How do adaptive learning expectations rationalize stronger monetary policy response in Brazil?[1]

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Abstract

This paper estimates a standard Dynamic Stochastic General Equilibrium (DSGE) model that includes a wage and price Phillip's curves with different expectation formation processes for Brazil and the USA. Other than the standard rational expectation process, we also use a limited rationality process, the adaptative learning model. We show that the adaptive learning model does a better job in fitting the data in both Brazil and the USA. Our estimation results show that expectations are more backward-looking and started to drift away sooner in 2021 in Brazil than in the USA. We then conduct optimal policy exercises that prescribe early monetary policy tightening in the context of positive output gaps and inflation far above the central bank target.

Resumo

Este artigo estima um modelo de equilíbrio geral dinâmico estocástico padrão que inclui curvas de Phillips para preços e salários com diferentes processos de formação de expectativas para o Brasil e EUA. Além de utilizar expectativas racionais, o trabalho também contém processos de racionalidade limitada, como o modelo de expectativas adaptativas com aprendizado. O modelo de expectativas adaptativas com aprendizado tem um desempenho superior em termos de qualidade de ajuste aos dados em ambos países de análise. Ademais, os resultados da estimação mostram que no Brasil as expectativas olham mais para o passado, com as expectativas sobre os níveis de preços em 2021 subindo antes no país que nos EUA. O artigo também discute política ótima, que recomenda aperto imediato da política monetária em um contexto de hiato de produto positivo e inflação acima da meta para lidar com expectativas adaptativas.

Keywords: DSGE, Inflation Dynamics, Optimal Monetary Policy, Forecasting and Simulation, Bayesian Estimation.

Palavras-chave: DSGE, Dinâmica da inflação, Política Monetária Ótima, Projeções e simulações, Estimação Bayesiana.

Área ANPEC: Macroeconomia, Economia Monetária e Finanças

JEL Codes: E2, E30, E52, E37, J6

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1 The views in this article are those of the authors and do not necessarily reflect the views of the IMF, or IMF policy.

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I. Introduction

After the introduction of inflation targeting regimes in many countries around the world, there was a sense that a relatively stable inflation and well-anchored inflation expectations ("the great moderation") would be the new normal in most countries. The pandemic shock and its repercussions accompanied a surge in global inflation not seen since the 1980s. As inflation spiked and remained high for several months, inflation expectations also started to drift higher. In this context, a better understanding of inflation dynamics and the implication for monetary policy is pressing at the current juncture.

In the second year of the pandemic, emerging market central banks tightened policies earlier in response to the bad inflation news. Is there any particular feature to these economies that justifies this earlier response? What kind of role do expectation formation processes play in shaping these decisions? We explore these questions through the lens of a canonical economic model of firm price setting and labor markets under different expectation formation processes.

Motivated by the need to better model the expectations formation and to match the inertia of macroeconomic variables, a recent growing literature proposed a deviation from the standard RE assumption. In particular, it is assumed that economic agents do not know about the underlying model that drives all the macroeconomic variables, and they form their expectations based on a simple statistical model with a smaller set of observed variables than the full information set. Those agents do update their beliefs about the underlying economic relations when new data becomes available (thus it is called adaptive learning, AL).

Our paper discussed the results of an estimated small DSGE model in which agents form expectations under AL. A comparison of the empirical performance of the AL model with the RE model shows that the AL model performs better in both in-sample and out-of-sample forecasts. The results also show that the AL mechanism creates inflation inertia that increases the cost of inflation stabilization.

There are two key results in our paper. In a context of positive output gap and rising inflation, we show that the central bank's future interest rate path would be steeper under AL expectations than under RE expectations because with the latter expectations provide an additional anchor and a self-enforcing tendency. Expectations in the AL model are always more inertial and harder to control once they start drifting away from the target. The second key result is that our estimation result showed that inflation expectations move by more in Brazil than the

USA when shocks hit the economy. Not only that, but the recent drift in beliefs captured by the model rationalized an earlier monetary policy response in Brazil.

Our paper also discusses the implication for optimal monetary policy when private agents form their expectations based on AL model. When inflationary shocks are present and the output gap is positive, it is optimal for monetary policy to respond sooner, more strongly and then ease. The goal is to avoid high inflation to become entrenched under AL. More aggressive policy actions are needed to anchor inflation and inflation expectations.

The rest of the paper is organized as follows: Section 2 discusses the related literature; Section 3 presents the model. In particular, we focus on the modeling of the expectation formation mechanism; Section 4 contains the data discussion and estimation results; Section 5 presents the results with scenario analysis in a context of positive output gap and inflation comparing the models for Brazil and the USA. Moreover, it discusses the optimal monetary policy and mechanisms through which a central bank affects inflation; and Section 5 concludes the paper.

II. Literature Review

Our way to model adaptive learning expectations and its estimation strategy mostly builds on the work by Slobodyan and Wouters (2012) and Slobodyan and Wouters (2012b). We extend their work in three dimensions. First, we use state-dependent conditional forecasting for our scenario analysis. Second, we apply optimal-control monetary policy on the model and compare the policy responses from an estimated reaction function. Third, we estimate a different benchmark model on both the U.S. and Brazil, which sheds light on the difference in expectations formation between an advanced economy and an emerging one.

