{j=1}^{N-n} E_j-kP\\ P(0)=P_0\\ \forall i\in\{1,2\dots n\}\end{gathered} \quad (10)$$

$$\max_{E_j} W_{NS} = \int_0^{\infty} e^{-rt} [(a-\rho_2) E_j - \frac{1}{2} E_j^2 + \rho_2 E_{NS0} - \gamma P] dt$$

$$s.t\ \dot{P} = \sum_{i=1}^{N} E_i + \sum_{j=1}^{n-N} E_j - kP \qquad (11)$$

$$P(0) = P_0$$

$$\forall\ j \in \{1, 2, \ldots, N-n\}$$

To solve the model presented in (10) and (11) we have to set-up the Hamilton-Jacobi-Bellman equation, since we are going to solve for the state-feedback strategies. We, then, arrive at:

$$rV_S(P) = \max_{E1, E2, \ldots, En} (a-\rho_1) \sum_{i=1}^{n} E_i - \frac{1}{2} \sum_{i=1}^{n} E_i^2 + n \rho_1 E_{S0} - n \gamma P$$
$$+ V_P{}' (\sum_{i=1}^{n} E_i + \sum_{j=1}^{N-n} E*_i - kP) \qquad (12)$$

For the signatories

$$rV_{NS}(P) = \max_{ej} (a-\rho_2) E_j - \frac{1}{2} E_j^2 + \rho_2 E_{NS0} - \gamma P$$
$$+ V_P{}' (\sum_{i=1}^{n} E*_i + \sum_{j \neq 1}^{N-n} E_j * + E_j - kP) \qquad (13)$$

For non-singatories. The asterisks denote the optimal response of the other player. That is, each seek a control (strategy) that maximize its own pay-off given that the other agents are doing the same.

Now are going to take a step-by-step approach to the solution. Performing the maximization indicated in (12) and (13), yields:

$$0 = a - \rho_1 - E_i + V_S{}'$$

$$0 = a - \rho_2 - E_j + V_{NS}{}'$$

Upon rearranging, we obtain the optimal emissions level for the signatories and non-signatories:

$$E_S = a - \rho_1 + V_S{}' \qquad (14)$$

$$E_{NS} = a - \rho_2 + V_{NS}{}' \qquad (15)$$

Remember that we have assumed that we are solving the model by backwards induction, therefore we have *n* signatories and *N-n* non-signatories. So we have a system of optimal controls to solve. For each signatory and non-signatory we have a similar problem since the countries are symmetric, hence we can impose symmetry condition and sum (3) and (4) from 1 to n and n+1 to N, respectively. We thus obtain:

$$\sum E_S = n(a - \rho_1 + V_S{}') \qquad (16)$$

$$\sum E_{NS} = (N - b)(a - \rho_2 + V_{NS}{}') \qquad (17)$$

To facilitate our subsequent algebraic manipulations, let us make the following modifications:

$$a-\rho_1=c_1$$
$$a-\rho_2=c_2$$
$$N-n=m$$

Now, the next step is to conjecture a value function that optimizes problem (12) and (13). Since the Hamilton-Jacobi-Bellman equation is linear in the variable P, we conjecture that the value function is also linear in P. This is a common approach to solve the HJB partial differential equation in differential games, see Breton (2010), Li (2013) and Basar (2018). We guess the following form for the value function for the signatories:

$$V_S=A_S+B_S P \tag{18}$$

And the following form for the non-signatories:

$$V_{NS}=A_{NS}+B_{NS} P \tag{19}$$

Taking the derivative with respect to P in (18) and (19), we obtain:

$$V_S{}'=B_S \tag{20}$$

$$V_{NS}{}'=B_{NS} \tag{21}$$

Substituting the results (20) in (14) and (16), and substituting (21) in (15) and (17). Again substituting the results of those substitutions in (12) and (13), along with (18) and (19) also in (12) and (13). If that was confusing we are just substituting the maximized values and the conjectured value function into the HJB-equation. We obtain:

$$\begin{gathered} r(A_S+B_S P)=c_1[n(c_1+B_S)]-\frac{1}{2}n[(c_1+Bs)]^2+n\rho_1 E_{S0}-n\gamma P \\ +B_S[n(c_1+B_S)+m(c_2+B_{NS})-kP] \end{gathered} \tag{22}$$

$$\begin{gathered} r(A_{NS}+B_{NS} P)=c_2(c_2+B_S)-\frac{1}{2}(c_2+Bs)^2+\rho_2 E_{NS0}-\gamma P \\ +B_{NS}[n(c_1+B_S)+m(c_2+B_{NS})-kP] \end{gathered} \tag{23}$$

