A BVAR note on the J-curve and the Marshall-Lerner condition for Brazil[1]

Francisco José Sales Rocha†

Marcos Renan Vasconcelos Magalhães‡

Átila Amaral Brilhante§

Resumo: Neste trabalho, foram testadas as hipóteses da Curva J e da condição de Marshall-Lerner para o Brasil no período de janeiro de 2003 a dezembro de 2019. A função impulso-resposta (FIR) e a decomposição da variância (DV), de um vetor autorregressivo Bayesiano (Minnesota *priors*), serviram de instrumentos para a verificação empírica das hipóteses supracitadas. Os resultados mostraram que o BVAR estimado respalda empiricamente as hipóteses em questão. No curto prazo, observa-se que uma depreciação real da moeda brasileira resulta, nos primeiros cinco meses, em um déficit na balança comercial. A partir do sexto mês, no entanto, o resultado da balança comercial se torna positivo e se mantém assim por mais de dez meses. Isto significa que não se pode rejeitar a hipótese da curva J. No longo prazo, verifica-se que a condição de Marshall-Lerner também não deve ser rejeitada. Ou seja, uma desvalorização cambial provoca um aumento no saldo da balança comercial por mais de 36 meses.

Palavras-chave: Curva J; condição de Marshall-Lerner; BVAR; Balança comercial; câmbio

Abstract: In the present work, the hypotheses of the J-curve and the Marshall-Lerner condition for Brazil from January 2003 to December 2019 were tested. The impulse-response function (IRF) and the variance decomposition (VD) of a Bayesian vector autoregressive model (Minnesota *priors*) served as instruments for the empirical verification of the above-mentioned hypotheses. The results showed that the estimated BVAR empirically supports the hypotheses in question. In the short term, it is observed that a real depreciation of the Brazilian currency results, in the first five months, in a deficit in the trade balance. However, as of the sixth month, the result of the trade balance becomes positive, and it remains like that for longer than ten months. This means that one cannot reject the J-curve hypothesis. In the long term, it was found that the Marshall-Lerner condition should not be rejected either. In other words, a currency devaluation causes an increase in the trade balance for longer than 36 months.

Keywords: J-curve; Marshall-Lerner condition; BVAR; trade balance; exchange rate

JEL Codes: F10; F31; F41

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[1] † Department of Applied Economics, Federal University of Ceará, Fortaleza, Brazil. E-mail: salesrocha@ufc.br; ‡ CAEN - Graduate Studies in Economics, Federal University of Ceará, Fortaleza, Brazil. E-mail: marcosrenan@caen.ufc.br; ‡ § ICA-Department of Philosophy, Federal University of Ceará, Fortaleza, Brazil. E-mail: atilaabrilhante@gmail.com.

1. Introduction

This paper aims to analyze, through a Bayesian vector autoregressive (BVAR) model, the dynamic interaction between the exchange rate and the trade balance. The theoretical basis of these co-movements for an open economy can be found in Ivrendi and Guloglu (2010). The focus of this article, however, will be to analyze whether the empirical evidence corroborate the hypotheses of the J-curve and the Marshall-Lerner condition in Brazil, from January 2003 to December 2019.

The J-curve[2] is generated, in a simplified way, by a currency depreciation that results, in the short term, in a trade deficit and, in the long term, in a trade surplus. According to Ramos Filho and Ferreira (2016), this phenomenon is explained by the relative rigidity in terms of exported and imported *quantum*, as a consequence of foreign exchange contracts[3]. The Marshall-Lerner condition, in its turn, establishes that an improvement in the trade balance will occur in response to a currency depreciation, if the volume of exports and imports is elastic in relation to the real exchange rate (Sonaglio, Scalco and Campos, 2010; Arruda and Martins, 2020).

This study provides a contribution to the specialized literature because, as far as we know, the BVAR models have been little used to analyze the aforementioned hypotheses, especially for Brazil.