The way that we set up agent's expectations situates our paper in the adaptive learning literature first advocated by Evans and Honkapohja (2001). The main idea from the learning literature is to replace the expected terms in intertemporal optimal conditions with an ad-hoc forecasting model that agents use to form expectations and update in every period using observed data, see also (Cho and Kasa (2015)) and (Eusepi and others (2019)). Our contribution to this literature is two-fold: first we discuss the introduction of optimal monetary policy in a model where expectations could de-anchor and point out the mechanism through which monetary policy can affect expectations. Second, we estimate the model for an emerging economy, Brazil, to compare how the learning mechanisms differ in those economies and what the implications are for the macroeconomic dynamics and monetary policy when a shock hits these economies.

In terms of the adaptive learning literature applied for Brazil, da Gama Machado (2012) assessed the role of asset price misalignment in monetary policy in an adaptive learning context, but using a calibrated model with constant coefficients. Instead, our paper estimates an adaptive learning model with time-varying learning parameters that depend on realized outcomes through a Kalman filter updating process. To the best of our knowledge, our paper is the first to estimate such models in the Brazilian context.

Our empirical result that the adaptive learning model outperforms the rational expectation model for both Brazil and the USA are similar to what Milani (2007) and Eusepi, Giannoni, and Preston (2018) show for the USA. Different from these papers, we estimate the model for an emerging economy and with time varying beliefs. Other than a better models performance, the adaptive learning model implies that forecast errors are correlated with forecast revisions, a feature of expectations documented empirically by Coibion and Gorodnichenko (2015).

Our results suggest that inflation expectations can be less well-anchored in Brazil than in the U.S., which warrants more aggressive monetary policy reactions in response to shocks. This is consistent with the finding that advanced economies tend to have better anchoring of long-term inflation expectations than emerging market economies (IMF (2016), Ha, Kose, and Ohnsorge (2019), Kamber, Mohanty, and Morley (2020), Bems and others (2021), among others). If as our empirical results show, inflation expectations are more 'adaptive' in certain economies, it raises the question of whether recent global factors that have put upward pressure on inflation (such as elevated commodity price and supply-chain shortages) could lead to a sustained period of high inflation levels and inflation variability in those places going forward.

Finally, our result that optimal monetary policy should respond more to inflation under adaptive learning when inflation is away from target has similarities with Orphanides and Williams (2004) that obtain that with non-rational expectations, monetary policy should respond more to inflation in order to subdue volatile expectations. However, when inflation are well-anchored, monetary policy should not respond as much and this has similarities with Eusepi, Giannoni, and Preston (2018) that find that monetary policy cannot and should not respond strongly to inflation fluctuations.

III. Model environment

Our model is based on Galí, Smets, and Wouters (2012), Smets and Wouters model. The main feature of this model is the existence of an Union that decides both the wage and household's labor supply decisions. In particular, each household has a simple consumption/saving decision to make based on the following problem:

$$\max_{C_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[log(C_t) - \int_0^1 \frac{l_{t,j}^{1+\vartheta}}{1+\vartheta} dj \right], \tag{1}$$

and the budget constraint of the household is given by

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + \int_0^1 W_{t,j} l_{t,j} dj + \pi_t, \qquad \text{for all } t \tag{2}$$

where P_t, is the price of consumption, B_t is savings, R_t is the real gross interest rates, $W_{t,j}$ for labor type $j \in (0,1)$ is the wage level chosen by the union, $l_{t,j}$ is the value implied by the demand curve for labor and π_t are profits net of lump sum government taxes. After linearizing the Euler equation around the efficient steady state, we obtain the familiar IS curve to be later estimated:

$$\hat{x}_t = E_t[\hat{x}_{t+1} - (\hat{i}_t - \hat{\pi}_{t+1})] + shk^y, \tag{3}$$

where $\hat{x}$ is the output gap, $\hat{i}_t$ and $\hat{\pi}_{t+1}$) are the interest rate and price deviations from steady state, respectively. The shock term, shk^y follows an $AR(1)$ process:

$$shk^y = \rho_y shk^y_{t+1} + \varepsilon_y, \tag{4}$$

The labor market is operated by perfectly competitive labor contractors that choose N_t and $l_{t,j}$ to maximize profits:

$$\max_{N_t, l_{t,j}} W_t N_t - \int_0^1 W_{t,j} l_{t,j} dj, \quad \text{subject to} \quad N_t = \left[\int_0^1 l_{t,j}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}}, \tag{5}$$

and that results in the labor demand:

$$l_{t,j} = N_t \left(\frac{W_t}{W_{t,j}} \right)^{\zeta}, \tag{6}$$

Given this labor demand, each union of type j negotiates wages to maximize the objectives of its members. In order to capture fluctuation in unemployment, we assume Calvo-style frictions to produce wage stickiness. Thus, we assume that there is a fraction $1 - \tau$ of firms that can optimize wages in the current period. For the non-optimizing unions, we assume that they use a simple indexation formula based on lagged nominal wage inflation $\pi_{w,t-1}$ and technology growth $\mu_{a,t-1}$:

$$W_{t,j} = \pi_{w,t-1}\mu_{a,t-1}W_{t,j}. \tag{7}$$

Meanwhile, the optimizing unions set $W_{t,j}$ to maximize the present value of the members' objectives:

$$\max_{W_t} E_t \sum_{i=0}^{\infty} (\beta\tau)^i \left[v_{t+i} W_t l_{t+i}^t - \frac{{l_{t+i}^t}^{1+\vartheta}}{1+\vartheta} \right], \qquad \text{subject to} \qquad l_t^{t+i} = N_{t+i} \left(\frac{W_{t+i}}{W_t} \right)^{\zeta}, \tag{8}$$

In this notation, l_t^{t+i} is the employment in time $t+i$ supplied by workers with the wage set in time t. v_{t+i} is household marginal utility of money at time $t+i$.