Our next step is to solve for the coefficients A_S , B_S , A_{NS} and B_{NS} , to do that lets consider the HBJ equation for the signatories (22), first. We expanding the quadratic terms, we obtain:

$$\begin{gathered} rA_S+rB_s P=nc^2+ncB_S-\frac{1}{2}n(c_1^2+2c_1 B_S+B_S^2)+n\rho_1 E_{S0}-n\gamma P \\ +nc_1 B_S+nB_s^2+mc_2 B_S+mB_S B_{NS}-kP \end{gathered} \tag{24}$$

The trick to find the coefficients is to notice that the left-hand side must equal the right-hand side for all values of P. That is equivalent of saying that:

$$X+YP=Z+W\,P$$

Where:

$$X = rA_S$$

$$Y = rB_S P$$

$$Z = nc_1^2 + nc_1 B_S - \frac{1}{2} n(c_1^2 + 2c_1 B_S + B_S^2) + n\rho_1 E_{S0}$$

$$+ nc_1 B_S + nB_s^2 + mc_2 B_S + mB_S B_{NS}$$

$$W = -n\gamma P - kP$$

Making $YP = WP$, yields:

$$rB_S P = -ndP - kB_S P$$

$$rB_S + kB_s = -n\gamma$$

$$B_S = -n\frac{\gamma}{r+k} \tag{25}$$

And making $X = Z$, yields:

$$rA_S = nc_1^2 + nc_1 B_S - \frac{1}{2} nc_1^2 - nc_1 B_S - \frac{1}{2} B_S^2 + n\rho_1 E_{NS0} + nc_1 B_S + nB_S^2 + mc_2 B_S + mB_S B_{NS}$$

After rearranging and making algebraic manipulations, we obtain the coefficient A_S as:

$$A_S = \frac{1}{2r} n(a - \rho_1 + B_S)^2 + (N-n)\frac{B_S}{r}(a - \rho_2 + B_{NS}) + n\frac{\rho_1}{r} E_{NS0} \tag{26}$$

Notice that we have substituted back the values defined for *c* and *m*.

For the non-signatories the problem is very similar, so we won't work everything again. We merely state that the coefficients B_{NS} and A_{NS} are given by:

$$B_{NS} = \frac{-\gamma}{r+k} \tag{27}$$

$$A_{NS} = \frac{1}{2r}(a - \rho_2 + B_{NS})^2 + (N-n-1)\frac{B_{NS}}{r}(B_{NS}+1) + \frac{nB_{NS}}{r}(a - \rho_1 + B_S) + \frac{\rho_2}{r} E_{NS0} \tag{28}$$

The value function for the signatories and non-signatories respectively are:

$$V_S = -n\frac{\gamma}{r+k} P + \frac{1}{2r} n\left(a - \rho_1 - n\frac{\gamma}{r+k}\right)^2 - (N-n)n\frac{\gamma}{r(r+k)}\left(a - \rho_2 - \frac{\gamma}{r+k}\right) + n\frac{\rho_1}{r} E_{NS0} \tag{29}$$

$$\begin{aligned} V_{NS} = \frac{-\gamma}{r+k} P + \frac{1}{2r}\left(a - \rho_2 - \frac{\gamma}{r+k} P\right)^2 - (N-n-1)\frac{\gamma}{r(r+k)}\left(1 - \frac{\gamma}{r+k}\right) \\ - n\frac{\gamma}{r(r+k)}\left(a - \rho_1 - n\frac{\gamma}{r+k}\right) + \frac{\rho_2}{r} E_{NS0} \end{aligned} \tag{30}$$

Now that we have established the optimal emission levels for the signatories and non-signatories and derived the value functions, we have to find the solution to the stock of pollution in equation (3), to do that we have to find the total level of emissions. Summing (16) and (17) and making the adequate substitutions yields:

$$\sum E_{NS} + \sum E_S = (N-n)(a-\rho_2 - \frac{\gamma}{r+k}) + n(a-\rho_1 - n\frac{\gamma}{r+k})$$

$$\frac{dP}{dt} = (N-n)(a-\rho_2 - \frac{\gamma}{r+k}) + n(a-\rho_1 - n\frac{\gamma}{r+k}) - kP \quad (31)$$

