The Macroeconomics Effects of Sovereign Wealth Funds in Small Open Economies

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Abstract

This paper assesses the macroeconomic effects of Sovereign Wealth Funds on commodity exporting small open economies. It develops a Dynamic Stochastic General Equilibrium model to assess the dynamic responses of main macroeconomic variables to commodity price shocks under two alternative fiscal rules: one in which the government uses all commodity related fiscal revenues to finance current government expenditures (spend-as-you-go rule), and the second all revenues are saved into a Sovereign Wealth Fund (prudent rule). The results show that under the prudent rule, government expenditure is less pro-cyclical and most macroeconomic variables are less volatile than under the spend-as-you-go rule. The size of government savings is the main mechanism behind the stabilization role played by Sovereign Wealth Fund. These results highlight the importance of adopting SWF as a stabilization mechanism in commodity exporting countries.

Keywords Business Cycles; Sovereign Wealth Funds; Fiscal Policy; Commodity price Shocks.

Resumo

Este artigo avalia os efeitos macroeconômicos dos Fundos Soberanos sobre pequenas economias abertas exportadoras de commodities. Desenvolve um modelo de Equilíbrio Geral Estocástico Dinâmico para avaliar as respostas dinâmicas das principais variáveis macroeconômicas a choques de preços de commodities sob duas regras fiscais alternativas: uma em que o governo usa todas as receitas fiscais relacionadas a commodities para financiar os gastos atuais do governo (*spend-as-you-go*), e no segundo todas as receitas são salvas em um Fundo Soberano (regra prudente). Os resultados mostram que sob a regra prudente, os gastos do governo são menos pró-cíclicos e a maioria das variáveis macroeconômicas são menos voláteis do que sob a regra *spend-as-you-go*. O tamanho da poupança do governo é o principal mecanismo por trás do papel de estabilização desempenhado pelo Fundo Soberano. Esses resultados destacam a importância da adoção de Fundos Soberanos como mecanismo de estabilização em países exportadores de commodities.

Palavras-chave: Ciclos de negócios; Fundos Soberanos; Politica fiscal; Choques de preços de commodities.

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1 Introduction

A common view is that unexpected (and more recently expected) commodity price shocks are one of the main sources of aggregate fluctuations in small open commodity exporting economies (Shousha, 2016; Fernández et al., 2017). The destabilizing effects of fluctuations in commodity prices are exacerbated when tax-related commodity revenues are a major source of government funding.[1] When this is the case, higher commodity prices can lead to higher fiscal revenues, which, depending on the institutional arrangements underlying the conduct of fiscal policy, may lead to higher government spending (exacerbating the pro-cyclical behavior of fiscal policy) and thus accentuating the cycles (Kaminsky et al., 2004; Pieschacon, 2012). This poses a major challenge in designing policy responses to commodity price fluctuations.

Sovereign Wealth Funds (SWFs) is an usual policy prescription for dealing with fluctuations in commodity prices in commodity exporting economies. Besides it has been used to diversify investments from non-renewable commodity exports, to increase savings for future generations, to fund economic development projects, among others objectives. According to the Sovereign Wealth Fund Institute, there are over 50 countries using some SWFs and the amount saved reached roughly US$ 7.9 trillions in 2018. The largest commodity related SWF is held by Norway, the Norway Government Pension Fund Global, that has over US$ 1 trillion in assets. There are developed and developing countries, oil and non-oil producers, and different institutional arrangements among the adopters (Mohaddes and Raissi, 2017).

This paper assess the macroeconomic impact of SWFs and international shocks in a representative small open commodity-exporting economy using a Dynamic Stochastic General Equilibrium (DSGE) model. The advantage of using such a model is that it identifies carefully the transmission mechanisms from shocks and the corresponding dynamic response of key domestic macroeconomic variables in a framework that is both micro-founded and consistent.

We use the model to assess the behavior of endogenous variables in response to commodity price shocks under two different fiscal policies. A spend-as-you-go rule, where the government spends all extra commodity revenue in current government consumption. The second is a prudent rule, where the government saves all extra commodity related fiscal revenue in a Sovereign Wealth Fund and spends only the expected real interest rate from the SWF.

The results show that under the prudent rule, government expenditure is less pro-cyclical and most macroeconomic variables are less volatile than under the spend-as-you-go rule. We also show that the main mechanism driving the responses of the endogenous variables to commodity price shocks is the size of government savings in the Sovereign Wealth Fund. The lower the fraction of the extra commodity revenue that the government saves in a SWF, the higher the volatility of the main aggregate variables. These results highlight the importance of adopting a Sovereign Wealth Fund as a stabilization mechanism in commodity exporting countries, helping to insulate the economy against fluctuations in commodity prices.

This paper relates to the literature that analyzes the design of fiscal policies and the effects of external shocks in commodity-exporting economies Garcia et al. (2011), Ojeda and Vargas (2014), Kirchner and van Wijnbergen (2012), Pieschacon (2012), Berg et al. (2013), Fornero et al. (2014), Medina and Soto (2014), Guerra-Salas (2014) and García-Cicco and Kawamura (2015).

The closest model is Pieschacon (2012) that develops a small open economy DSGE model and perform two counterfactual policy experiments. One in which she replaces the behavioral equations that describe fiscal policy in Mexico by the ones describing Norway and vice versa. She shows that "Norwegianizing" Mexico insulates the Mexican economy from oil price shocks. Similarly, "Mexicanizing" Norway has the opposite effects. She concludes that fiscal policy is an important mechanism driving the pass-through from oil price shocks to the domestic economy. Therefore, fiscal discipline can be a valuable tool to regulate the size of oil price shocks.

Our paper differs in a few dimensions. First, we decide to treat commodity production endogenously. This allows us to better capture the spillover effects from a booming commodity sector. Higher commodity prices generate a wealth effect that raises the demand for nontradable goods, creating a scarcity effect, increasing the relative price of nontradable goods (real exchange rate appreciation), lowering the tradable sector competitiveness. With endogenous commodity production, higher commodity prices increase the demand for labor and capital in the commodities sector, raising wages and capital rental rates across all sectors. There is also a cost channel as commodities are used as inputs in the final tradable good production. Second, we decide to explicitly model the rules that describe the fiscal policy in our representative small open economy. This allows us to perform policy experiments that shed some light on the mechanism driving the size of the responses of our endogenous variables to commodity price shocks. As we show, the size of government savings in the Sovereign Wealth Fund is the main mechanism driving the responses of our endogenous variables to commodity price shocks.[2]

The rest of this paper is organized as follows. Section 2 presents the model and the two fiscal rules that will be

[1] In Norway, commodity related tax revenues accounts for roughly 40% of fiscal revenues. Meanwhile, in Mexico, oil revenues account for about one third of government revenues (Pieschacon, 2012).

[2] Guerra-Salas (2014) also model the fiscal rules as we do in this paper, but in his model, commodity is an endowment as in Pieschacon (2012). Besides the model in Guerra-Salas (2014) does not present several real frictions that are important transmission mechanisms. García-Cicco and Kawamura (2015) discuss fiscal policy alternatives to deal with Dutch-disease problems as a result of commodity price fluctuations. Their model features Ricardian and non-Ricardian Households, real and financial frictions in a SOE DSGE model. They do not discuss the role of Sovereign Wealth Funds. They also treat commodity production as an endowment.

examined. Section 3 describes the solution method and the calibration strategy. Section 4 presents and discuss the main results, while section 5 performs additional analysis. Finally, section 6 presents some concluding remarks.

2 The Model

The model economy is a small open economy in the lines of Mendoza (1995) composed by households, firms and the government. There are three types of goods: a tradable, a nontradable and a commodity good. The commodity is either exported or used as input in the tradable production. This feature captures a cost channel from commodity price shocks. Firms are subject to a working capital constraint whereby shocks that affect the interest rate faced by the domestic economy in international financial markets affect the cost of borrowing. The government levies taxes on labor income, consumption of goods and on commodity revenue. Besides it can issue bonds in international markets. The government uses theses revenues to finance its expenses.

2.1 Households

There is a continuum of infinitely-lived households that maximizes a lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t - \tau c_{t-1}, h_t\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, c_t is consumption at t and h_t are hours of work. Households are subject to habit formation where $\tau \in (0,1)$ governs the degree of internal habit formation. Households own all physical capital and can issue bonds in the international financial markets in order to smooth out consumption. They are the firm owners. Thus their period-by-period budget constraint, in terms of the *numeraire* tradable good, is given by

$$\begin{aligned}(1+\tau_c)[c_t^T + p_t^N c_t^N] + \sum_{i=T,N,CM} \left(I_t^i + \Phi_i\left(K_{t+1}^i, K_t^i\right)\right) + r_{t-1}D_{t-1}^H = (D_t^H - D_{t-1}^H)+ \\ + (1-\tau_k)\sum_{i=T,N,CM} u_t^i K_t^i + (1-\tau_w)\sum_{i=T,N,CM} w_t^i h_t^i + (\pi_t^T + \pi_t^N + \pi_t^{CM}),\end{aligned} \tag{2}$$

where c_t^T and c_t^N denotes consumption of tradable and nontradable goods, p_t^N is the relative price of nontradable consumption goods, I_t^i is the investment in sector i, where $i = T, N, CM$. K_t^i denotes physical capital in sector i, r_t is the domestic real interest rate, D_{t-1}^R is the stock of real debt carried out from previous period, π_t^i are profits from the non-traded, traded and resource sectors, respectively. w_t^i is the real wage in sector i, u_t^i is the capital rental rate. τ^c, τ^k and τ^w are tax rates, and Φ_i denotes capital adjustment costs.