The solution to this problem is the wage Phillips curve that is used in the simulations we use in the next section.

$$\pi_{w,t} = \kappa_1 y_t - \kappa_2 \hat{\hat{w}}_t + \beta\pi_{w,t+1} + \varepsilon_{pi_w,t}, \tag{9}$$

where $\pi_{w,t}$ is nominal wage inflation, that is $\pi_{w,t} = \hat{\hat{w}}_t - \hat{\hat{w}}_{t-1} + \hat{\pi}_t$. Real wages are measured as deviations from technological growth, that is $\bar{w} = w_t - a_t$, and y_t is output gap.

On the production side, we also assume Calvo price-setting frictions. The final good firms are perfectly competitive and maximize profits:

$$\max_{Y_t} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj, \qquad \text{subject to} \qquad Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{10}$$

The solution to this problem delivers the familiar demand curve for the i^{th} intermediate good monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\epsilon}. \tag{11}$$

In this simple model, the production function of the intermediate firm is just $Y_{i,t} = N_{i,t}$. Finally, we assume that at every period there is a fraction of firms $1 - \theta$ that can re-optimize their prices while a fraction θ just keep their prices fixed $P_{i,t} = P_{i,t-1}$. The optimizing intermediate good firms then choose a price to solve the following problem:

$$\max_{P_t} E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left[v_{t+j} P_t Y_{i,t+j} - P_{t+j} Y_{i,t+j} s_{t+j} \right], \qquad \text{subject to} \quad Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\epsilon}, \tag{12}$$

After log-linearizing the solution to this problem around the steady state, one obtains the NK Phillips curve:

$$\pi_t = \kappa_p \hat{w}_t + \beta \pi_{t+1} + \varepsilon_{pi,t}, \tag{13}$$

We close the model with a standard monetary policy reaction function that features interest rate smoothing and estimated responses to inflation and output deviations:

$$\hat{i_t} = \rho \widehat{i_{t-1}} + (1-\rho)[\rho_\pi \hat{\pi}_t + \rho_x x_t] + \varepsilon_{i,t}, \tag{14}$$

where $\hat{i_t}$ is the nominal 1-year ahead policy rate as deviation from the neutral rate and ε_i are monetary policy shocks.

A. Expectation formation processes

This section is going to zoom in on the role that expectation formation processes have on the dynamic of main macroeconomic variables. The strategy is to estimate the model described in III under different expectation formation processes.

Monetary policy in the RE version of similar models has been studied extensively, for example in Svensson (1999) and Clarida, Gali, and Gertler (1999). This section compares the model dynamics under the standard RE formation process with the one implied by limited rationality models. The rational expectations model assume that households use all the information available in the model, including all parameters and variables, to form their expectations. In other words, expectations are model consistent and in the absence of further unexpected shocks $E_t[y_{t+1}] = y_{t+1}$. The limited rationality model we use is the adaptative learning expectations as developed in Slobodyan and Wouters (2012b) and Slobodyan and Wouters (2012) framework. In this model, households use and update statistical models with a smaller set of variables at every period. Households learn from mistakes and use their forecasts errors to update parameter values with a Kalman filter.

Households use a limited information set, X_j, and form their expectations linearly with:

$$y^j_{t+1} = X^j_t \beta^j_t, \tag{15}$$

for all the variables j that appear with leads in our equilibrium equations. In the terminology of the learning literature, this linear equation is called the Perceived Law of Motion (PLM). While any kind of linear model would work in this framework, the one with the best out-of-sample forecast performance is a simple univariate $AR(2)$ model. That is, the information set X_j contains a constant and two lags of y^j_{t+1}. With this model, the leading variables of the model can be cast in a SURE format:

$$\begin{pmatrix} Y^1_t \\ Y^2_t \\ Y^3_t \\ \vdots \\ Y^m_t \end{pmatrix} = \begin{pmatrix} X^1_t & 0 & \dots & 0 \\ 0 & X^2_t & \dots & 0 \\ 0 & 0 & X^3_t & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X^m_t \end{pmatrix} \begin{pmatrix} \beta^1_t \\ \beta^2_t \\ \beta^3_t \\ \vdots \\ \beta^m_t \end{pmatrix} + \begin{pmatrix} \eta^1_t \\ \eta^2_t \\ \eta^3_t \\ \vdots \\ \eta^m_t \end{pmatrix} \tag{16}$$