This is a first-order non-homogeneous linear differential equation, whose solution is:

$$P = \frac{A}{k} + C_1 e^{-kt} \quad (32)$$

Where:

$$A = (N-n)(a-\rho_2 - \frac{\gamma}{r+k}) + n(a-\rho_1 - n\frac{\gamma}{r+k})$$

$$C_1 = P_0 - \frac{A}{k}$$

2.3 The participation game

Now that we have derived the optimal emissions and the corresponding optimal trajectory of the emission game, we have to find the size of the self-enforcing coalition. To do that we substitute the optimal emission from the signatories and non-signatories with the optimal trajectory in (10) and (11) and integrate.

Let us recall how those expressions look like.

$$\sum_{i=1}^{n} W_S = \int_0^{\infty} e^{-rt}[(a-\rho_1)\sum_{i=1}^{n} E_i - \frac{1}{2}\sum_{i=1}^{n} E_i^2 + n\rho_1 E_{S0} - n\gamma P]dt$$

For the signatories, and:

$$W_{NS} = \int_0^{\infty} e^{-rt}[(a-\rho_2)E_j - \frac{1}{2}E_j^2 + \rho_2 E_{NS0} - \gamma P]dt$$

For the non-signatories:

The optimal emissions level are:

$$\sum E_S = n(a-\rho_1 - n\frac{\gamma}{(r+k)})$$

For the signatories, and:

$$E_{NS} = a - \rho_2 - \frac{\gamma}{(r+k)}$$

For the non-signatories.

Let us first calculate the indefinite integral. Making the corresponding substitutions, yields:

$$\int e^{-rt}[(a-\rho_1)n[(a-\rho_1) - n\frac{\gamma}{(r+k)}] - \frac{1}{2}(a-\rho_1 - n\frac{\gamma}{(r+k)})^2 + \rho_1 n E_{NS0} - n\gamma P]dt$$

$$\int e^{-rt}[(a-\rho_2)[(a-\rho_2)-\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-\frac{\gamma}{(r+k)})^2+\rho_2 E_{NS0}-dP]dt$$

Those two expression look quite complicated to integrate, but they really aren't. Let us make some modifications:

$$W_S=\int e^{-rt}[M1+m1P]dt \tag{33}$$

$$W_{NS}=\int e^{-rt}[M2+m2P]dt \tag{34}$$

Where:

$M1=(a-\rho_1)n[(a-\rho_1)-n\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-n\frac{\gamma}{(r+k)})^2+\rho_1 nE_{S0}$

$m1=-n\gamma$

$M2=(a-\rho_2)[(a-\rho_2)-\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-\frac{\gamma}{(r+k)})^2+\rho_2 E_{NS0}$

$m2=-\gamma$

Now, substituting the solution to our differential equation in (32), yields

$$W_S=\int e^{-rt}[M1+m1(\frac{A}{k}+C_1e^{-kt})]dt$$

$$W_{NS}=\int e^{-rt}[M2+m2(\frac{A}{k}+C_1e^{-kt})]dt$$

We can improve this by doing:

$$W_S=M1\int_0^\infty e^{-rtdt}+[m1(\frac{A}{k})]\int_0^\infty e^{-rt}dt+m1C_1\int_0^\infty (e^{-(r+k)}t)dt$$

$$W_{NS}=M2\int_0^\infty e^{-rtdt}+[m2(\frac{A}{k})]\int_0^\infty e^{-rt}dt+m2C_1\int_0^\infty (e^{-(r+k)}t)dt$$

The definite integral from zero to infinite is:

$\int_0^\infty e^{-rtdt}=\frac{1}{r}$

$\int_0^\infty e^{-(r+k)t}dt=\frac{1}{(r+k)}$

So the discounted pay-off for the signatories and non-signatories are:

$$W_S=[M1+m1(\frac{A}{k})]\frac{1}{r}+\frac{m1C_1}{(r+k)} \tag{35}$$

$$W_{NS} = [M2 + m2(\frac{A}{k})]\frac{1}{r} + \frac{m2C_1}{(r+k)} \quad (36)$$

If we substitute back every coefficient, this expression will look too complicated and intractable, so we have to find the self-enforcing size of the coalition numerically.

