Bubble detection in Bitcoin and Ethereum and their relationship with volatility regimes

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Resumo

Nosso objetivo é testar a existência de bolhas para preços diários das criptomoedas Bitcoin e Ethereum e verificar se há relação entre bolhas e regimes de volatilidade. A análise empírica usa dados diários de janeiro de 2013 até dezembro de 2019 para Bitcoin e de agosto de 2015 até dezembro de 2019 para Ethereum. Nós testamos a presença de bolhas com os testes supremo do Dickey-Fuller aumentado (SADF) e SADF generalizado (GSADF) usando valores críticos simulados por Monte Carlo e procedimentos de bootstrap de Gutierrez (2011), Harvey et al. (2016) e Pedersen and Schütte (2020). Os resultados do teste GSADF indicam a presença de bolhas para ambas as criptomoedas. Simular valores críticos por wild-bootstrap, que é robusto a volatilidade não-estacionária, leva a um maior número de bolhas em ambas as moedas digitais. Adicionalmente, baseado nas estimativas dos modelos de variação condicional com mudanças de regime, nós encontramos que bolhas identificadas são associadas com um regime de baixo retorno e volatilidade, indicando uma mudança no trade-off entre risco e retorno quando os preços das criptomoedas diferem de seus valores fundamentais.

Palavras-chave: Bolhas especulativas; Criptomoedas; Volatilidade; Modelos MSGARCH

Abstract

Our aim is to test the existence of bubbles for the daily prices of cryptocurrencies Bitcoin and Ethereum and verify if there is a relationship between bubbles and volatility regimes. The empirical analysis uses daily data from January 2013 to December 2019 for Bitcoin and from August 2015 to December 2019 for Ethereum. We test the presence of bubbles with the supremum augmented Dickey-Fuller (SADF) and the generalized SADF (GSADF) tests using critical values simulated by Monte Carlo and by the bootstrap procedures of Gutierrez (2011), Harvey et al. (2016) and Pedersen and Schütte (2020). The results of the GSADF test indicate the presence of bubbles for both cryptocurrencies. Simulating critical values by wild-bootstrap, which is robust to non-stationary volatility, leads to the highest number of bubbles in both digital coins. In addition, based on the estimates of conditional variance models with regime changes, we find that the bubbles identified are associated with a regime of low returns volatility, indicating a change in the trade-off between risk and return when the prices of cryptocurrencies differ from their fundamental values.

Keywords: Speculative bubbles; Cryptocurrencies; Volatility; MSGARCH models

JEL codes: G15; C58

1 Introduction

The beginning of the cryptocurrencies is dated by the creation of Bitcoin in 2008. The creation of Bitcoin aimed to implement a decentralized and anonymous global banking system, possible due to the blockchain technology. Bitcoin has its open source code, which may have encouraged the creation of other alternative

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1 Blockchain is a decentralized transaction logging technology. It is a type of logbook, distributed and operated by thousands of computers in a peer to peer network, where each one has an equal copy of all transactions, preventing the manipulation of a central entity.
cryptocurrencies (altcoins). Ethereum is one of them. Unlike Bitcoin and other altcoins, Ethereum does not limit its use to the form of currency, but also extends to smart contracts and other decentralized applications. Although there may be other cryptocurrencies with this functionality, Ethereum introduces this concept to the cryptocurrency market. Ethereum’s main difference from Bitcoin is allowing its users to create their own cryptocurrencies, personalized through tokens on the Ethereum platform. This attracted startups and other agents interested in financing by Initial Coin Offering (ICO). Considering market capitalization, Bitcoin and Ethereum are the most liquid cryptocurrencies. On December 1, 2019, Bitcoin held 66.4% of all cryptocurrency market capitalization, that is, US$ 134.2 billion. The Ethereum held 8.13%, equivalent to US$ 16.4 billion - roughly double the capitalization of the third cryptocurrency with greater capitalization, Ripple (XRP).

The rise of cryptocurrencies popularity has led to an increase in the variance of their market prices, allowing for speculative behavior. According to Chaim and Laurini (2018), cryptocurrencies have relatively high unconditional volatility and prices subject to sudden fluctuations. Liu and Tsyvinski (2018) conclude that risk-return ratio of cryptocurrencies is distinct from other traditional assets, such as traditional currencies, stocks and precious metals. The market prices of Bitcoin and Ethereum have grown sharply and disproportionately compared to other financial assets. It generated the narrative of bubble presence in that market. The presence of a bubble in an asset leads to the negotiation of market prices higher than what a rational agent would pay for them (fundamental value). In turn, the fundamental value of a financial asset is calculated by the expected cash flow discounted at a constant rate (Campbell et al., 1997).

After the collapse of real estate and stock prices in the years following the 2008 financial crisis, the formation of speculative bubbles in financial asset markets has become an important research topic. Consequently, the literature related to bubble detection tests has expanded. Considering the features of the cryptocurrencies mentioned earlier, we investigate whether there is evidence of a speculative bubble in the Bitcoin and Ethereum markets, and whether any relationship between the bubble (if any) and returns volatility regimes can be verified. The interest in testing the presence of bubbles in the prices of Bitcoin and Ethereum increased due to the sharp growth of their prices in 2016, and the consequent abrupt drop in prices at the end of 2017. Thus, the present work seeks to answer this question through an empirical analysis that investigates whether there is the presence of explosive or unsustainable behavior for the daily prices of Bitcoin between January 2013 and December 2019 and Ethereum between August 2015 and December 2019. In order to examine whether Bitcoin and Ethereum cryptocurrencies have speculative bubbles and to date the bubble period (if any), we use the unit root tests with rolling windows such as the supremum augmented Dickey-Fuller (SADF) test by Phillips et al. (2011) and the generalized supremum augmented Dickey-Fuller (GSADF) test by Phillips et al. (2015). We evaluate both tests based on different bootstrapping methods for simulating critical values, such as the methodologies of Gutierrez (2011), Harvey et al. (2016), and Pedersen and Schütte (2020).

Phillips et al. (2015) developed the GSADF test for low frequency data, which increases the chance of false bubble detection according to Harvey et al. (2016). This issue can negatively affect the profitability of any asset management strategy (Milunovich et al., 2019). To avoid false bubble detection, we consider the wild-bootstrap method to produce robust critical values for heteroscedasticity and the presence of non-stationary volatility. To the best of our knowledge, there are no studies that test and delimit the explosive or unsustainable behavior for the daily prices of Bitcoin and Ethereum by the SADF and GSADF tests using the bootstrap method to simulate critical values from the procedures of Harvey et al. (2016) or Pedersen and Schütte (2020). These bootstrapping procedures are robust to heteroscedasticity, a stylized fact in financial time series. Further, the advantage of Harvey et al. (2016) procedure is the robustness to non-stationary volatility, a phenomenon also expected for cryptocurrencies data according to Hafner (2020).

In addition, our purpose is also to analyze whether or not periods of dated bubbles – if verified by the SADF and GSADF tests – are related to the corresponding volatility regimes of Bitcoin and Ethereum price returns. Thus, we evaluated the possibility of the existence of patterns in the dynamics of conditional variance in the moments of explosive bubbles based on Markov regime switching generalized autoregressive conditional heteroskedasticity model (MSGARCH) estimates. The hypothesis consists of verifying a dynamic of greater volatility in the presence of bubbles as a response to the price increase (returns), maintaining the trade-off of the positive relationship between return and risk in finance.