The use of a BVAR model to analyze the hypotheses of the J-curve and the Marshall-Lerner condition stems from two main points: (i) Bayesian VAR solves the problem of over-parameterization, so common in VAR models that make use of classical econometrics. This problem results in the lack of robustness in classical VAR techniques, that is, large asymptotic variances. Therefore, the BVAR model provides a more reliable analysis of the predictions about model variables, impulse-response functions and variance decomposition (Doan, Litterman, Sims, 1983; Banbura, Giannone, Reichlin, 2010); (ii) Bayesian VAR eliminates the problem of the order of integration of model variables and also decreases the relevance of sample size (Sims, Uhlig, 1991).

There are several studies for the Brazilian economy, outside the scope of Bayesian analysis, that carried out empirical verifications of the hypotheses of the J-curve and the Marshall-Lerner condition. Arruda and Martins (2020) analyzed[4] the impacts of a currency depreciation on total net exports of basic and industrialized goods in a panel for the Brazilian states in the period from January 1999 to December 2015. Using Panel Vector Autoregressive (PVAR) models, their results indicate the occurrence of the J-curve for the industrialized goods. Finally, using Panel Dynamic Ordinary Least Squares (PDOLS) estimators, the authors identified empirical evidence that validate the existence of the Marshall-Lerner condition, since the response of Brazilian states' net exports to a currency depreciation was positive.

Marçal et al. (2009) test the hypotheses of the J-curve and the stability of the relationship between the real exchange rate and the trade balance from the first trimester of 1980 to the fourth trimester of 2004 for Brazil. The results show that there is stability between the exchange

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[2] An aggregate analysis of the J-curve in accordance with the works of Neves and Lélis (2007), Marçal et al. (2009) and Fligenspan (2009) will be carried out. Based on the results of the estimated BVAR, it can be observed that the problem of the aggregation bias has no influence on the quality of the model, since IRF showed a statistically significant relation of a real exchange rate devaluation on the trade balance.

[3] For further explanation on the J-curve, see Krugma and Obtsfeld (2000) and Kulkarni and Clarke (2009).

[4] From the perspective of regional economic models, according to Rickman (2010).

rate and the trade balance, but they do not present favorable evidence of the existence of the J-curve. On the other hand, Neves and Lélis (2007) estimate price and income elasticities of exports between 1980 and 2004 with a Panel Data approach and conclude that all Brazilian states present inelastic export demand in relation to price and income, except for São Paulo, which has a significant participation of high value-added products.

Ramos Filho and Ferreira (2016) test the J-curve hypothesis for selected sectors of the Brazilian manufacturing industry, with annual data from 1996 to 2012. The authors used an ARDL model with cointegration (Pesaran et al., 2001) which presented results that only four sectors, inherent to those of high and low technological intensity, incompletely exhibit the J-curve effect. Thus, the authors conclude that the occurrence of the J-curve is not associated with the level of technological intensity of the sectors of the economy.

Finally, it is worth mentioning that there are several studies conducted for different countries that test the hypotheses of the J-curve and the Marshall-Lerner condition. Bahmany-Oskooee and Fariditavana (2016), for example, test the J-curve hypothesis for Canada, China, the United States and Japan using ARDL and NARDL models, with quarterly data from 1973 to 2014: the J-curve hypothesis is empirically supported for the United States via ARDL, while for China, via NARDL. Turkay (2014) uses the VEC approach to test the Marshall-Lerner condition hypothesis in Turkey's trade balance compared to the rest of the world between 1980 and 2012. The results obtained validate the hypothesis in question. Lastly, Nusair (2017) tests the J-curve hypothesis for 16 Eastern European countries, with quarterly data from 1994 to 2005. The author uses ARDL (linear) and NARDL (nonlinear) models and concludes that the nonlinear model is the most suitable to test the existence of the J-curve, given that it had no empirical support in Eastern European countries when using linear model, however, when using nonlinear model, the J-curve hypothesis is not rejected for 12 of the 16 countries analyzed.

Besides this introduction, the following section discusses the Bayesian VAR methodology. In the third section, the analysis of the results is presented, followed by the final conclusions.

