Comparing Consumption-Based Asset Pricing Models: New evidence from Brazil

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Abstract

We evaluate several parametric specifications for preferences in the consumption-based asset pricing framework and compare their empirical performance in explaining asset market data from Brazil. Besides the traditional CRRA, habit formation and recursive Kreps-Porteus preferences in consumption, we also include utility functions in the Savings-CAPM and preferences with labor income. We perform a formal statistical comparison of these models and rank them based on the conditional and unconditional Hansen-Jagannathan (HJ) distances. Finally, we study the existence of the equity premium puzzle in Brazil. The results suggest that the recursive utility model has the best performance according to our empirical evidence. The second best is the Saving-CAPM and the third best is the model with labor income preferences. No evidence of equity premium was found in the Brazilian data.

1. Introduction

The risk of macroeconomic variables is considered an important factor in investor decisions in most economies where there is link between production and the stock market. An assessment of asset prices using different theories that capture specific characteristics of the co-movements of the aggregate economy and asset markets could help support the decisions made by market agents and could lead to a different role for central banks, as well as government intervention in response to potential asset market failure such as economic downturns.

In this paper we investigate the implication of asset market data in several consumption-based capital asset pricing frameworks. We evaluate various parametric specifications for preferences based on consumption-based asset pricing models, and compare their empirical performance in explaining asset market data from Brazil. Besides the traditional preference functions that have constant relative risk aversion (CRRA), consumer habits and recursive utility of Krep-Porteus (1978), we also include other functional forms such as the Saving-CAPM (Cordenonssi, 2006) and labor income (Davis and Martin, 2009). We also develop and estimate a model that captures habits in savings, aiming to capture the habit formation in savings.

The consumption-based capital asset pricing model (CCAPM) was originally introduced by Rubinstein (1976), Lucas (1978) and Breeden (1979) for the purpose of linking consumption to the stock market. However, it gained importance from the work of Lucas (1978), who worked with Euler equations to provide a very useful way to obtain an empirical framework for the analysis of consumption-based asset pricing tests. Most authors have used the CCAPM framework to estimate and test aggregate nonlinear rational expectation models directly from stochastic Euler equations. Some examples are Hansen and Jagannathan (1991), Mehra and Prescott (1985) and Hansen and Singleton (1982, 1984). Other extensions have been proposed on the theoretical side in order to improve its empirical performance, including (1) habit formation (Abel, 1990; Campbell and Cochrane, 1999; Constantinides, 1990); and (2) recursive preference (Epstein and Zin, 1989, 1991; Weil, 1989). The common characteristic of these models is the non-time-separable specification for lifetime utility, that is, utility functions in which past consumption plays a role in determining current utility.

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More recently, some authors have incorporated the housing utility function into the CCAPM (e.g., Piazzesi et al. 2007). In this model, the representative agent is not only concerned with the volatility of consumption, but also with the risk related to the relative participation of housing services as part of the preferences. Other authors have introduced labor income and domestic production into the model to improve asset price prediction (see Santos and Veronesi, 2006; Davis and Martin, 2009).

The literature in Brazil has used the CCAPM mainly to test the models of aggregate consumption with traditional CRRA, habit formation and recursive Kreps-Porteus preferences in consumption. We can mention the works of Cavalcanti (1993), Reis et al. (1998), Issler and Rocha (2000), Issler and Piqueira (2000), Gomes and Paz (2004), Gomes (2004), Gomes (2010), Costa and Carrasco (2015), and Silva and Carrasco (2019). Papers addressed in testing the existence of the equity premium puzzle are: Sampaio (2002), Bonomo and Domingues (2002), Issler and Piqueira (2000), Cysne (2005), Catalão and Yoshino (2006), Samanez and Santos (2007).

This paper attempts to complement the literature by providing a comparison of model performance with data from the Brazilian stock market. Our paper unfolds in the following ways: (i) we attempt to complement the literature by providing estimates of several asset pricing models and comparing their empirical performance to explain asset market data from Brazil; (ii) we consider in the study utility functions that incorporate saving behavior and labor income; (iii) in contrast to existing studies, we perform a formal statistical comparison of these models and rank them based on metrics of the Hansen-Jagannathan (HJ) distance; and (iv) we also develop and estimate a model which aims to capture the habit formation in savings.

The methodological procedure uses the generalized method of moments (GMM) to estimate the structural parameters of the models, also making it possible to test the restrictions arising from the Euler equations of each model using a huge range of instruments. As for the comparison of the models, we employ two criteria: specification errors with the residuals obtained from Euler equations with the optimized estimated parameters; and the Hansen-Jagannathan distance (1997) with nonlinear equations, including the conditional version, considering the instruments adopted based on the observed variables. Comparing the performance of the models seems natural to us, since each one captures a specific characteristic of the driving forces that move asset prices. Estimating models in different contexts also provides evidence of the diversity of the behavioral profile of representative agents.

The main results show that the recursive utility model has the best performance according to our empirical evidence and model. The Saving-CAPM and labor income models were the second and third best models respectively. No evidence of equity premium was found in Brazilian data.

The rest of the paper is organized as follows. Section 2 introduces the conceptual framework of the basic pricing equation and develops the models. Section 3 describes data sources and the results, including structural estimation, model comparison and preliminary assessment of the existence or not of the equity premium puzzle in Brazil. Section 4 concludes.

2. Models

2.1 Consumption-Based Capital Asset Pricing Model (CCAPM)

Lucas (1978) presented the CCAPM in an economy in which a representative agent chooses how much to consume and how much to invest to maximize the expected present value of his future utility function, constrained by the evolution of his stock of wealth. The optimal choice problem of this agent for separable utility is represented as follows:

$$\underset{[C_{t+s},\theta_{t+s}]_{s=0}^{\infty}}{Max} \mathrm{E_t}\left[\sum_{s=0}^{\infty}\beta^s\, u_{t+s}(C_{t+s})\right] \tag{1}$$

$$C_t + \theta_{t+1}P_t = \theta_t P_t + \theta_t d_t + Y_t;\;\; C_t, \theta_{t+1} \geq 0, \forall t$$

Where $u_{\mathrm{t}}(\,.\,)$ is the instantaneous utility function at t, β is the intertemporal discount coefficient, C_t is the aggregate consumption of households that have optimizing behavior, θ_t is the vector of assets, P_t is the vector of asset prices in each period, d_t is the vector of dividends paid by the assets, and Y_t is the exogenous income received in each period by agents. The Euler condition for this problem results in:

$$P_t^i = \mathrm{E_t}\left[\beta\,\frac{(\partial u_{t+1}/\partial C_{t+1})}{(\partial u_t/\partial C_t)}\left(P_{t+1}^i + d_{t+1}^i\right)\right], \qquad i = 1,2,\dots,N \tag{2}$$

Where the payoff of the *i*-th asset at $t+1$ is defined as $x_{t+1}^i = P_{t+1}^i + d_{t+1}^i$ and the marginal rate of intertemporal substitution of consumption as $M_{t+1} = \beta\,\frac{(\partial u_{t+1}/\partial C_{t+1})}{(\partial u_t/\partial C_t)}$. Equation (2) results in

$$P_t^i = E_t\left[M_{t+1}x_{t+1}^i\right] \tag{3}$$

which is the pricing equation established by Harrison and Kreps (1979), Hansen and Richard (1987) and Hansen and Jagannathan (1991, 1997) associated with the stochastic discount factor (SDF), which relies on the pricing equation. $E_t(\cdot)$ denotes the conditional expectation given the information available at time t, M_{t+1} is the stochastic discount factor (SDF). This pricing equation means that the market value today of an uncertain payoff tomorrow is represented by the payoff multiplied by the discount factor, also taking into account different states of nature by using the underlying probabilities.

2.2 Utility preferences

Table 1 presents the six models we applied, estimated and compared regarding performance according to the Brazilian data in this paper, including the extension of the SCAPM to incorporate the saving habit of the representative agent, which we call the saving habit formation model.