Where η are the errors with a non-diagonal variance-covariance matrix Σ. In every period, the learning update to the B vector (the stacked vector containing the β for all models) is done with a Kalman filter mechanism:

$$B_{t|t} = B_{t|t-1} + P_{t|t-1} X_{t-1} [\Sigma + X'_{t-1} P_{t|t-1} X_{t-1}]^{-1} \left(y^j_{t+1} - X^j_t B_{t|t-1} \right), \tag{17}$$

with the transition equation

$$B_{t+1|t} = \bar{B} + F \left(B_{t|t} - \bar{B} \right), \tag{18}$$

where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal. Finally, the corresponding covariance matrix and its transition are given by:

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} X_{t-1} [\Sigma + X'_{t-1} P_{t|t-1} X_{t-1}]^{-1} \left(X'_{t-1} P_{t|t-1} \right), \tag{19}$$

and

$$P_{t+1|t} = F P_{t|t} F' + V. \tag{20}$$

Once the coefficients for the believes are updated, $B_{t|t-1}$, the households form their expectations for the lead variables as in (15). If we replace these lead variables in the model solution, we obtain a time-dependent backward-looking representation of the model:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \alpha_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t. \tag{21}$$

Differently from the rational expectations solution, the matrices α_t, T_t and R_t are time dependent. They depend on the parameters that define policy function and on the forecast model summarized by the vector B_t. The system described in (21) is the Actual Law of Motion (ALM) of the model.

IV. Data and Model Estimation

The model described in Section 3 is estimated with quarterly macroeconomic data from 2000Q1 to 2019Q4 for Brazil and the USA. The set of variables included in the estimation are the output gap, the real wage gap, annualized quarterly price inflation deviation from target, and the policy rate. The specific variables used are described in the appendix.

As previously documented in Howard, Rich, and Tracy (2022), measures of average wage per worker in the USA suffered from changes in the workforce composition. Lower wage workers suffered larger employment losses than high wage workers. That created an artificial increase in average wage through this composition effect. The same qualitative change in workforce composition happened in Brazil. Housework and informal workers are the least paid category and the one that suffered the largest employment losses (Fig 1). Since our model does not have enough structure to explain this workforce composition change, we use the composition-constant real wage calculated by Howard, Rich, and Tracy (2022) for the USA and use the same logic to calculate a composition-constant real wage for Brazil. The adjustment at this aggregate level does not completely correct for the workforce composition change, but it lessen its effect on the real average wage series that we use for Brazil (Fig 2).

There are many possible filters to use in the data. We use standard filters, such as the HP filter and a linear filter, to calculate the output gap and the real wage gap[1]. We use the methodology developed in (Sun and Tsang (2019)) to make our filter selection. Thus, the results presented in the next section use the HP filter, which was chosen because the model has better in-sample fit (Table 1) and, in general, better out-of-sample forecast performance for wages and prices (Table 2). Given that the neutral interest rate in Brazil seemed to have changed during the

[1]Other filters can also be used and this is an area of robustness to be further explored

period (Perreli and Roache (2014)), we calculate $\hat{i}_t$ as the policy rate deviation from its linear trend. To take into account a possible neutral interest rate mismeasurement, we add an interest rate measurement equation to the Brazil's estimated model.

Figure 1. Brazil: Low wage workers suffered larger employment losses

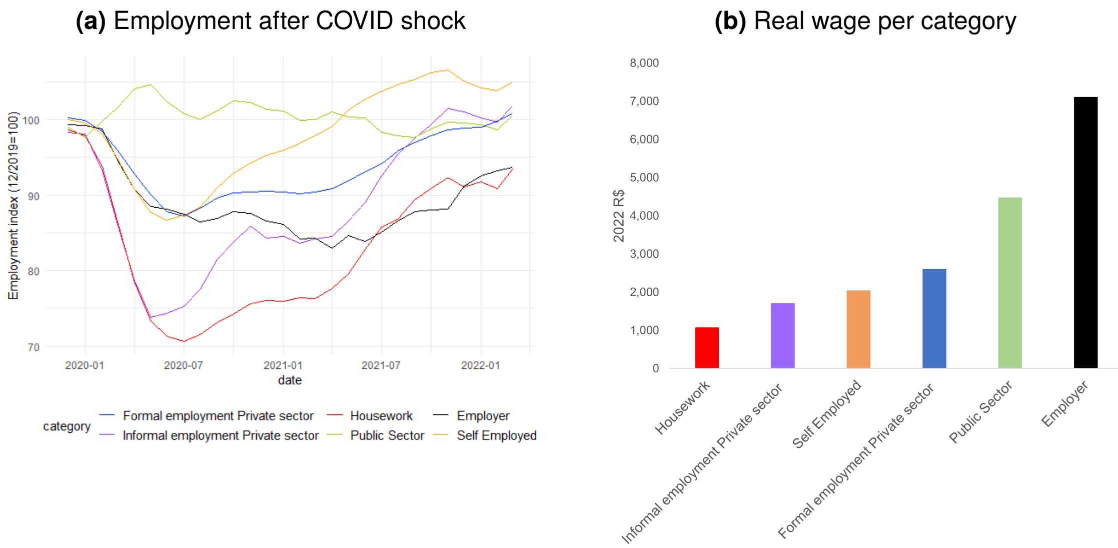
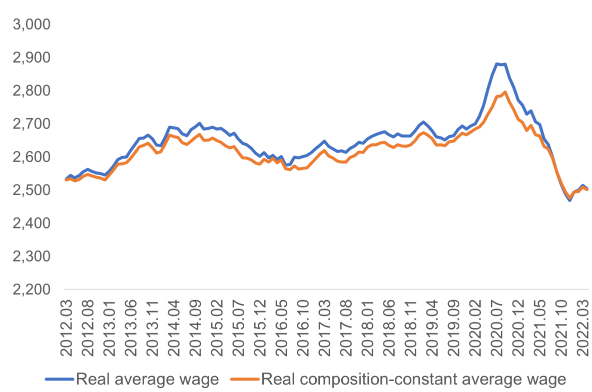