Empirical results detected the presence of bubbles in Bitcoin and Ethereum prices by the GSADF test at a 5% statistical significance. The SADF test also detected bubbles but only for Ethereum and based on the

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3We extract the data from the CoinMarketCap website homepage.
wild-bootstrap method. In the case of Bitcoin and Ethereum prices, simulating critical values by wild-bootstrap leads to a greater number of bubbles (11 for both cryptocurrencies). This result is contrary to that found by Harvey et al. (2016) and Hafner (2020) in the sense that the evidence in favor of bubbles is weaker when considering a robust procedure in the presence of non-stationary volatility.

In general, Monte Carlo and sieve-bootstrap methods by Gutierrez (2011) dated bubbles similarly. Dating by the wild-bootstrap technique is less related to other methods, as this methodology is robust to non-stationary volatility. The sieve-bootstrap procedure of Pedersen and Schütte (2020) dated the bubbles similarly to the Monte Carlo and Gutierrez (2011) sieve-bootstrap methods but only in the case of Bitcoin.

In particular, there are periods of increase in the cryptocurrencies price standard deviation, which supports the narrative of bubbles in this market. The wild-bootstrap method by Harvey et al. (2016) detects the bubbles for Bitcoin in late 2013, mid and late 2016, mid 2017, early 2018 and mid 2019. For instance, the end of 2013 was marked by the Mt. Gox incident. The same method dates bubbles in Ethereum for the beginning and mid of 2016, beginning, mid and end of 2017, beginning and end of 2018, and for the year 2019 almost entirely.

Another bubble moment for Ethereum and Bitcoin covers the year of 2017 and the first months of 2018, when the popularity of cryptocurrencies became even greater, leading to a sudden and explosive appreciation. One of the possible explanations for these advances in price levels may be the increase in agents’ expectations about future appreciations, which is a typical characteristic of rational bubbles, as stated by Campbell et al. (1997).

Concerning the results from the MSGARCH models, we verified the presence of two volatility regimes for both Bitcoin and Ethereum. Each regime can be associated with low and high volatility regimes based on the unconditional volatility level estimates. GSADF test dated bubble periods – regardless of the method of estimating critical values – that are related with a low volatility regime for Bitcoin and Ethereum in approximately 85% of cases. Hence, we observed that the dynamics of conditional variance does not provide additional information about the identification of bubbles in prices of the considered digital coins. This indicates the absence of maintaining the relationship between risk and return in periods when market prices differ from their intrinsic values – bubbles.

This paper is divided as follows. Section 2 presents an empirical literature review that explores speculative bubble identification including for the cryptocurrency markets, but based on different techniques. The methodology is detailed in section 3. In the following, section 4 reports and discusses the results. Finally, we present the conclusion and indications of future research are presented.

2 Literature Review

After the collapse of stock and real estate prices in the years following the 2008 financial crisis, the topic of research on the formation of speculative bubbles in financial asset markets was highlighted. Many empirical studies have been developed and have served as a basis for the evaluation of this theme in different types of assets. Following the classic theory of finance, bubble assets are those that are traded at higher market prices than what a rational agent would pay for them (fundamental value or intrinsic value). The fundamental value of a financial asset is calculated by the expected cash flow discounted at a constant rate (Campbell et al., 1997).

Regarding the tests to detect bubble formation, the work of Phillips et al. (2011) (PWY) leads to an evolution. The proposed test is a change in the Augmented Dickey-Fuller (ADF) unit root test, in which the alternative hypothesis is explosive behavior – instead of stationarity. That is, the test is one-tailed on the right – and not on the left like the ADF test. The null hypothesis is the presence of unit root as the traditional ADF test.

Homm and Breitung (2012) test and compare several econometric methods for detecting speculative bubbles, concluding that the PWY procedure works satisfactorily in detecting the dot-com bubble in the Nasdaq index. In 2015, Phillips et al. (2015) (PSY) present a new tool for consistent delimitation of the origin and the end of bubbles in real time. This method is the GSADF test and is able to identify and date evidence of multiple bubbles in the S&P 500 index.

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3Mt. Gox was launched in 2010 and is considered as the first Bitcoin broker. In 2011, the platform was responsible for intermediating about 7 out of 10 transactions in this digital coin. The company suddenly suspended transactions permanently in February 2014 claiming to have suffered hacker attacks, which supposedly stole around 850,000 Bitcoins (Pollock, 2018).

4The dot-com bubble occurred approximately between 1994 and 2000 and was marked by the strong appreciation of shares of companies connected to the Internet and a sudden reversal of their prices in March 2000.
With the introduction of Bitcoin in 2008, the era of cryptocurrencies began. Decentralization is one of several innovations in relation to traditional currencies, which was allowed by the use of blockchain. This new technology attracted the interest of users, who started to develop the Bitcoin protocol and created other cryptocurrencies derived from its original source code (altcoins). The platforms emerged and contained other proposals. Ethereum is among them, which unlike Bitcoin and other altcoins, does not limit its use to the form of a currency, but also extends to smart contracts and other decentralized applications. The difference of Ethereum in relation to Bitcoin is the possibility for its users to create their own personalized cryptocurrencies, which attracted startups and other agents interested in financing through ICO. Events involving the attack by hackers – as is the case with the Mt. Gox incident –, the increase in the popularity of these assets and their strong price fluctuations led to the narrative of the existence of a bubble in this market (Chaim and Laurini, 2018).

Until then, the literature has analyzed the cryptocurrency markets as warned by Cheah and Fry (2015). The authors use the methodology of Johansen et al. (2000) to estimate Bitcoin’s fundamental value and obtain evidence of bubble in the Bitcoin market between 2010 and 2014. For the same period, using the method of Phillips et al. (2011), Cheung et al. (2015) identify a series of short-lived bubbles between 2011 and 2014, and three large bubbles between 2011 and 2013, with durations ranging from 66 to 106 days. The end of the Mt. Gox Exchange marks the last and largest of them.

Ciaian et al. (2016) is the first work that analyzes the formation of Bitcoin prices. The authors consider common determinants of traditional currencies prices and specific Bitcoin factors to study the formation of Bitcoin prices. The work concludes that Bitcoin’s market forces – demand and supply – have a significant impact on its price. However, they have not ruled out the hypothesis that speculative behavior influences cryptocurrency prices. Liu and Tsyvinski (2018) conclude that the risk-return relationship of cryptocurrencies is distinct from other traditional assets, such as currencies, stocks and precious metals. According to the authors, cryptocurrency returns have low exposure to the stock, currency and commodity markets, which may be a sign of the influence of specific factors in the digital currency markets. Despite this, the work concludes that cryptocurrencies have the potential to affect various sectors on the real side of the economy, which generates an economic and social concern about the digital asset markets.

The year of 2017 marks a new period of high volatility and appreciation in the cryptocurrency markets. The market price of Bitcoin rose more than 1,700% this year for example, bringing further discussions of the existence of a bubble. Souza et al. (2017) identify the presence of a speculative bubble in all major cryptocurrencies until the beginning of 2017 – including Bitcoin and Ethereum – by using four versions of the tests by Phillips et al. (2011) and Phillips et al. (2015): RtADF (Augmented Dickey-Fuller test for right tail), RADF (Residual Augmented Dickey-Fuller test), ADF and SADF. However, they simulate the critical values for the four tests using the Monte Carlo method – which does not address the problem of heteroscedasticity of the residuals in the historical series. Corbet et al. (2018) identify speculative bubbles in the Bitcoin and Ethereum markets for the period between 2009 and 2017 using the method of Phillips et al. (2011), and also examine the fundamental determinants of their prices. Su et al. (2018) perform the GSADF test method and highlight the existence of explosive behavior in the North American and Chinese Bitcoin markets in various periods, especially throughout 2017. However, Chaim and Laurini (2019a) do not find evidence of a Bitcoin bubble for the end of 2017 under the theory of strict local martingale, even though the results point to the existence of a bubble for the period between 2013 and 2014.