Table 1 - A summary of various intertemporal asset pricing models

Utility preferences	Description
CRRA	Constant relative risk aversion
Habit Formation	External habit formation of consumption
Recursive Utility	Kreps-Porteus, Epstein–Zin–Weil recursive utility
SCAPM	Cordenonssi model with utility CRRA of saving
Saving Habit Formation Model	SCAPM with external habit formation of saving
Labor income Model	Adapted version of the model of Santos and Veronesi (2006)

2.1 Constant relative risk aversion (CRRA)

The CRRA is defined as $u(C_t) = \frac{C_t^{1-\rho}-1}{1-\rho}$, where $\rho \geq 0$ and $\rho \neq 1$ are the coefficient of relative risk aversion coefficient and at the same time represents the intertemporal elasticity of substitution of consumption. The Euler equation resulting from the optimization process results in:

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} R_{t+1}^i \right] \quad \text{for i} = 1, 2, \dots, N \quad \forall t \tag{4}$$

Thus, the stochastic discount factor is:

$$M_{t+1}^{CRRA} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \tag{5}$$

2.2 Habit formation

We consider a simple version of the habit formation model a la Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999), among many others. The representative agent's expected lifetime utility is the same as in (5) but a current consumption ratio C_t is incorporated for a benchmark or consumption level of habit C_{t-1}^k in the utility function, as follows:

$$u(C_t, C_{t-1}) = \frac{1}{1-\rho} \left(\frac{C_t}{C_{t-1}^k} \right)^{1-\rho} \tag{6}$$

Solving the Bellman equation result in an Euler equation for asset return with the SDF yields:

$$M_{t+1}^{Habit} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{C_t}{C_{t-1}} \right)^{k(\rho-1)} \tag{7}$$

Compared with CCAPM, the habit formation mechanism in this model introduces time-non-separability into preferences and the SDF ends up having lagged consumption growth as an additional risk factor. When $k = 0$, there is no consumption habit, so the CCAPM emerges as a special case.

2.3 Recursive utility

Epstein, Zin and Weil (1989, 1991) developed an alternative model based on the recursive preferences of Kreps and Porteus (1978) in which the representative agent faces a trade-off between the utility of the current period and the utility to be derived from all future periods. Thus, the actions of the agent today can affect the evolution of opportunities in the future. The recursive utility function is very flexible and complex, where the degree of risk aversion (σ) is not directly related to the inverse of the intertemporal substitution elasticity of consumption (ρ). In addition, Euler's equation of the recursive utility model employs the return of two assets, one of which is necessarily the return of the optimal consumer portfolio. The recursive utility is defined as:

$$V_t = \left[(1-\beta) C_t^{1-\rho} + \beta (E_t[V_{t+1}^{1-\sigma}])^{(1-\rho)/(1-\sigma)} \right]^{1/(1-\rho)} \tag{8}$$

where $u(V_{t+1}) = (E_t[V_{t+1}^{1-\sigma}])^{1/(1-\sigma)}$ is the certainty equivalent of future utility or continuation value; $0 < \beta < 1$ is the discount factor; $\sigma \geq 0$ is the coefficient of relative risk aversion; and $\rho \geq 0$ and $\rho \neq 1$ denote the inverse elasticity of intertemporal substitution. With recursive preferences, the representative agent is no longer indifferent between the timing of resolving uncertainty. In this structure, Epstein, Zin and Weil (1991) derived the following Euler equations

$$1 = E_t \left[\left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right]^{\frac{1-\sigma}{1-\rho}} \left[\frac{1}{B_{t+1}} \right]^{\frac{\sigma-\rho}{1-\rho}} R_{t+1}^i \right] \tag{9}$$

where B_{t+1} is the gross return of the optimal portfolio. When $\sigma = \rho$, B_{t+1} becomes irrelevant and under such restriction the recursive utility model becomes the CCAPM with CRRA utility, i.e., the model with CRRA utility is a special case of the model with recursive utility. This implies a SDF with:

$$M_{t+1}^{Recursive} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\sigma}{1-\rho}}\left[\frac{1}{B_{t+1}}\right]^{\frac{\sigma-\rho}{1-\rho}} \quad (10)$$

2.4 Saving-CAPM

Cordenonssi (2013) presented the saving-CAPM (SCAPM) as a proposed solution of the equity premium puzzle described by Mehra and Prescott (1985), incorporating savings in the agent's utility function. The SCAPM assumes that the agents of the economy obtain satisfaction not only by consuming, but also by saving, because what is saved today will be consumed in the future (Cordenonssi, 2013). The model's contribution consists of changing the assumption of a subsistence economy, so that the private sector's available income (Y_t) is not fully consumed (C_t), but also saved (S_t). The SCAPM assumes that representative agents feel satisfaction from current and future consumption, the latter being represented by current savings. So, the agents solve the following optimization problem:

$$\begin{gathered} \max U(C_t, S_t) \\ s.a.\ Y_t = C_t + S_t \end{gathered} \quad (11)$$

Applying the Lagrange multiplier method, we have:

$$\mathcal{L} = U(C_t, S_t) + \lambda(Y_t - C_t - S_t) \quad (12)$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{\partial U(C_t, S_t)}{\partial C_t} - \lambda = 0 \Rightarrow \frac{\partial U(C_t, S_t)}{\partial C_t} = \lambda \quad (13)$$

$$\frac{\partial \mathcal{L}}{\partial S_t} = \frac{\partial U(C_t, S_t)}{\partial S_t} - \lambda = 0 \Rightarrow \frac{\partial U(C_t, S_t)}{\partial S_t} = \lambda \quad (14)$$

Thus, from equations (13) and (14), the first-order correlation between consumption and savings is derived as:

$$\frac{\partial U(C_t, S_t)}{\partial C_t} = \frac{\partial U(C_t, S_t)}{\partial S_t} \quad (15)$$

Cordenonssi (2006) formulate the marginal utility function of consuming and saving for the following reasons: (1) saving, not consumption, determines demand for financial assets; (2) in macroeconomics, the main theories on private consumption establish a strong link with income; and (3) savings is the link between consumption and wealth, since it is incremental wealth. The author assumed that the utility in saving follow a traditional CRRA:

$$u(S_t) = \frac{S_t^{1-\rho} - 1}{1-\rho} \quad (16)$$

Solving the Bellman equation and imposing the equilibrium condition result in an Euler equation with the SDF being:

$$M_{t+1}^{SCAPM} = \beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho} \quad (17)$$

2.5 Saving habit formation

We developed the Saving-Habits model from the procedure developed by Cordenonssi (2006) for the Saving-CAPM. The model arises from the idea of testing whether the habit of saving has an influence on the return on financial assets. We believe that saving depends not only on future income changes and income risk, but also on past saving. The return on saving in Brazil has a certain relationship with the economy's basic interest rate, making it possible for investors to transition to the financial market depending on the economic and political situation of the domestic market compared to the rest of the world. We define the utility function of saving habit as:

$$u(S_t, S_{t-1}) = \frac{1}{1-\rho}\left(\frac{S_t}{S_{t-1}^k}\right)^{1-\rho} \tag{18}$$

$$u_t^{'} = \frac{\partial u(S_t, S_{t-1})}{\partial S_t} = \left(\frac{S_t}{S_{t-1}^k}\right)^{-\rho}\frac{1}{S_{t-1}^k} \tag{19}$$

We use the equation (18) and replace the first derivative in the utility function (19) in the Euler equation (20) and we calculate the stochastic discount factor of the model (21) as result:

$$1 = E_t\left[\beta\frac{u_{t+1}'}{u_t'}R_{t+1}^i\right] \tag{20}$$

$$1 = E_t\left[R_{t+1}\beta\frac{\left(\frac{S_{t+1}}{S_t^k}\right)^{-\rho}\frac{1}{S_t^k}}{\left(\frac{S_t}{S_{t-1}^k}\right)^{-\rho}\frac{1}{S_{t-1}^k}}\right] = E_t\left[R_{t+1}\beta\left(\frac{S_{t+1}}{S_t^k}\right)^{-\rho}\left(\frac{S_t}{S_{t-1}^k}\right)^{\rho}\left(\frac{S_{t-1}}{S_t}\right)^{k}\right]$$

$$1 = E_t\left[R_{t+1}\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho}\left(\frac{S_t}{S_{t-1}}\right)^{k\rho}\left(\frac{S_t}{S_{t-1}}\right)^{-k}\right] = E_t\left[R_{t+1}\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho}\left(\frac{S_t}{S_{t-1}}\right)^{k(\rho-1)}\right]$$

$$M_{t+1}^{\text{Saving habit}} = \beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho}\left(\frac{S_t}{S_{t-1}}\right)^{k(\rho-1)} \tag{21}$$