Figure 2. Real composition-constant wages did not increase as much



The model is estimated using the Bayesian likelihood methods with standard priors as in Smets and Wouters (2007). Some parameters have weak identification and are calibrated using standard values in the literature. Those parameters that are related to the steady state values of the observed variables of the model are also calibrated.

In Table 1, we report the in-sample forecast performance of the model for Brazil. The AL model outperforms the RE model regardless of our choice of the filter (linear or HP). The AL model with the HP filter has the best fit overall.

Table 1. In-Sample Forecast Performance for RE and AL Models

Log marginal likelihood	**RE**	**AL**
Linear Filter	-98.9	-96.4
HP Filter	-95.2	-93.9

In Table 2, we compare the out-of-sample forecast performance of the model in terms of root mean squared errors. We report results at different forecast horizons for the real wage gap, policy rate and inflation. Even though the RE model performs better in forecasting the next quarter inflation, the AL model outperforms in forecasting 4-quarters ahead. The AL approach also shows better forecasts for the policy rate in the near term, independent of the choice of the filter. The forecasts for the real wage gap is also better under AL even at the 1-quarter-ahead horizon. Part of the explanation for this better performance might be the high persistence of this variable. As forecast horizon lengthens, the forecast performance for the real wage gap of both models deteriorates, with RE deteriorating at a faster speed.

Table 2. Out-of-Sample Forecast Performance for RE and AL Models

	Real Wage Gap		**Policy Rate**		**Inflation**	
Linear Filter	**RE**	**AL**	**RE**	**AL**	**RE**	**AL**
1-quarter ahead RMSE	0.58	0.18	0.05	0.01	0.21	0.29
4-quarter ahead RMSE	0.46	0.71	0.4	0.21	0.50	0.32
8-quarter ahead RMSE	0.75	0.58	0.44	0.44	0.4	0.24
HP Filter	**RE**	**AL**	**RE**	**AL**	**RE**	**AL**
1-quarter ahead RMSE	0.44	0.21	0.031	0.02	0.08	0.2
4-quarter ahead RMSE	0.50	0.49	0.21	0.11	0.51	0.34
8-quarter ahead RMSE	0.88	0.51	0.19	0.39	0.2	0.22

V. Model simulation results

A. Comparison of the estimation results for Brazil and the USA

The historical shock decomposition sheds light about the different timing of monetary policy responses in the two countries. After very large negative inflation shocks in 2020, Brazil experienced large cost-push shocks in 2021 that peaked in 2021Q4 and that have been offset by negative real wage shocks recently (Fig 3a). Meanwhile, the USA has been experiencing large real wage shocks that explain about a third of the recent inflation (Fig 4a). Finally, there is a stark difference in monetary policy responses in the two countries. While Brazil had diminishing negative contributions from interest rates shocks starting in 2021Q2 that became positive in 2021Q4 (Fig 3b), the USA went in the opposite direction with increasing negative contributions from interest rate shocks in 2021 (Fig 4b).

One of the motivation questions in the introduction was if there is any particular feature to an emerging economy that justified this earlier response by Brazil Central Bank and what role expectations played in shaping these decisions. Our estimation results, shown in figure 5, help to answer this question. If inflation expectations are well anchored, we would expect the lag inflation coefficients in the household's forecasting models to be small and the mean inflation to be zero [2].

The first striking result in figure 5 is that expectations in Brazil depend a lot more on past outcomes than in the US. This can be seen when adding the estimated coefficients on the first two lags of both inflation and wages. The case of the estimated real wages expectations are even more striking as they seem substantially more persistent in Brazil. That is, any shock that increases real wages takes a lot longer to dissipate. The second result to highlight from this figure is the coefficient stability over the last ten years before the pandemic. The coefficient reflecting the mean expected inflation was zero as households expected inflation to be at the central bank target. The pandemic challenged this stability in both countries, but earlier and by more in Brazil. Mean expected inflation surged in the first quarter of 2021 in Brazil, as the country started seeing inflation outcomes above target. Meanwhile, mean expected inflation in the US increased by less and only by the third quarter of 2021.

[2]Inflation here is measured at an annual rate and in deviations from target.

