Área escolhida: Área 4 – Macroeconomia, Economia Monetária e Finanças.

Título: MODELLING BRAZILIAN FEDERAL GOVERNMENT FISCAL REACTION IN THE TIME-FREQUENCY DOMAIN

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Resumo:

Utilizamos wavelet não condicional para abordar a discussão sobre a reação fiscal no Brasil no período de 2001 a 2022. O Wavelet Power Spectrum (WPS) apresenta maior intensidade de volatilidade durante a crise fiscal (2016-2017) e período de pandemia. Cronologicamente, primeiro identificamos a insolvência de médio prazo (ciclos de 1 a 2 anos) da dívida líquida no período 2003-2004. Em seguida, encontramos comovimentos antifase (curto e médio prazo) liderados pelo saldo primário na crise do subprime, durante a crise fiscal 2016-2017 e na recente pandemia. Há essa mesma evidência com baixa frequência (2 a 4 anos) entre 2013 e 2019. São sinais fiscais graves.

Abstract:

We use unconditional wavelet to address the discussion on fiscal reaction in Brazil over the period from 2001 to 2022. The Wavelet Power Spectrum (WPS) shows a higher volatility intensity during fiscal crisis (2016-2017) and pandemic period. Chronologically, we first identify medium-term insolvency (cycles of 1 to 2 years) of net debt in the period 2003-2004. Next, we find antiphase (short and medium term) co-movements led by the primary balance in the subprime crisis, during the 2016-2017 fiscal crisis and in the recent pandemic. There is this same evidence with low frequency (2 to 4 years) between 2013 and 2019. These are serious fiscal signs.

Palavras-chaves: Solvência da dívida, Wavelet, Resultado Primário.

Keywords: Debt Solvency; Wavelet coherency; Primary balance.

JEL classification: C6, E62, H63.

1. Introduction

Since the 1970s – mainly from Barro (1974) – the economic literature has recognized that public debt plays an important role in theoretical analysis of monetary and fiscal effects. This issue has become more relevant after the increase in public indebtedness in several countries in the 80s. For instance, total public debt as a ratio of GDP in U.S. rose from 31.16% in December 1980 to 56.04% a decade later. In this context of sharply elevated public debts, Bohn (1998) argues that although a nonstationary debt-income ratio would be implied by some models of optimal government finance, a high and growing debt level should be seem as worrisome, based on macroeconomic models with limited taxation, as Blanchard (1984).

About this discussion, however in empirical terms, Reinhart and Rogoff (2010) propose studying the longer-term macroeconomic implications of much higher public debt, based on a simple exercise using a multi-country historical dataset. Regarding emerging markets, they perform an exercise for 24 economies, and they find that for 1900–2009, median and average GDP growth hovers around 4% and 4.5% for levels of debt below 90% of GDP, but median growth falls markedly to 2.9% for high debt (above 90%). Because emerging markets often depend on external borrowing, they also employ more recent data on external debt, including debt owed both by governments and by private entities. When gross external debt reaches 60% of GDP, annual growth declines by about 2%, and for levels of external debt in excess of 90% of GDP, growth rates fall roughly in half.

Studying the behavior of debt in emerging countries seems complex because higher debt levels are usually associated with higher inflation. Specifically in the post-pandemic period, this line of research is vital, as such economies are registering high and unprecedented levels of indebtedness together with high inflation: a combination that can be quite dangerous, and perhaps in some economies irreversible in the short term. In this context, studying Brazil is relevant since it is an emerging economy among the 10 largest in the world, and also because this country has gone through a recent fiscal crisis, compromising the austerity of state governments. Observing Brazilian net debt to GDP, it ranges between 30% and 40% between 2001 and 2006. From 2007 it registered values between 20% and 30% until the end of 2016. During the fiscal crisis, this indebtedness started to grow again in the years of 2017 and 2019, reaching again the level of 40%. The concern is in the unprecedented behavior due to the pandemic in the years 2020 and 2021: debt to GDP rose from 31.23% (Mar/20) to 48.34% (Apr/22). There was only a strong increase comparable to that in the 2002 election period.

The discussion on the relationship between debt, inflation and economic growth in Brazil is still a little scarce, and in this sense, we can highlight the recent contribution of Matos et al. (2022). The more specific discussion about the solvency of the federal government's debt is much broader. Rocha (1997) finds an intertemporal budget balance only until 1990 when financial assets are frozen. Issler and Lima (1998) and Cavalcanti and Silva (2010) find evidence of stationarity in the public deficit with adjustment always via taxes, with expenditures being exogenously determined, while Lima and Simonassi (2005) find that seigniorage affects deficit management and a fiscal reaction is taken when the deficit becomes greater than 2.2%.