The capital stock for each sector evolves according to:

$$K_{t+1}^i = (1-\delta)K_t^i + I_t^i, \tag{3}$$

for $i = \{T, N, CM\}$, and δ is the depreciation rate.

Households choose contingent plans for consumption of tradable goods (c_t^T), nontradable goods (c_t^N), labor supply for each sector (h_t^T, h_t^N, h_t^{CM}), capital stock in the next period for each sector ($K_{t+1}^T, K_{t+1}^N, K_{t+1}^{CM}$) and debt-holdings ($D_t^H$) by maximizing a discounted expected utility (14) subject to the budget constraint (2), the laws of motion of capital (3) for each sector and a non-Ponzi game constraint of the form:

$$\lim_{j\to\infty} \mathrm{E}_t \frac{d_{t+j+1}}{\prod_{s=0}^{j}(1+r_s)} \leq 0.$$

Defining λ_t as the Lagrange multiplier associated to the budget constraint (2), the first order conditions associated to the households' problem are (2), (3) for each sector $i = \{T, N, CM\}$, all holding with equality, and

$$[U'_{c_t}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) + \beta\,\mathrm{E}_t(U'_{c_t}(c_{t+1} - \tau c_t, h_{t+1}^T, h_{t+1}^N, h_{t+1}^{CM}))]A'_{c_t^T}(c_t^T, c_t^N) = (1+\tau_c)\lambda_t, \tag{4}$$

$$[U'_{c_t}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) + \beta\,\mathrm{E}_t(U'_{c_t}(c_{t+1} - \tau c_t, h_{t+1}^T, h_{t+1}^N, h_{t+1}^{CM}))]A'_{c_t^N}(c_t^T, c_t^N) = (1+\tau_c)p_t^N\lambda_t, \tag{5}$$

$$-U'_{h_t^T}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) = \lambda_t(1-\tau_w)w_t^T \tag{6}$$

$$-U'_{h_t^N}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) = \lambda_t(1-\tau_w)w_t^N, \tag{7}$$

$$- U'_{h_t^{CM}}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) = \lambda_t(1-\tau_w) w_t^{CM}, \tag{8}$$

$$\lambda_t[1 + \Phi'_{K_{t+1}^T}(K_{t+1}^T, K_t^T)] = \beta E_t\{\lambda_{t+1}[(1-\delta) - \Phi'_{K_{t+1}^T}(K_{t+2}^T, K_{t+1}^T) + (1-\tau_k) u_{t+1}^T]\}, \tag{9}$$

$$\lambda_t[1 + \Phi'_{K_{t+1}^N}(K_{t+1}^N, K_t^N)] = \beta \, \mathrm{E}_t\{\lambda_{t+1}[(1-\delta) - \Phi'_{K_{t+1}^N}(K_{t+2}^N, K_{t+1}^N) + (1-\tau_k) u_{t+1}^N]\}, \tag{10}$$

$$\lambda_t[1 + \Phi'_{K_{t+1}^{CM}}(K_{t+1}^{CM}, K_t^{CM})] = \beta \, \mathrm{E}_t\{\lambda_{t+1}[(1-\delta) - \Phi'_{K_{t+1}^{CM}}(K_{t+2}^{CM}, K_{t+1}^{CM}) + (1-\tau_k) u_{t+1}^{CM}]\}, \tag{11}$$

$$\lambda_t = \beta(1+r_t) \, \mathrm{E}_t \, \lambda_{t+1}, \tag{12}$$

where

$$U'_{c_t}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) = \left[c_t - \tau c_{t-1} - \frac{(h_t^T)^{\omega^T}}{\omega^T} - \frac{(h_t^N)^{\omega^N}}{\omega^N} - \frac{(h_t^{CM})^{\omega^C}}{\omega^{CM}} \right]^{-\sigma},$$

$$U'_{c_t}(c_{t+1} - \tau c_t, l_{t+1}^T, l_{t+1}^N, l_{t+1}^{CM}) = (-\tau) \left[c_{t+1} - \tau c_t - \frac{(h_{t+1}^T)^{\omega^T}}{\omega^T} - \frac{(h_{t+1}^N)^{\omega^N}}{\omega^N} - \frac{(h_{t+1}^{CM})^{\omega^C}}{\omega^{CM}} \right]^{-\sigma},$$

$$U'_{h_t^i}(c_t - \tau c_{t-1}, h_t^T, h_t^N, h_t^{CM}) = -(h_t^i)^{(\omega_i - 1)} \left[c_t - \tau c_{t-1} - \frac{(h_t^T)^{\omega^T}}{\omega^T} - \frac{(h_t^N)^{\omega^N}}{\omega^N} - \frac{(h_t^{CM})^{\omega^C}}{\omega^{CM}} \right]^{-\sigma},$$

$$\Phi'_{K_{t+1}^i}(K_{t+1}^i, K_t^i) = \phi_i(K_{t+1}^i - K_t^i),$$

$$\Phi'_{K_{t+1}^i}(K_{t+2}^i, K_{t+1}^i) = -\phi_i(K_{t+2}^i - K_{t+1}^i),$$

for $i = \{T, N, CM\}$, and

$$A'_{c_t^T}(c_t^T, c_t^N) = \chi \left(\frac{c_t}{c_t^T} \right)^{\frac{1}{\varphi}},$$

$$A'_{c_t^N}(c_t^T, c_t^N) = (1-\chi) \left(\frac{c_t}{c_t^N} \right)^{\frac{1}{\varphi}}.$$

2.1.1 Consumption and labor

Aggregate consumption good is a basket of tradable (c_t^T) and nontradable goods (c_t^N) given by

$$c_t = \left[(1-\alpha_c)^{\frac{1}{\varphi}} (c_{T,t})^{\frac{\varphi-1}{\varphi}} + \alpha_c^{\frac{1}{\varphi}} (c_{N,t})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \tag{13}$$

where $\varphi \in (0,1)$ governs the share of nontradables in aggregate consumption, φ is the elasticity of substitution. Labor in the utility function is given by

$$h_t = H(h_t^T, h_t^N, h_t^{CM}) = \frac{(h_t^T)^{\omega^T}}{\omega^T} + \frac{(h_t^N)^{\omega^N}}{\omega^N} + \frac{(h_t^{CM})^{\omega^{CM}}}{\omega^{CM}},$$

where h_t^T, h_t^N, h_t^{CM} are, respectively, hours worked in the tradable sector, nontradable sector and commodity sector, and $\omega_T, \omega_N, \omega_{CM}$ are the Frisch elasticity of labor supply for each sector. This formulation implies labor is not perfectly mobile across sectors.

2.1.2 Functional forms

Each household have preferences described by utility function featuring internal habit formation in consumption:

$$U(c_t, h_t^T, h_t^N, h_t^{CM}) = \frac{[c_t - \tau c_{t-1} - h_t]^{1-\sigma} - 1}{1-\sigma}, \tag{14}$$

where $\sigma > 0$ is the coefficient of relative risk aversion and the parameter $\tau \in (0,1)$ denotes the intensity of internal habit formation.[3]

Capital adjustment costs take the form:

$$\Phi_i(K^i_{t+1}, K^i_t) = \frac{\phi_i}{2}\left(K^i_{t+1} - K^i_t\right)^2,$$

where ϕ denotes the parameter that governs the size of the adjustment cost.

2.2 Firms

There are three types of sectors in our model economy: a commodity sector, a final tradable good sector and a final nontradable good sector. They all operate under perfect competition. Within each sector there is a large number of identical firms.

2.3 Commodity Sector

Commodity firms use a cobb-douglas technology that combines labor and physical capital:

$$Y^{CM}_t = Z^{CM}_t (K^{CM}_t)^{\alpha^{CM}} (h^{CM}_t)^{1-\alpha^{CM}}, \tag{15}$$

where Z^{CM}_t is Total Factor Productivity (TFP) in the commodity sector, h^{CM}_t are hours worked and K^{CM}_t are capital services employed in the commodity production, and $\alpha^N \in (0,1)$ denotes the capital share.

Firms face a working capital constraint that requires them to hold an amount n^j of their wage and capital bill in a non-interest bearing asset to finance a fraction of the total production factor cost each period Neumeyer and Perri (2005); Uribe and Yue (2006); Shousha (2016). The working capital constraint takes the form

$$n^{CM}_t \geq \eta^{CM}[w^{CM}_t h^{CM}_t + u^{CM}_t K^{CM}_t], \tag{16}$$

$\eta^{CM} \geq 0$ represents the fraction of the total production factor bill that firms must hold. u^{CM} and w^{CM} denotes the rental rate of physical capital and the wage rate.