2.6: Labor income

Davis and Martin (2009) introduced leisure to the model of Piazzesi et al. (2007). We apply a simplified version of this model for comparison purposes. Thus, the representative agent chooses $\{C_t, X_{t+1}, N_t\}_{t=0}^{\infty}$ to maximize his utility:

$$V(x_t) = \max_{\{X_{t+1}, N_t\}_{t=0}^{\infty}}\left\{\frac{(C_t L_t^v)^{1-\rho}}{1-\rho} + \beta E_t[V(X_{t+1})]\right\} \tag{24}$$

$$s.a: \quad C_t = X_t - \frac{X_{t+1}}{R_{t+1}} + W_t N_t$$

where $0 < \beta < 1$ is the discount factor; C_t is real consumption of nondurables and services; X_t is the wealth portfolio that delivers the entire consumption stream C_t as the dividend, W_t is the real wage; N_t is the working hours; L_t is the leisure time; and R_{t+1} is the gross rate of return of the wealth portfolio. The hours of the representative agent are normalized to 1. Thus, the Euler equation and the stochastic discount factor of the model can be obtained:

$$1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho+v(1-\rho)}\left(\frac{W_{t+1}}{W_t}\right)^{-v(1-\rho)}R_{t+1}^i\right] \tag{25}$$

$$M_{t+1}^{Labor\ income} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho+v(1-\rho)}\left(\frac{W_{t+1}}{W_t}\right)^{-v(1-\rho)} \tag{26}$$

Some models are particular cases of the canonical form of the CCAPM, making their stochastic discount factors equal, including: (1) when $(k = 0)$ for the habit formation model; (2) when $(\sigma = \rho)$ for the recursive utility model; and (3) when $(v = 0)$ for the labor income model. On the other hand, the saving habit formation model is a particular case of the SCAPM and its stochastic discount factors are equal when $(k = 0)$. Table 2 presents a summary of the models, their utility functions and the stochastic discount factors for each one.

Table 2 - Utility functions and stochastic discount factors of the models

Model	Utility function	Stochastic discount factor
CRRA	$U_t(C_t) = \frac{C_t^{1-\rho} - 1}{1-\rho}$	$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}$
Habit Formation	$U_t(C_t, C_{t-1}) = \frac{1}{1-\rho}\left(\frac{C_t}{C_{t-1}^k}\right)^{1-\rho}$	$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{C_t}{C_{t-1}}\right)^{k(\rho-1)}$
Recursive Utility	$U_t(C_t, U_{t+1}) = \left[(1-\beta)C_t^{1-\rho} + \beta(E_t[U_{t+1}^{1-\sigma}])^{\frac{(1-\rho)}{(1-\sigma)}}\right]^{\frac{1}{(1-\rho)}}$	$M_{t+1} = E_t\left[\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\sigma}{1-\rho}}\left[\frac{1}{B_{t+1}}\right]^{\frac{\sigma-\rho}{1-\rho}}\right]$
SCAPM	$U_t(S_t) = \frac{S_t^{1-\rho} - 1}{1-\rho}$	$M_{t+1} = \beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho}$
Saving Habit Formation	$U_t(S_t) = \frac{1}{1-\rho}\left(\frac{S_t}{S_{t-1}^k}\right)^{1-\rho}$	$M_{t+1} = \beta\left(\frac{S_{t+1}}{S_t}\right)^{-\rho}\left(\frac{S_t}{S_{t-1}}\right)^{k(\rho-1)}$
Labor income	$U_t(C_t, L_t) = \frac{(C_t L_t^v)^{1-\rho}}{1-\rho}$	$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho+v(1-\rho)}\left(\frac{W_{t+1}}{W_t}\right)^{-v(1-\rho)}$

3. Data and structural estimation

3.1 Data

The sample period extends from 1996:1 to 2019:4, for 96 observations. The variables used in our model comparison exercise are: (1) gross market rate of return, constructed from the Brazilian stock market index[3] (R_m); (2) gross rate of return on risk-free assets, constructed from the SELIC (benchmark) interest rate (R_f); (3) growth rate of per capita consumption; (4) growth rate of per capita saving; and (5) wage growth. All variables are measured in real terms (2018:4 constant prices) after inflation adjustment[4].

i. Population: The quarterly population series was built by interpolating the original annual series. Data source: IBGE.
ii. Consumption (C_t): Final household consumption per capita was the gross growth rate of per capita consumption (C_{t+1}/C_t) deflated by the gross inflation rate. We used the original series and seasonally adjusted series using X-13. Data source: Institute of Applied Economic Research (IPEA).
iii. Saving (S_t): The series was derived from the difference between the series "gross national disposable income" and the series "final household consumption" scaled by population in each quarter. The series used to estimate the models was the gross growth rate of per capita saving (S_{t+1}/S_t). Both series were deflated by the gross inflation rate. We used the original series and seasonal adjustment carried out by X-13. Data source: IPEA.
iv. Gross rate of market return (R_m): We used the IBOVESPA index to reflect the market return and collected the data with daily frequency from Yahoo Finance, calculating the cumulative deflated gross return for each quarter deflated by the gross inflation rate.

[3] IBOVESPA index.
[4] IPCA (Comprehensive Consumer Price Index). Inflation: The (gross) inflation rate was defined as the gross growth of the Comprehensive Consumer Price Index (IPCA) (2018Q4= 100). Data source: Brazilian Institute of Geography and Statistics (IBGE).

v. Gross rate of return on risk-free assets (R_f): We adopted the SELIC rate (the benchmark rate) to reflect the risk-free asset in Brazil, using the rate referenced to the 2018Q4 period, calculating the cumulative deflated gross return for each quarter necessary to estimate the models. Data source: IPEA and Central Bank of Brazil.

vi. Wages (W_t): We employed the extended salary mass (MSA), which is understood as the sum of income from work or pension benefits and/or other social protection benefits, which are transferred by the government to households. The series used for the models was the quarterly gross wage growth (W_{t+1}/W_t), deflated by the gross inflation rate. We used the original series and seasonally adjusted series using X-13. Data source: IBGE.

Table 3 reports summary statistics of these variables used in the empirical study. The gross market rate of return variance (R_m) is much greater than the other variables due to the volatility of the Brazilian financial market. As a limitation, we do not have data for all macroeconomic series since 1996 in Brazil. Thus, the series on growth in savings and wages have shorter time frames than the other series. Details about data source and variable definition can be found in Appendix A.

Table 3 - Summary statistics (1996:1–2019:4)

Key variables	Size	Period	Mean	Std error	Variance	Min	Max
Market return (R_m)	95	1996:1 – 2019:4	1.0387	0.1548	0.0240	0.6650	1.5595
Risk-free return (R_f)	95	1996:1 – 2019:4	1.0201	0.0167	0.0003	0.9855	1.0792
Consumption growth	95	1996:1 – 2019:4	1.0070	0.0318	0.0010	0.9160	1.0782
Saving growth	79	2000:1 – 2019:4	1.0081	0.0699	0.0049	0.8420	1.1368
Wage growth	63	2004:1 – 2019:4	1.0111	0.0508	0.0026	0.8906	1.1219

Notes: (i) All variables are measured in real terms (2018:4 constant prices). (ii) The two asset returns, consumption growth, saving growth and wage growth are measured in gross rate (=1 + net rate) per quarter.