The self-enforcing criteria is also a game where agents have two strategies: join the agreement or do not join the agreement. When deciding if joining the agreement is going to be profitable, the reasoning is that, if he is a cooperator, what would be his pay-off if he deviated, that is, if he left the agreement. Similarly, a non-cooperator would compare his pay-off against the pay-off he would receive if the joined the agreement. This reasoning is the following inequalities:

$$\begin{array}{l} W_S(n*) \geq W_{NS}(n*-1) \\ W_{NS}(n*) \geq W_S(n*+1) \end{array} \quad (37)$$

The first inequality is the internal stability condition, the second one is the external stability condition.

To proceed in analyzing our numerical results we consider initially three cases: a baseline case with no permits, a weak treaty with a higher cap level entailing a higher supply of permits and a stronger treaty with lesser supply of pollution permits. In the voluntary market, since we assume that they are smaller, we assume that the supply of permits is always less than the regulated market, but at a smaller price as well.

Our model will be calibrated with numerical parameters. It is important to highlight that those parameters do not reflect any real data. The literature follows this pattern, so we follow it as well. But we need to choose parameter values so as to avoid nonsensical results. When we parameterize our model we want to establish two condition: the level of emissions is always positive, thus avoiding zero or negative emission rates; We want also that the steady state level of pollution to be non negative as well. Those two conditions are necessary in order that our model avoid nonsensical results. Some parameters are directly taken from the literature, namely the discount rate and the rate of decay of pollution are directly taken from Rubio and Casino (2005), the damage parameter is taken from Breton et al (2010). To make sure our model is well calibrated, we make the remark that when absent the permit market, our model reduces to the model proposed by Rubio and Casino (2005), therefore a baseline result is present to reproduce the results achieved by Rubio and Casino (2005), then we proceed to add the permit market on top of the baseline model, rested assured that we have provided well selected parameters. And to make sure that our results are not random, we will provide some robustness checks, but we relegate those results to an appendix and based on those results discuss some possible limitations of our model.

2.4 Baseline results

Our model parameters are:

N = 10;

a = 200;

γ = 0.3;

ρ_1 = 1.4;

ρ_2 = 0.8;

E_{S0} = 160 (weak agreement), 96 (stringent agreement);

E_{NS0} = 186;

r = 0.025;

k = 0.05;

P0 = 0

The small number for the parameter *N* is to make easier to present the results, and also one can suppose that each value of *n* represent a block of countries that sign the agreement. For the baseline results the values of ρ_1 and ρ_2 are set to zero. The parameters *r* and *k* where inspired by Rubio and Casino(2005). Given these parameters the self-enforcing coalition is:

Table 1:The baseline calibration

n	W_S	W_{NS}
1		4.8608e+05
2*	4.864e+05	4.8736e+05
3*	4.8736e+05	4.8992e+05
4	4.8896e+05	4.9376e+05
5	4.912e+05	4.9888e+05
6	4.9408e+05	5.0528e+05
7	4.976e+05	5.1296e+05
8	5.0176e+05	5.2192e+05
9	5.0656e+05	5.3216e+05
10	5.12e+05	

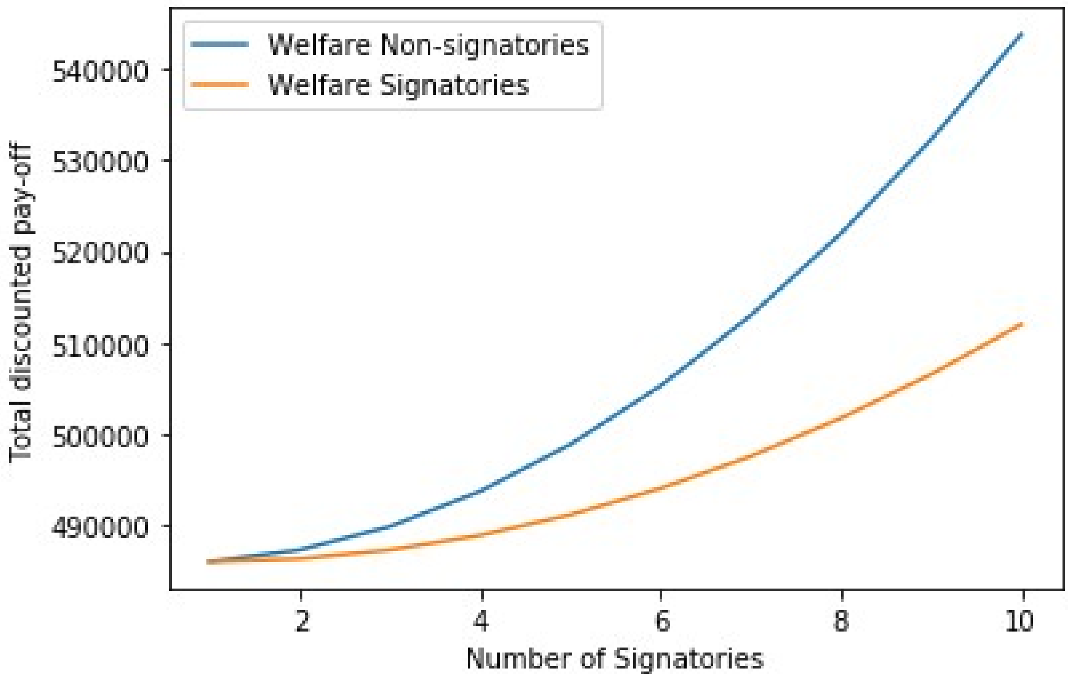
Source: research results.