Firms can borrow from international financial markets to finance their working capital expenses at a cost r_t, the interest rate faced by the domestic country in international financial markets. Profits,π^{CM}, are given as:

$$\pi^{CM}_t = (1-\tau^{CM})p^{CM}_t Y^{CM}_t + (D^{CM}_t - D^{CM}_{t-1}) - w^{CM}_t h^{CM}_t - u^{CM}_t K^{CM}_t - (n^{CM}_t - n^{CM}_{t-1}) - r_{t-1}D^{CM}_{t-1}, \tag{17}$$

where τ^{CM} is the tax rate on commodity revenues, p^{CM}_t is the commodity price, and D^{CM}_t stands for the debt position of the firm in period t. Let firms' net liabilities (a^{CM}_t) in period t be defined as $a^{CM}_t \equiv (1+r_t)D^{CM}_t - n^{CM}_t$. Then profits can be rewritten as

$$\pi^{CM}_t = (1-\tau^{CM})p^{CM}_t Y^{CM}_t + \frac{a^{CM}_t}{1+r_t} - a^{CM}_{t-1} - w^{CM}_t h^{CM}_t - u^{CM}_t K^{CM}_t - \left(\frac{r_t}{1+r_t}\right) n^{CM}_t. \tag{18}$$

The international interest rate is positive at all times, so the working capital constraint will bind in every period. Thus, using (16) holding with equality in equation (18) to eliminate n^{CM}_t, we get:

$$\pi^{CM}_t = (1-\tau^{CM})p^{CM}_t Y^{CM}_t - w^{CM}_t h^{CM}_t\left[1 + \eta^{CM}\left(\frac{r_t}{1+r_t}\right)\right] - u^{CM}_t K^{CM}_t\left[1 + \eta^{CM}\left(\frac{r_t}{1+r_t}\right)\right] + \frac{a^{CM}_t}{(1+r_t)} - a^{CM}_{t-1}. \tag{19}$$

The firms' objective is to choose a^{CM}_t, h^{CM}_t and K^{CM}_t to maximize the present value of the stream of profits discounted using households' marginal utility of wealth, who are the firms owners, subject to the (flow) budget constraint, to the production technology, and to a non-Ponzi game borrowing constraint. Formally,

$$\text{Max} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \pi^{CM}_t \tag{20}$$

subject to (15), (19) and

[3] This type of utility function is described in Greenwood et al. (1988). A key feature of GHH preference is that there is no income effect on households labor supply.

$$\lim_{j\to\infty} \mathrm{E}_t \frac{a^{CM}_{t+j+1}}{\prod_{s=0}^{j}(1+r_s)} \leq 0 \tag{21}$$

The first order conditions associated with K^{CM}_t and h^{CM}_t are

$$u^{CM}_t \left[1 + \eta^{CM}\left(\frac{r_t}{1+r_t}\right)\right] = \alpha^{CM}(1-\tau^{CM})\frac{p^{CM}_t Y^{CM}_t}{K^{CM}_t} \tag{22}$$

$$w^{CM}_t \left[1 + \eta^{CM}\left(\frac{r_t}{1+r_t}\right)\right] = (1-\alpha^{CM})(1-\tau^{CM})\frac{p^{CM}_t Y^{CM}_t}{h^{CM}_t} \tag{23}$$

When $\eta^{CM} = 0$ we obtain the usual first order conditions, whereby firms choose K^{CM}_t and h^{CM}_t such that the marginal productivity of K^{CM}_t and h^{CM}_t equals the capital rental rate and the wage rates, respectively. Therefore, the introduction of the working capital constraint creates a wedge between the marginal productivity of K^{CM}_t and h^{CM}_t and their marginal cost, introducing a direct channel through which interest rate changes affect the real economy.

Any process a^{CM}_t satisfying equations (15), (19) and (21) is optimal. Therefore, assuming that firms start with no liabilities, then an optimal plan is holding no liabilities at all times, that is

$$\frac{a^{CM}_t}{1+r_t} = a^{CM}_{t-1}.$$

This implies firms will borrow only the amount needed to finance their working capital expenses and hence their debt holdings in period t equals

$$D^{CM}_t = \eta^{CM}\left[\frac{w^{CM}_t l^{CM}_t + u^{CM}_t K^{CM}_t}{1+r_t}\right]. \tag{24}$$

Thus, the optimal conditions are (22), (23), (24) and (15) holding with equality.

2.4 Nontradable Sector

The nontradable good is produced using a Cobb-Douglas production function that takes labor and physical capital as inputs:

$$Y^N_t = Z^N_t (K^N_t)^{\alpha^N} (h^N_t)^{1-\alpha^N}, \tag{25}$$

where Z^N_t is TFP in the commodity sector, h^N_t are hours worked and K^N_t are capital services employed in the commodity production, and $\alpha^N \in (0,1)$ is the capital share.

They also face a working capital constraint that requires them to hold an amount n^N of their production factor bill in a non-interest bearing asset to finance a fraction of the total production factor cost each period. The working capital constraint takes the form

$$n^N_t \geq \eta^N [w^N_t h^N_t + u^N_t K^N_t], \tag{26}$$

where $\eta^N \geq 0$ denotes the fraction of the total production factor bill that firms must hold.

Nontradable firms are allowed to borrow from international financial markets to finance their expenses with production factors. Profits of nontradable firms are:

$$\pi^N_t = p^N Y^N_t + (D^N_t - D^N_{t-1}) - w^N_t l^N_t - u^N_t K^N_t - (n^N_t - n^N_{t-1}) - r_{t-1} D^N_{t-1}, \tag{27}$$

where p^N_t is the price of nontradable goods and D^N_t stands for the debt position of the firm in period t. Let firms' total liabilities (a^N_t) in period t be defined as $a^N_t = (1+r_t)D^N_t - n^N_t$. Then, profit of the firms can be rewritten as

$$\pi^N_t = p^N_t Y^N_t + \frac{a^N_t}{1+r_t} - a^N_{t-1} - w^N_t l^N_t - u^N_t K^N_t - \left(\frac{r_t}{1+r_t}\right) n^N_t, \tag{28}$$

As in the commodity sector, the working capital constraint will bind in every period (for positive interest rates). Thus, using (26) holding with equality in (28) to eliminate n^N_t, we get:

$$\pi^N_t = p^N_t Y^N_t - w^N_t l^N_t \left[1 + \eta^N\left(\frac{r_t}{1+r_t}\right)\right] - u^N_t K^N_t \left[1 + \eta^N\left(\frac{r_t}{1+r_t}\right)\right] + \frac{a^N_t}{(1+r_t)} - a^N_{t-1}, \tag{29}$$

The firms' objective is to choose a^j_t, l^j_t and K^j_t to maximize the present value of the stream of profits discounted using households' marginal utility of wealth subject to the (flow) budget constraint, to the production technology, and to a non-Ponzi game borrowing constraint. Formally,

$$\text{Max} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \pi_t^N, \tag{30}$$

subject to (25), (29), and to

$$\lim_{m\to\infty} \mathrm{E}_t \frac{a_{t+m+1}^N}{\prod_{s=0}^{m}(1+r_s)} \leq 0. \tag{31}$$

The first order conditions associated with K_t^N and h_t^N are, respectively

$$u_t^N \left[1+\eta^N \left(\frac{r_t}{1+r_t}\right)\right] = \alpha^N \frac{p_t^N Y_t^N}{K_t^N}, \tag{32}$$

$$w_t^N \left[1+\eta^N \left(\frac{r_t}{1+r_t}\right)\right] = (1-\alpha^N)\frac{p_t^N Y_t^N}{h_t^N}. \tag{33}$$

As in the commodity sector, any process a_t^N satisfying equations (25), (29) and (31) is optimal. Therefore, assuming that firms start with no liabilities, then an optimal plan is holding no liabilities at all times. This implies that firms borrow only the amount needed to finance their working capital:

$$D_t^N = \eta^N \left[\frac{w_t^N l_t^N + u_t^N K_t^N}{1+r_t}\right]. \tag{34}$$

Thus, the optimal conditions are (32), (33), (34) and (25) holding with equality.

2.5 Tradable Sector

The final tradable good is produced using a Cobb-Douglas technology that combines capital, labor services, and commodity goods as inputs in the production process:

$$Y_t^T = Z_t^T (K_t^T)^{\alpha^T} (CM_t^T)^{\gamma^T} (h_t^T)^{1-\alpha^T-\gamma^T}, \tag{35}$$

where Z_t^T is TFP in the commodity sector, h_t^T are hours worked, K_t^T are capital services and CM_t^T are commodity inputs, $\alpha^T, \gamma^T \in (0,1)$ are the capital and commodity share, respectively.

The working capital constraint takes the form:

$$n_t^T \geq \eta^T [w_t^T l_t^T + u_t^T K_t^T + p_t^{CM} CM_t^T].$$

where η^T governs the share of production factors bill that firms must hold over the production process.

Firms in this sector can also borrow from international financial markets to cover their working capital expenses. The problem of the firm is to choose physical capital (K_t^T), labor services (l_t^T), commodity goods (CM_t^T) and liabilities (a_t^T) in order to maximize their discounted expected stream of profits, that is:

$$\text{Max} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \pi_t^T, \tag{36}$$

subject to (35) and

$$\pi_t^T = Y_t^T - w_t^T l_t^T \left[1+\eta^T\left(\tfrac{r_t}{1+r_t}\right)\right] - u_t^T K_t^T \left[1+\eta^T\left(\tfrac{r_t}{1+r_t}\right)\right] - p_t^{CM} CM_t^T \left[1+\eta^T\left(\tfrac{r_t}{1+r_t}\right)\right] + \tfrac{a_t^T}{(1+r_t)} - a_{t-1}^T,$$

$$\lim_{m\to\infty} \mathrm{E}_t \frac{a_{t+m+1}^T}{\prod_{s=0}^{m}(1+r_s)} \leq 0, \tag{37}$$

where p_t^{CM} denote the price of commodity goods.