Figure 1 - Market and risk-free returns Quarterly Gross Returns

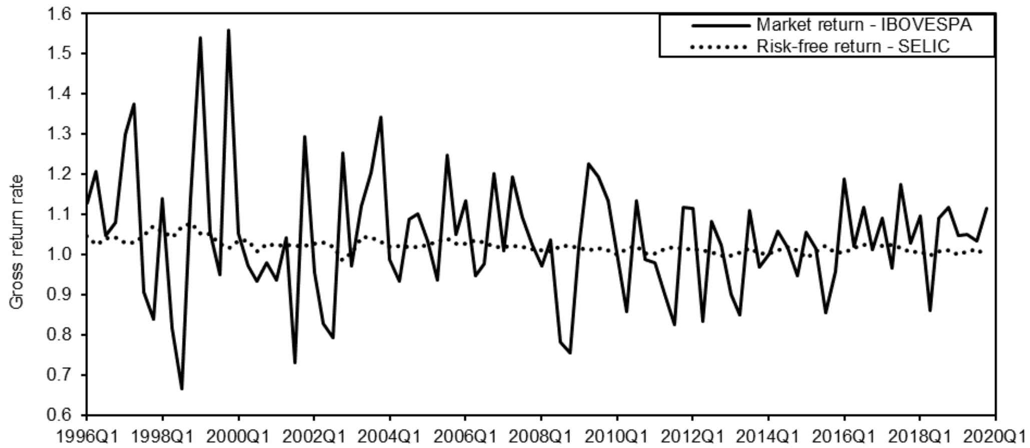


Figure 1 shows a period of great volatility at the beginning of the series, mainly due to the 1997 Southeast Asian monetary crisis and later the implementation of the "macroeconomic tripod" in the domestic economy in 1999, consisting of three measures: floating exchange rate; fiscal targets; and inflation targets. It is also possible to verify the impact of the American subprime crisis in 2007, generating a period of large losses in the Brazilian stock market and a strong outflow of foreign capital from the country.

3.2 Structural estimation

We apply GMM to estimate the six intertemporal models of asset prices. The conditional moment conditions that we use in the GMM estimation are two Euler equations - one for market return and the other for risk-free return - derived from each model, as shown in Section 2. That is, for each model, the GMM procedure identifies a set of best-fitting structural parameter estimates that can explain both market and risk-free returns.

Table 4 - Structural parameters

Parameter	Interpretation	Appears in
β	Discount factor	All models
ρ	Serves doubly, as relative risk aversion and inverse of elasticity of intertemporal substitution	CCAPM with CRRA, SCAPM, Habit Formation, Saving Habit Formation and Labor income
	Inverse of elasticity of intertemporal substitution	Recursive Utility
σ	Relative risk aversion	Recursive Utility
k	Benchmark or level of habit	Habit Formation and Saving Habit Formation
v	Leisure share in utility function	Labor income

Estimates are performed for the original series and seasonally adjusted[5] and presented with their respective instruments. The GMM estimation results for the original series are reported in Table 5 and the estimations of seasonally adjusted series are reported in Table 6. We define significant results as validating the instrument when the *p*-value of the J-statistic is greater than 5%, at the same time that the *p*-value of the parameter estimate is less than 5%. It can be seen from the estimation results that in general the models are internally consistent - the Hansen overidentification J-statistics are all insignificant at the conventional level, suggesting valid moment conditions. Most of the discount factors estimated in the models are in the interval (0.95; 1), which is consistent with the literature on the theoretical discount factor. Aversion to relative risk values belong to the range (0; 10), which also corresponds to the literature.

Table 5 - GMM system estimation of structural parameters for the original series.

	(1) CCAPM CRRA	(2) Habit formation	(3) Recursive utility	(4) SCAPM	(5) Saving habit formation	(6) Labor income
$\hat{\beta}$	0.9985**	0.9854**	0.9766**	0.9865**	0.9868**	0.9917**
	(0.0063)	(0.0069)	(0.0358)	(0.0019)	(0.0155)	(0.0076)
$\hat{\rho}$	1.5553**	1.3055	0.4332	0.2209**	1.1754*	1.4649
	(0.4260)	(1.0523)	(6.0392)	(0.1049)	(0.5008)	(3.0545)
$\hat{k}$		2.5984			3.0112	
		(6.3792)			(7.4038)	
$\hat{\sigma}$			0.8230			
			(2.2583)			
$\hat{v}$						5.3858
						(18.2811)
J-statistic	11.3419	6.7995	2.7540	1.2069	3.0533	2.3079
[*p*-value]	0.0784	0.2360	0.4311	0.8770	0.3835	0.5110
Sample size	95	94	95	79	78	63
Instrument	t to t-2	t to t-2	t to t-2	t to t-2	t to t-2	t to t-2
MSE	0.2187	0.2216	0.1508	0.1422	0.269	0.1781
MAE	0.5531	0.5599	0.4631	0.4288	0.661	0.5259
Uncond. HJ dist.	121.2689	112.2494	20.4244	82.5232	107.5000	103.1887
Cond. HJ dist.	11.5987	9.6734	0.9847	11.3618	13.2081	14.2631

Notes: (i) Standard errors in parentheses. (ii) * 5% significance; ** 1% significance. (iii) Both market and risk-free return equations are included in the GMM system. (iv) Each model was estimated with a number of instruments ranging from 8 to 10 to generate robustness. (v) The table shows the median of the estimated values of each model.

[5] Seasonal adjustment carried out by X-13ARIMA-SEATS.

Table 6 - GMM system estimation of structural parameters of the seasonally adjusted series.

	(1) CCAPM CRRA	(2) Habit formation	(3) Recursive utility	(4) SCAPM	(5) Saving habit formation	(6) Labor income
$\hat{\beta}$	0.9907**	0.9705**	0.9798**	0.9866**	0.9912**	0.9950**
	(0.0071)	(0.0583)	(0.0145)	(0.0038)	(0.0097)	(0.0162)
$\hat{\rho}$	1.3384	1.5800	1.0070	0.3911	0.8854	0.8071
	(1.0470)	(4.0933)	(2.5304)	(0.7047)	(1.0031)	(1.8418)
$\hat{k}$		3.8604			2.3666	
		(36.2852)			(20.1486)	
$\hat{\sigma}$			1.0016			
			(0.5961)			
$\hat{v}$						3.2018
						(39.6434)
J-statistic	9.5406	4.5125	4.0887	6.3144	7.7902	3.6072
[p-value]	0.1454	0.2112	0.5367	0.1769	0.0506	0.6072
Sample size	95	94	95	79	78	63
Instrument	t to t-2	t to t-2	t to t-2	t to t-2	t to t-2	t to t-2
MSE	0.1789	0.2840	0.1514	0.1540	0.1972	0.1531
MAE	0.4797	0.6503	0.4539	0.4640	0.5488	0.4725
Uncond. HJ dist.	115.7281	117.8148	8.6691	84.3415	94.0362	122.0455
Cond. HJ dist.	10.981	12.2546	0.2907	11.5706	13.9137	16.5753

Notes: (i) Standard errors in parentheses. (ii) * 5% significance; ** 1% significance. (iii) Both market and risk-free return equations are included in the GMM system. (iv) Each model was estimated with a number of instruments ranging from 8 to 10 to generate robustness. (v) The table shows the median of the estimated values of each model.

3.3 Model comparison

The previous section showed that the six asset pricing models in general are not rejected by the data according to the GMM estimation results. The models are different in terms of their empirical performance in explaining the asset and stock markets in Brazil. However, GMM cannot distinguish which model performs better, so we need to adopt other criteria for model comparison. We employ two model comparison criteria in this section: (1) the comparison of pricing errors based on measures of errors in asset price using the Euler equations obtained in each model; and (2) the Hansen and Jagannathan (1997) distance (HJ distance) in two versions: unconditional HJ distance (the traditional version of the measure) and conditional HJ distance (which incorporates the instruments used in the GMM estimates). The two criteria focus on different characteristics of a theoretical model, and we can view them as complementary to each other. The pricing errors consider the errors obtained by the model in pricing but fail to measure the set of assets simultaneously. Thus, we also apply the HJ distance method, which is specifically designed to measure Euler equation errors with all structural characteristics preserved.

3.3.1 Pricing errors

The error measures that we use to account for the pricing errors are: mean squared errors (MSE) and mean absolute errors (MAE). Clearly, the MSE tends to "punish" big mistakes, while the MAE tends to treat errors of different magnitudes in a more symmetrical way. Thus, the classification of models by these two criteria can lead to different results, and therefore offer robustness when analyzed together. The measures are widely disseminated in the literature and are obtained by the following equations:

Table 7 - Mean squared errors (MSE) and Mean absolute errors (MAE)

Mean squared errors (MSE)	*Mean absolute errors (MAE)*
$MSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e_i^2}$	$MAE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \lvert e_i \rvert}$

Note: e_i denotes the errors of Euler's equations.