Figure 3. Brazil: Shock decomposition

(a) Inflation **(b)** Interest rates

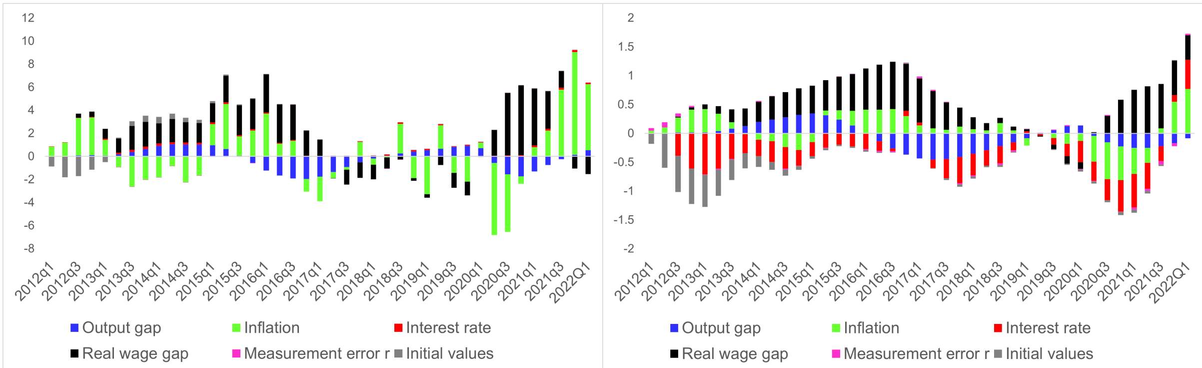
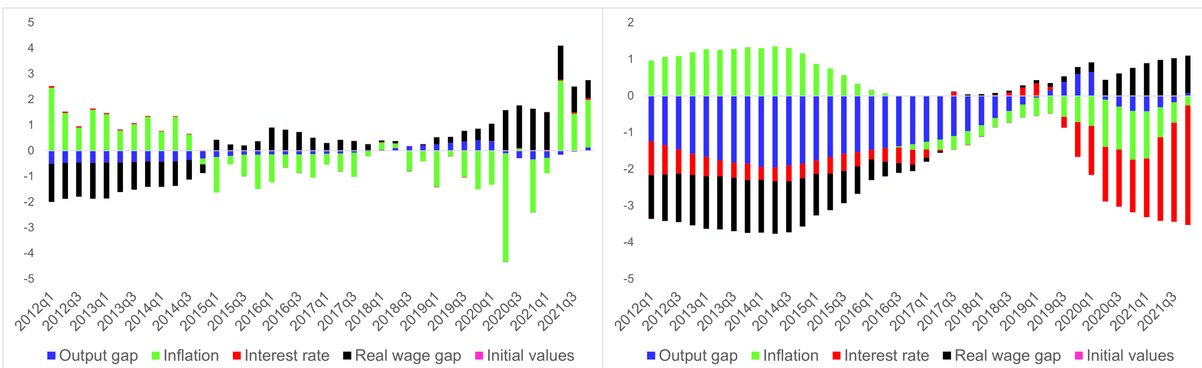


Figure 4. USA: Shock decomposition

(a) Inflation **(b)** Interest rates



Looking at the models' impulse response functions at the last data point (2021Q4) is also informative about the different role that AL expectations play in the two countries model dynamics (see figures 6 and 7). First thing to note is that inflation expectations in Brazil always move by more than in the US, regardless of the shock. As inflation expectations respond more to past inflation outcomes, as reflected in figure 5, there is a feedback of inflation to inflation expectations that keep inflation higher for longer for all the shocks in the model. This higher inflation response happens despite the stronger monetary policy response.

Figure 5. Inflation and wages expectations respond to past values by more in Brazil

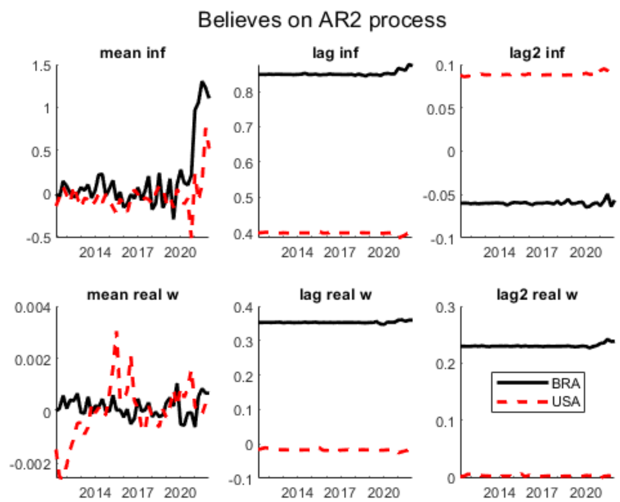
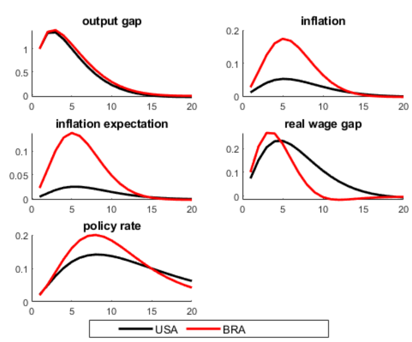
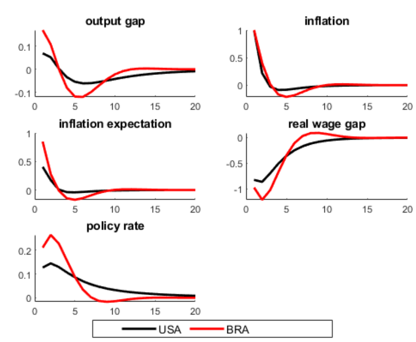