Simonassi (2007) finds a structural change in fiscal policy from 1995 onwards able to modify sustainable behavior in a low public sector response to deficit increases. Aligned methodologically, Mendonça et al. (2009) find a post-2000 regime change and corroborate the previous results of low fiscal reaction to deficits. More recently, Campos and Cysne (2019) estimate the fiscal reaction for Brazil using Kalman filter, spline smoothing and co-integration and they find a reduction in the variation of the primary deficit in response to the debt to GDP.

We add to this discussion by reviewing the fiscal reaction of the Brazilian federal government over the past two decades, now using wavelet-based coherency. This framework enables us to study time- and frequency-varying co-movements between primary balance and debt to GDP cycles. We are methodologically aligned to Lo Cascio (2015) which suggests the wavelet approach to verify fiscal sustainability in the U.S. between the years 1795 to 2012. The author finds evidence of fiscal sustainability in the long term, however only until 1995.

This paper is structured as follows. In Section 2 we have a theoretical discussion, and in Section 3 we show the wavelet framework. Section 4 illustrates the setup of the empirical model and reports our main findings, while Section 5 is devoted to the final discussion.

2. Debt Theory

The usual starting point of any discussion on sustainability of a public debt issue by a large national government is the model proposed by Barro (1979). The author assumes that the government can finance its expenditure through current taxation and public debt issue.[1] The government's budget in each period is

$$G_t + rb_{t-1} = \tau_t + (b_t - b_{t-1}) \tag{1}$$

In this model, real government expenditure in t, aside from interest rate payment on debt, is given by G_t, real tax revenue in t is denoted by τ_t, real stock of public debt outstanding ate the end of period t is denoted by b_t, and r means the rate of return on public and private debt. It is usual analyzing this same relationship powered by GDP. Solving the equation forward in time, this implies that the debt ratio in the future depends on the initial debt ratio, current and future interest and growth rates, and current and future primary balances. If the debt ratio is to remain constant, it is straightforward deriving a steady state relation between debt and the primary balance ratio. However, aiming to discuss on what debt level might be sustainable,[2] we need to know more about the behavior of the primary balance. We follow Blanchard et al. (2021) by believing that a reassuring theoretical and empirical answer was given in an influential paper by Bohn (1998).

Bohn (1998) proposes an empirical approach trying to address two relevant questions: How do governments react to the accumulation of debt? Do they take corrective measures when the debt-GDP ratio starts rising, or do they let the debt grow? In a very didactic way, the author suggests that one can find direct evidence for corrective actions by examining the response of the primary (noninterest) budget surplus to changes in the debt-income ratio. A positive

[1] He does not deal with currency issue, although this type of finance could be included as one form of current taxation.
[2] We assume that debt is sustainable so long as the probability of a debt explosion, and thus debt default, remains very low.

response shows that the government is taking actions—reducing noninterest outlays or raising revenue—that counteract the changes in debt. The idea is to search for a systematic relationship between debt to GDP (or debt-income ratio) and primary balance to GDP (or primary balance-income ratio), called fiscal reaction, by estimating the following regression:

$$s_t = \rho b_t + \alpha \boldsymbol{Z}_t + \varepsilon_t \qquad (2)$$

where s_t means the primary balance to GDP, i.e., taxes minus noninterest spending, and $\boldsymbol{Z}_t$is a set of other determinants and ε_t an error term. We follow Lo Cascio (2015) exercise applied to U.S. by proposing to revisit the fiscal reaction to Brazil, based on an unconditional mathematical framework in the time-frequency domain.

3. Methodology

The Fourier analysis can be considered one of the most important bases for the wavelet transform development. This analysis is a powerful tool to modelling time series on frequency domain. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands. Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\, \psi^*_{\tau,s}(t) dt \tag{3}$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series in space and scale. To capture different frequencies of the signal, we use a mother wavelet that is stretched and shifted:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{4}$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequencies movements. So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) dt \tag{5}$$

The first wavelet measure that we will present it's the wavelet power spectrum (WPS), which reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \tag{7}$$

To compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GWPS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \tag{8}$$