The first order conditions are given by equation (35) holding with equality and

$$u_t^T \left[1+\eta^T \left(\frac{r_t}{1+r_t}\right)\right] = \alpha_T \frac{Y_t^T}{K_t^T}, \tag{38}$$

$$p_t^{CM} \left[1+\eta^T \left(\frac{r_t}{1+r_t}\right)\right] = \gamma^T \frac{Y_t^T}{CM_t^T}, \tag{39}$$

$$w_t^T \left[1+\eta^T \left(\frac{r_t}{1+r_t}\right)\right] = (1-\alpha^T-\gamma^T)\frac{Y_t^T}{h_t^T}, \tag{40}$$

Firms borrow only the amount needed to finance their working capital, therefore the debt position is given by:

$$D_t^T = \eta^T \left[\frac{w_t^T l_t^T + u_t^T K_t^T + p_t^{CM} CM_t^T}{1 + r_t} \right]. \tag{41}$$

2.6 International Capital Markets

The economy can borrow any amount at the country-specific interest rate r_t in international financial markets. The interest rate faced by the domestic economy (r_t) is given by:

$$r_t = \overline{r} + \text{v}^d \left[e^{\left(D_t - \overline{D}\right)} - 1 \right] \tag{42}$$

where $\overline{r}$ is the steady state level of the country-specific interest rate, v^d is a parameter that governs the interest rate sensitivity to debt deviations from its steady-state values, D_t is the total debt position of the economy in period t and $\overline{D}$ is its steady state level. The last term is a a debt-elastic interest rate premium, which is needed to close the model (Schmitt-Grohé and Uribe, 2003). It captures, in reduced form, the idea that the risk premium is increasing with the debt position relative to its steady state level: the more the aggregate debt position is above its steady state level, the higher is the country-specific interest rate in international financial markets.

2.7 Government

The government taxes consumption, labor and the commodity sector, and uses these resources to finance government consumption and savings in a Sovereign Wealth Fund, F_t. The period by period government budget constraint is given by

$$T_t^{CM} + \tau_c(c_t^T + P_t^N c_t^N) + \tau_k \sum_{i=T,N,CM} u_t^i K_t^i + \tau_w \sum_{i=T,N,CM} w_t^i h_t^i + (1 + r_f)F_{t-1} = p_t^N G_t + F_t \tag{43}$$

where $T_t^{CM} = \tau_t^{CM} P_t^{CM} Y_t^{CM}$ is the commodity tax revenue. Fiscal policy will follow two set of rules that it will be described shortly. For simplicity, we abstract from government borrowing.

2.7.1 Fiscal Rules: Sovereign Wealth Funds

The government chooses between two different rules: a **spend-as-you-go rule** and a **prudent rule**. Under the **spend-as-you-go rule**, the government uses all commodity and non-commodity revenues to finance its current expenses. That is, government spending is determined by

$$p_t^N G_t = T_t^{CM} + \tau_c(c_t^T + P_t^N c_t^N) + \tau_k \sum_{i=T,N,CM} u_t^i K_t^i + \tau_w \sum_{i=T,N,CM} w_t^i h_t^i \tag{44}$$

And government savings in the Sovereign Wealth Fund are:

$$F_t = 0 \ \forall \ t \tag{45}$$

This rule implies that fluctuations in government revenues are transmitted directly to government spending.

Alternatively, the government could choose the **prudent rule** and save all commodity-related tax revenue in a Sovereign Wealth Fund, F_t. Under this rule, the government can spend only the expected real interest rate, r_f, accrued on the SWF, F_{t-1}. The period by period government budget constraint becomes

$$(1 - s_f)TCM_t + r_f F_{t-1} + \tau_c(c_t^T + P_t^N c_t^N) + \tau_w(w_t^T h_t^T + w_t^N h_t^N + w_t^{CM} h_t^{CM}) = p_t^N G_t \tag{46}$$

where the Sovereign Wealth Fund evolves according to:

$$F_t = (1 - r_f)F_{t-1} + s_f TCM_t \tag{47}$$

where $0 \leq s_f \leq 1$ is a parameter governing the size of government savings in the SWF. This **prudent rule** is similar to the fiscal rule in place in Norway, which transfers all oil revenue to a Sovereign Wealth Fund, the Norway Government Pension Fund Global. The Norwegian SWF invests all resources abroad, helping to insulate the Norwegian economy from oil price fluctuations. This is relevant as oil constitutes an important source of tax revenues, 40% of fiscal revenue, and income, 20% of GDP in 2006 (García-Cicco and Kawamura, 2015).

We simulate the model under these two different rules and compare their impacts on the macroeconomy.

2.8 Exogenous Processes

There are four exogenous processes: a commodity price process and three productivity shocks (one for each sector). Commodity price follows an autoregressive process:

$$\log\left(p_t^{CM}\right) = (1-\rho_1^{CM}-\rho_2^{CM})\log\left(\overline{p}^{CM}\right) + \rho_1^{CM}\log\left(p_{t-1}^{CM}\right) + \rho_2^{CM}\log\left(p_{t-2}^{CM}\right) + \varepsilon_t^{PCM}, \tag{48}$$

where $\sigma_t^{CM} \sim i.i.d.(0, \sigma^{PCM})$ is a zero mean Gaussian process with variance σ^{PCM}. The technology process for each sector is given by

$$\log\left(Z_t^N\right) = (1-\rho^N)\log\left(\overline{Z}^N\right) + \rho^N\log\left(Z_{t-1}^N\right) + \varepsilon_t^N, \tag{49}$$

$$\log\left(Z_t^T\right) = (1-\rho^T)\log\left(\overline{Z}^T\right) + \rho^T\log\left(Z_{t-1}^T\right) + \varepsilon_t^T, \tag{50}$$

$$\log\left(Z_t^{CM}\right) = (1-\rho^{CM})\log\left(\overline{Z}^{CM}\right) + \rho^{CM}\log\left(Z_{t-1}^{CM}\right) + \varepsilon_t^{CM}, \tag{51}$$

where $\overline{Z}^i$ represents the steady state level for productivity in each sector $i = \{T, N, CM\}$, ρ^i denotes the persistence parameter and $\varepsilon_t^i \sim i.i.d.(0, \sigma^i)$ is a zero mean Gaussian process with variance σ^i.

2.9 Market Clearing and Additional Definitions

The market clearing condition for the Nontradable sector is given by

$$Y_t^N = c_t^N + G_t, \tag{52}$$

Nontradable goods are either consumed by households or purchased by the government. For the tradable final good sector, the equilibrium condition is given by

$$Y_t^T = c_t^T + \sum_i [I_t^i + \Phi^i(K_{t+1}^i, K_t^i)] + TB_t^T, \tag{53}$$

where TB_t^T is the trade balance for the tradable final good sector. Similarly, for the tradable commodity sector we obtain

$$p_t^{CM}(Y_t^{CM} - CM_t^T) = TB_t^{CM}, \tag{54}$$

where TB_t^{CM} is the trade balance in the commodity sector. As we abstract from government debt, we define the aggregate debt position as

$$D_t = D_t^H + D_t^T + D_t^N + D_t^{CM}, \tag{55}$$

The balance of payments is given by

$$(D_t - D_{t-1}) - (F_t - F_{t-1}) = r_{t-1}D_{t-1} - r_f F_{t-1} - TB_t, \tag{56}$$

where $TB_t \equiv TB_t^T + TB_t^{CM}$ is the aggregate trade balance. Finally, we define the real exchange rate as

$$RER_t = \left[\chi + (1-\chi)(p_t^N)^{1-\varphi}\right]^{\frac{1}{1-\varphi}} \tag{57}$$

where p_t^N is the relative price of nontradable goods in terms of the numeraire (tradable final good).

2.10 Competitive Equilibrium

Given initial conditions $K_0^T, K_0^N, K_0^{CM}, D_{-1}^H$, stochastic disturbances $\varepsilon_t^T, \varepsilon_t^N, \varepsilon_t^C$ and ε_t^{CM} and an exogenous commodity price (p_t^{CM}), a competitive equilibrium consists of a set of allocations $\{c_t^T, c_T^N, K_{t+1}^T, K_{t+1}^N, K_{t+1}^{CM}, I_t^T, I_t^N, I_t^{CM}, h_t^T, h_t^N, h_t^{CM}, D_t^H, Y_t^T, Y_t^N, Y_t^{CM}, CM_t\}_{t=0}^{\infty}$, and prices $\{\lambda_t, r_t, p_t^N, w_t^T, w_t^N, w_t^{CM}, u_t^T, u_t^N, u_t^{CM}\}_{t=0}^{\infty}$, such that

1. The allocations $\{c_t^T, c_T^N, K_{t+1}^T, K_{t+1}^N, K_{t+1}^{CM}, I_t^T, I_t^N, I_t^{CM}, h_t^T, h_t^N, h_t^{CM}, D_t^H\}$ solve the households' problem;
2. Given the prices, the allocations $\{K_t^T, K_t^N, K_t^{CM}, h_t^T, h_t^N, h_t^{CM}, CM_t^T, D_t^{CM}, D_t^N, D_t^T\}$ solve the firms' problem;
3. The government chooses its expenditure $\{G_t\}$ and the amount of its savings in the Sovereign Wealth Fund $\{F_t\}$ such that its budget constraint and the fiscal rule are satisfied; and
4. The market clearing conditions for tradable, nontradable and commodity goods, capital, labor are satisfied.

3 Solution Method and Econometric Methodology

We compute a first order Taylor approximation of the linear equilibrium conditions around the non-stochastic steady state and we use the method proposed by Sims (2002) to find the model solution. Table 1 present the parameters that are calibrated.