2. Econometric Methodology and Database

A variety of Bayesian *priors*[5] were developed to be used in vector autoregressive models, such as Litterman/Minnesota, Wishart-Normal, Sims-Zha Normal Wishat, Sims-Zha Norma Flat, and others. We opted for the Litterman/Minnesota method. In this method, β *prior* is usually distributed and conditional to matrix Σ_e. Therefore, the Bayesian method can find *posteriors* for several different types of *priors* via simulation. However, for simplicity, assume that the prior of parameter β of the above regression model is normally distributed, that is:

$$p(\beta) \sim N(\underline{b}, \underline{V}) \tag{1}$$

So the *posteriors* will also be normally distributed:

$$p(\beta|\Sigma_e, z) \sim N(\overline{b}, \overline{V}) \tag{2}$$

[5] The Bayesian estimation method, by treating the parameters as random variables, imposes important probability distributions on *priors* of a VAR model complete set of coefficients, in order to obtain parsimony in the number of parameters to be estimated. Thusly, when using a BVAR, the estimators are more robust and, consequently, there is an estimated model with a better predictive power. However, a proper choice of *priors* should impose some structure on the VAR that reflects the nature and process of generating data. This article closely follows the work on Bayesian VAR by Ouliaris et al. (2018).

β mode and estimated mean will be given by a matrix of the weighted average of ordinary least squares estimates and the priors established in the research, whose *posterior* for β is given by the following expression:

$$\overline{b} = \left[\underline{V}^{-1} + \Sigma_e^{-1} \otimes (X'X)\right]^{-1} \left[\underline{V}^{-1}\underline{b} + (\Sigma_e^{-1} \otimes X')y\right] \quad (3)$$

From equation (3), one can observe that the $\overline{b}$ estimator depends on the variance-covariance matrix of random errors, $\sum_e$. . It is necessary to make an estimate for this matrix. This can be done using Ordinary Least Squares (OLS) information, as follows[6]:

$$\overline{V} = \left[\underline{V}^{-1} + \Sigma_e^{-1} \otimes (X'X)\right]^{-1} \quad (4)$$

It is essential, however, to establish some hypotheses on this variance-covariance matrix of random errors and on how it is estimated. There are three possibilities: (i) to use the residual variance estimates of an adjusted AR(1) model for each series; (ii) to replace Σ_e by its estimate, $\hat{\Sigma}_e$, in which the diagonal elements of this matrix, σ_i^2 , correspond to the OLS estimates of the error variances of a VAR. In the present work, this procedure was used to obtain the estimated variance-covariance matrix of random errors; or (iii) to use the Σ_e estimates of a complete VAR model.[7]

Once established how matrix Σ_e will be estimated, then β *priors* should be calibrated: $\mu_1, \lambda_1, \lambda_2$,and λ_3 is a set of hyperparameters. μ_1 is $\underline{b}$ prior mean. In some cases, one can wish this prior to be equal to 1 or very close to 1, to capture the persistence in I(1) economic and financial time series. However, if the VAR series are in difference or in growth rate, then a choice of $\mu_1 = 0$ would be more appropriate. λ_1 is the global adherence over the variance (first lag) and it controls the global adherence of β prior. λ_1 should be close to zero if there is more certainty about the *prior*, that is, when $\lambda_1 = 0.1$ is established, the prior information is allowed to dominate the sample information. In this situation, the prior is relatively strong.[8] λ_2 represents the relative adherence of other variables' variance. In other words, λ_2 controls the importance of the lag of variable j in the i-th BVAR equation, with $i \neq j$, and it is called cross-variable weights. If cross-lags play a relevant role in each equation of the model, then $\lambda_2 \cong 1$, otherwise $\lambda_2 \cong 0$.[9]. λ_3>0 represents the relative adherence of the variance of the lags and, as a result, the decline rate of these lags. If λ_3=1, then there is a linear decline in lags (Moreira et al, 2015). After calibration of BVAR *priors*, the quality of the *priors* will be verified via robustness analysis.

In this work, the variables have monthly periodicity, from January 2003 to December 2019[10]. The model consists of four variables: one external variable (Source: FRED): i) U.S. imports, such as *proxy* for the income of the rest of the world (WI)[11]; and three domestic variables (Source: IPEADATA): ii) index of economic activity in Brazil/IBC-Br, such as domestic income *proxy* (GDPBR); iii) exports and imports (US$ FOB) that enable the

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[6] Another way to get information about matrix Σ_e is to produce Bayesian estimates of it, which will require establishing priors on the variance matrix in question.
[7] However, this possibility is not highly recommended, since the estimated matrix may be singular. That is, there may not be enough information when the number of variables and lags in the VAR are too large.