We examine the prediction errors separately for each asset to build the measure considering the market asset and risk-free asset. Based on the MSE and MAE measures, we created the comparison criteria between the models estimated in this work based on the medians and minima of each measure for the models with data with and without seasonal adjustment. In addition, we measure the difference between the MAE and MSE measurements of the different models. A large difference indicates that model 1 is an improvement over model j, $MAE_1 - MAE_j$. We also examine the percentage gain of each model in relation to the model that presented the best result according to each criterion, to measure the improvement of model 1 as a percentage reduction in the pricing error in relation to model j, $1 - MAE_1/MAE_j$. The results of the measurements are used as a criterion to rank the models according to Table 8.

In order to better understand the nature of the pricing errors, we plot the time series of the absolute pricing errors for each model in Figures 2 and 3 with the same scale to facilitate visual comparison, which shows that the recursive utility model produces smaller prediction errors. Now, based on these graphs and drawing on our knowledge about the history of Brazil's economy during the sample period, we provide the following remarks on prediction errors:

(1) Except for the recursive utility model, all other models have relatively bigger absolute prediction errors in the period between 1996 and 1999. These were related to some large economic or political issues marked by policies to stop currency outflow after the Mexican crisis in 1994, such as a change in exchange rate policy with interventions by the Central Bank of Brazil to keep the administered exchange rate within a band, as well as a sharp rise in the basic interest rate[6].

(2) Notice that among all the models considered, only the recursive utility model separates the elasticity of intertemporal substitution and the degree of risk aversion, mainly because it allows the presence of a small predictable long-term component in the growth of consumption, which enables the model to explain several characteristics of the observed asset prices according to Attanasio and Weber (2010). The model of Epstein and Zin (1989) is the most accepted in the literature, and for Brazilian data has excellent performance.

(3) Labor income and SCAPM are models that present intermediate pricing errors when compared to other models. CCAPM, habit formation and saving habit formation are the models with the highest intermediate pricing errors.

3.3.2 Hansen-Jagannathan distance

Hansen and Jagannathan (1997) developed a measure that gained great popularity as a tool to select asset pricing models. This measure, called the Hansen-Jagannathan distance (HJ distance), calculates the minimum distance of errors between the models' stochastic discount factors and the set of admissible SDFs that price all assets. The Hansen-Jagannathan distance may or may not incorporate negative SDFs. Li, Xu and Zhang (2010) strongly recommended the use of the restricted model (SDF> 0) in empirical studies, since they found that the restricted metric has greater power in detecting models with poor specification. The Hansen-Jagannathan distance relates the distance between the true pricing kernel (m), which prices all assets, and the model's pricing kernel proxy (y), where $m \in M$ is the set of all pricing kernels that correctly price the economy's assets. Hansen and Jagannathan (1997) observed that, given a pricing model, it will be false when $y \neq m$ and there will then be a strictly positive distance between the two factors, as follows:

$$\delta = min\|y - m\|$$
$$E[mR] = p$$

[6] SELIC rate: basic interest rate of the Brazilian economy.

The problem can be rewritten as a Lagrange minimization problem:

$$\delta^2 = \min \sup E[y - m]^2 + 2\lambda'(E[mR] - p) \tag{27}$$

The value of δ represents the minimum distance between the model proxy and the true pricing kernel. Hansen and Jagannathan (1997) solved the Lagrange minimization problem by assuming that $\bar{m}$ and $\bar{\lambda}$ are solutions, thus defining $y - \bar{m}$ as the minimum adjustment to y that makes it the true pricing kernel, and found:

$$y - m = \lambda R$$
$$\lambda = E[RR']^{-1}E[yR - p]$$

The Hansen-Jagannathan distance can then be written as:

$$\delta = \|y - \bar{m}\| = \|\bar{\lambda}'R\| = \sqrt{\bar{\lambda}'E[RR']\bar{\lambda}} = \sqrt{E[yR - p]'E[RR']^{-1}E[yR - p]} \tag{28}$$

The HJ distance can be used to compare the models adopting the parameters estimated by GMM in this work. The unconditional HJ distance uses $g_N(v;\theta)$ and G_N, as defined in equations (18) and (19), and is the traditional form of the metric defined by Hansen and Jagannathan:

$$g_N(v;\theta) \equiv \frac{1}{N}\sum_{i=1}^{N} h_i\,(v;\theta) \equiv \frac{1}{N}\sum_{i=1}^{N}\left[M_t(\theta)R_t^i - 1_N\right] \tag{29}$$

$$G_N \equiv \frac{1}{N}\sum_{t=1}^{N} R_{i,t}R_{j,t}\ , para\ i,j = 1,\dots,n\ ativos \tag{30}$$

We also computed a conditional version of the distance metric that incorporates z_t to condition information, as proposed by Chen, Favilukis and Ludvigson (2013). In this case, the way in which $g_N(v;\theta)$ and G_N are calculated changes according to equations (33) and (34), where $\otimes$ denotes the Kronecker product.

$$g_N(v;\theta) \equiv \frac{1}{N}\sum_{i=1}^{N}[(M_i(\theta)R_t - 1_N) \otimes z_t] \tag{31}$$

$$G_N \equiv \frac{1}{N}\sum_{i=1}^{N}(R_{t+1} \otimes z_t)(R_{t+1} \otimes z_t)' \tag{32}$$

The HJ distance, whether conditional or unconditional, is defined using the parameters estimated by GMM $\hat{\theta}$:

$$\delta = HJ_{distance}(\hat{\theta}) \equiv \sqrt{\left[g_N(v;\hat{\theta})\right]'_{1\times N}\,[G_N^{-1}]_{N\times N}\left[g_N(v;\hat{\theta})\right]_{N\times 1}} \tag{33}$$

The HJ distance also provides an incorrect specification measure, that is, it provides a minimum square distance between the model's stochastic factor and the closest point to the space of all the stochastic discount factors that correctly price the assets. If the model is correctly specified, both conditional and unconditional HJ distance measures are zero. We compute the HJ distance for each model to build the measure considering the market asset and the risk-free asset simultaneously. Starting from the two versions of the HJ distance, we created the comparison criteria between the models estimated in this work based on the medians and minima of each measure for the models with data with and without seasonal adjustment. In addition, we measured the difference between the measures of the different models. A large difference indicates that model 1 is an improvement over model j, $\delta_1 - \delta_j$. We also examined the percentage gain of each model in relation to the model that presented the best result according to each criterion, allowing measuring the improvement of model 1 as a percentage reduction in relation to model j, $1 - \delta_1/\delta_j$. The results of the measurements were used as a criterion to rank the models according to Table 9.

3.3.2 Comparison and ranking of models

While it is natural to expect different model rankings when different approaches are employed, it is instructive to discuss the methodological differences of the two model comparison methods. The pricing errors with the MSE and MAE measures do not consider the structural characteristics of the models and also measure errors separately for each asset. In contrast, the HJ distance preserves all the structural characteristics of the Euler equation, including those that come from specification errors and oversimplified assumptions in the theoretical model, besides contemplating the assets simultaneously as well as the instruments used in the estimation by GMM in its conditional version. This means that if the theoretical models are considered literally to be ‘‘correct’’— in the sense that they are indeed the data generating mechanisms – it is easier and arguably more appropriate to interpret the results based on the HJ distance. The ranking of the models for the various criteria adopted in this work are presented in Tables 8 and 9.

Table 8 - Model comparison based on pricing errors.