Despite being indicative, large valuations and high volatility do not necessarily constitute a speculative bubble. Bariviera et al. (2017) explore some statistical characteristics of Bitcoin and conclude that the cryptocurrency has a high volatility, which has been decreasing over time, as well as that its long-term memory is not related to market liquidity. Chaim and Laurini (2018) identify two periods of high volatility in the Bitcoin market. The first – associated with the Mt. Gox incident – is between 2013 and 2014, and the second is the year of 2017. According to the authors, the Bitcoin has high unconditional volatility and occasional wide price movements. Average jumps affect returns and are relevant to portfolio analysis and risk management procedures. Like Bitcoin, the other big cryptocurrencies - including Ethereum - also have relatively high volatility with sudden price movements. Chaim and Laurini (2019b) find that the component of permanent (unconditional) volatility appears to be driven by the development of these markets and their levels of popularity.

In general, the literature focuses on procedures based on Phillips et al. (2011) and Phillips et al. (2015), but does not simulate critical values by bootstrap as Harvey et al. (2016) and Pedersen and Schütte (2020), which is
robust in the presence of heteroscedasticity in the series. In addition, the proposal of Harvey et al. (2016) is robust to the presence of non-stationary volatility, as is expected for cryptocurrencies according to Hafner (2020).

3 Methodology

In order to obtain empirical evidence about the presence of bubbles in Bitcoin and Ethereum prices, we use unit root tests such as (i) SADF by Phillips et al. (2011) and (ii) GSADF by Phillips et al. (2015). We calculate the critical values simulated by Monte Carlo and the bootstrap methods of Gutierrez (2011), Harvey et al. (2016) and Pedersen and Schütte (2020). Additionally, we present the method to estimate conditional variance models with regime changes for the purpose of identifying possible patterns about the relationship between bubble and level of volatility.

3.1 Data description

We use the daily closing price series of Bitcoin and Ethereum in US dollars. We extract data from website CoinMetrics. Our samples include the period from January 2013 to December 2019 for Bitcoin prices with 2,556 observations and the period from August 2015 to December 2019 for Ethereum prices, that is, 1,607 observations.

3.2 Bubble detection tests

The bubble tests, that is, explosive behavior, are based on the right tail of the ADF unit root test for the series \( y_t \) on day \( t \) as

\[
\Delta y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \epsilon_t \tag{1}
\]

in which \( \mu \) is the constant, \( p \) is the maximum number of lags, \( \phi_i \) is the coefficient of the lag of the dependent variable for \( i = 1, ..., p \), and the error term \( \epsilon_t \) is a white noise. In the case of bubble tests, the null hypothesis is the presence of a unit root (\( \rho = 0 \)), and the alternative hypothesis is a mildly explosive bubble with the autoregressive coefficient as \( \rho > 0 \). We use the natural logarithm of the chosen cryptocurrency price as the series \( y \).

To illustrate the tests, we consider a sample interval of \([0, 1]\), which represents a normalization of the sample with size \( T \). We represent the coefficient estimated by equation (1) as \( \rho_{r_1, r_2} \), in which \( r_1 \) and \( r_2 \) are the first and the last observation used in the regression. The ADF statistic on the normalized sample \([r_1, r_2]\) is represented by \( ADF_{r_1, r_2} \). In addition, \( r_w \) denotes the window size of the regression, defined by \( r_w = r_2 - r_1 \), and \( r_0 \) is the fixed starting point of the window.

3.2.1 SADF tests

The SADF test by Phillips et al. (2011) is based on recursively estimating the ADF statistic with a fixed starting point and an expansion window. The first observation in the sample is the starting point for each estimation window, \( r_1 \), that is, \( r_1 = 0 \). We define the final point of the initial estimate window, \( r_2 \), according to the choice of the minimum window size, \( r_w \). The regression is recursively estimated, while changing the window size. In the last step, our estimate is based on the entire sample, that is, \( r_2 = 1 \).

The SADF test statistic is the supreme value of the sequence \( ADF_{r_2} \) for \( r_2 \in [r_0, 1] \) as \( SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_{r_2} \). This test allows us to delimit the beginning and end moments of the bubble period, therefore, its duration. The null hypothesis of the SADF test is the presence of a unit root. However, the alternative hypothesis is that you have only one bubble in the period.

3.2.2 GSADF tests

Phillips et al. (2015) develop the GSADF test. In this test, the estimation windows are more flexible, allowing to recursively vary both the start point and the end points. As in the SADF test, we can obtain the start and
We use the initial values $\mu_1$ with replacement of innovations is based on centered residuals, $\mu_2$. And we follow steps 3 to 5 like the Harvey et al. (2016) procedure. The purpose of the Gutierrez (2011) procedure is another alternative is to use the bootstrap method by Harvey et al. (2016). This method mitigates the potential influence of the unconditional heteroscedasticity of the residuals. We describe below the procedure based on five steps as detailed in Phillips and Shi (2020).

Consider $\tau_0 = \lfloor \tau_0 \rfloor$ and $\tau_b$ as the number of observations of the window with size that we must control. 

**Step 1:** using the complete sample period, we estimate the Equation (1) imposing the null hypothesis that $\rho = 0$ and we obtain the estimated residuals $\hat{\epsilon}_t$. 

**Step 2:** with a sample of size $\tau_0 + \tau_b - 1$, we generate a bootstrap sample given by

\[
\Delta y^b_t = \frac{\Delta y^b_{t-j}}{\hat{\phi}_j} + \epsilon_t^b
\]

We use the initial values $y^b_t = y_t$ with $i = 1, ..., j + 1$, in which $\hat{\phi}_j$ are the OLS estimates obtained from Equation (1) in step 1. $\epsilon_t^b = \hat{\omega}_i e_t$ are the residuals, in which we obtain $\hat{\omega}_i$ randomly from the standard normal distribution and we get $\epsilon_t^b$ randomly by replacing the estimated residuals $\hat{\epsilon}_t$. This step implements the wild-bootstrap procedure, which considers the heteroscedasticity of the residuals. 

**Step 3:** using the bootstrapped series, we calculate the test sequence as $\{GSADF^b_t\}_{t=n-1}^n$ and the maximum value of the test statistic sequence is 

\[
M^b_t = \max_{t \in [n_0, n_0 + \tau_b - 1]} GSADF^b_t
\]

**Step 4:** we repeat steps 2 and 3 $B$ times. 

**Step 5:** finally, we obtain the critical value of the test through the 95th percentile of $\{M^b_t\}_{t=1}^B$. 

The steps 3 through 5 replicate the recursive test procedure and obtain the critical values as Phillips and Shi (2020).

**3.2.3 Critical values simulation by bootstrap procedure of Harvey et al. (2016)**

We can simulate the critical values of the SADF and GSADF tests. One option is the Monte Carlo simulation. Another alternative is to use the bootstrap method by Harvey et al. (2016). This method mitigates the potential influence of the unconditional heteroscedasticity of the residuals. We describe below the procedure based on five steps as detailed in Phillips and Shi (2020).

Consider $\tau_0 = \lfloor \tau_0 \rfloor$ and $\tau_b$ as the number of observations of the window with size that we must control. 

**Step 1:** using the complete sample period, we estimate the Equation (1) imposing the null hypothesis that $\rho = 0$ and we obtain the estimated residuals $\hat{\epsilon}_t$. 