[8] If $\lambda_1 \geq 10$ is established, the prior is said to be uninformative/uncertain and the generated estimates will be close to the estimated coefficients of an unrestricted VAR. For more details, see Ouliaris et al. (2018).
[9] If $\lambda_2 = 0$, , then VAR collapses to a single-variable model .
[10] The analysis ends in December 2019, due to the shock of COVID-19.
[11] Imports of U.S. goods from the rest of the world in millions of dollars. The variable is made available by FRED already deseasonalized. Later, it was deflated by the U.S. Consumer Price Index.

construction of the trade balance, given by the ratio exports/imports (XM); iv) and real effective exchange rate (XR).[12,13]

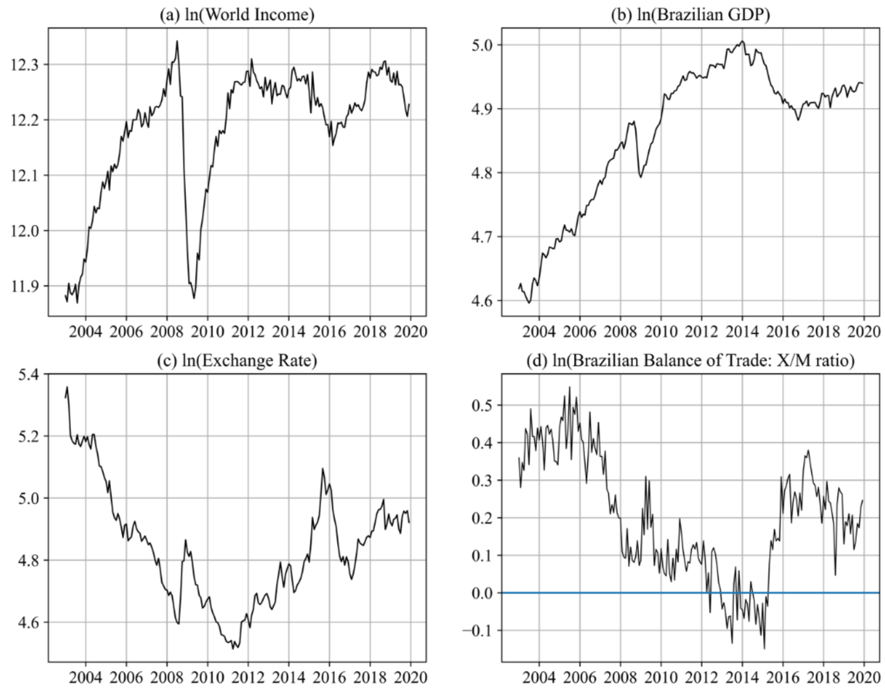
In a simplified way, the variables that make up the BVAR are shown in vector Y_t, represented by equation 5.

$$Y_t \;= (\mathrm{WI}, \mathrm{GDPBR}, \mathrm{XR}, \mathrm{XM}) \tag{5}$$

The variables will be used in level, although the four variables that make up the model are integrated of order one, $I(1)$, and that there is cointegration. This seeks to avoid imposing possibly incorrect restrictions on the model, according to Sims and Ulig (1991) and Stock and Watson (1990). In other words, even with $I(1)$ variables, the residuals continue to be stationary, given the inclusion of variables in level differences in the model (Hamilton, 1994).[14] The model will be adjusted using the natural logarithm of the variables and the ordering of the variables follows the same as the one exposed in equation 5.

Figure 1 shows the model's endogenous variables in logarithm from January 2003 to December 2019. The first, panel (a), refers to the *proxy* to capture the world income, as already highlighted. One can notice that, from January 2003 to July 2008, the variable showed significant growth, reaching a global maximum in the last month. However, with the subprime financial crisis, in less than a year, the variable went from its global maximum to a figure close to its global minimum, in May 2009.

Figure 1: Model variables



Note: Except for the exchange rate, all other variables are deseasonalized.