To study the dependencies between two original time series $x(t)$ and $y(t)$ in time-scale/frequency plane, Torrence and Webster (1999) were the first to define the wavelet coherence. The measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by:

$$W_{xy}(\tau, s) = W_x(\tau, s) W_y^*(\tau, s) \tag{9}$$

where $W_x(.)$ and $W_y(.)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and * denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $|W_{x,y}(\tau, s)|$. It computes the local covariance between two signals at each scale. The squared wavelet coherence is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherence is denoted as:

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{x,y}(\tau, s)\right)\right|^2}{S(s^{-1}W_x(\tau, s)^2)S(\,s^{-1}W_y(\tau, s)^2)} \tag{10}$$

where $S(.)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization fator ensuring the conversion to an energy density. Torrence and Webster (1999) note that in numerator of the squared wavelet coherence, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it's ensured that $0 \leq R^2 \leq 1$.

Hence, the main advantage of the wavelet coherence on XWT is the common measure unit to examine several combinations of signals. Torrence and Compo (1999) reveal that once the wavelet transforms conserves variance, the wavelet coherence is a good representation of the normalized covariance between two-time series, where the closer to zero (one) the coherence, the weaker (stronger) the local correlation between the time-series. The wavelet coherence has not theoretical distribution known; hence we follow the approach of Aguiar-Conraria and Soares (2011) deriving the confidence interval using Monte Carlo methods.

Although the wavelet coherence computes the degree of local linear correlation between two signals, it does not reveal patterns of lead-lag relationship neither if the movements are positives or negatives. To deal with these limitations, the phase-difference is commonly used to examine the delays in the fluctuations between the two time-series. Following Torrence and Webster (1999) we define the phase difference as:

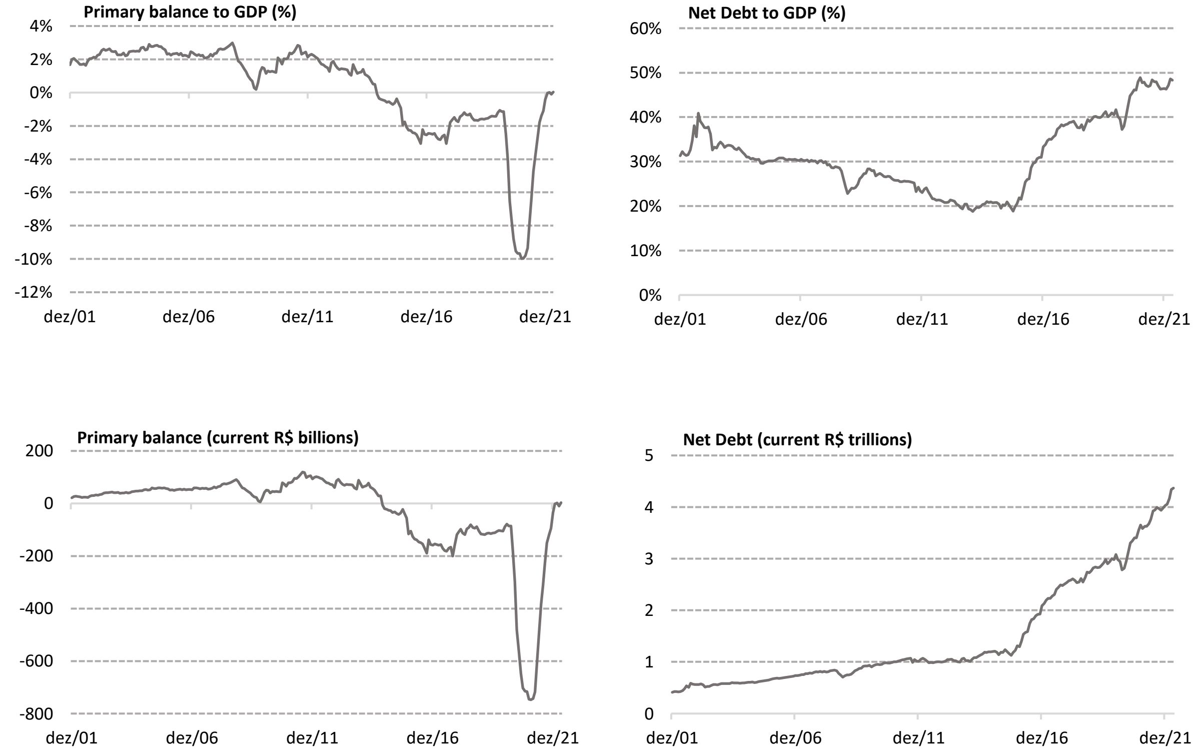
$$\phi_{xy}(\tau, s) = \tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1}W_{x,y}(\tau, s)\right)\right\}}{\Re\left\{S\left(s^{-1}W_{x,y}(\tau, s)\right)\right\}}\right) \tag{11}$$