Figure 6. Impulse response functions for specific shocks

(a) Demand shock



(b) Inflation shock

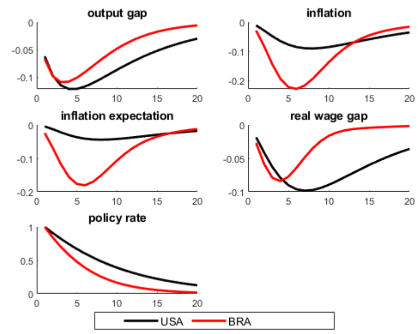


B. Adaptive learning expectations could increase the cost of stabilization

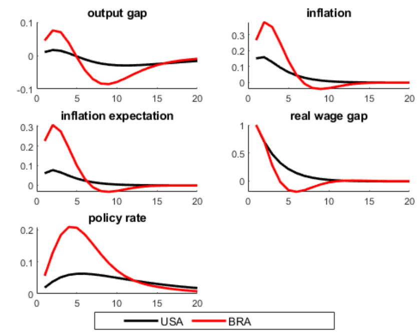
This section produces conditional forecasting scenarios to isolate the role of different expectations formations in the current inflationary environment. The scenarios described in figure 8 share a common set of shocks. In particular, we assume that, in both scenarios, Brazil faces an unexpected cost-push shock that takes actual inflation to what we observed in 2022Q2 and with a half-life of 6.5 quarters. Moreover, we assume that the output shock filtered by the models just unwind according to the estimated $AR(1)$ process in equation 4. With this strat-

Figure 7. Impulse response functions for specific shocks

(a) Mon policy shock

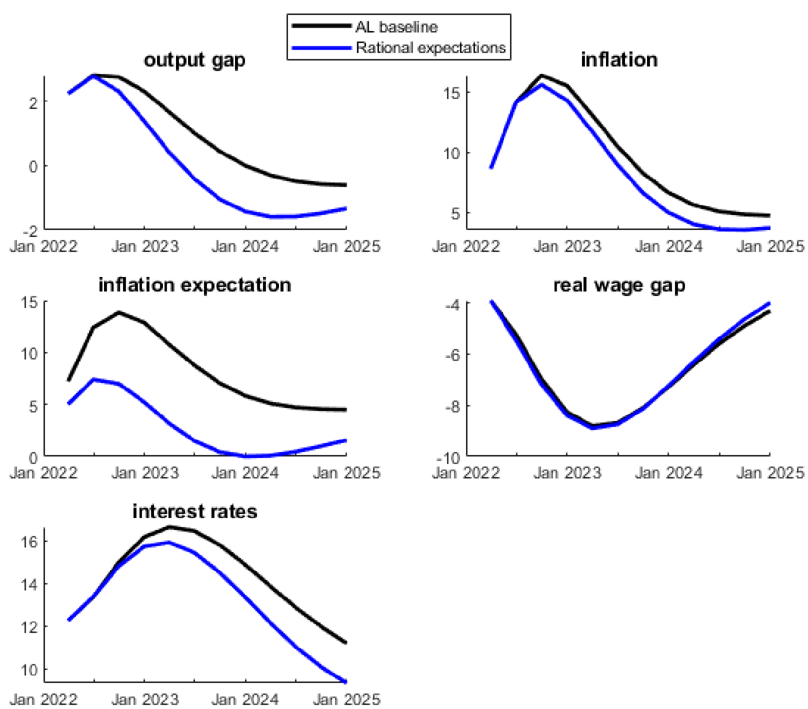


(b) Real wage shock



egy, we aim to isolate as much as possible the role of expectations formations in the dynamics going forward.

Figure 8. Inflation is stickier when expectations are adaptive learning



Inflation outcomes are more favorable under RE expectations (the blue line in figure 8) and are back to target at the beginning of 2024. The unexpected inflation shocks result in inflation outcomes that are substantially higher than inflation expectations, which peak in the second quarter of 2022. These lower expectations contribute to the decisive fall in inflation and interest rates over 2024.

The inflation inertia created under the estimated adaptive learning scenario (the black line in figute 8) results in inflation well above Brazil's central bank target for the next three years. Inflation remains at an annual rate of 4.8 percent by 2024Q4, despite tighter monetary policy and a negative output gap from the first quarter of 2024. Inflation starts to come down as the negative real wage gap more than offsets the remaining cost push shocks.

The main reason for this negative outcome is the sharp response in inflation expectations as a result from high inflation outcomes. Households start to believe that future inflation will run a lot higher than the target and these expectations contribute to keep inflation above target despite a negative output gap.

Our modelling strategy also contributes to the current debate initiated by Cochrane (2022) on adaptive expectations. We show how important it is to include labor market developments when discussing the effect of expectation formations. A price Phillip's curve that only includes the output gap, as in Cochrane (2022), and not a labor sector would predict that, under adaptive expectations, the only way to lower inflation is with a negative output gap. We show that the negative real wage gaps, meaning that real wage does not keep up with labor productivity, enable an anchoring of inflation even with fully adaptative expectations.