Table 1: Calibrated parameter values

Parameter	Value	Source/Target Value
Frisch elasticity of labor supply	$\omega_T = \omega_N = \omega_{CM} = 1.455$	Mendoza (1991)
Relative Risk aversion	$\sigma = 2$	Mendoza (1991)
Elasticity of substitution	$\varphi = 0.5$	Akinci (2011)
Depreciation rate	$\delta = 0.025$	10% per year
Interest rate	$\overline{r} = 0.015$	Schmitt-Grohé and Uribe (2015)
Discount factor	$\beta = 0.98$	$\beta = 1/(1+\overline{r})$
Interest rate sensitivity to debt deviations from its SS	$v^D = 0.0007$	Schmitt-Grohé and Uribe (2003)
Capital share ratio	$\alpha^T = \alpha^{CM} = 0.35$	Na (2015)
Capital share ratio	$\alpha^N = 0.25$	Uribe (1997)
Commodity input share	$\gamma^T = 0.05$	Commodity inputs = 5%
Consumption basket parameter	$\chi = 0.35$	Share of nontradable output = 40%
Capital adjustment cost (tradable)	$\phi_T = 4.6$	Shousha (2016)
Capital adjustment cost (nontradable)	$\phi_N = 9.3$	Shousha (2016)
Capital adjustment cost (commodity)	$\phi_{CM} = 10.3$	Shousha (2016)
Consumption tax rate	$\tau_c = 0.2$	Guerra-Salas (2014)
Labor income tax rate	$\tau_w = 0.1$	Guerra-Salas (2014)
Commodity tax rate	$\tau^{CM} = 0.3$	García-Cicco and Kawamura (2015)
AR coefficient of Commodity price	$\rho_1^{CM} = 1.026$	AR(2) estimates
AR coefficient of Commodity price	$\rho_2^{CM} = -0.39$	AR(2) estimates

We follow Mendoza (1991) and set $\omega_T = \omega_N = \omega_C = 1.455$ and the relative risk aversion paramenter (elasticity of intertemporal substitution) in $\sigma = 2$. We set the elasticity of substitution between tradable and nontradable (φ) to 0.5, according to Akinci (2011). We set the depreciation rate at 2.5%, which is a standard value in the literature. The country interest rate ($\overline{r}$) is set to 1.5%, a value used by Schmitt-Grohé and Uribe (2015) and we calibrate the discount factor β accordingly. We set the parameter that governs the interest rate sensitivity to debt deviations from its steady-state to $\mathrm{v}^d = 0.0007$ as in Schmitt-Grohé and Uribe (2003). The parameter χ is set to 0.35 implying a nontradable good production-to-output ratio around 40 percent. Following Na (2015), we set $\alpha^{CM} = \alpha^T = 0.35$, while $\gamma^T = 0.05$. We follow Uribe (1997) and define the labor share in the nontraded sector in 0.75, this implies setting $\alpha^N = 0.25$. We follow Shousha (2016) and set the parameters of the capital adjustment cost in each sector to $\phi_T = 4.6$, $\phi_N = 9.3$ and $\phi_{CM} = 10.3$. We follow Guerra-Salas (2014) and set the consumption and labor income tax rates to $\tau_c = 0.2$ and $\tau_w = 0.1$. The commodity tax rate is set to $\tau^{CM} = 0.3$, a value close to García-Cicco and Kawamura (2015). For the persistence parameters of the commodity process (ρ_1^{CM} and ρ_2^{CM}) we use MLE estimates from an AR(2) process.

The steady state values of commodity price, debt position and trade balance are set to match long-run relations. The $\overline{p}^{CM}$ is set to get a long run value for commodity exports-to-output ratio of 10%. The steady state value for debt position ($\overline{D}$) is set to get a trade-balance-to-output ratio of 1%.

We use Bayesian methods to estimate the parameters that govern the persistence of the productivity shocks (ρ^T, ρ^N, ρ^C), and the standard deviations of the exogenous processes (ε^T, ε^N, ε^C and ε^{CM}). We proceed our estimation using Bayesian techniques. We employ a Metropolis-Hastings algorithm to generate draws from the posterior kernel and integrate the joint posterior distributions of the model parameters. For the choice of the prior distributions, we follow the related literature. In particular, for the persistence of commodity and technology processes, we choose a Beta distribution with a prior mean of 0.5 and standard deviation of 0.2. For the standard deviations of the shocks (ε^T, ε^N, ε^C and ε^{CM}), we use a Inverse Gamma distribution with prior mean of 0.10 and standard deviation of 1.

3.1 Data used in the estimation

The model parameters are estimated using real oil price and real GDP measures for the tradable sector, nontradable sector and oil sector. Our dataset consists of quarterly data for Mexico over the period 1993:Q3-2019:Q4.

As Pieschacon (2012) real GDP in the tradable sector includes agriculture, manufactures and non-oil mining. The non-tradable includes construction and tertiary activities. The data source is the Instituto Nacional de Estadística y Geografia—INEGI. Real GDP measures are in constant 2003 pesos.

Oil price is the price of West Texas Intermediate crude oil, the reference price for Mexican oil. US GDP deflator is used to obtain real oil prices. The source of oil prices and US GDP deflator is the Federal Reserve Economic Data (FRED) database.

All data are seasonally adjusted using X-13 ARIMA-SEATS and are in log deviations from trend. We also demean each variable separately. We use an one-sided HP filter to detrend each variable.

4 Results

4.1 Results of the Bayesian Estimation

Table 2 presents the prior distributions, prior means and prior standard deviations used for each estimated parameter and also their respective posterior results.

Table 2: Estimated parameters

Parameter	Prior Distribution	Prior Mean	Prior s.d.	Posterior Mean	Posterior s.d.
ρ^T	beta	0.500	0.2000	0.8363	0.0518
ρ^N	beta	0.500	0.2000	0.7786	0.0620
ρ^C	beta	0.500	0.2000	0.8637	0.0487
σ^T	invg	0.100	1.0000	0.0141	0.0010
σ^N	invg	0.100	1.0000	0.0118	0.0003
σ^C	invg	0.100	1.0000	0.0518	0.0037
σ^{CM}	invg	0.100	1.0000	0.1090	0.0073

Notes: Posterior statistics are generated using 250 thousand draws from the posterior distribution.

The values are obtained from the joint posterior distribution of parameters using a Metropolis Hastings algorithm. We have performed 1 million draws from the posterior kernel to obtain the posterior distribution. The average acceptance ratio along the chains was approximately 28%, and we have assessed convergence following Brooks and Gelman (1998). We have discarded the first half of the draws to assure independence of initial conditions. The statistics of interest are computed from the ergodic joint posterior distributions of the deep parameters.

4.2 The effects of commodity price shocks under the two rules

We present the results under the two fiscal rules. Figures 1 and 2 display the dynamic responses of our model variables in response to a commodity price shock. Solid (black) lines represent the Impulse Response Function to one standard deviation commodity price shock under the spend-as-you-go rule, while dashed (red) lines represent the Impulse Response Functions to the same shock under the prudent rule.

Higher commodity prices raises output, hours and investment in the commodity sector. Wages and capital rental rates also increase after the shock. There are spillover effects to the tradable and nontradable sectors. As commodity prices increases this leads to a real exchange rate appreciation and to an increase in the relative price of nontradables. This raises output in the nontradable sector, raising hours worked and investment. The tradable sector is adversely affected by the real exchange rate appreciation. Besides higher commodity prices represent a negative supply shock to the tradable sector as commodities are used as inputs in the tradable production process.

Altogether these responses lead to higher aggregate output, consumption, investment and hours worked. The trade balance improves after a commodity price shock despite of the real exchange rate appreciation. The country debt position improves after the shock and this leads to a decline in the real interest rate faced in international financial markets.

What are the effects on government spending? Under both rules, government spending increases at impact and stays above its steady-state value for almost 40 quarters. However, under the spend-as-you-go rule, the response is much higher vis-à-vis the prudent rule.