The smoothed real ($\Re$) and imaginary ($\Im$) parts should already be calculated in the wavelet coherence function. Both $R^2(\tau, s)$ and $\phi_{xy}(\tau, s)$ are functions of the position index (τ) and scale (s). We also need the information on the signs of each part to determine the value of $\phi_{xy} \epsilon [-\pi, \pi]$. A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{xy} \epsilon \left(0, \frac{\pi}{2}\right)$ the series move in phase, but the time-series y leads x, while if $\phi_{xy} \epsilon \left(-\frac{\pi}{2}, 0\right)$ then it is x that is leading. A phase-difference of $\phi_{xy} = \pm\pi$ indicates an anti-phase relation. Finally, if $\phi_{xy} \epsilon \left(\frac{\pi}{2}, \pi\right)$, then x is leading and time-series y is leading if $\phi_{xy} \epsilon \left(-\pi, -\frac{\pi}{2}\right)$.

4. Empirical Exercise

Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN. In Fig. 1, we report the related temporal series.

Fig. 1. Primary Balance and Net Debt.



Source: Data can be found in the Brazilian Central Bank's Time Series Management System (SGS). Variable codes are: 4468 - Public Sector Net Debt (R$); 4503 - Public Sector Net Debt (% GDP); 5068 - NFSP Flow accumulated in 12 months without currency devaluation (R$); 5783 - NFSP without currency devaluation (% GDP).

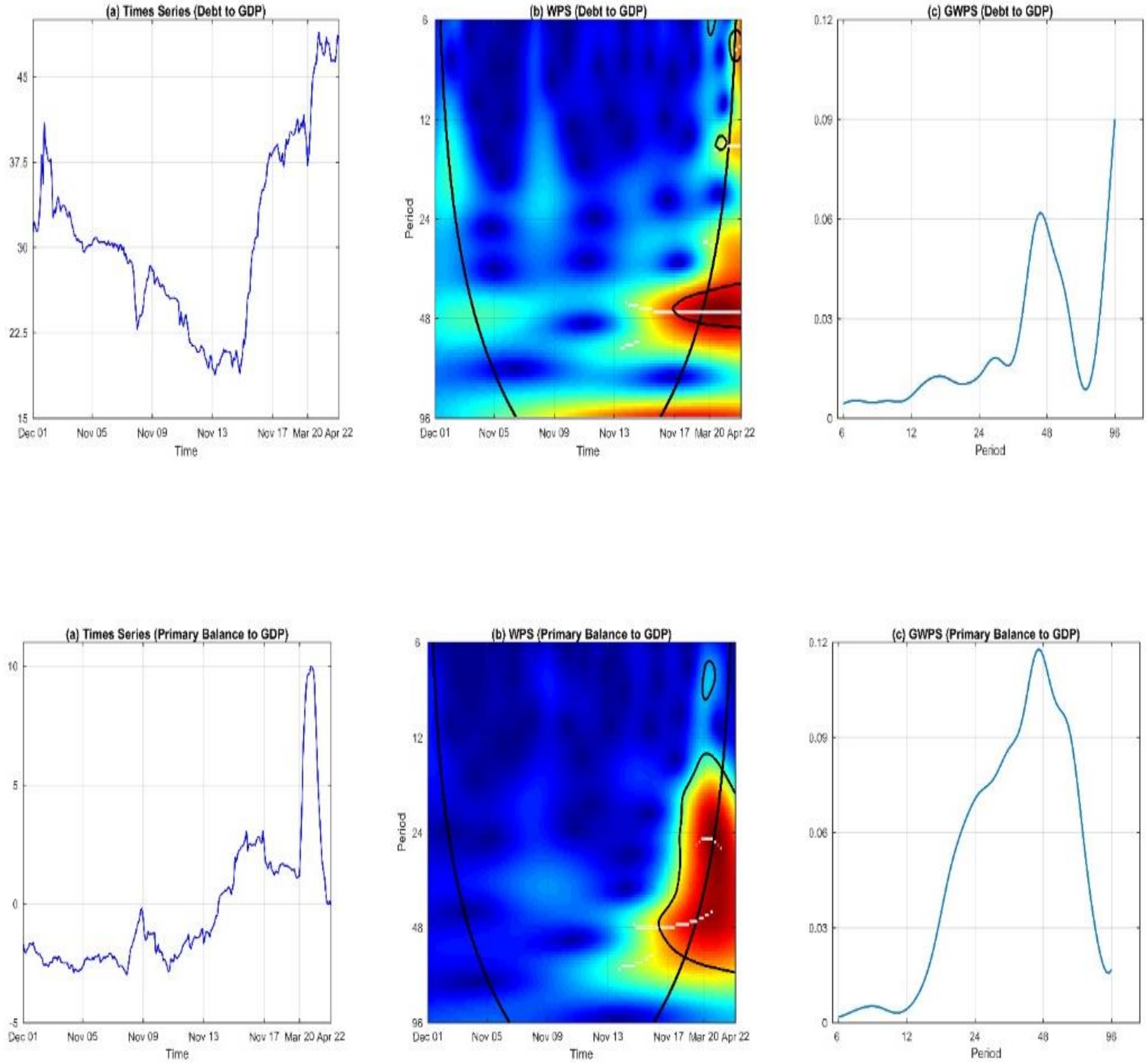
The analysis of indebtedness in nominal values suggests three periods with very clear characteristics. Between December 2001 and late 2015, there is an apparently linear growth trend, from 2016 to March 2020 another trend with greater slope. In the last 2 years of the sample, there is a third phase of evolution, with more accentuated growth and with an additional element: high inflation. In this third moment, this net debt rose from R$ 2.8 trillion in March 2020 to R$ 4.4 trillion in April 2022. When weighted by GDP, the behavior presents different characteristics. After a strong increase of almost 10% at the beginning of the time window, the debt-GDP ratio reduces from 40.91% (September 2002) to around 19% at the end of 2013. In this period, there was just one unusual swing during the subprime crisis. We also find a stability around 20% in 2014 and 2015, and a subsequent growth trend, breaking the 40% level at the end of 2019. The concern is in the unprecedented behavior due to the pandemic: debt to GDP rose from 31.23% (March 2020) to 48.34% (April 2022).