Given the negative inflation outlook under AL expectations, we can then ask what a central bank minimizing a welfare function would do in this scenario, which is addressed in the next section.

C. Optimal monetary policy discussion

In this section, we use the same strategy as in Alichi and others (2015) and instead of using the estimated monetary policy function as in equation (14), we assume that the central bank chooses a path for interest rate so as to minimize the following welfare loss function:

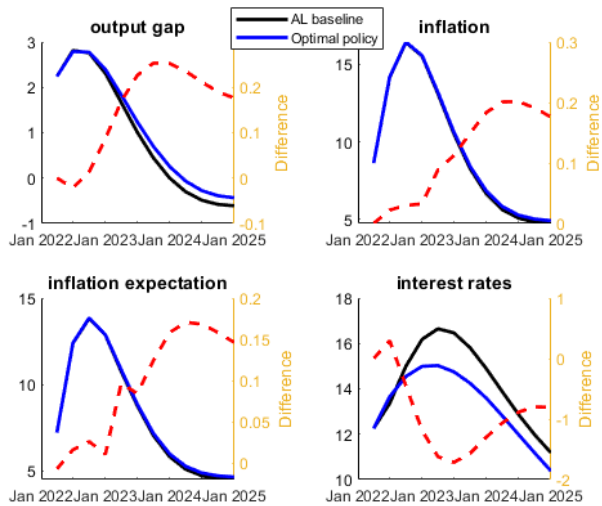
$$\min_{\hat{i}_t} E_t \sum_{t=0}^{\infty} \beta^t \left(0.75(\hat{i}_t - \hat{i}_{t-1}) + \hat{y}_t^2 + \hat{\pi}_t^2\right), \tag{22}$$

note that we assume equal weights for output gap $\hat{y}$ and inflation deviations from target $\hat{\pi}$. We also assume a role for interest rate smoothing. Thus, we define the optimal monetary policy path as the interest rate path that minimizes this function. In this section simulations, we further assume that the central bank has full knowledge of the current shocks hitting the economy, know all the future shocks that will hit the economy and also have full knowledge of how their actions impact expectations.

In our model, the central bank has three channels to influence inflation. The standard direct channel in which a tighter policy cools off demand, lowering the output gap and hence inflation. The other two channels operate through inflation expectations. By tightening policy, the central bank lowers current inflation that enters in the $AR(2)$ inflation expectations equation, lowering next period expectations. Finally, the central bank can affect households' learning process (the coefficients in the $AR(2)$ equation). By seeing less inflation this period than they have expected, households update their model of how past inflation matters for future inflation. The combined effect of these three channels can be seeing in figure 9.

The optimal policy prescribes front-loading the interest rate tightening and then easing compared to the estimated monetary policy reaction function. The blue line in figure 9 shows the optimal interest rate path and the secondary axis shows the difference between the two lines. The optimal path has policy tighter by 35 basis points in the 2022Q2 but then looser over the next two years and a half. With that policy, the output gap does not fall as much and inflation is a bit higher than in the estimated monetary policy reaction function. Note that it takes time for the deviation in monetary policy to influence inflation and that the difference gets higher over time. The more aggressive tightening of the estimated monetary policy reaction function is also a result of not incorporating the full path of inflation shocks in the interest rate decisions.

Figure 9. The central bank should front-load tightening and then ease



VI. Conclusions

After a long period of stable inflation and inflation expectations, the COVID-19 crises ramifications produced a surge in global inflation not seen since the 1980s. In this context, it is important to understand how expectations are formed and how they can affect the macroeconomic outlook going forward.

This paper introduced a standard New Keynesian model that includes a wage and prices Phillip's curves but differ in the way that households form expectations. We move away from the standard assumption of rational expectations and include "limited rationality" backward-looking expectation formation process in which households learn from previous forecasting mistakes. This modelling strategy creates a new mechanism through which central banks can affect inflation. In particular, central banks can affect households' learning process. Our first important result is that a standard DSGE model with AL expectations outperform the model with RE in terms of its in-sample and out-of-sample forecasting performance.

Our modelling strategy also shows how important it is to include labor market developments when discussing the effect of expectation formations. A price Phillip's curve that only includes the output gap and not a labor sector would predict that, under AL expectations, the only way to lower inflation is with a negative output gap. In a model with labor markets, this condition is not necessary and a negative real wage gap can act as an anchor to inflation.

Our results also rationalize why Emerging Market central banks tightened monetary policy earlier than Advanced Economies in response to bad inflation news. We argue that inflation expectations are more backward-looking and less anchored in Brazil than in the U.S., which warrants more aggressive monetary policy reactions in response to shocks. Moreover, the recent drift in beliefs captured by our learning model also rationalized an earlier monetary policy response in Brazil.

We then simulated scenarios to represent the current situation with a positive output gap and rising inflation. We show that it is optimal for monetary policy to respond sooner and more strongly under AL expectations than under RE expectations. The self-enforcing tendency to the target in RE expectations model provide an additional anchor. Finally, the optimal monetary policy response seeks to influence the learning process and avoid that believes of higher inflation lead to higher costs of disinflation.

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