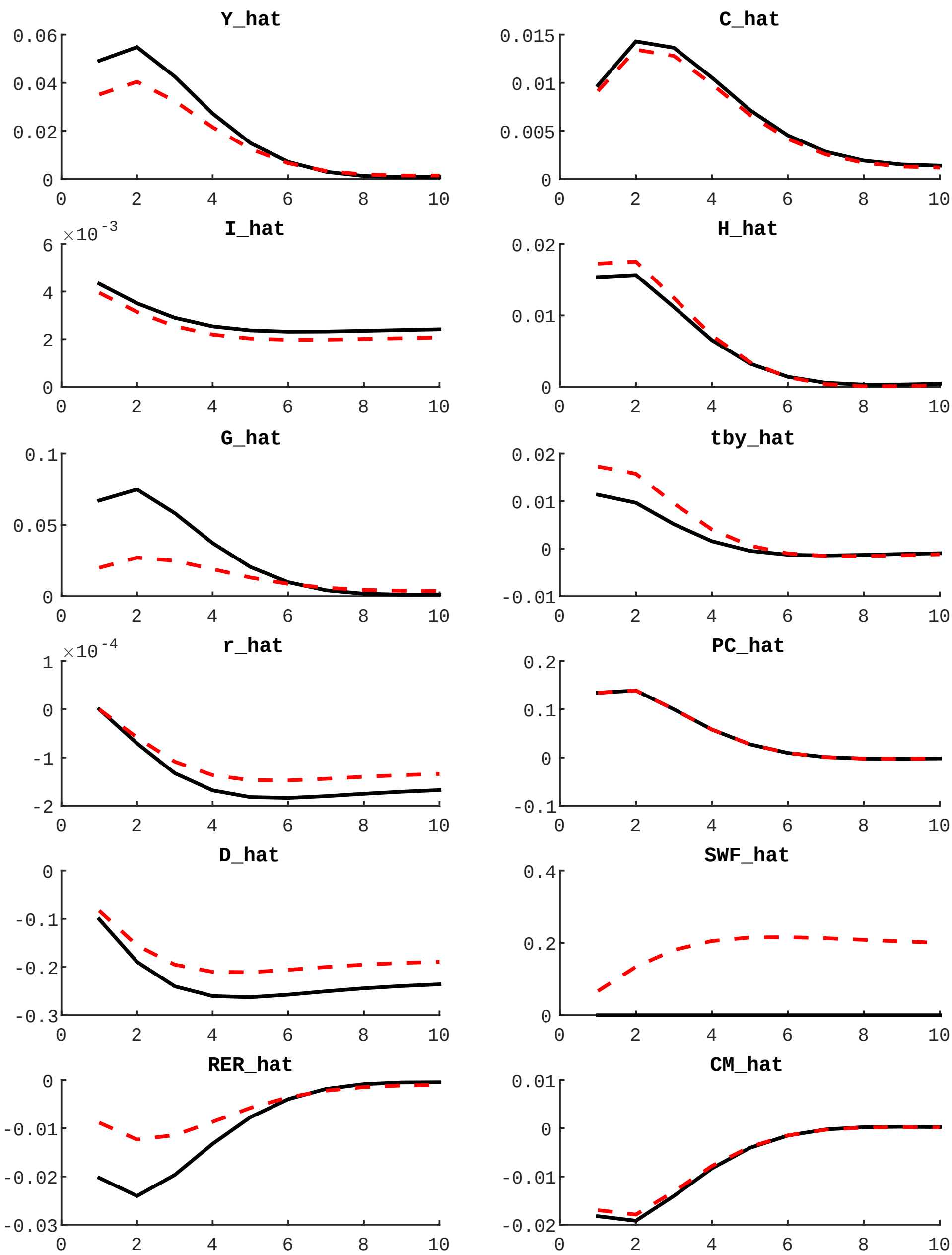

Figure 1: **Impulse Response Functions(IRFs) to one standard deviation commodity price shock**. Solid black lines represent the IRFs to one standard deviation commodity price shock under the spend-as-you-go rule. Dashed red lines represent the IRFs to the same shock under the prudent rule. Variables are in log-deviations from the steady-state, except for the trade balance-to-output ratio and interest rate that are linearized (they are in absolute deviations from the steady-state).

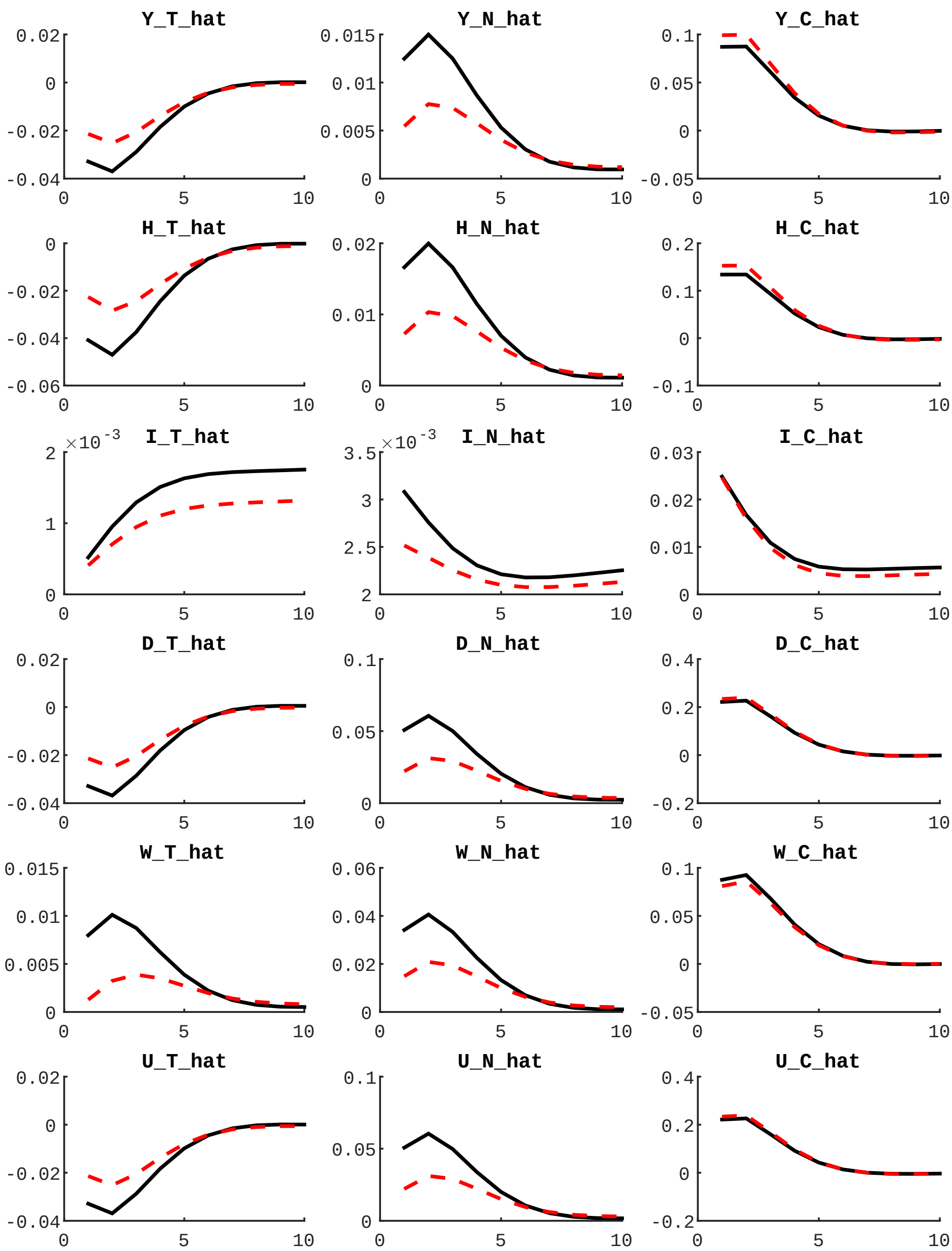

Figure 2: **Impulse Response Functions(IRFs) to one standard deviation commodity price shock**. Solid black lines represent the IRFs to one standard deviation commodity price shock under the spend-as-you-go rule. Dashed red lines represent the IRFs to the same shock under the prudent rule. Variables are in log-deviations from the steady-state, except for the trade balance-to-output ratio and interest rate that are linearized (they are in absolute deviations from the steady-state).

4.3 Does the prudent rule lower volatility?

In this section, we investigate whether the adoption of the prudent rule lower the volatility of the main macroeconomic variables. We use two methods to compute the volatility: one is the unconditional standard deviation of our endogenous variables computed at the model solution. For this, we simulated the model for 2,000 periods, drop the first 1,950 periods and compute the standard deviations of the simulated variables. This method takes into account all underlying shock processes. The second is computed as the discounted squared response function of our endogenous variable to the commodity price shock, as follows:

$$\sigma_x^{irf} \equiv \Sigma_{j=0}^{40} \beta^j \left(\frac{\partial X_j}{\partial \epsilon^{CM}} \right)^2$$

While the former is a measure of unconditional volatility, the latter is a measure of volatility conditional to the underlying commodity price shock. That is considering only the dynamic response of each variable to the commodity price shock. Table 3 displays the volatility under the two rules using the first method.

Table 3: Unconditional Standard Deviations under the two policies

Variable	Spend-as-you-go [A]	Prudent [B]	Ratio [C=B/A]
σ_Y	0.085	0.064	0.75
σ_C	0.029	0.028	0.97
σ_I	0.008	0.007	0.88
σ_H	0.028	0.03	1.07
σ_G	0.115	0.046	0.40

Notes: The table shows the unconditional standard deviations of aggregate output σ_Y, aggregate consumption σ_C, aggregate investment σ_I, hours worked σ_H and Government expenditure σ_G. We simulate the model for 2,000 periods, drop the first 1,850 periods and compute the standard deviations of the simulated variables.

Under the prudent rule, the volatility of output, consumption, investment and government spending are lower relative to the spend-as-you-go rule. Output volatility drops by 25%, consumption by 3%, investment by 12% and government spending by 60%. Hours display slightly higher volatility under the prudent rule (7% higher). Next we compute our measure of conditional volatility (conditional to the commodity price shock). Table 4 displays the volatility under the two rules using the second method.

Table 4: Volatility under the two policies

Variable	Spend-as-you-go [A]	Prudent [B]	Ratio [C=B/A]
σ_Y^{irf}	0.0691	0.0385	0.56
σ_C^{irf}	0.0057	0.005	0.88
σ_I^{irf}	0.0007	0.0005	0.77
σ_H^{irf}	0.0055	0.0069	1.25
σ_G^{irf}	0.1289	0.0204	0.16

Note: The table shows the volatility of aggregate output σ_Y^{irf}, aggregate consumption σ_C^{irf}, aggregate investment σ_I^{irf}, hours worked σ_H^{irf} and Government expenditure σ_G^{irf} conditional to the commodity price shock.