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\]

We use the initial values $y^b_t = y_t$ with $i = 1, ..., j + 1$, in which $\hat{\phi}_j$ are the OLS estimates obtained from Equation (1) in step 1. $\epsilon_t^b = \hat{\omega}_i e_t$ are the residuals, in which we obtain $\hat{\omega}_i$ randomly from the standard normal distribution and we get $\epsilon_t^b$ randomly by replacing the estimated residuals $\hat{\epsilon}_t$. This step implements the wild-bootstrap procedure, which considers the heteroscedasticity of the residuals. 

**Step 3:** using the bootstrapped series, we calculate the test sequence as $\{GSADF^b_t\}_{t=n-1}^n$ and the maximum value of the test statistic sequence is 

\[
M^b_t = \max_{t \in [n_0, n_0 + \tau_b - 1]} GSADF^b_t
\]

**Step 4:** we repeat steps 2 and 3 $B$ times. 

**Step 5:** finally, we obtain the critical value of the test through the 95th percentile of $\{M^b_t\}_{t=1}^B$. 

The steps 3 through 5 replicate the recursive test procedure and obtain the critical values as Phillips and Shi (2020).

**3.2.4 Critical values simulation by bootstrap procedure of Gutierrez (2011)**

After step 1 of the previous subsection, Gutierrez (2011) proposes resampling $\hat{\epsilon}_t$. We obtain $\hat{\epsilon}_t^b$ resampling with replacement of innovations is based on centered residuals, $\hat{\epsilon}_t - \hat{\epsilon}_t$, where $\hat{\epsilon}_t = \sum_{i=1}^{j} \hat{\epsilon}_i / T$ and we generate $\Delta y^b_t$ using the OLS estimates $\hat{\phi}_j$ as

\[
\Delta y^b_t = \sum_{j=1}^{p} \hat{\phi}_j \Delta y^b_{t-j} + \epsilon_t^b
\]

And we follow steps 3 to 5 like the Harvey et al. (2016) procedure. The purpose of the Gutierrez (2011) procedure is to correct the critical value if you have small samples. However, according to Harvey et al. (2016), Gutierrez (2011) adopts a resampling scheme that is not robust to non-stationary volatility.

**3.2.5 Critical values simulation by bootstrap procedure of Pedersen and Schütte (2020)**

Alternatively, Pedersen and Schütte (2020) simulate the critical value of the SADF and GSADF tests by bootstrap considering the autocorrelated errors and without assuming that the series has a unit root, unlike the procedure by Harvey et al. (2016). The Pedersen and Schütte (2020) procedure has the following steps:

**Step 1:** based on the estimate using all the sample of the equation (1) by OLS, we obtain the estimates $\hat{\phi}_i$ and the residuals:

\[
\hat{\epsilon}_t = y_t - \hat{\alpha} - \hat{\rho} y_{t-1} - \sum_{i=1}^{k} \hat{\phi}_i \Delta y_{t-i}
\]

the end of each bubble period. However, the greater flexibility of the GSADF test guarantees consistency of results in sampling periods that include multiple episodes of bubbles, unlike the SADF test.
for $t = k^* + 1, ..., T$, in which we obtain the optimal lag $k^*$ through the Bayesian Information Criteria (BIC) from a chosen maximum lag $k_{max}$.

**Step 2:** later, an independent and identically distributed sample errors by bootstrapping is generated from the random selection with replacement of residuals $\hat{\varepsilon}_t$ obtaining $\hat{\varepsilon}_t^b$ according to:

$$\hat{\varepsilon}_t^b = \hat{\varepsilon}_t - (T - k^*)^{-1} \sum_{i=1}^{T} \hat{\varepsilon}_i$$

(6)

**Step 3:** we construct $\hat{\mu}_t^b$ recursively from $\hat{\varepsilon}_t^b$ obtained in step 2 as:

$$\hat{\mu}_t^b = \sum_{i=1}^{k^*} \phi_i \hat{\mu}_{t-i}^b + \hat{\varepsilon}_t^b$$

(7)

The sample generated by bootstrap has to eliminate the initialization effect and also has size $T$. So we draw $(T - k^*) + b$ bootstrap errors in step 2 and we drop the first $b - k^*$ values of $\hat{\mu}_t^b$. Based on $\hat{\mu}_t^b$, we obtain $\hat{\gamma}_t^b$ as $\hat{\gamma}_t^b = \hat{\gamma}_{t-1}^b + \hat{\mu}_t^b$, where $t = 1, ..., T$ with $\hat{\gamma}_0^b = 0$. The purpose of step 3 is to control the autocorrelation in the residuals.

**Step 4:** we generate the bootstrap test statistics using $\hat{\gamma}_t^b$ as

$$SADF^r(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_{r_2}^{r_2}$$

(8)

$$GSADF^r(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_{r_2}^{r_2}$$

(9)

We calculate the critical values of the tests SADF and GSADF by bootstrap for nominal significance level $q$ repeating step 2 to 4 $B$ times considering the q-quantile of the bootstrap test statistics, in which $B$ is the number of replications.

The bootstrap method of simulating critical values by Harvey et al. (2016) has an improvement over the SADF and GSADF tests when considering the effects of heteroskedasticity. However, these authors do not examine the effects of autocorrelation about such tests. In this sense, Pedersen and Schütte (2020) propose to simulate the critical values of the GSADF test. Their proposal has finite sample size properties moderately better in the absence of non-stationary volatility, as well as considering the presence of autocorrelated errors and without assuming the presence of a unit root for $y$.

3.2.6 Strategy for defining bubble periods

Our strategy for detecting bubble periods (from the SADF and GSADF tests) is based on the proposal by Phillips et al. (2011) and Phillips et al. (2015). We compare each sequence $ADF_{r_2}$ statistics to the respective critical value of the right tail to identify the start of the bubble at the moment $T_{r_2}$. The estimate of the bubble’s point of origin can be denoted by $T_r$, when the statistic $ADF_{r_2}$ becomes greater than the critical value. Thus, when the statistic $ADF_{r_2}$ becomes less than the critical value, we obtain the final period of $T_{r_2}$. Formally, the bubble period estimates – as fractions of the sample – are defined by:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} r_2 : ADF_{r_2} > c_{\beta_{r_2}}^{\beta_{r_2}}$$

(10)

$$\hat{r}_f = \inf_{r_2 \in [r_0, 1]} r_2 : ADF_{r_2} < c_{\beta_{r_2}}^{\beta_{r_2}}$$

(11)

where $c_{\beta_{r_2}}^{\beta_{r_2}}$ corresponds to the critical value $100(1 - \beta_{r_2})\%$ of the standard ADF statistic based on $[T_{r_2}]$ observations.

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5In our case, if non-stationary volatility is verified – which is expected for cryptocurrencies according to Hafner (2020) –, it is more appropriate to consider the wild-bootstrap with the strategy of Pedersen and Schütte (2020), what the authors call the recolored version of Harvey et al. (2016).
3.2.7 Volatility models with regime changes

To verify whether the periods with bubbles for the analyzed cryptocurrencies present a common pattern according to the associated volatility dynamics from MSGARCH, an approach proposed by Haas et al. (2004) are estimated. MSGARCH models account for regime changes in the conditional variance dynamics of time series, expressed by a family-GARCH structure in different regimes, being able to capture differences in the process that generates volatility in periods of low and high returns conditional variance.

Let \( y_t = \ln(P_t) - \ln(P_{t-1}) \) be the log-returns of digital currencies prices \( P \) at time \( t = 1, \ldots, T \). Denoting \( \mathcal{I}_{t-1} \) the information set observed up to time \( t - 1 \), i.e. \( \mathcal{I}_{t-1} = \{ y_{t-i}, i > 0 \} \), the general Markov-switching GARCH specification can be specified by:

\[
y_t | (s_t = k, \mathcal{I}_{t-1}) \sim \mathcal{D}(0, h_{k,t}, \epsilon_k)
\]

where \( \mathcal{D}(0, h_{k,t}, \epsilon_k) \) is a continuous distribution with zero mean, time-varying variance \( h_{k,t} \) and additional shape parameters contained in vector \( \epsilon_k \). \( s_t \) is an integer-valued variable, defined on the discrete space \( \{1, \ldots, K\} \) and \( K \) is the number of regimes. Standard innovations are defined by \( \eta_{k,t} = y_t / h_{k,t}^{1/2} \sim i.i.d. \mathcal{D}(0, h_{k,t}, \epsilon_k) \).