Original series				Seasonally adjusted series			
Model ranking	Median MSE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Median MSE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel A: Median of MSE							
1. SCAPM	0.1422			1. Labor income	0.1531		
2. Recursive	0.1693	0.0271	16%	2. SCAPM	0.1540	0.0009	1%
3. Labor income	0.1781	0.0359	20%	3. CRRA	0.1789	0.0258	14%
4. CRRA	0.2106	0.0684	32%	4. Saving habit	0.1976	0.0445	23%
5. Habit	0.2151	0.0729	34%	5. Recursive	0.2240	0.0709	32%
6. Saving habit	0.2764	0.1342	49%	6. Habit	0.3783	0.2252	60%
Model ranking	Minimum MSE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Minimum MSE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel B: Minimum of MSE							
1. Labor income	0.1332			1. Labor income	0.1260		
2. Saving habit	0.1346	0.0014	1%	2. SCAPM	0.1370	0.0110	8%
3. SCAPM	0.1347	0.0015	1%	3. Saving habit	0.1393	0.0133	10%
4. Recursive	0.1508	0.0176	12%	4. Recursive	0.1514	0.0254	17%
5. CRRA	0.1707	0.0375	22%	5. CRRA	0.1699	0.0439	26%
6. Habit	0.1773	0.0441	25%	6. Habit	0.2449	0.1189	49%
Model ranking	Median MAE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Median MAE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel C: Median of MAE							
1. SCAPM	0.4640			1. SCAPM	0.4288		
2. Recursive	0.4725	0.0085	2%	2. Labor income	0.4697	0.0409	9%
3. Labor income	0.4797	0.0157	3%	3. CRRA	0.5259	0.0971	18%
4. CRRA	0.5304	0.0664	13%	4. Recursive	0.5388	0.1100	20%
5. Habit	0.5496	0.0856	16%	5. Saving habit	0.5490	0.1202	22%
6. Saving habit	0.7529	0.2889	38%	6. Habit	0.6721	0.2433	36%
Model ranking	Minimum MAE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Minimum MAE	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel D: Minimum of MAE							
1. Recursive	0.3839			1. Labor income	0.4048		
2. Saving habit	0.4022	0.0183	5%	2. Recursive	0.4053	0.0005	0%
3. SCAPM	0.4051	0.0212	5%	3. SCAPM	0.4150	0.0102	2%
4. Labor income	0.4403	0.0564	13%	4. Saving habit	0.4198	0.0150	4%
5. CRRA	0.4501	0.0662	15%	5. CRRA	0.4464	0.0416	9%
6. Habit	0.4696	0.0857	18%	6. Habit	0.5976	0.1928	32%

Notes: (i) $\delta_1 - \delta_j$ means the absolute difference between the metrics of model j and model 1. (ii) $1 - \frac{\delta_1}{\delta_j}$ means the percentage difference between the MAE and MSE metrics of model j and model 1.

Figure 2 - Original series of absolute (in-sample) prediction errors of market returns

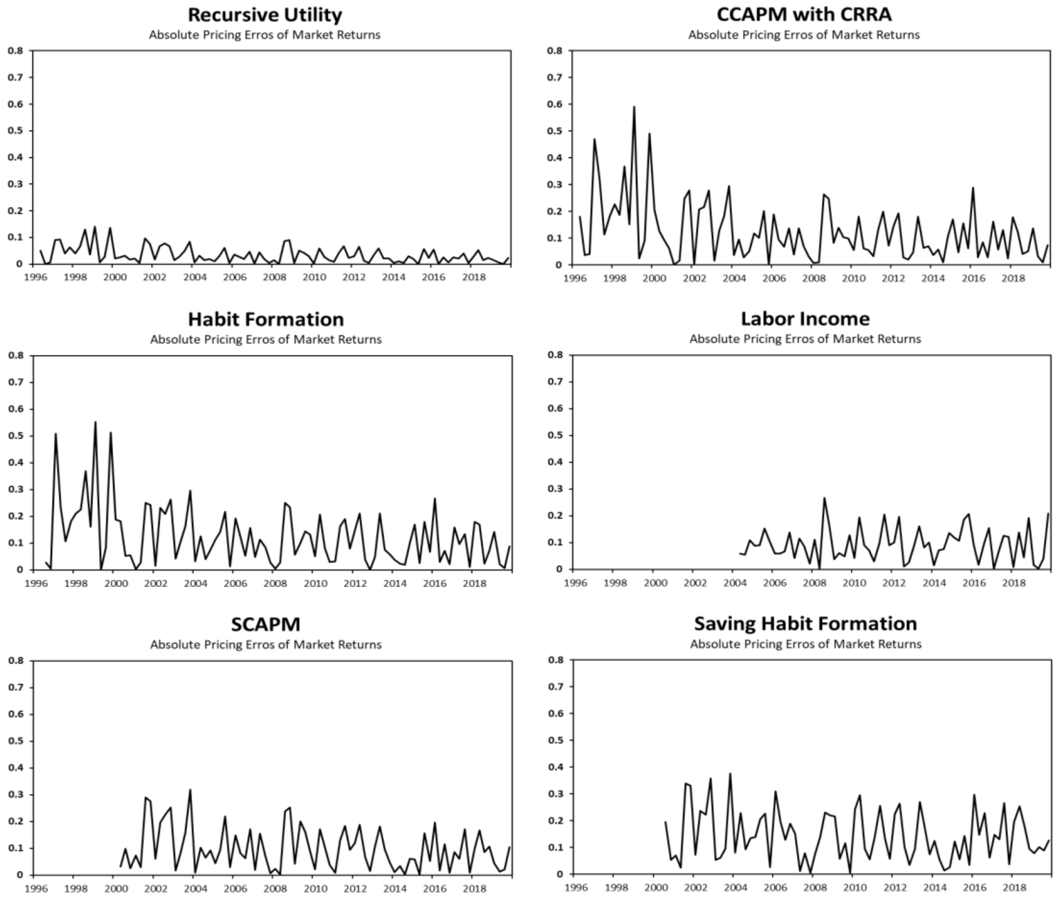


Figure 3 - Seasonally adjusted series of absolute (in-sample) prediction errors of market returns

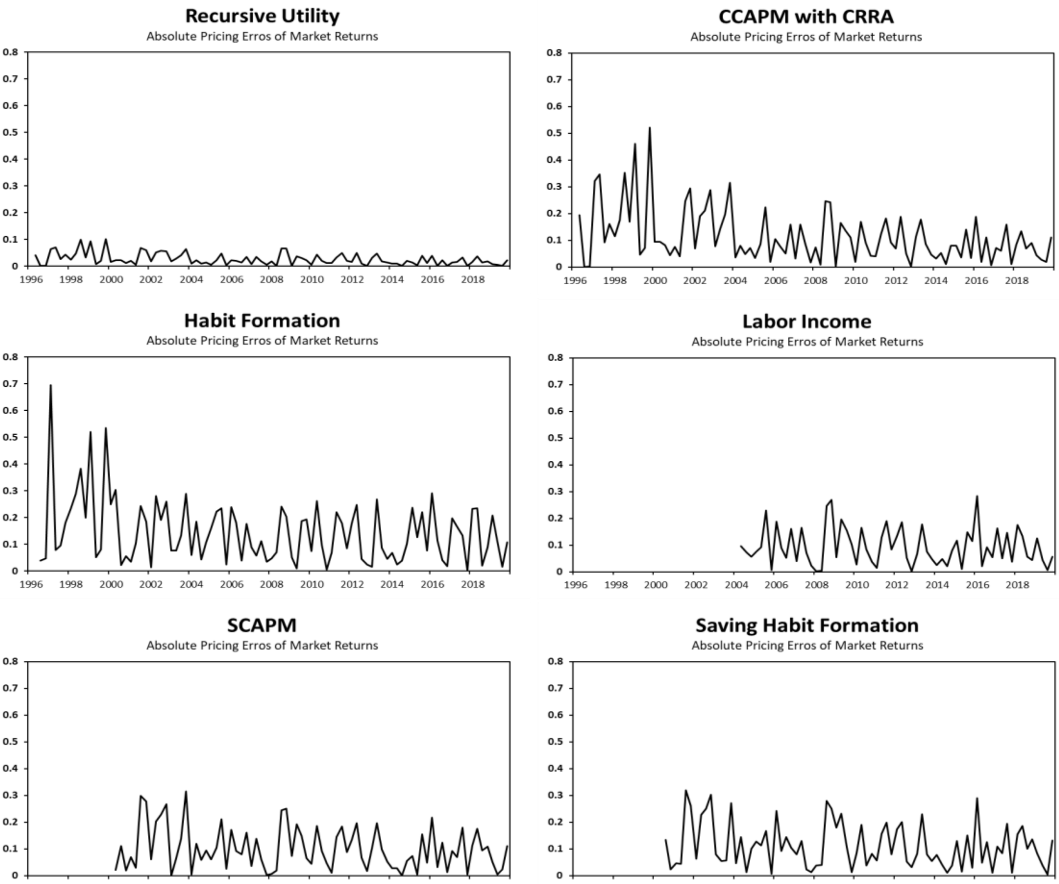


Table 9 - Model comparison based in the HJ distance.