[12] Real Effective Exchange Rate - National Consumer Price Index (NCPI) - Exports: index (2010 average = 100). The real effective exchange rate is a weighted arithmetic mean of Brazil's bilateral real exchange rates relative to 23 trading partners selected. This series was calculated using NCPI as domestic price index.

[13] The seasonal adjustment in the trade balance and in the IBC-Br was carried out through X-13/ARIMA-SEATS.

[14] By using variables in level, however, the possibility of cointegration between the variables is implicitly allowed (Peersman and Smet, 2001), since the purpose of the analysis using autoregressive vector, VAR, SVAR or BVAR, is to determine the co-movements between the variables and not the estimated parameters. Therefore, the cointegration structure established between the variables is not an obstacle for the analysis of IRFs and VD of an estimated autoregressive vector (Sims, Stock and Watson, 1990).

The second variable, presented in panel (b), is the IBC-Br and it captures well the dynamics of the Brazilian economy. The Brazilian growth in the first decade of the millennium, as well as the 2014-16 internal crisis, can be visually verified. The third variable, panel (c), in its turn, is an index for the real effective exchange rate for Brazil's main trading partners. Finally, the last variable, in panel (d), is the log-difference between Brazilian exports and imports.

The impulse-response function and the variance decomposition of the estimated model showed that the co-movements between the variables are, in general, in accordance with the conventional economic theory. It is worth mentioning that even with $I(1)$ and cointegrated variables, there are no spurious relations between the variables that make up the BVAR.

3. Estimation and Results

Bayesian VAR estimation process was performed following the conventional routine of multivariate time series studies. The unit root test, KPSS, was performed, and it showed that all variables are non-stationary[15]; the Johansen test for cointegration was also performed and the conclusion drawn from it was that there is a cointegrating vector in the model[16]; besides, the optimal number of lags used in the model was determined taking into account the information criteria of Schwartz, Akaike and Hannan-Quin and the elimination of the residual autocorrelation problem, which established that a BVAR with four lags would be the most suitable.[17]

BVAR was estimated based on equation (5). The following *priors* were used: $\mu_1 = 1$, for the series are integrated of order one, I(1); $\lambda_1 = 5$, given that, in the present work, both the *priors'* information and the sample information are of vital importance in explaining the co-movements of the variables that make up the behavior of Brazil's trade balance; $\lambda_2 = 0.99$, for it is considered that cross-lags have a relevant role in each equation of the model; and λ_3=1, since a linear decline in the lags of the variables that make up the model is allowed (MOREIRA et al, 2015).

Since the adjusted BVAR did not present autocorrelated residuals, and it is stationary and robust[18], then it can be used to analyze the co-movements of the variables that compose it via impulse-response function (IRF) and variance decomposition (VD) of forecast errors.

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[15] In unit root and cointegration tests a significance level of 5% is used.

[16] According to Álvarez and Ballabriga (1994), Litterman/Minnesota priors have a good performance in the presence of cointegration. The authors showed, in a Monte Carlo experiment, that the addition of long-term constraints to the priors does not improve, in non-asymptotic samples, the performance, predictive analysis, IRFs and VD of a Bayesian VAR. Using variables in level, however, the possibility of cointegration between the variables is implicitly allowed (Peersman and Smet, 2001). The purpose of the analysis using autoregressive vector, VAR, SVAR or BVAR, is to determine the co-movements between the variables and not the estimated parameters. Therefore, the cointegration structure established between the variables is not an obstacle for the analysis of IRFs and VD of an estimated autoregressive vector (Sims, Stock and Watson, 1990).

[17] The results of the tests of unit root, cointegration, residual autocorrelation and Bayesian VAR stability can be consulted in the Appendix of this work.