In MSGARCH models, regimes \( s_t \) evolves based on Markov chain process with a \( K \times K \) transition probability matrix \( P \):

\[
P = \begin{bmatrix}
p_{1,1} & \cdots & p_{1,K} \\
\vdots & \ddots & \vdots \\
p_{K,1} & \cdots & p_{K,K}
\end{bmatrix}
\]

(13)

where \( p_{i,j} = P[s_t = j | s_{t-1} = i] \) measures the probability of a transition from state \( s_{t-1} = i \) to state \( s_t = j \).

Given the parametrization of \( \mathcal{D}(\cdot) \), we have \( E[y^2 | s_t = k, \mathcal{I}_{t-1}] = h_{k,t} \), and \( h_{k,t} \) is the variance of \( y_t \) conditional on the realization of \( s_t = k \). In MSGARCH method, conditional variances \( h_{k,t} \), for \( k = 1, \ldots, K \), are assumed to follow \( K \) GARCH-type processes. Hence, conditional to regime \( s_t = k \), the conditional variance is a function of past returns \( y_{t-1} \), past volatilities \( h_{k,t-1} \) and a vector \( \theta_k \) which encodes the parameters related to each regime, that is:

\[
h_{k,t} = h(y_{t-1}, h_{k,t-1}, \theta_k)
\]

(14)

where \( h(\cdot) \) function that defines the conditional variance process, i.e. the family-GARCH conditional heteroskedasticity function.

We consider the GARCH model of Bollerslev (1986) so that \( h_{k,t} \) is specified by:

\[
h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k h_{k,t-1}
\]

(15)

for \( k = 1, \ldots, K \) and \( \theta_k = (\alpha_{0,k}, \alpha_{1,k}, \beta_k) \).

On the other hand, to consider the so-called leverage effect where past negative observations have a larger influence on the conditional volatility than past positive observations of the same magnitude, EGARCH (Exponential GARCH) specification of Nelson (1991), also considered in this work, is defined by:

\[
\ln(h_{k,t}) = \alpha_{0,k} + \alpha_{1,k}(|\eta_{k,t-1}| - E[|\eta_{k,t-1}|]) + \alpha_{2,k} \eta_{k,t-1} + \beta_k \ln(h_{k,t-1})
\]

(16)

for \( k = 1, \ldots, K \) and \( \theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k) \). If parameter \( \alpha_{2,k} \) is significant, volatility distinctly responds to negative shocks (bad news) and positive shocks (good news).

MSGARCH models identification includes the definition of the heteroskedastic function (GARCH or EGARCH), the number of regimes (\( K \)), and the setting of the conditional distribution of the standardized innovations (e.g. Normal or t-Student). Hence, we estimate the parameters by maximum likelihood. We test different MSGARCH models concerning four different specification for conditional variance (GARCH, EGARCH, GJR-GARCH and TARCH), Normal and t-Student distributions, and the presence of up to three regimes, within a total of 24 models for each cryptocurrency. We select the best structure based on BIC.
4 Results

We divide the results section into two parts. The first subsection addresses the results of bubble detection tests for the Bitcoin and Ethereum series. The second subsection presents the possible patterns of the volatility regimes in the periods with the presence of bubbles.

4.1 Bubble tests

This subsection presents the results of the SADF and GSADF tests with the critical values simulated by Monte Carlo and by bootstrap for the daily prices of Bitcoin and Ethereum. We adopt the strategy of testing the model specification using the common ADF test initially, using equation (1) for the natural logarithm of the daily cryptocurrency price.

As these are daily series, we define the maximum period of lags of 30 days. We define the initial size of the estimation windows for the SADF and GSADF tests as \( \frac{n}{2} + 1.8/\sqrt{T} \), with \( n \) representing the total number of observations, as the window size increases the power of the tests according to Phillips et al. (2015) – the equivalent of 117 observations for the Bitcoin series and 88 observations for Ethereum. We use BIC as the criteria for choosing the number of lags for the two cryptocurrencies. Finally, we calculate the critical values based on 1,000 simulations in each method (Monte Carlo and bootstrapping) following the specification for each model in the simple ADF test for the natural price logarithm of each of the series.\(^6\)\(^7\)

According to Harvey et al. (2016) and Hafner (2020), these bubble detection tests are biased in the presence of non-stationary volatility. The evidence in favor of bubbles is weaker in the presence of non-stationary volatility, so we should correct the size of the tests, which disfavors the use of the test with the critical values simulated by Monte Carlo. The sieve-bootstrap method by Gutierrez (2011) adopts a resampling scheme that is not robust to non-stationary volatility according to Harvey et al. (2016). Assuming that volatility is non-stationary for cryptocurrencies as expected by Hafner (2020), the most suitable method for dating and testing for bubble presence is the wild-bootstrap. Next, we present the results of the bubble tests for Bitcoin and Ethereum in sequence.

4.1.1 Bitcoin

Table 1 presents the statistics of the SADF and GSADF tests and the respective p-values for the Monte Carlo and bootstrap methods to calculate the critical values for the Bitcoin series. We do not reject the null hypothesis of unit root by the SADF test at 5% significance.

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Statistic</th>
<th>P-value</th>
<th>Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SADF</strong></td>
<td></td>
<td></td>
<td><strong>GSADF</strong></td>
<td></td>
</tr>
<tr>
<td>Monte Carlo</td>
<td>0.488958</td>
<td>0.4690</td>
<td>5.412436***</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sieve-bootstrap (Gutierrez, 2011)</td>
<td>0.488958</td>
<td>0.4540</td>
<td>5.412436***</td>
<td>0.0000</td>
</tr>
<tr>
<td>Wild-bootstrap</td>
<td>0.488958</td>
<td>0.3410</td>
<td>5.412436**</td>
<td>0.0210</td>
</tr>
<tr>
<td>Sieve-bootstrap (Pedersen and Schütte, 2020)</td>
<td>0.488958</td>
<td>0.4520</td>
<td>5.412436***</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Note: ***, ** and * represent respectively statistic significance at 1%, 5% and 10%.

Table 1: Unit root test statistics for Bitcoin prices according to different procedures to obtain the critical values

On the other hand, we reject the null hypothesis of unit root at 5% significance by the GSADF test, regardless of the method used to calculate the critical values. That is, the result of the GSADF test indicates that the Bitcoin series has multiple bubble periods (under the alternative hypothesis).

The next step is to analyze the dating of the bubble periods indicated by the GSADF test at 5% significance. The number of bubbles indicated by the GSADF test – with the critical values simulated by the different methods – varies from 27 to 36 bubbles in the period. Table 2 shows the number of bubbles and how many are durable for the different methods of calculating critical values. The advice of Phillips et al. (2015) is that we can

\(^6\)We obtain that the constant is statistically significant at 5% significance when reducing the test equation from the general to the particular for the ADF test for both series.

\(^7\)The reason for having only 1,000 simulations is the time we need to simulate the critical values. We take up to 13 processing days for one of the bootstrap procedures on an i7-7500U CPU, 2.9 GHz, with 8 GB RAM and 256 GB SATA HD SSD and Windows 10.
discard shorter periods than the size of the natural logarithm of the series because this is not lasting bubbles.