Original series				Seasonally adjusted series			
Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel A: Median of unconditional HJ-Distance ($\times 10^{-3}$)							
1. Recursive	43.0058			1. Recursive	68.1291		
2. SCAPM	82.5232	39.5174	48%	2. SCAPM	85.7380	17.6089	21%
3. Labor income	103.1887	60.1829	58%	3. Saving habit	94.0928	25.9637	28%
4. Saving habit	109.1240	66.1182	61%	4. CRRA	115.7281	47.5990	41%
5. Habit	111.7693	68.7635	62%	5. Labor income	119.9977	51.8686	43%
6. CRRA	120.8440	77.8382	64%	6. Habit	123.8050	55.6759	45%
Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel B: Median of conditional HJ-Distance ($\times 10^{-3}$)							
1. Recursive	6.0756			1. Recursive	8.3812		
2. Habit	9.6734	3.5978	37%	2. Habit	10.4169	2.0357	20%
3. SCAPM	11.3618	5.2862	47%	3. CRRA	10.9560	2.5748	24%
4. CRRA	11.5510	5.4754	47%	4. SCAPM	11.7308	3.3496	29%
5. Saving habit	14.2949	8.2193	57%	5. Saving habit	13.9204	5.5392	40%
6. Labor income	14.4150	8.3394	58%	6. Labor income	16.5321	8.1509	49%
Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel C: Minimum of unconditional HJ distance ($\times 10^{-3}$)							
1. Recursive	6.2970			1. Recursive	8.6691	2.2869	21%
2. CRRA	11.5510	5.2540	45%	2. CRRA	10.9560	73.7721	89%
3. Labor income	74.1753	67.8783	92%	3. SCAPM	82.4412	79.4563	90%
4. SCAPM	80.2403	73.9433	92%	4. Saving habit	88.1254	82.7515	91%
5. Saving habit	83.3485	77.0515	92%	5. Labor income	91.4206	83.7781	91%
6. Habit	108.4766	102.1796	94%	6. Habit	92.4472	2.2869	21%
Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$	Model ranking	Metric	$\delta_1 - \delta_j$	$1 - \frac{\delta_1}{\delta_j}$
Panel D: Minimum of conditional HJ distance ($\times 10^{-3}$)							
1. Recursive	0.4821			1. Recursive	0.2907		
2. Habit	9.5089	9.0268	95%	2. Habit	8.5154	8.2247	97%
3. CRRA	11.2071	10.7250	96%	3. CRRA	9.7310	9.4403	97%
4. SCAPM	11.3525	10.8704	96%	4. SCAPM	11.3525	11.0618	97%
5. Saving habit	12.6802	12.1981	96%	5. Saving habit	11.8660	11.5753	98%
6. Labor income	14.4150	13.9329	97%	6. Labor income	16.5321	16.2414	98%

Notes: (i) $\delta_1 - \delta_j$ means the absolute difference between the metrics of model j and model 1. (ii) $1 - \frac{\delta_1}{\delta_j}$ means the percentage difference between the HJ distance metrics of model j and model 1.

Table 10 - Ranking of models using original series.

Criteria	Ranking of models
Panel A: Conditional HJ Distance	
Median	Recursive utility ≽ Habit formation ≽ [SCAPM, CRRA] ≽ [Saving habit formation, Labor income]
Minimum	Recursive utility ≽ [Habit formation, CRRA, SCAPM, Saving habit formation, Labor income]
Panel B: Unconditional HJ Distance	
Median	Recursive utility ≽ SCAPM ≽ Labor income ≽ [Saving habit formation, Habit formation, CRRA]
Minimum	Recursive utility ≽ CRRA ≽ [Labor income, SCAPM, Saving habit formation, Habit formation]
Panel C: MSE	
Median	SCAPM ≽ [Recursive utility, Labor income] ≽ [CRRA, Habit formation] ≽ Saving habit formation
Minimum	[Labor income, Saving habit formation, SCAPM] ≽ Recursive utility ≽ [CRRA, Habit formation]
Panel D: MAE	
Median	SCAPM ≽ Recursive utility ≽ [Labor income, CRRA, Habit formation] ≽ Saving habit formation
Minimum	Recursive utility ≽ [Saving habit formation, SCAPM] ≽ [Labor income, CRRA, Habit formation]

Notes: (i) A ≽ B means A outperforms B. (ii) [A, B] means A and B belong to the same model confidence set (percentage difference less than 5%), so their performance is indistinguishable.

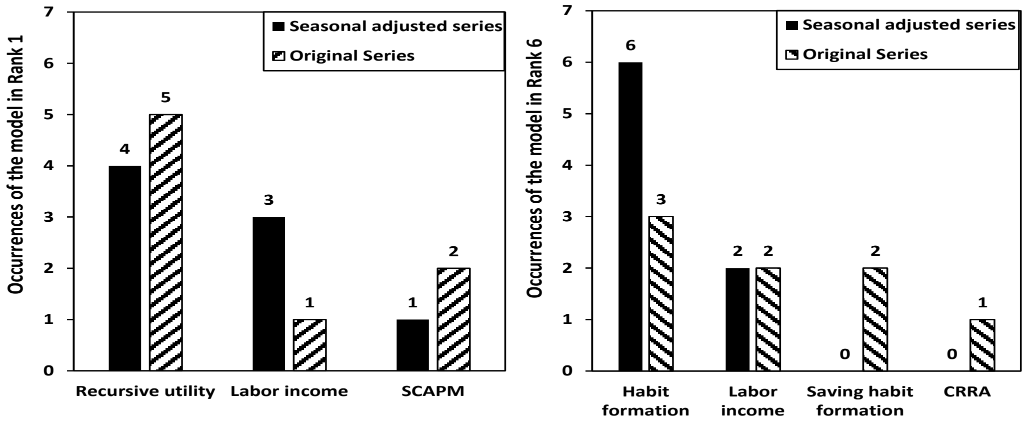
Table 11 - Ranking of models using seasonally adjusted series.

Criteria	Ranking of models
Panel A: Conditional HJ Distance	
Median	Recursive utility ≽ [Habit formation, CRRA] ≽ SCAPM ≽ Saving habit formation ≽ Labor income
Minimum	Recursive utility ≽ [Habit formation, CRRA, SCAPM, Saving habit formation, Labor income]
Panel B: Unconditional HJ Distance	
Median	Recursive utility ≽ SCAPM ≽ Saving habit formation ≽ [CRRA, Labor income, Habit formation]
Minimum	Recursive utility ≽ CRRA ≽ [SCAPM, Saving habit formation, Labor income, Habit formation]
Panel C: MSE	
Median	Labor income ≽ SCAPM ≽ CRRA ≽ Saving habit formation ≽ Recursive utility ≽ Habit formation
Minimum	Labor income ≽ [Saving habit formation, SCAPM] ≽ Recursive utility ≽ CRRA ≽ Habit formation
Panel D: MAE	
Median	SCAPM ≽ Labor income ≽ [CRRA, Recursive utility, Saving habit formation] ≽ Habit formation
Minimum	[Labor income, Recursive utility, SCAPM, Saving habit formation] ≽ SCAPM ≽ Habit formation

Notes: (i) A ≽ B means A outperforms B. (ii) [A, B] means A and B belong to the same model confidence set (percentage difference less than 5%), so their performance is indistinguishable.

Figure 4 shows that the model that best fits Brazilian data was the one that uses the recursive utility function (Kreps and Porteus), as developed by Epstein and Zin (1989), ranked first in 4 of the 8 criteria in the models with seasonal adjustment of variables and in 5 of the 8 criteria for models without seasonal adjustment. Considering only the criteria based on the HJ distances, the recursive utility model of consumption is ranked first in all 8 criteria presented.

Figure 4 - Occurrences of each model in Rank 1 and Rank 6



3.3.3 Discussion of results

Recursive preferences focus on the tradeoff between current period utility and the utility to be derived from all future periods. A recursive utility function can be constructed from two components: (1) a time aggregator, which completely characterizes preferences in the absence of uncertainty; and (2) a risk aggregator, which defines the certainty equivalent function that characterizes preferences over static gambles and is used to aggregate the risk associated with future utility. We looked for natural candidates for each of these components and give an example of how Bellman's equation can be used to characterize optimal plans in a dynamic stochastic environment when agents have recursive preferences. Epstein and Zin (1989) extended the preferences in Kreps and Porteus (1978) to allow for a stationary infinite horizon model and for non-expected utility certainty equivalents (Backus, 2005). Since an agent's actions today can affect the evolution of opportunities in the future, it is useful to summarize the future consequences of these actions with a single index, i.e., future utility. We believe that in Brazil, agents have to adjust decisions regarding investing in assets given the various changes in public policies. This instability generates a concern of agents to guarantee future utility, by calibrating the degree of risk aversion and elasticity of temporal substitution of consumption over an infinite horizon. In this scenario, we believe that the utility function represents the representative agent in Brazil well, making it the model that performs the best according to the criteria developed in this paper.

