A Behavioral New Keynesian Model For Brazil

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Abstract
We build and estimate a behavioral New Keynesian model for the Brazilian economy to study monetary- and fiscal-policy interactions. Inattention parameters for consumers and firms are estimated via Bayesian methods using the model. Our results suggest parameters $M$ and $M'$, as given in Gabaix (2020), are 0.7580 and 0.8649 for consumers and firms, respectively. In addition, we simulate monetary and fiscal dominance contexts to infer economic lessons under different policy regimes. We find the monetary authority should pursue its inflation target in monetary dominance situations and it should accommodate for public debt trajectory in fiscal dominance contexts.

Keywords: DSGE, New Keynesian, behavioral, monetary and fiscal dominance.

Resumo
Neste artigo, construímos um modelo Novo Keynesiano comportamental para a economia brasileira a fim de estudar as interações entre as políticas fiscal e monetária. Parâmetros de “inattenção” para consumidores e firmas são estimados através de métodos Bayesianos usando o modelo. Nossos resultados sugerem que os parâmetros $M$ e $M'$, como dados por Gabaix (2020), são 0.7580 e 0.8649, respectivamente. Ainda mais, simulamos contextos de dominância monetária e fiscal para inferir lições econômicas sob os diferentes regimes de política econômica. Encontramos que a autoridade monetária deve perseguir sua meta de inflação em situações de dominância monetária e deve acomodar para a trajetória da dívida pública em contextos de dominância fiscal.

Palavras-chave: DSGE, Novo Keynesiano, comportamental, dominância monetária e fiscal.

JEL Codes: E52, E58, E61, E62, E71

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1 Introduction

Brazil’s fiscal conditions have deteriorated continuously over the past years. Taking a look at Treasury official data, government expenditures have grown in average 5.4% p.a. in real values over the last twenty years (STN, 2017). In addition, the Brazilian government has experienced systemic primary budget deficits since 2014 (-0.4% of GDP in 2014, -2% in 2015, -2.6% in 2016 and -1.9% in 2017). As a consequence, government gross debt has increased from 52% pf GDP in 2013 to around 74% of GDP in the end of 2017. Worse, debt trajectory has been ascending and with little room for stabilization without more politically burdensome fiscal reforms — such as administrative and reform.

In this context, relevant questions surrounding the interaction between fiscal and monetary policies arise under Brazil’s increasingly fiscally unbalanced environment. Sargent and Wallace (1981) were pioneers in exploring the theoretical consequences of unbalanced budgets in a monetarist economy, with no exciting results for monetary authorities desiring to fight inflation in such conditions. In particular, the authors conclude that sooner or later budget deficits cause the central bank to issue currency, thus generating inflation.

Woodford (2001) also studied the interactions between fiscal and monetary policies reaching similar conclusions, though with different causalities. According to the author, monetary policy has fiscal effects and vice-versa — and both of them cannot be ignored. Otherwise, the interaction between policies is not fully understood. In addition, Woodford shows a possible direct link between the price level and government debt — concluding that, possibly, fiscal policy determines the efficacy of monetary policy.

These theoretical foundations and the fiscally unbalanced context in Brazil are motivations for this paper. In a few words, we study the interaction between fiscal and monetary policies through a dynamic stochastic general equilibrium (DSGE) — a New Keynesian — model for Brazil. In particular, our model incorporates micro founded inattention behavior by consumers and firms — it also incorporates elements of Brazilian economic institutions, such as a rule for government primary budget surplus and an inflation targeting rule. Our focus is on trying to understand how policy interactions may affect product and inflation variances under regimes of monetary and fiscal dominance.

To better understand how the interactions of fiscal and monetary policies can generate results in terms of economic performance and welfare, we take into account behavioral aspects for consumers and firms in the construction and estimation of our model for the Brazilian economy — it becomes a behavioral New Keynesian model, so to speak. In particular, we develop a model inspired by Gabaix (2020), who micro founded the notion of bounded rationality through inattention for economic agents in an enrichment of the traditional New Keynesian model. We then estimate it using Bayesian methods and simulate monetary and fiscal dominance contexts to assess the best policy options for a range of parameter values for fiscal and monetary authorities.

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1 We utilize data up to 2017 in our analysis to shield from the current Brazilian national administration, 2018-2022 (expected), which has been considerably impacted by the Covid-19 Pandemic.
2 Gross government debt in Portuguese: Dívida Bruta do Governo Geral (DBGG).
3 We define the dominant policy as being determined independently by its responsible authority. For example, monetary dominance is characterized by the independent setting of interest rates by the central bank.
4 We use variance values for output and inflation as proxies for welfare. In this case, more variance means less welfare.
We find interesting results. First, we estimate the inattention parameters of Gabaix (2020) for the Brazilian economy. We find $M = 0.758$ for consumers and $M_f = 0.865$ for firms for our period of analysis — ranging from 2000 to 2017. In principle, such parameters represent the average attention of consumers and firms relative to the fully rational agents (who would have $M = M_f = 1$). To the best of our knowledge, we are the first to perform such estimation for Brazil. Furthermore, our simulations show another two main results: (i) under a monetary dominance context, it is desirable for the central bank to pursue an inflation target; and (ii) under a fiscal dominance context, the more the monetary authority fights inflation, the higher the variances it generates.