In the case of the Bitcoin price series, we rule out periods of less than eight days so that we have between 10 and 11 bubbles with different methods for the critical value of the GSADF test. We do not have much difference for the number of lasting bubbles between the methods, in which both sieve-bootstrap methods lead to more bubbles that are not lasting.

<table>
<thead>
<tr>
<th>Procedure</th>
<th># of bubbles</th>
<th># of lasting bubbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>Sieve-bootstrap (Gutierrez, 2011)</td>
<td>34</td>
<td>10</td>
</tr>
<tr>
<td>Wild-bootstrap</td>
<td>28</td>
<td>11</td>
</tr>
<tr>
<td>Sieve-bootstrap (Pedersen and Schütte, 2020)</td>
<td>36</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 2: Number of bubbles and number of lasting bubbles for Bitcoin prices by the GSADF test according to the different procedures to obtain the critical values

We consider a dummy variable that is equal to 1 in bubble periods for Bitcoin prices, and zero otherwise. Table 3 presents the correlation matrix of this dummy variable using the different methods to simulate the critical values of the tests for Bitcoin prices. The dating of the bubbles by the sieve-bootstrap method of Gutierrez (2011) and by Monte Carlo is similar, given the correlation of 0.968. In general, the sieve-bootstrap procedures of Gutierrez (2011) and Pedersen and Schütte (2020) date the bubbles in a similar way to the Monte Carlo method. Dating using the wild-bootstrap method is less related to other methods – with correlations of at most 0.75 – and this is the procedure that is robust to non-stationary volatility.

<table>
<thead>
<tr>
<th></th>
<th>Monte Carlo</th>
<th>Sieve-bootstrap (Gutierrez, 2011)</th>
<th>Wild-bootstrap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>1.000</td>
<td>0.968</td>
<td>1.000</td>
</tr>
<tr>
<td>Sieve-bootstrap (Gutierrez, 2011)</td>
<td>0.968</td>
<td>1.000</td>
<td>0.752</td>
</tr>
<tr>
<td>Wild-bootstrap</td>
<td>0.754</td>
<td>0.752</td>
<td>1.000</td>
</tr>
<tr>
<td>Sieve-bootstrap (Pedersen and Schütte, 2020)</td>
<td>0.876</td>
<td>0.863</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Table 3: Correlation matrix of bubble dummy variables for Bitcoin prices

We obtain periods that are dated like bubbles by the different procedures, changing the day marginally. The end of the year 2013, the half and the end of the year 2016, and the half of 2019 are dated as bubbles for Bitcoin in all methods.

There is a narrative of speculative bubbles for periods of explosive behavior at the end of 2013. As discussed in section 2, the Mt. Gox Exchange incident marked the end of 2013, when the institution claimed to have suffered invasions from hackers, which culminated in the definitive closure of its operations a few months later. In addition, there was the arrest of a large number of Bitcoin on the Silk Road site. During this period, the unit price of Bitcoin exceeded US$ 1,000 and reverted to US$ 400 in a few months. Corbet et al. (2018), Chaim and Laurini (2019a), Cheah and Fry (2015), Cheung et al. (2015), Souza et al. (2017), and Su et al. (2018) also identify this period as a speculative bubble.

One possible explanation for the dating of the bubbles in late 2016 is that in September of the same year there was a halving of Bitcoin. At this point, there is a half reduction in the reward to miners, which decreases the supply of new cryptocurrencies. After September 2016, there was an increase in the price of Bitcoin, which can be from the halving in September 2016.

In the Appendix, Figure 3 (4) presents, respectively, the critical values at 95% of confidence level (in blue) and calculated (in red) for the GSADF test based on the simulations by Pedersen and Schütte (2020) with wild-bootstrap (sieve-bootstrap). The shaded area in the Figure is the bubble dated according to the adopted bootstrap procedure. The wild-bootstrap method for calculating the critical value of the test, which seems to be the most appropriate, presents the following dates as lasting bubbles: end of 2013, mid and end of 2016, mid of 2017, beginning of 2018, and mid of 2019.
4.1.2 Ethereum

Table 4 presents the statistics of the SADF and GSADF tests and the respective p-values for the Monte Carlo and bootstrapping procedures to calculate the critical values for the Ethereum series. The result of the SADF test indicates the rejection of the null hypothesis of unit root at 10% of statistical significance. However, we reject the null hypothesis of unit root at 5% of statistical significance by the SADF test using only the wild-bootstrap. As the GSADF test improves the power to detect bubbles according to Phillips et al. (2015), we focus on the results of the GSADF test.

<table>
<thead>
<tr>
<th>Method</th>
<th>SADF Statistics</th>
<th>SADF P-value</th>
<th>GSADF Statistics</th>
<th>GSADF P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>1.517*</td>
<td>0.062</td>
<td>4.571***</td>
<td>0.000</td>
</tr>
<tr>
<td>Sieve-bootstrap (Gutierrez, 2011)</td>
<td>1.517*</td>
<td>0.054</td>
<td>4.571***</td>
<td>0.000</td>
</tr>
<tr>
<td>Wild-bootstrap (Harvey et al., 2016)</td>
<td>1.517**</td>
<td>0.026</td>
<td>4.571**</td>
<td>0.049</td>
</tr>
<tr>
<td>Sieve-bootstrap (Pedersen and Schütte, 2020)</td>
<td>1.517*</td>
<td>0.066</td>
<td>4.571***</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: ***, ** and * represent respectively statistic significance at 1%, 5% and 10%.

Table 4: Unit root test statistics for Ethereum prices according to different procedures to obtain the critical values.

We reject the null hypothesis of unit root at 5% of statistical significance based on the GSADF test, regardless of the procedure to calculate the critical value. That is, the result of the GSADF test indicates that the Ethereum series presents multiple periods of bubble (under the alternative hypothesis).

The next step is to analyze the dating of the bubble periods indicated by the GSADF test at 5% significance, in addition to the SADF test, with the critical values simulated by wild-bootstrap. The number of bubbles indicated by the SADF test is 5 bubbles at 5% of statistical significance using critical values simulated by wild-bootstrap. Among these 5 bubbles, 2 are long-lasting considering the recommendation of Phillips et al. (2015). In the case of the Ethereum price series, we disregard periods of less than eight days as bubbles.

Table 5 reports the number of bubbles and how many are long-lasting for the different methods to calculate the critical values for Ethereum prices. The GSADF test indicates a number of bubbles that varies from 27 to 33 bubbles in the period with the critical values simulated by the different methods. We detect the smallest number of bubbles from the sieve-bootstrap simulations by Gutierrez (2011) and Monte Carlo with 27 and 28 bubbles respectively. The evidence using both Harvey et al. (2016) wild-bootstrap and Pedersen and Schütte (2020) sieve-bootstrap methods is a total of 33 bubbles. However, the total of lasting bubbles varies between 6 and 11 considering the recommendation of Phillips et al. (2015). The wild-bootstrap method leads to the highest number of episodes of lasting bubbles, which is 11 periods. This method also detects the highest number of episodes of lasting bubbles in the case of Bitcoin (see Table 2). Monte Carlo’s simulation of the critical value results in the lowest number of lasting bubbles for Ethereum.

<table>
<thead>
<tr>
<th>Method</th>
<th># of bubbles</th>
<th># of lasting bubbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>28</td>
<td>6</td>
</tr>
<tr>
<td>Sieve-bootstrap (Gutierrez, 2011)</td>
<td>27</td>
<td>7</td>
</tr>
<tr>
<td>Wild-bootstrap</td>
<td>33</td>
<td>11</td>
</tr>
<tr>
<td>Sieve-bootstrap (Pedersen and Schütte, 2020)</td>
<td>33</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5: Number of bubbles and number of lasting bubbles for Ethereum prices by the GSADF test according to the different procedures to obtain the critical values.