The number of the stable coalition is given by the stability condition given in (37), we must compare the total discounted pay-off for a signatory at given size of coalition *n* with the total discounted pay-off of a non-signatory at size of a coalition "*n*-1". So we always compare the pay-off at *n,* say 5, and compare to the pay-off of the non-signatory directly below 5, which is 4. We proceed this way until we find an *n* which gives the largest pay-off for the signatories in that, if an additional agent were to join the agreement, his pay-off would decline, so he chooses to be a non-signatory.

The number of the self-enforcing coalition is marked with an asterisk, therefore is two and three. It is possible to inspect that WS(2) = 4.864e+05 and WNS(1) = 4.8608e+05, so WS(2) > WNS(1), so two countries form a bilateral agreement, at WS(3) = WNS(2) marks a point of indifference, but WS(4) < WNS (3), so at most three countries sign the agreement. This is coherent with the results achieve by Barrett (1994) and Rubio and Casino (2005), in fact the baseline case is a replication of those results, so our model has indeed arrived at the same baseline results although through a slightly different approach.

To better illustrate the results, a diagram helps the visualization:

Figure 1: The baseline calibration



Source: research results

The graph above is a visualization of the table we have just presented, each entry in Table 1 is plotted in the graph, so we can see when the pay-off of a signatory is higher when compared to the pay-off of a non-signatory. Remember, we must compare the welfare of for a given number of signatories, say 3, with the pay-off of a non-signatory at 2, this is the correct way of reading the graph. The blue line is the welfare of the non-signatory countries while the orange line is the welfare of the signatory countries. Notice that as the number of signatories increase, the gap between the orange line and the blue line increases, this difference is the size of incentive to free-ride. Also notice that the pay-off from cooperation is increasing, the pay-off from full cooperation is also higher than the pay-off from the fully non-cooperative case. We have illustrated then how the baseline calibration reflects the classical prisoner's dilemma nature of our problem.

2.4.1 Results with a permit system

Now, lets suppose that a pollution permit system exists. That is, agents will be endowed with an initial amount of pollution permits such that, if they exceed their cap established by their initial endowment they have to buy additional permits in a market at an exogenous price.

To decide how to set the initial endowments, we can consider the emissions at the full cooperative outcome and set how much we would like to reduce from that.

Reminding us that the emission of signatories is given by:

$$E_{NS}=a-\rho_2-n\frac{\gamma}{(r+k)}$$

And setting the previously given parameters, we have that the cooperative level of emission is:

$$E_{NS}=200-\left(\frac{10(0.3)}{0.025+0.05}\right)=160$$

Now, let us suppose two sort of agreements, a weak and a more stringent one. In the weak agreement agents are distributed pollution permits that are equal to the full cooperative outcome of the game without a permit market, that is, if they pollute more than the full cooperative outcome, agents musty buy permits to compensate for the excess of emissions. In the strong agreement agents are given 60% of the permits of the weak agreement, that is, the incentive to pollute decreases as agents incur in higher costs of emitting pollutants. In the voluntary markets we assume that agents, due to they own environmental concern, set a cap of 5% of their business-as-usual level of emissions. Performing this calculation using the optimal emissions level for the non-signatories we find that their self-imposed cap is equal to 186,2. To work with round numbers we set their cap at 186.