Our work relates to several strands of the economics literature. It is firstly associated with several papers which built DSGE models for Brazil (e.g., Silveira, 2008; Nunes & Portugal, 2009; Castro, Gouvea, Minella, Santos, & Sobrinho, 2011; Portugal & Ornelas, 2011; Gadelha & Divino, 2013). In addition, it is also associated with a few authors who have focused on understanding the trade-offs in implementing fiscal policies in the country (e.g., Carvalho & Valli, 2010; Mussolini & Telles, 2012). Furthermore, our work relates to papers on fiscal dominance situations, such as Loyo (1999) — who built a dynamic stochastic general equilibrium model to identify evidences of fiscal dominance in Brazil during the 1980s — and Blanchard (2005) — who also investigated a fiscal dominance context in the country using an open-economy model to investigate the impacts of strong exchange rate depreciation on the inflation targeting-regime. Importantly, our paper is also closely associated with Andrade, Cordeiro, and Lambais (2019), who estimated behavioral inattention parameters parametrically and analyzed identification issues in a behavioral New Keynesian model.

Our model is also related to part of the behavioral economics literature. Kahneman (2003) presents a summary of his and Amos Tversky’s main contributions in introducing psychological elements into economic decision-making analysis, such as intuitive beliefs and bounded rationality. Such contributions gave the foundations for Gabaix (2014), who built a model where economic agents have bounded rationality. By introducing the idea of “sparse” maximization (the sparse max operator), the author is able to write a behavioral version of the traditional microeconomics framework — consumer theory and competitive equilibrium, for example. The meaning of sparse, in this case, is the same of a sparse vector or matrix, full of zeroes. Economically, this makes behavioral agents pay less attention to certain variables in comparison with fully rational agents. For Gabaix (2014), consumer theory is developed on the basis of the following maximization problem:

$$
\max_{c_1, ..., c_n} u(c_1, ..., c_n) 
$$

subject to a budget constraint $p_1c_1 + ... + p_nc_n \leq w$, where $u(.)$ is the utility function, $c_n$ is the consumption of good $n$, and $p_n$ is the price of good $n$.

In his proposed sparse maximization transformation, the agent maximizes her utility the same way, like the fully rational agent. However, she does so based on her perceived prices for goods — because the agent does not pay full attention to all prices, since she has limited cognitive capacities to process information (even if it is fully available). The agent then faces an objective reality — the way reality actually behaves — and a perceived

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5To understand this in a simple way is to think of the fully rational agent as having a vector full of ones (that is, she pays attention to all variables) while the sparse agent has many zero values in it (she does not pay attention to all variables).
reality — that is, the perception of reality by the inattentive agent. This is possible to capture by rewriting the problem above as:

$$s \max_a u(a, x)$$

(2)

subject to $b(a, x) \geq 0$, where $u(.)$ is the utility function, $b$ is a constraint and $s$ stands for sparsity. This maximization problem is less than the fully attentive version of the max operator, as the author puts it. Here one can notice that the departure from rational expectations is subtle. The agent stills maximizes inter-temporally, though with her perception of prices. The latter might not be the same as the actual prices — indeed, they will often differ, since the agent does not pay attention to all prices. This makes the agent be partially imperceptive to her objective reality.

There are many behavioral principles condensed in the sparse max: inattention, disproportionate salience, and the use of defaults being among them. The way to represent sparsity by few parameters that are nonzero or differ from the usual state of affairs makes it possible to calibrate a behavioral vector $m$ such that $m = 0$ represents zero attention while $m = 1$ represents a fully rational agent, as Gabaix (2014) shows. Thus, rational expectations can be understood as a particular result of the sparsity model.

Sparse maximization described above underpins the micro foundations for the behavioral DSGE model built in the next section, just as Gabaix (2020) showed. It is used to derive macro parameters of inattention for the Euler equation and the Phillips curve, thus making consumers and firms “behavioral” in our New Keynesian framework. The rest of this paper is organized as follows: in the next section we present our model, followed by the empirical evidence, estimation results, simulations and conclusion.

2 Model

There are four players in this model: households, firms, monetary authority, and fiscal authority. The model is first derived in the traditional way, following the New Keynesian model presented in Gali (2008). The behavioral transformation is subsequently applied, using the “sparse” max foundation laid in the previous section with the DSGE transformation given by Gabaix (2020).

Households act as consumers and receive their incomes from firms. They pay lump-sum taxes to government. They can also invest in one-period public bonds and receive interest in the next period. Firms utilize labor to make final goods, which will then be consumed by households and government. The monetary authority follows a rule for setting interest rates, and its parameters change depending on the policy regime (monetary or fiscal dominance). The government collects taxes and sells bonds, paying for its consumption and its debt. It follows a primary budget surplus rule, so that its final objective is to stabilize public debt trajectory.

2.1 Households

This section follows closely on the developments of Gali (2008), with minor changes. Households are intertemporal optimizers. Let us assume there is a continuum of households indexed by $j \in [0,1]$. Consumers have the same preferences and utility functions,
which allows for the use of a representative household.