The primary balance is the inverse of the public sector financing need (NFSP). Observing its accumulated value (12 months), the federal government used to record a surplus until

October 2014. Even during the subprime crisis, the primary result was compromised, but there was no deficit. The first deficit was recorded in November 2014, followed by new deficits, with emphasis on the deficit value exceeding 200 billion in October 2017, during the fiscal crisis in the country. The recovery that began on that date was completely compromised by the pandemic. We highlight the record deficit of almost R$ 750 billion in January 2021. The recovery was rapid, and it was possible to identify a deficit in December 2021 lower than the registered before the pandemic. The behavior of the GDP-weighted series is visually very similar. It is important to highlight the value of approximately 10% of GDP associated with the record deficit at the beginning of 2021.

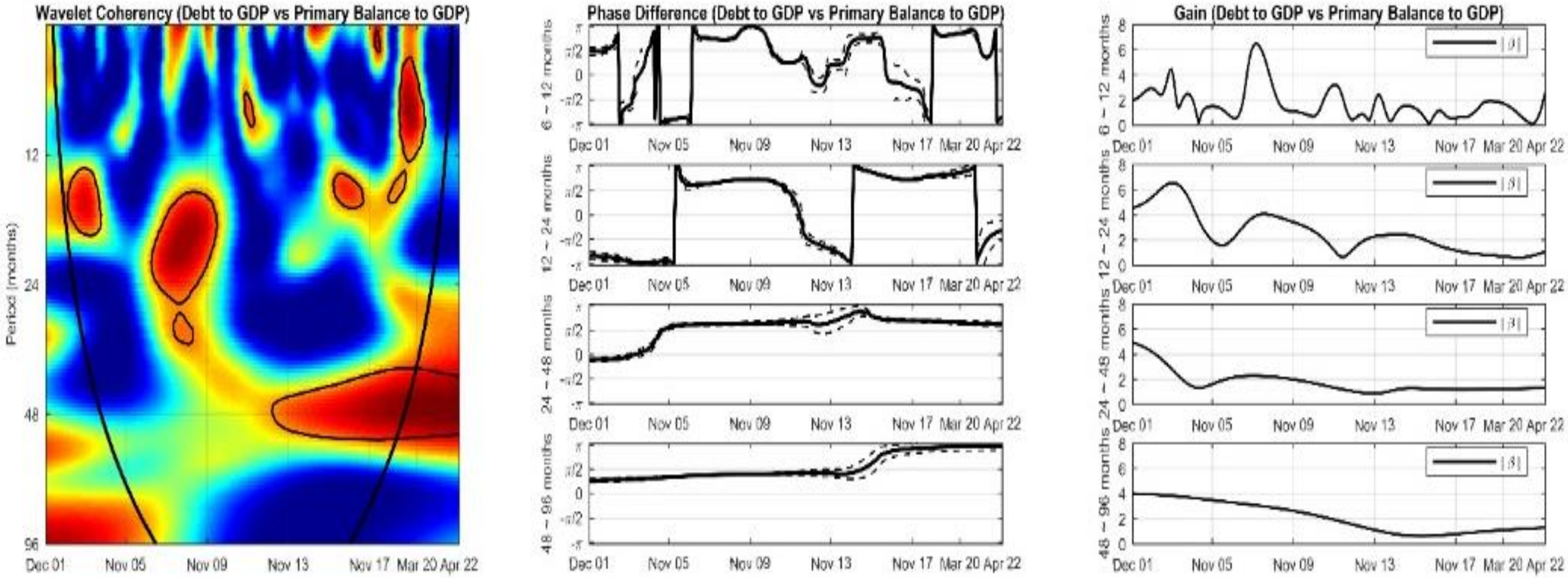
Fig. 2 shows WPS and GWPS for all series used here, while in Fig 3. We report the results on wavelet coherency, phase-difference and gain.