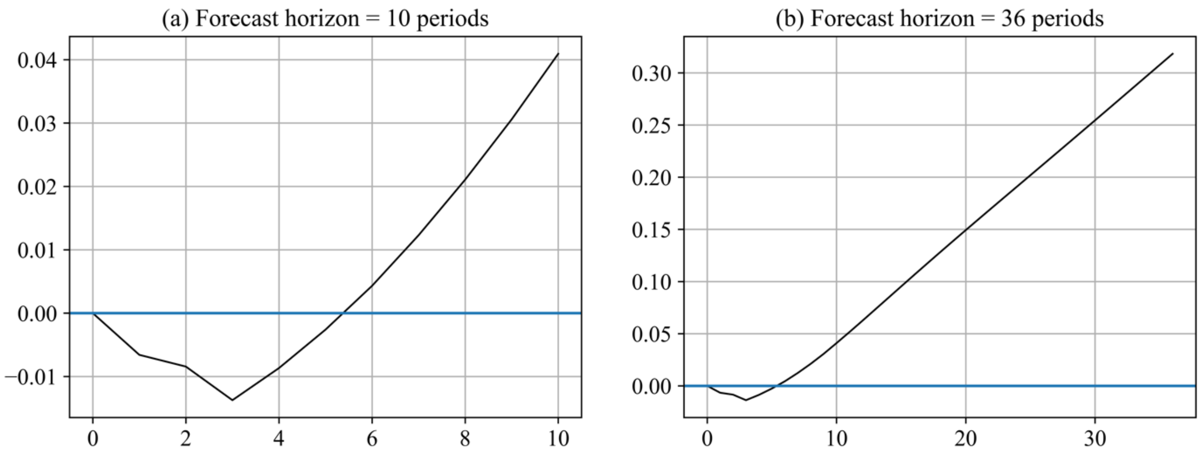
[18] The robustness test will be presented in section 3.3. This test consists of introducing the price of *commodities* in the BVAR and analyzing IRFs' stability or lack thereof. If the introduction of this variable results in significantly different IRFs when compared to the IRFs of the original model, then the VAR is not robust.

3.1. Impulse-Response Function

In Figure 2, the accumulated impulse-response function[19], via generalized method[20] (Generalized IRF), shows the trade balance (XM) response to an innovation of a standard deviation in real exchange rate (XR). The aim is to understand the short-term – panel (a) – and the long-term – panel (b) – dynamic behavior of the trade balance, given a currency devaluation.

In the short term, panel (a), it can be observed that a real depreciation of the Brazilian currency results, in the first five months, in a deficit in the trade balance. However, as of the sixth month, the result of the trade balance becomes positive and it remains like that for longer than ten months. This means that one cannot reject the J-curve hypothesis. In the long term, panel (b), it can be verified that the Marshall-Lerner condition should not be rejected either. In other words, a currency devaluation causes an increase in the trade balance for longer than 36 months. Therefore, the estimated BVAR empirically supports the existence of the J-curve and the Marshall-Lerner condition for Brazil from January 2003 to December 2019.

Figure 2: Trade balance accumulated responses to an innovation of one standard deviation in the real exchange rate for 10 and 26 months



Source: made by the authors.

3.2. Variance Decomposition

Figure 3 shows the forecast error variance decomposition (VD) for four different forecast horizons. That is, for two, four, twelve and eighteen months. Each column shows, for the response variable - trade balance, the forecast error proportion that is explained by the structural shocks of each of the four variables that make up the model, duly listed on the left side of the table. Therefore, for each forecast horizon, the sum of the entries in each line is equivalent to 100%.

The trade balance forecast errors, in the first four months, are explained by their own shocks, on average 96%. However, in the medium term or 18 months, its forecast errors are explained by its own shocks, around 69%, by world income shocks, 4%, by Brazilian GDP shocks, 8%, and by the real exchange rate, 18%. It can be observed that the real exchange rate, over time, significantly expands its participation in explaining the trade balance forecast errors.

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[19] The non-cumulative IRF is available with the authors. It presents a similar dynamic to the accumulated IRF. The latter was the one chose to be presented in the work, as it provides a better idea of the J-curve.

[20] IRFs generalized method is robust to changes in the order variables enter the BVAR (Pesaran and Shin, 1998).