Table 2:Weak agreement calibration

n	W_S	W_{NS}
1	4.852552e+05	4.814152e+05
2	4.856711e+05	4.827911e+05
3	4.867271e+05	4.854472e+05
4	4.884232e+05	4.893832e+05

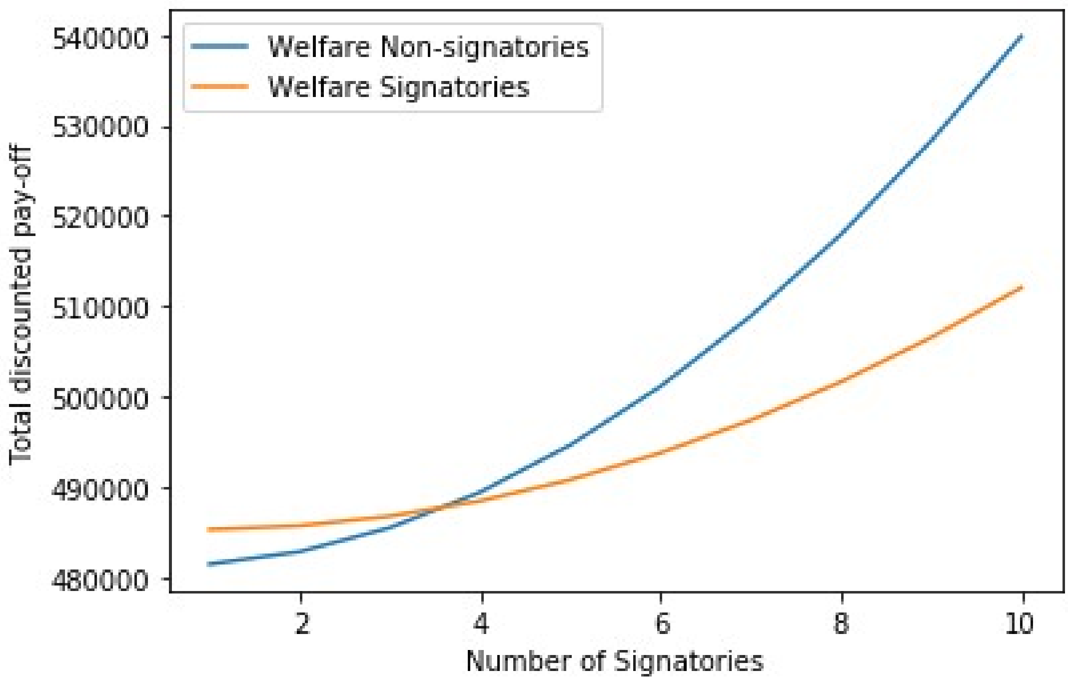
5*	4.907591e+05	4.945991e+05
6	4.937352e+05	5.010952e+05
7	4.973511e+05	5.088711e+05
8	5.016071e+05	5.179272e+05
9	5.065032e+05	5.282631e+05
10	5.120391e+05	5.398791e+05

Source: Research results

As one can observe, the size of the self-enforcing coalition is 5, an improvement over the baseline model. The intuition behind this result is that with the permit system the agents have an incentive to cooperate, as the number of agents in the agreement increase, they jointly cut emissions, by coordinating their action through the cap and the permit price, they are able to approximate their emission levels closer to the full cooperative outcome.

The graph for the model with a permit system is given below.

Figure 2:Weak agreement calibration



Source: research results

Notice that the incentive to defect is still present and we cannot achieve full participation. We must highlight the fact that the size of the self-enforcing coalition is not when the two lines intersect. Remember that an agent compares his pay-off of joining the coalition with the pay-off of a coalition of size *n-1*.

Now we report the results for the strong agreement, that is, with a smaller initial endowment of permits:

Table 3:Stringent agreement calibration

n	W_S	W_{NS}
1	4.818952e+05	4.79495e+05
2	4.823111e+05	4.808711e+05

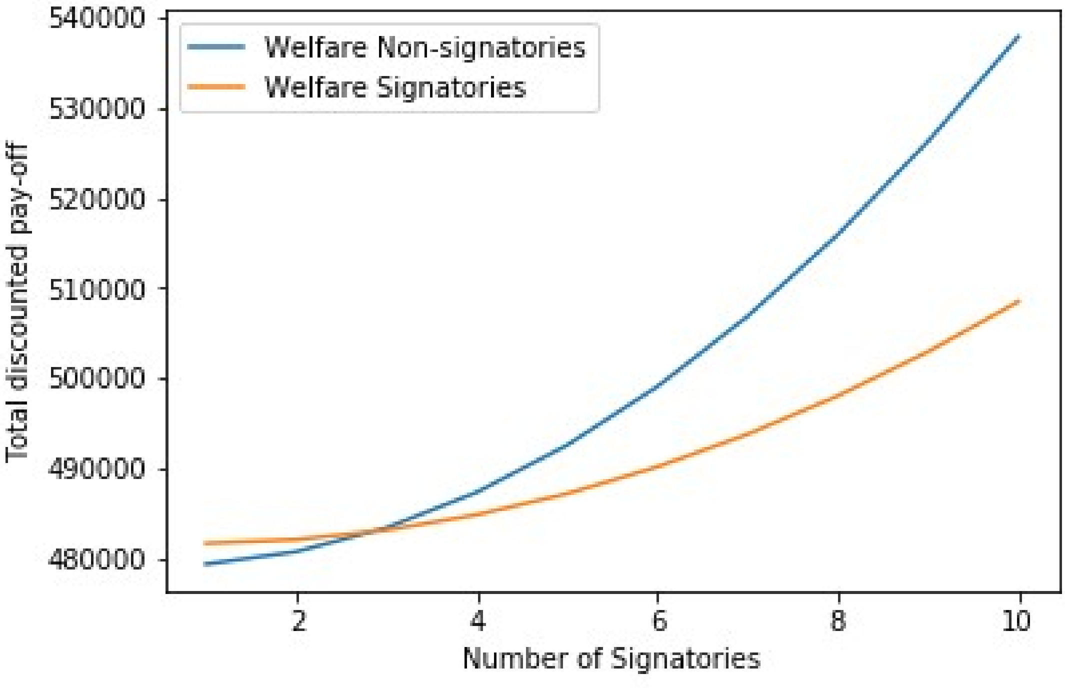
3	4.833671e+05	4.835272e+05
4*	4.850632e+05	4.874632e+05
5	4.873991e+05	4.926791e+05
6	4.903752e+05	4.991752e+05
7	4.939911e+05	5.069511e+05
8	4.982471e+05	5.160072e+05
9	5.031432e+05	5.263431e+05
10	5.086791e+05	5.379591e+05

Source: research results

In the strong agreement, the size of the self-enforcing coalition has dropped to 4. This indicates that as the cap on the emission level decreases, countries will find more profitable to deviate from the agreement. Fewer pollution permits penalizes agents when they can in fact stay bellow the emissions cap, we can conclude that, fewer permits will create an incentive to emit more pollutants as opposed to cutting back on emissions and profit from the reduction in pollution emission.

Bellow we show the graph for the strong agreement:

Figure 3:Stringent agreement calibration



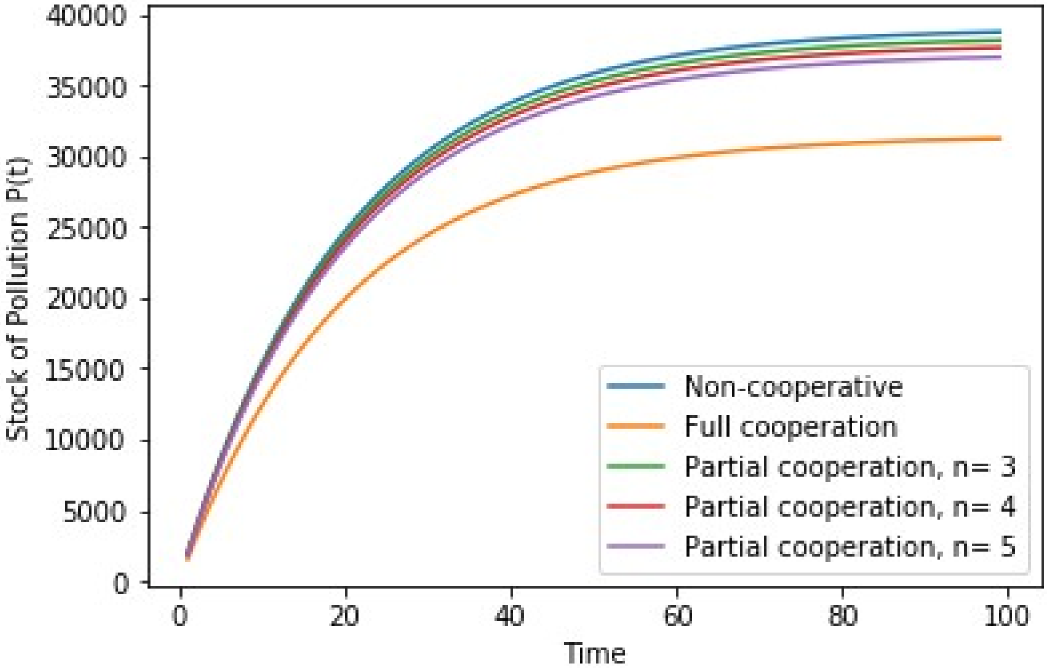
Source: Research results

We again see that the incentive to join the agreement has decreased when compared to the weak agreement but we still could find a higher cooperation level when compared to the baseline model with no permit system.

We also would like to stress that cooperation is fundamental, it is possible to observe that in all cases the welfare from total cooperation is higher than in the non-cooperative case. Also, the steady-state stock of pollution under cooperation is always lower than in the non-cooperative case. So, from our model we can also conclude that it is unlikely that strictly decentralized markets can cope with a global externality problem such as climate change. Cooperation is fundamental along with mechanism that provide the correct market incentives.

Given that we have found the size of the stable self-enforcing coalitions for the baseline case, the weak and strong treaty, as well as the full cooperative outcome, which is just given by the maximum number of players, we can compare the steady-state stock of pollution for the various outcomes. Below, a graph that shows how the steady-state stock of pollution varies for each outcome considered in our numerical explorations:

Figure 4:Steady-state comparison



Source: research results

The different colors of the lines represent a different outcome of the model, we can observe that the smallest level of the stock of pollution occur for the full-cooperative outcome, this strengthens the general conclusion that cooperation is vital to tackle the problem of climate change. The non-cooperative outcome achieves the highest level of stock pollutants, showing how if each player behaves non-cooperatively there is a tendency to over exploit common global natural resources. Although the existence of the voluntary market could reduce somewhat the emission level, the result pales in comparison with the cooperative outcome. We can also observe that partial cooperation diminishes the steady state level of stock pollutant for increasing levels of participation. We can establish then, based on our results the following order relationship

$$SS_{NC} < SS_{PC} < SS_{FC} \quad (38)$$

Where:

SS_{NC} = Non-cooperative steady-state of the pollution stock;

SS_{PC} = Partial cooperation steady-state of the pollution stock;

SS_{FC} = Full cooperation steady-state of the pollution stock.

3 Conclusions

We set for ourselves in this work the task of analyzing the effect of a permit market on the size of stable coalitions in an international environmental agreement game in a dynamic context. Our main hypothesis was that a permit market system would increase the level of participation in the agreement but if the agreement was too stringent on the level of emissions, participation would decrease. To investigate the hypothesis we have solved a two-stage differential game, where we found the optimal emissions level for the players, the optimal trajectory of the pollution stock and we have applied the stability condition in order to find the stable number of signatories of the agreement. The proposition in the hypothesis turned out to be true for our numerical experiments. We have proposed a weak agreement, where the number of permits was higher when compared with the strong agreement. The number of signatories on the weak agreement was 5, while in the strong agreement was 4. This is an improvement when compared to the baseline case, where the permit market in not taken in consideration. The interpretation of this result is that a market mechanism is a useful tool to aid agents to coordinate their action in reducing emissions, when a price is put on the excess emission agents had an additional incentive to approximate their level of emission closer to the full cooperative level. However, if the number of permits was too distant from the full cooperative outcome, the incentive to leave the agreement became present. We can conclude that the design of the permit market is of relevance when proposing an agreement to reduce emission of pollutants. From a policy perspective, then, we can conclude that the number of pollution permits is an important tool to generate enough incentives for countries to join an international environmental agreement. A design which could be considered too ambitious (too few permits), could reduce the number of willing participants when compared to a more modest target for reduction given by the number of permits.

A final conclusion is that cooperation is fundamental, by analyzing the trajectory of the optimal stock of pollution we could verify that the full cooperation led to the lowest level of the stock of pollution, while the full non-cooperative outcome yielded the highest stock of pollution. This imply that, even though we considered that non-signatories had access to a voluntary market for carbon permits the level of emission was still high enough to generate a worse outcome when compared to the full cooperative solution. Partial cooperation reduced the stock of pollution when compared to the full noncooperative outcome but stayed well above the full cooperative solution. This reinforces our conclusion that cooperation is fundamental to deal with global environmental externalities.

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