Fig. 2. Debt to GDP and primary balance time series (a), respective WPS (b) and Global WPS (c).



Notes: The black contours on the Wavelet Power Spectrum (WPS) plot refers to 5% significance and is theoretically obtained considering an AR (1) as the null hypotheses. In the heatmap, colder colors represent lower power while warmer colors represent higher power. The shaded area outside the Cone of Influence is subject to edge effects.

Fig. 3. Multivariate coherency: Debt to GDP vs Primary Balance



Notes: The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain of the explanatory variable, primary balance.

Remember that WPS reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane, while GPWS takes the average of wavelet power spectrum over all times. The WPS for primary balance shows a great power intensity between 2017 and 2020 with medium and low frequency, which is not recorded in the other periods. Regarding debt to GDP, we observe that the WPS has high power at lower frequencies only between 2016 and 2017 (fiscal crisis), and in the other periods the variance is low. The GWPS shows peaks only at long frequency, close to 48 months, between 6% and 9% for primary balance, between 9% and 12% for debt. We find that the spectrum varies along the time and frequency, indicating the non-stationarity of data and suitability of the method employed.

The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The purpose is modelling the linkage between primary balance and debt cycles. For a didactic analysis, we define as short-term or high-frequency an analysis with co-movements lasting less than 12 months, i.e., 1 year. Medium-term analyzes correspond to a frequency between 12 and 24 months (from 1 to 2 years). Long-term or low-frequency analyzes are associated with the interval between 24 to 48 months, 2 to 4 years, or in some cases, even more than 4 years.

We can highlight five periods in which the fiscal cycles show strong interaction.

Analyzing these moments in chronological order, we are able to identify medium-term (cycles of 1 to 2 years) insolvency of Brazilian government debt in the period 2003-2004, period characterized by electoral turmoil followed by a new government administration. We can measure this insolvent behavior with an elasticity ranging between 4 and 6. Next, we find (short and medium term) co-movements with opposite direction led by the primary balance in the subprime crisis, during the 2016-2017 fiscal crisis and in the recent pandemic. There is this same evidence – antiphase leadership by primary balance – with low frequency (2 to 4 years) between 2013 and 2019. This finding on debt cycles anticipated by primary balance cycles can be measured or characterized by elasticities lower than 4. All of our evidence is robust when using gross debt to GDP. We do not report these results, to save space, but we can disclose them if requested.

5. Conclusion

The literature that discusses strict tax rules has become obsolete. The current concern is focused on the stochastic analysis of sustainability in indebtedness. The concept of sustainability is focused on the future, developing methods and techniques that point in this direction will help in the construction of more assertive scores on the behavior of fiscal management in countries. Our paper does not suggest predictions for the future, but analyzes in greater detail the information hidden in past data. Looking at the past in terms of averaging hides patterns of interaction between crucial variables in the model. In other words, the wavelet methodology allows us to examine with more freedom how the variables affect each other and in different intensities. In this sense, we invite researchers to revisit this issue by using instruments, other techniques, other fiscal variables, or even other theoretical relationship useful to discuss on debt solvency, mainly in emerging economies.

Conflict of Interest

The authors declare that he has no conflict of interest.

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