The simulation of critical values by wild-bootstrap leads to the largest number of bubbles for Bitcoin and Ethereum prices. This result is different from that obtained by Hafner (2020) and Harvey et al. (2016), in which the evidence of bubbles is weaker when considering a robust test for the presence of non-stationary volatility.

Table 6 presents the correlation matrix of the dummy variable for bubble periods between the different methods to simulate the critical values for Ethereum prices. The dating of bubbles in Ethereum prices by the sieve-bootstrap method of Gutierrez (2011) and Monte Carlo is similar due to the correlation of 0.937. This pattern also occurs for dating bubbles in Bitcoin prices (see Table 3). The sieve-bootstrap methods of
Gutierrez (2011) and Pedersen and Schütte (2020) do not date bubbles in similar periods for Ethereum prices. The wild-bootstrap and sieve-bootstrap of Pedersen and Schütte (2020) methods date the bubbles with a smaller relationship than using other methods in general, with correlations between 0.70 and 0.76.

<table>
<thead>
<tr>
<th></th>
<th>Monte Carlo</th>
<th>Sieve-bootstrap</th>
<th>Wild-bootstrap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sieve-bootstrap</td>
<td></td>
<td>0.937</td>
<td>1.000</td>
</tr>
<tr>
<td>(Gutierrez, 2011)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wild-bootstrap</td>
<td>0.715</td>
<td>0.735</td>
<td>1.000</td>
</tr>
<tr>
<td>(Pedersen and Schütte, 2020)</td>
<td>0.731</td>
<td>0.747</td>
<td>0.760</td>
</tr>
</tbody>
</table>

Table 6: Correlation matrix of bubble dummy variables for Ethereum prices

The different methods for calculating critical values similarly date the bubbles in Ethereum prices in early 2016, mid and late 2017, and in early and late 2018. These dates even agree with bubble periods using Monte Carlo because this method leads to the smaller number of bubbles for Ethereum. The dating of lasting bubbles in early 2016 is in line with Corbet et al. (2018).

In the Appendix, Figures 5 and 6 present the critical values at 95% of confidence level (in blue) and calculated values (in red) for the GSADF test based on the sieve – of Pedersen and Schütte (2020) – and wild-bootstrap simulations. The shaded area in the Figures indicates bubble periods according to the specific simulation methodology. The wild-bootstrap method for calculating the critical value – which seems to be the most appropriate – presents the following periods as lasting bubbles: beginning and mid of 2016, beginning, mid and end of 2017, beginning and end of 2018, and almost the entire year of 2019. Corbet et al. (2018) also document a bubble for Ethereum in 2017.

4.2 The presence of bubbles and the volatility dynamics

To assess possible patterns in the volatility dynamics of the cryptocurrencies considered in this study for the periods with the presence of bubbles, we estimate MSGARCH models. There are a total of 24 estimated models for each digital currency, different according to the definition of the conditional variation structure (GARCH, EGARCH, GJR-GARCH and TARCH), innovations probability distribution (Normal and t-Student), and with the presence up to three regimes. Based on the BIC criterion, we select an EGARCH model with 2 regimes and t-Student distribution for Bitcoin, while for Ethereum, the most parsimonious model presents a GARCH structure, with two regimes and a t-Student distribution. Table 7 shows the MSGARCH parameters estimates for each cryptocurrency.

According to the estimates in Table 7, the identified regimes can be interpreted as regimes of low and high volatilities for Bitcoin and Ethereum, as the levels of returns unconditional variance are different in the two regimes. This evidence is greater for Ethereum, in which the annualized unconditional volatility in regime 1 (or low volatility regime) is 35.40%, while in regime 2 (high volatility regime) it is 369.78%. In addition, for the two digital currencies, the probabilities of staying in regime 1 ($p_{1,1}$) are higher, which indicates that the regime of low volatility is the most persistent, implying that periods of high volatility do not usually have a long duration (Table 7).

For Bitcoin, in both regimes, the parameters associated with volatility asymmetry ($\alpha_{2,1}$) were significant, indicating the presence of different effects for negative and positive shocks. In regime 1, the parameter $\alpha_{2,1}$ is positive, indicating that a negative (positive) shock in volatility has the effect of reducing (increasing) the level of volatility. In regime 2, $\alpha_{2,1}$ is negative, indicating that negative shocks are associated with an increase in the level of volatility (Table 7).

Figure 1 shows the temporal evolution of Bitcoin and Ethereum returns conditional volatility for the period considered in this paper, estimated by the MSGARCH models with regime changes. We note that the average level of volatility over time for Ethereum is higher than Bitcoin, which confirms the results of unconditional volatility levels (Table 7). In addition, Ethereum presents a greater number of "volatility spikes", indicating a greater variation in prices, when compared to Bitcoin (Figure 1).

The smoothed transition probabilities for Bitcoin and Ethereum, over the respective periods, are shown in Figure 2. In general, if the transition probability for regime 1 is greater than 0.5 (less than 0.5), we say that the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
<th>t Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{0,1} )</td>
<td>-0.0882</td>
<td>0.0004</td>
<td>-215.2775</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{0,1} )</td>
<td>0.1979</td>
<td>0.0006</td>
<td>321.5585</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{0,1} )</td>
<td>0.0680</td>
<td>0.0004</td>
<td>190.4003</td>
<td>0.00</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.9907</td>
<td>0.0000</td>
<td>19914.5478</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon_1 )</td>
<td>3.0183</td>
<td>0.0070</td>
<td>429.4248</td>
<td>0.00</td>
</tr>
<tr>
<td>Inc. Vol.</td>
<td>29.33% (annualized)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_{0,2} )</td>
<td>-2.6311</td>
<td>0.0202</td>
<td>-130.5002</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{1,2} )</td>
<td>-0.0005</td>
<td>0.0032</td>
<td>-0.1502</td>
<td>0.44</td>
</tr>
<tr>
<td>( \alpha_{2,2} )</td>
<td>-0.2808</td>
<td>0.0010</td>
<td>-276.1443</td>
<td>0.00</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.5239</td>
<td>0.0036</td>
<td>144.2773</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon_2 )</td>
<td>17.1660</td>
<td>0.4174</td>
<td>41.1265</td>
<td>0.00</td>
</tr>
<tr>
<td>Inc. Vol.</td>
<td>102.89% (annualized)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{1,1} )</td>
<td>0.9569</td>
<td>0.0015</td>
<td>654.3570</td>
<td>0.00</td>
</tr>
<tr>
<td>( p_{2,2} )</td>
<td>0.2211</td>
<td>0.0005</td>
<td>469.3630</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Panel B: Ethereum