Table 3: Variance Decomposition – Proportion of variance precision error per variable

Structural shocks (innovation)	Forecast (months)	World Income	Brazilian GDP	Real Exchange Rate	Trade balance
Trade balance	1	0.06	1.19	0.95	97.79
	4	1.15	1.63	1.63	95.59
	12	4.89	5.72	10.00	79.41
	18	4.33	8.11	18.57	68.98

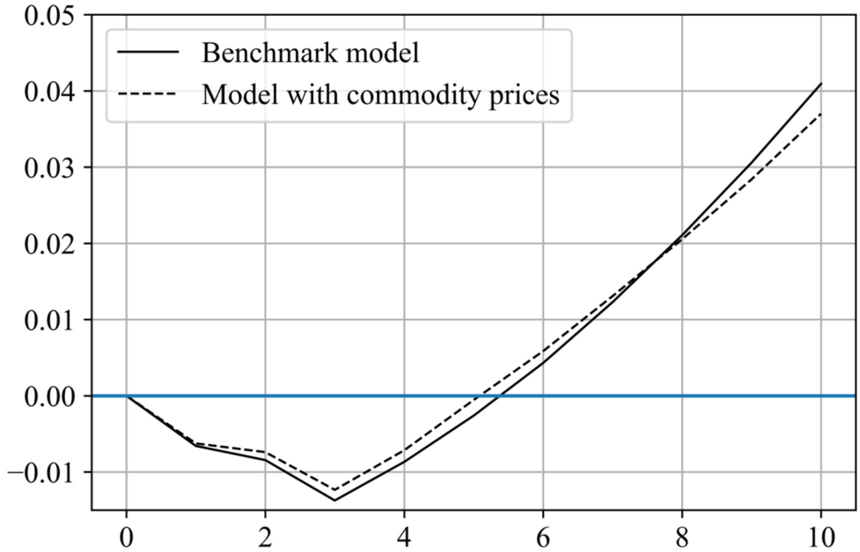
Source: made by the authors.

3.3. Robustness analysis

The robustness analysis was performed by including the commodity price variable (PCOMM) in the estimated BVAR[21]. In summary, the model was estimated assuming that the commodity price variable is the most exogenous of the variables that make up the BVAR, resulting in the following ordering of the autoregressive vector: i) PCOMM, ii) WI, iii) GDPBR, iv) XR and v) XM. It is worth mentioning that the variable that captures commodity prices is the Consumer Price Index, which considers all commodities around the world and is made available by the U.S. Bureau of Labor Statistics.[22]

Four lags and the same *priors* established in the model were used. Finally, it was analyzed whether there are significant changes in the dynamics of impulse-response functions (generalized) for BVARs with and without commodity prices.

Figure 3: Comparison of the trade balance accumulated responses to an innovation of one standard deviation in the real exchange rate without (original model) and with the inclusion of commodity prices



Source: made by the authors.

[21] For a robustness analysis in an autoregressive vector, see Luporini (2008).

[22] The same variable was incorporated into a SVAR model by Rocha, Magalhães and Brilhante (2022), who sought to investigate, among other things, the extent of the influence of commodity prices on credit cycles in Brazil. The study points out that commodity price index and U.S. GDP are relevant to consistently capture the dynamic interaction between credit and important domestic variables in Brazil, namely, GDP, inflation, interest rate and exchange rate.

Figure 3 compares the model after the inclusion of commodity prices with the original model. It can be verified that the inclusion of commodity prices in the BVAR did not result in significant changes in the (generalized) IRF, which captures real exchange rate shocks on the Brazilian trade balance. Therefore, it is concluded that the model is robust and the results presented in the study are relevant for the empirical verification of the hypotheses of the J-curve and the Marshall-Lerner condition in Brazil.

4. Conclusion

In this work, the hypotheses of the J-curve and the Marshall-Lerner condition were tested through the impulse-response function (IRF) and the variance decomposition (VD) of a Bayesian vector autoregressive model (Minnesota *priors*). The results showed that the BVAR estimated for Brazil empirically supports the hypotheses in question from January 2003 to December 2019. A depreciation of the Brazilian real results, in the first five months, in a deficit in Brazil's trade balance. However, from the sixth month onwards, the trade balance changes its behavior, remaining positive for longer than a year. The present study contributes to the specialized literature by using Bayesian VAR model, since it has been little used in studies that test the aforementioned hypotheses in Brazil.