3.4 The equity premium puzzle

Finally, it is worthwhile to discuss the existence or not of the *equity premium puzzle* in the context of the Brazilian economy. A detailed investigation of this puzzle is beyond the scope of this paper. Nonetheless, a preliminary investigation of the matter is appropriate. Suppose that for equation (3), two Brazilian assets are considered: market return (R_m) and risk-free return (R_f). Combining both equations, we have:

$$0 = E_t\left[M_{t+1}\left(R_{m,t+1} - R_{f,t+1}\right)\right] \quad (36)$$

If we knew the value for population parameters, we could simply test whether $M_{t+1}\left(R_{m,t+1} - R_{f,t+1}\right)$ is zero on average, which is a simple statistical test. Although we do not know the population values for parameters, we use in this test their respective median values estimated for each model (see Tables 5 and 6). We also perform a test for three different periods within the historical series to check if there is evidence of the puzzle in any period. The results are shown in Table 12.

Table 12 shows there is no equity premium puzzle in Brazil. According to our preliminary investigation, none of the six models estimated in this paper, whether or not with seasonally adjusted data, in any analyzed period, pointed to the existence of the puzzle in Brazil. This absence is due to the fact that the Brazilian equity premium is very volatile, and the volatility was different in the sub-periods of the time series used in this paper. Issler and Piqueira (2000) found no equity premium puzzle in Brazil because there is no equity premium.

Table 12 - The equity premium puzzle in Brazil

	Period	$mean(t)$	$sd(t)$	t-statistic	p-value	Is there EPP?
Panel A: seasonally adjusted series						
CRRA	1996-2019	0.0179	0.1527	11.444	0.2554	No
(1)	1996-2003	0.0285	0.2176	0.7408	0.4644	No
	2004-2011	0.0125	0.1228	0.5753	0.5693	No
	2012-2019	0.0126	0.0914	0.7694	0.4477	No
Habit formation	1996-2019	0.0158	0.1497	10.255	0.3078	No
(2)	1996-2003	0.0223	0.2131	0.5829	0.5643	No
	2004-2011	0.0159	0.1247	0.7097	0.4834	No
	2012-2019	0.0189	0.0841	12.294	0.2288	No
Recursive utility	1996-2019	0.0012	0.1513	0.0756	0.9399	No
(3)	1996-2003	-0.0035	0.2099	-0.0955	0.9246	No
	2004-2011	0.0012	0.1296	0.0505	0.9600	No
	2012-2019	0.0061	0.0941	0.3585	0.7225	No
SCAPM	1996-2019	0.0100	0.1235	0.7175	0.4752	No
(4)	1996-2003	0.0131	0.1242	0.5956	0.5558	No
	2004-2011	0.0128	0.0913	0.7804	0.4413	No
	2012-2019	0.0110	0.1241	0.7825	0.4363	No
Saving habit formation	1996-2019	0.0160	0.1251	0.7105	0.4829	No
(5)	1996-2003	0.0195	0.0867	12.349	0.2268	No
	2004-2011	0.0130	0.1088	0.9453	0.3482	No
	2012-2019	0.0133	0.1243	0.6058	0.5491	No
Labor income	1996-2019	0.0126	0.0921	0.7608	0.4527	No
(6)	1996-2003	0.0179	0.1527	11.444	0.2554	No
	2004-2011	0.0285	0.2176	0.7408	0.4644	No
	2012-2019	0.0125	0.1228	0.5753	0.5693	No
	Period	mean(t)	sd(t)	t-statistic	p-value	Is there EPP?
Panel B: original series						
CRRA	1996-2019	0.0188	0.1532	11.994	0.2334	No
(1)	1996-2003	0.0306	0.2191	0.7900	0.4355	No
	2004-2011	0.0125	0.1212	0.5850	0.5628	No
	2012-2019	0.0132	0.0921	0.7996	0.4302	No
Habit formation	1996-2019	0.0174	0.1519	11.087	0.2704	No
(2)	1996-2003	0.0253	0.2189	0.6437	0.5247	No
	2004-2011	0.0161	0.1225	0.7331	0.4692	No
	2012-2019	0.0199	0.0857	12.721	0.2134	No
Recursive utility	1996-2019	0.0031	0.1505	0.2013	0.8409	No
(3)	1996-2003	0.0001	0.2091	0.0028	0.9978	No
	2004-2011	0.0025	0.1284	0.1122	0.9114	No
	2012-2019	0.0068	0.0935	0.4044	0.6888	No
SCAPM	1996-2019	0.0103	0.1231	0.7427	0.4599	No
(4)	1996-2003	0.0133	0.1233	0.6080	0.5476	No
	2004-2011	0.0131	0.0911	0.8009	0.4295	No
	2012-2019	0.0135	0.1231	0.9658	0.3372	No
Saving habit formation	1996-2019	0.0162	0.1221	0.7399	0.4651	No
(5)	1996-2003	0.0224	0.0870	1.4080	0.1698	No
	2004-2011	0.0112	0.1067	0.8349	0.4070	No
	2012-2019	0.0103	0.1219	0.4802	0.6345	No
Labor income	1996-2019	0.0121	0.0904	0.7470	0.4609	No
(6)	1996-2003	0.0188	0.1532	11.994	0.2334	No
	2004-2011	0.0306	0.2191	0.7900	0.4355	No
	2012-2019	0.0125	0.1212	0.5850	0.5628	No

Notes: (i) $t = [M_{t+1}(R_{m,t+1} - R_{f,t+1})]$; (ii) $\text{t} - \text{statistic} = mean(t)/\,(sd(t)/\sqrt{n})$, where n is the sample size; (iii) p-value greater than 0.10 means that $[M_{t+1}(R_{m,t+1} - R_{f,t+1})]$ is not significantly different from zero at the usual levels of 1%, 5% and 10%; (iv) EPP: Equity premium puzzle

4. Conclusion

To assess the capacity of the consumption-based asset pricing models to simultaneously explain stock market and aggregated macroeconomic variables, we estimate and compare six variants of consumption-based asset pricing models with asset market data from Brazil. They include the CRRA, habit formation model, recursive utility model and labor income model; their variants including SCAPM and saving habit formation model. The estimates using the GMM were performed for quarterly data from 1996 to 2019, both in the original version as published in the sources and also with seasonal adjustment.

The results based on the comparison criteria between the models demonstrate that the CRRA with recursive utility (Kreps and Porteus), developed by Epstein and Zin (1989), best fit the Brazilian stock market. That model is also the most accepted in the literature, mainly because it allows the presence of a small predictable component in the long-term consumption growth, which enables the model to explain several characteristics of the observed asset prices, according to Attanasio and Weber (2010). In Brazil, several studies have used the habit formation model, but in this paper, we show that the recursive utility model can be an alternative with better adjustment to Brazilian data.

Our empirical results show:

(1) To understand the most important driving force in the asset markets, we ranked the models by two performance criteria: the average size of prediction errors and the Hansen-Jagannathan distance framework. Models not much explored in the literature, such as Saving-CAPM and labor income, show good performance in asset pricing;

(2) The estimates made with data without seasonal adjustment had a greater number of statistically significant parameters than the estimates for data with seasonal adjustment; and

(3) The estimated parameters pointed to a lower risk aversion of Brazilians in addition to a slightly greater willingness to delay consumption compared with the parameters estimated in research with previous periods.

Additionally, we conclude there is no equity premium puzzle in Brazil. None of the six models estimated in this paper, whether or not with seasonally adjusted data, in any analyzed period, pointed to the existence of the puzzle in Brazil. This result corroborates most previous findings. As a contribution to future studies, the following are suggestions: extension of the approach contained in this work to other types of assets and the use of portfolios to better characterize the Brazilian stock market; and comparative analysis between models in different regions of Brazil, since the country has continental dimensions and poor income distribution, so the recursive utility model may not apply in all regions of the country in the same way.

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