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
<th>t Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{0,1} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>45.2519</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{0,1} )</td>
<td>0.0409</td>
<td>0.0006</td>
<td>64.4808</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{0,1} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.9009</td>
<td>0.0005</td>
<td>1777.8503</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon_1 )</td>
<td>3.4712</td>
<td>0.0239</td>
<td>145.0765</td>
<td>0.00</td>
</tr>
<tr>
<td>Inc. Vol.</td>
<td>35.40% (annualized)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_{0,2} )</td>
<td>0.0005</td>
<td>0.0000</td>
<td>64.9932</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{1,2} )</td>
<td>0.1457</td>
<td>0.0051</td>
<td>28.4713</td>
<td>0.00</td>
</tr>
<tr>
<td>( \alpha_{2,2} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.8459</td>
<td>0.0002</td>
<td>3881.8573</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon_2 )</td>
<td>20.4118</td>
<td>0.6302</td>
<td>32.3871</td>
<td>0.00</td>
</tr>
<tr>
<td>Inc. Vol.</td>
<td>369.78% (annualized)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{1,1} )</td>
<td>0.8696</td>
<td>0.0028</td>
<td>305.9781</td>
<td>0.00</td>
</tr>
<tr>
<td>( p_{2,2} )</td>
<td>0.2094</td>
<td>0.0015</td>
<td>42.0255</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 7: Parameters estimates of MSGARCH models for Bitcoin and Ethereum returns volatility modeling.
Figure 1: Price returns conditional variance temporal evolution of Bitcoin and Ethereum estimated by MS-GARCH models.
dynamic of conditional variance is in regime 1 (regime 2), that is, in the low volatility regime (high volatility regime). For Bitcoin, in Figure 2a, there is a greater distinction between high and low volatility regimes, with a predominance of a less volatile regime (regime 1). The periods in which the high volatility regime is observed in Figure 2a are associated with the volatility peaks in general (see Figure 1a). On the other hand, for Ethereum, the transition probabilities show a greater variation in Figure 2b, indicating a greater change in volatility dynamics between the two regimes.

![Graph showing smoothed probabilities for Bitcoin and Ethereum](image)

**Figure 2**: Temporal evolution of smoothed transition probabilities between low (regime 1) and high (regime 2) volatility regimes for Bitcoin and Ethereum estimated by MSGARCH models.

Based on the results of the different tests to identify periods with the presence of bubbles, we evaluated the association of these periods with the respective volatility regime – low volatility regime (regime 1) and high volatility regime (regime 2). Table 8 shows the number of days of the bubbles were identified by the GSADF test and its association with the volatility dynamics for Bitcoin and Ethereum. In general, for both evaluated cryptocurrencies, we observe that the periods identified as bubbles are associated with the low volatility regime (Regime 1). Such results are similar for the different ways of determining critical values (via Monte Carlo simulation and via bootstrapping methods). We obtain greater bubble observance in low volatility regimes in the GSADF test for Bitcoin and Ethereum (Table 8). In general, the presence of bubbles associated with low volatility regimes contradicts Ciaian et al. (2016), which stated that bubbles in Bitcoin would be associated with periods of greater volatility as speculative activity by investors affects the price of Bitcoin.

The results observed in Table 8 show a paradoxical relationship, not expected. In periods of explosive bubble, we have associated an increase in asset prices and, therefore, a higher associated return. On the other
Table 8: Association of periods (days) with bubbles for Bitcoin and Ethereum, identified by the GSADF test, supported by Monte Carlo and bootstrap (sieve- and wild-bootstrap simulations of the critical values, with the associated volatility regime, determined according to the transition probabilities obtained by the MSGARCH models

| Regime | Bitcoin | | | | | Ethereum | | | | |
|--------|---------|--------|--------|--------|--------|---------|--------|--------|--------|
|        | Monte Carlo | Sieve-bootstrap | Wild-bootstrap | Sieve-bootstrap | (Pedersen and Schütte, 2020) |
| Bubbles | # Bubbles | 561 | 530 | 318 | 673 |
| Regime 1 | # Bubbles | 70 | 69 | 47 | 83 |
| Regime 2 | Bubbles Regime 1 (%) | 12.48% | 13.02% | 14.78% | 12.33% |
| Regime 2 | Bubbles Regime 2 (%) | 87.52% | 86.98% | 85.22% | 87.67% |

This observed relationship, in an alternative way, has already been pointed out in the financial literature. Sornette et al. (2018), for example, showed the relation between a lower volatility and periods associated with major financial market crashes. Additionally, based on bubble tests such as those considered in this work, Harvey et al. (2016) extended the SADF test to the case of the presence of non-stationary volatility. The authors showed less evidence of explosive bubbles when the test considers a dynamic conditional variance for the series under analysis, confirming the idea that volatility does not provide additional information for the detection of bubbles in the prices of financial assets. Similar results were also observed in the work of Hafner (2020).

5 Conclusions

Our goal is to test for the presence of a speculative bubble in Bitcoin’s daily prices between January 1, 2013 and December 31, 2019, and for Ethereum prices between August 2015 and December 2019. In addition, we expect to delimit the beginning and end of the periods in which the price series report explosive behavior (if any) and relate these bubble periods to the dynamics of the volatility of returns. We use the SADF and GSADF tests simulating the critical values by Monte Carlo and bootstrap. Our volatility analysis is through GARCH models with Markov regime changes.

The introduction of blockchain technology constitutes cryptocurrencies as a new payment system and mainly a new financial asset for trading. Therefore, cryptocurrencies are subject to speculative behavior like most financial instruments. The rise in popularity of digital currencies in recent years and the increase in cryptocurrency prices generate the narrative of the existence of a speculative bubble in this market. Bitcoin and Ethereum are the main cryptocurrencies considering market capitalization. The formation of speculative bubbles in financial asset markets becomes a growing topic of academic research in the years following the 2008 financial crisis. The works of Phillips et al. (2011) and Phillips et al. (2015) develop unit root tests based on the right tail that are tools for exploring this theme.

We find that the test results with the critical values simulated by bootstrap are different from those
simulated by Monte Carlo. Especially in the case of Ethereum, the results differ in terms of the number of bubble periods and their duration. According to Hafner (2020), we expect the volatility of cryptocurrencies to be non-stationary, the wild-bootstrap method seems to be the most suitable to calculate the critical value for robustness to non-stationary volatility.

Finally, tests with critical values simulated by bootstrap of Harvey et al. (2016) identify the strongest evidence of bubbles when compared to those simulated by Monte Carlo or even by bootstrap of Gutierrez (2011). This is contrary to Hafner (2020) and Harvey et al. (2016) that the evidence in favor of bubbles is weaker when using the robust bootstrap method in the presence of non-stationary volatility.

We detect the presence of bubbles in Bitcoin and Ethereum prices from the GSADF tests, while the SADF test detects only for Ethereum using wild-bootstrap at 5% of statistical significance. The GSADF test with critical values by the wild-bootstrap has the highest number of lasting bubbles for Bitcoin and Ethereum, which are 11. However, this evidence is contrary to Hafner (2020) that Bitcoin presents stronger evidence for the presence of a bubble.

In general, the Monte Carlo and sieve-bootstrap by Gutierrez (2011) date bubbles similarly. The dating of the wild-bootstrap is the one that has less relation with the other procedures, precisely the procedure robust to non-stationary volatility. The use of the sieve-bootstrap critical values from Pedersen and Schütte (2020) dates bubbles similarly to the simulation by Monte Carlo and sieve-bootstrap from Gutierrez (2011) only in the case of Bitcoin. In addition, volatility models with regime changes indicate that the bubble periods are mostly associated with periods of low volatility. Therefore, the positive theoretical relationship between risk and return does not hold when the market prices of cryptocurrencies differ from their respective fundamental values. Future work includes, for example, the inclusion of other cryptocurrencies in the analysis of bubbles, since there is a growth in the number and liquidity of cryptocurrencies traded.

References


A Appendix
Figure 3: Time evolution of the GSADF test statistics for Bitcoin price with critical values simulated by wild-bootstrap

Figure 4: Time evolution of the GSADF test statistics for the Bitcoin price with the critical values simulated by sieve-bootstrap of Pedersen and Schütte (2020)
Figure 5: Time evolution of the GSADF test statistics for Ethereum price with critical values simulated by wild-bootstrap

Figure 6: Time evolution of the GSADF test statistics for the Ethereum price with the critical values simulated by sieve-bootstrap of Pedersen and Schütte (2020)