The robustness analysis was performed by including the commodity price variable (PCOMM) in the estimated BVAR. It can be observed that the inclusion of commodity prices in the BVAR did not result in significant changes in the (generalized) IRF, which captures the real exchange rate shocks on the trade balance. Thus, it is concluded that the estimated BVAR model is robust and the results presented in the study in question are relevant for a good understanding of the dynamic relationship between trade balance and real exchange rate.

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Appendix

Table A1: BVAR stability test - Roots of Characteristic Polynomial

Root	Modulus
0.983278-0.015984i	0.983408
0.983278+0.015984i	0.983408
0.899755	0.899755
0.880430-0.156696i	0.894266
0.880430+0.156696i	0.894266
-0.477445-0.566948i	0.741204
-0.477445+0.566948i	0.741204
0.286879-0.592305i	0.658122
0.286879+0.592305i	0.658122
0.217421-0.509736i	0.554168
0.217421+0.509736i	0.554168
-0.492544	0.492544
-0.380773-0.304396i	0.487489
-0.380773+0.304396i	0.487489
0.308264	0.308264
-0.076766	0.076766

Note 1: No root lies outside the unit circle., which implies the stability of the model.
Note 2: Lag specification = 4.
Note 3: Endogenous variables: ln(GDPBR), ln(XR), ln(XM) and ln(WI).

Table A2: KPSS test for unit roots

Variable		ln(GDPBR)*	ln(XR)*	ln(XM)*	ln(WI)*	ln(WI)**
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.421	0.398	0.340	0.117	0.837
Asymptotic critical values:	1% level	0.216	0.216	0.216	0.216	0.739
	5% level	0.146	0.146	0.146	0.146	0.463
	10% level	0.119	0.119	0.119	0.119	0.347

Note 1: Null Hypotesis: the column variable is stationary.
Note 2: Bandwidth: 11 (Newey-West automatic) using Bartlett kernel.
Note 3: * indicates that the constant and linear trend were used to compute the test. On the other hand, ** indicates that just the constant was used to run the test.

Tables A3 and A4 report that the model is well specified, that is, it does not suffer autocorrelation with respect to residuals.

Table A3: Likelihood ratio test – residuals

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	17.7265	16	0.3402	1.111655	(16, 538.3)	0.3402
2	12.74121	16	0.6916	0.795359	(16, 538.3)	0.6916
3	19.76718	16	0.2309	1.241964	(16, 538.3)	0.231
4	21.76587	16	0.1509	1.370065	(16, 538.3)	0.151
5	21.72926	16	0.1522	1.367714	(16, 538.3)	0.1522

Note 1: Null hypothesis: No serial correlation at lag h.
Note 2: *Edgeworth expansion corrected likelihood ratio statistic.

Table A4: Likelihood ratio test – residuals

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	17.7265	16	0.3402	1.111655	(16, 538.3)	0.3402

2	31.6626	32	0.4836	0.990551	(32, 635.9)	0.4839
3	45.97562	48	0.5562	0.957421	(48, 649.2)	0.5569
4	81.41414	64	0.07	1.289892	(64, 644.3)	0.0707
5	102.1476	80	0.0482	1.298948	(80, 633.6)	0.0491

Note 1: Null hypothesis: No serial correlation at lags 1 to h.
Note 2: *Edgeworth expansion corrected likelihood ratio statistic.

Table A5: Trace and maximum eigenvalue tests

Number of cointegration vectors	Eigenvalue	Trace test statistic	Critical value of trace test	p-value
None*	0.188685	74.02245	63.8761	0.0055
At least 1	0.082937	32.41184	42.91525	0.3665
At least 2	0.048738	15.18252	25.87211	0.5594
Number of cointegration vectors	Eigenvalue	Trace test statistic	Critical value of trace test	p-value
None*	0.188685	41.61061	32.11832	0.0026
At least 1	0.082937	17.22932	25.82321	0.4386
At least 2	0.048738	9.943125	19.38704	0.6249

Note: the optimal specification of the Johasen test was established based on the Akaike criterion. This specification is composed of intercept and trend in the error correction vector (1st part of the table) and no intercept in the VAR (2nd part of the table). Trace and maximum eigenvalue tests show, at a significance level of 5%, that there is only one cointegration equation in the model.