The results show that the use of the rule lowers volatility for the majority of the variables considered. Output volatility drops by roughly 44%, consumption by 12% and investment by 23%. For government spending, under the prudent rule, volatility is lower by 84%. Therefore, a rule that saves all extra revenue from commodity price booms can act as an insulation mechanism for small open commodity-exporting economies.

5 Additional Analysis

5.1 The importance of endogenous commodity consumption

In the baseline model, commodity goods are used as an input in tradable goods production. As a result, higher commodity prices also affect the tradable sector through this cost channel. Therefore, we assess the importance of this channel by setting the parameter that governs the size of commodity usage in the tradable sector to a small value, from $\gamma^T = 0.05$, in the baseline calibration, to $\gamma^T = 0.000001$, in the alternative calibration.

Figure 3 presents the Impulse Response Functions to one standard deviation commodity price shock under the baseline (spend-as-you-go) rule and the prudent rule (saving in a Sovereign Wealth Fund) both using the baseline calibration for $\gamma^T = 0.05$, and under the prudent rule using the small value for this parameter.

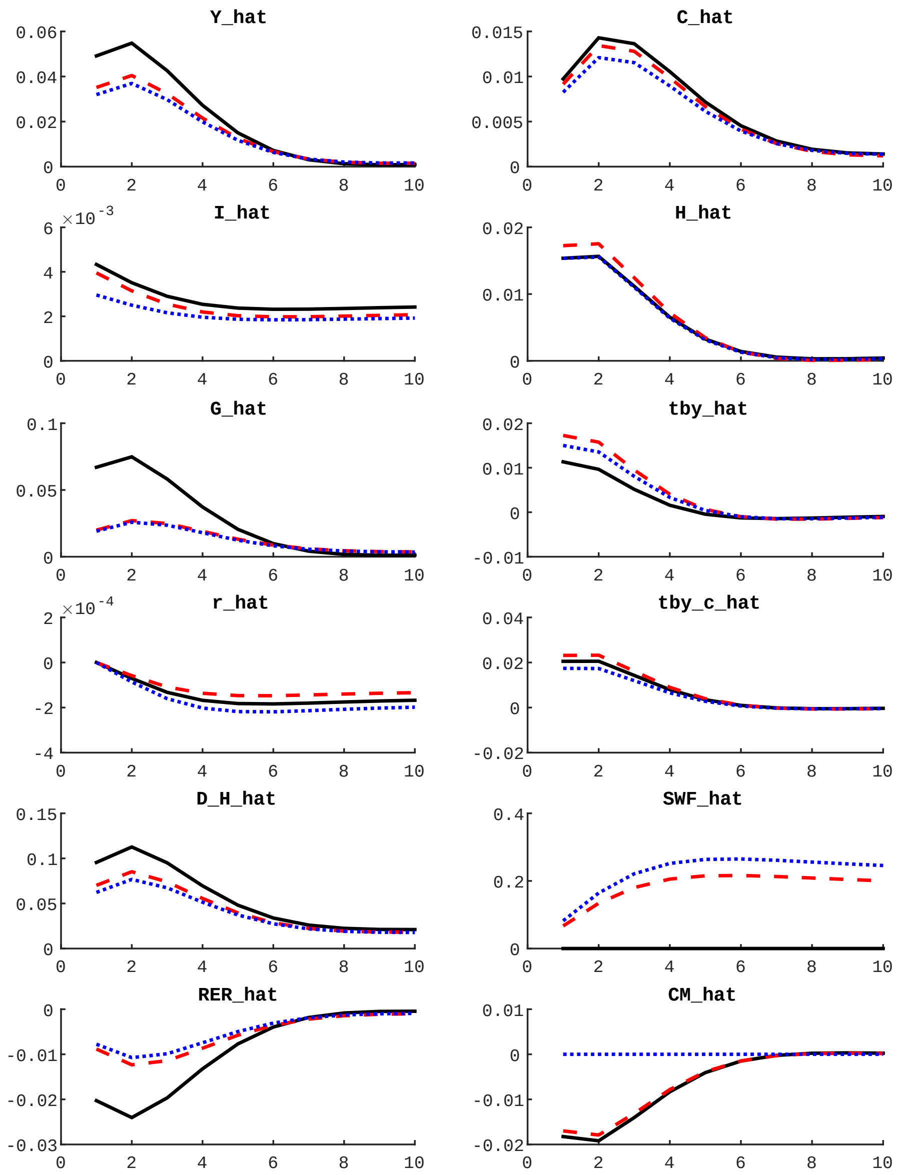

Figure 3: **Impulse Response Functions(IRFs) to one standard deviation commodity price shock**. Solid black lines represent the IRFs to one standard deviation commodity price shock under the spend-as-you-go rule when there is endogenous commodity consumption. Dashed red line represents the IRFs to the same shock under the prudent rule when there is endogenous commodity consumption, and the dotted blue line represents the IRFs to the same shock under the prudent rule when endogenous commodity consumption is close to zero ($\gamma^T = 0.000001$). Variables are in log-deviations from the steady-state, except for the trade balance-to-output ratio and interest rate that are linearized (they are in absolute deviations from the steady-state).

Turning off the commodity cost channel does not alter our main results. The qualitative response of our main aggregate variables remain unchanged. Output, consumption, investment and hours still increase after the commodity price shock. The lower response of investment vis-à-vis the baseline calibration (under both rules) is driven by lower levels of investment in all sectors. The commodity sector trade balance to output ratio improves by less in response to commodity price shocks.

In the tradable sector, output drops by less when we turn off the commodity cost channel. The drop in output occurs only as a result of the competitiveness channel as higher commodity prices lead to a real appreciation.

5.2 The size of government savings in the Sovereign Wealth Fund

Under the prudent rule, the government saves all extra commodity revenue in a Sovereign Wealth Fund. As we have seen, this fiscal rule lowers the response of government spending to commodity price shocks. We play with the parameter governing the size of the government savings in a SWF. Under this new simulation, we assume that the government saves only half of the extra commodity revenue in a SWF and spends the rest.

Figure 4 presents the Impulse Response Functions to one standard deviation commodity price shock under the baseline (spend-as-you-go) rule, the prudent rule (saving all extra revenue in a Sovereign Wealth Fund), and under the alternative rule, where the government saves only half of the extra revenue.

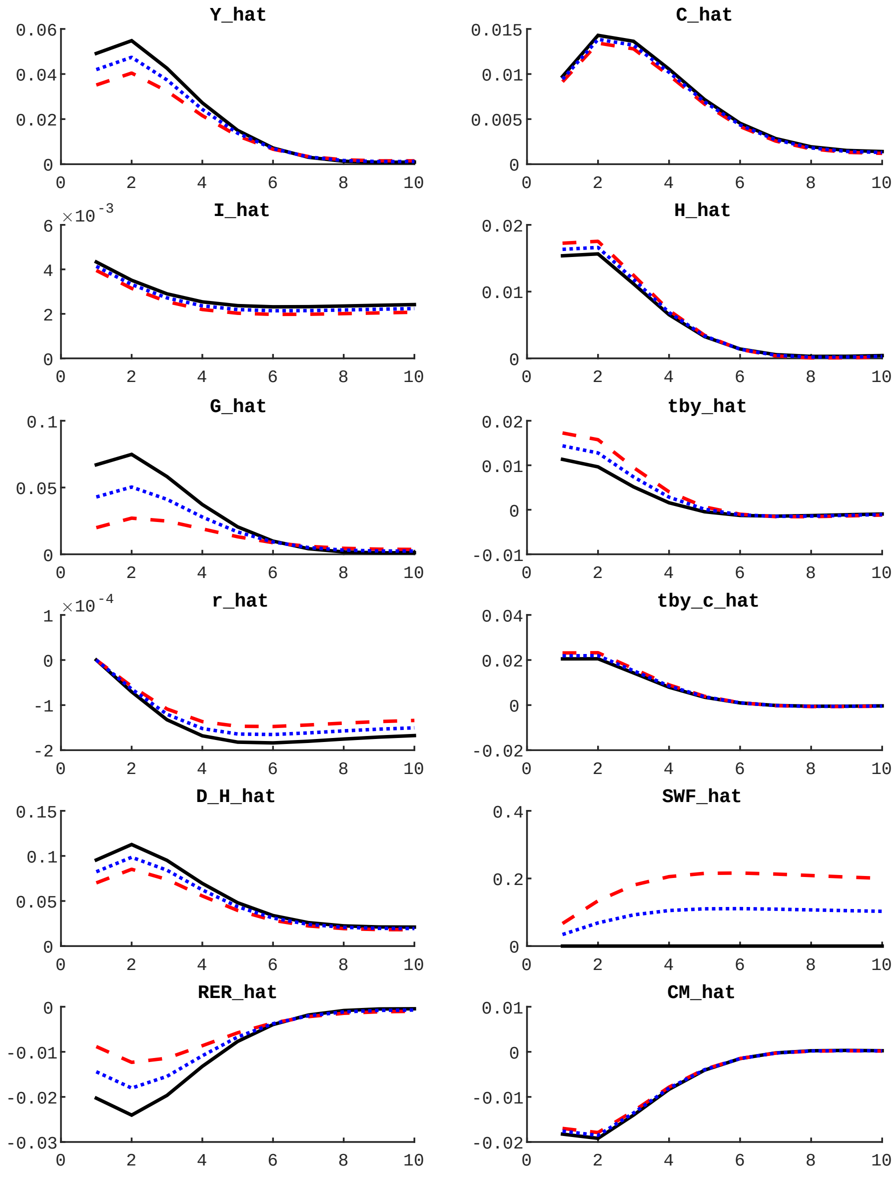

Figure 4: **Impulse Response Functions(IRFs) to one standard deviation commodity price shock**. Solid black lines represent the IRFs to one standard deviation commodity price shock under the spend-as-you-go rule. Dashed red line represents the IRFs to the same shock under the prudent rule when the government saves all commodity extra revenue in a SWF, and the dotted blue line represents the IRFs to the same shock under the prudent rule when the government saves only half of the extra revenue in a SWF. Variables are in log-deviations from the steady-state, except for the trade balance-to-output ratio and interest rate that are linearized (they are in absolute deviations from the steady-state).

The size of government savings in a SWF drives the size of the responses of our main aggregate variables to a commodity price shock. Under the alternative rule, the IRFs are much closer to the IRFs under the spend-as-you-go rule. The results show that when the government saves a lower fraction of the extra revenue output, consumption, investment and hours responses are much larger in comparison to the prudent rule when the government saves all extra revenue in a SWF.

We also compute the conditional volatilities under the alternative rule in response to the commodity price shock. Table 5 presents the results.

Table 5: Volatility under the three policies

Variable	Spend-as-you-go [A]	Prudent [B]	Ratio [C=B/A]	Alternative [D]	Ratio [E=D/A]
σ_Y^{irf}	0.069	0.039	0.56	0.052	0.76
σ_C^{irf}	0.006	0.005	0.88	0.005	0.94
σ_I^{irf}	0.001	0.001	0.77	0.001	0.88
σ_H^{irf}	0.006	0.007	1.25	0.006	1.12
σ_G^{irf}	0.129	0.02	0.16	0.061	0.47

Note: The table shows the volatility of aggregate output σ_Y^{irf}, aggregate consumption σ_C^{irf}, aggregate investment σ_I^{irf}, hours worked σ_H^{irf} and Government expenditure σ_G^{irf} conditional to the commodity price shock. In the **alternative** policy, the government saves only half of the extra commodity revenue in a SWF and spends the rest.

The results show that as the size of government savings decline, the volatility of our main aggregate variables increases. Under the alternative rule, where the government saves only half of the extra revenue and uses the rest to finance current expenditure, output volatility drops by 24% relative to the spend-as-you-go rule. This is smaller than under the prudent rule, where output volatility drops by 44%. Similarly, under the alternative, government spending volatility drops by 53% relative to the spend-as-you-go case. Meanwhile, under the prudent rule, the the same volatility is reduced by 84%. Therefore, the size of government savings in the Sovereign Wealth Fund is the main mechanism driving the response of our endogenous variables to commodity price shocks. These results are important as they highlight that the size of government savings is an important policy decision, as it governs the role of Sovereign Wealth Funds as an insulation mechanism against commodity price shocks in small open economies.

5.3 The role of the tax rate on the commodity sector

What are the effects of raising the tax rate on the commodity sector? In our baseline calibration, we set the tax rate on the commodity sector at 30%. We simulate the model under the prudent rule using three different tax rates. One as in the baseline calibration, one setting the tax rate at a lower value, $\tau_L^{CM} = 10\%$, and one at a higher value, $\tau_H^{CM} = 50\%$. Figure 5 presents the Impulse Response Functions to one standard deviation commodity price shock under these scenarios.

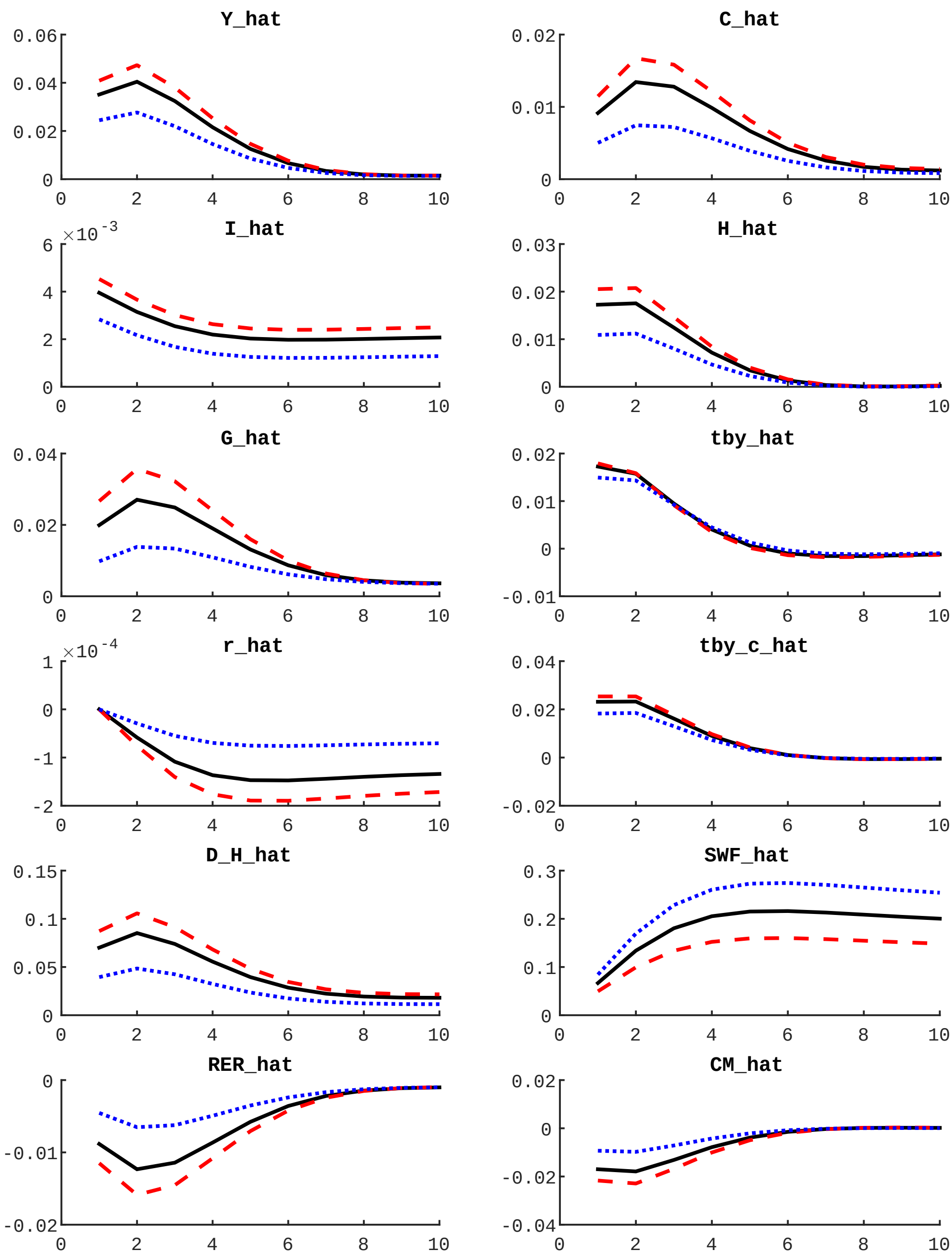

Figure 5: **Impulse Response Functions(IRFs) to one standard deviation commodity price shock**. Solid black lines represent the IRFs to one standard deviation commodity price shock under the prudent rule when the tax rate $\tau^{CM} = 30\%$. Dashed red line represents the IRFs to the same shock under the prudent rule when when the tax rate $\tau_L^{CM} = 10\%$, and the dotted blue line represents the IRFs to the same shock under the prudent rule when the tax rate $\tau_H^{CM} = 50\%$. Variables are in log-deviations from the steady-state, except for the trade balance-to-output ratio and interest rate that are linearized (they are in absolute deviations from the steady-state).

Raising the tax rate on commodity goods and saving all extra revenue into a Sovereign Wealth Fund lowers the responses for most of our aggregate variables. Output, consumption, investment, hours and government spending display lower responses to a commodity price shock when the tax rate is high, $\tau_H^{CM} = 0.5$ (blue dotted line). The opposite is true when the tax rate is low $\tau_L^{CM} = 0.1$ (dashed red line). As for the Sovereign Wealth Fund, as the tax rate increases, the response of F_t to a commodity price shock also increases. Therefore, the government can use the tax policy, by increasing the tax rate on commodity goods, as an instrument to stabilize the domestic economy in response to commodity price shocks.

6 Concluding Remarks

In this paper, we develop a Dynamic Stochastic General Equilibrium model to assess the macroeconomic impact of Sovereign Wealth Funds. In particular, we investigate the behavior of our endogenous variables in response to commodity price shocks under two different fiscal policies. A spend-as-you-go rule, where the government uses all extra commodity revenue in current government consumption, while the second is a prudent rule, where the government saves all extra commodity revenue in a Sovereign Wealth Fund and spends only the real interest rate accrued on the Sovereign Wealth Fund.

We show that under the prudent rule, government expenditure and most macroeconomic variables are less volatile than under the spend-as-you-go rule. The main mechanism driving the responses to commodity price shocks is the size of government savings in the Sovereign Wealth Fund. The lower the fraction of the extra commodity revenue that the government saves in a SWF, the higher the volatility of our main aggregate variables. These results highlight the importance of adopting a Sovereign Wealth Fund as a stabilization mechanism in commodity exporting countries.

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