Let \( C_{j,t} \) and \( N_{j,t} \) represent consumption and supply of labor for the representative consumer \( j \) at period \( t \). Utility is represented by \( U(\cdot) \). The agent seeks to maximize the following:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_{j,t}, N_{j,t})
\]  

subject to:

\[
P_t C_{j,t} + R_t^{-1} B_{j,t} \leq B_{j,t-1} + W_t N_{j,t} + D_{j,t} + T_{j,t}
\]

where \( \beta \in (0,1) \) is the agent’s discount factor, \( E_0 \) is the expectancy operator, \( U(\cdot) \) is the utility, \( P_t \) is the price level at period \( t \), \( B_{j,t} \) are the one-period public bonds held by agent \( j \) at period \( t \), \( R_t \) is the nominal interest rate paid on public bonds at period \( t \) defined by the monetary authority, \( W_t \) represents wages received by households at period \( t \), \( D_{j,t} \) represents profits received by consumer \( j \) from firms at period \( t \), and \( T_{j,t} \) are lump-sum taxes on agent \( j \) at period \( t \). In this case, agent \( j \) chooses \( C_{j,t}, N_{j,t} \) and \( B_{j,t} \) in order to maximize equation \( 3 \).

There is also the classic Non-Ponzi condition, which dictates that the representative agent \( j \) will not hold any bonds in the afterlife:

\[
\lim_{t \to \infty} E_t[B_{j,t}] \geq 0 \quad \forall \ t
\]

Following Gali (2008), \( C_t \) is a consumption index given by \( C_t = (\int_0^1 C_t(i) \frac{1-\delta}{\delta} di) \). where there exists a continuum of goods \( i \in [0,1] \) and \( \epsilon \) represents the demand elasticity. At every period, agent \( j \) is required to maximize \( C_{j,t} \) for any expenditure level \( \int_0^1 P_t(i) C_{j,t}(i) di \). Solving the problem yields \( C_{j,t}(i) = \frac{P_t(i)}{P_t} C_t \forall i, j \in [0,1] \). Notice that since the model works with a representative agent it is possible to aggregate all of them as \( C_t(i) = [\int_0^1 C_{j,t}(i) dj] \). Considering this aggregation, one arrives at \( C_t(i) = \frac{P_t(i)}{P_t} C_t \forall i \in [0,1] \) where \( P_t = [\int_0^1 P_t(i)^{1-\epsilon} di] \).

The utility function for representative agent \( j \) is given by:

\[
U(C_{j,t}, N_{j,t}) = \frac{(C_{j,t})^{1-\sigma}}{1-\sigma} Z_t^C - \frac{(N_{j,t})^{1+\phi}}{1+\phi}
\]

where \( Z_t^C \) is a stochastic shock in consumption, \( \sigma \) is the consumer’s intertemporal elasticity of substitution and \( \phi \) is the Frisch elasticity of labor supply.

The first order conditions (FOCs) for this problem are:

\[
C_{j,t} : \quad \lambda_t = \frac{\beta^t (C_{j,t})^{-\sigma}}{P_t} Z_t^C
\]

\[
N_{j,t} : \quad \lambda_t = \frac{\beta^t (N_{j,t})^\phi}{W_t}
\]

\[
B_{j,t} : \quad 1 = \beta R_t E_t\left[ \frac{(C_{j,t+1})^{-\sigma} Z_{t+1}^C}{(C_{j,t})^{-\sigma} Z_t^C P_t^{1+\epsilon}} \right]
\]

where \( \lambda_t \) is the Lagrangian multiplier at time \( t \). Notice that substitutions are already
made in equation 9 so that it gives the intertemporal substitution of consumption by the representative household — her Euler equation.

2.2 Firms

We also closely follow Gali (2008) for the firms’ case. Assume there is a continuum of firms indexed by $i \in [0,1]$ with the production function below:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (10)$$

where technology is commonly shared and represented by $A_t$. Firms have the following demand schedule $C_t(i) = \left[ \frac{P_t(i)}{P_t} C_t \right]^{-\epsilon}$ and a portion of them readjusts prices each period, according to Calvo (1983). Firms are identical, which allows the model to work with a representative firm.

First, it is important to understand the price setting mechanism of firms. The way companies seek to maximize their current expected value in this model is via price adjustments. Following Calvo, since $1 - \theta$ of firms re-optimize their prices each period, the aggregate price level is given by:

$$P_t = \left[ \int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1 - \theta)(P^*_t)^{1-\epsilon} \right] \quad (11)$$

which can be rewritten as:

$$P_t = \left[ \theta P_{t-1}(i)^{1-\epsilon} + (1 - \theta)(P^*_t)^{1-\epsilon} \right] \quad (12)$$

where $S(t) \subset [0,1]$ is the set of firms which do not readjust their prices at period $t$. $P^*_t$ is the reset price at $t$. Dividing both sides of the above equation by $P_{t-1}$:

$$\Pi_t^{1-\epsilon} = \theta + (1 - \theta)\left[ \frac{P^*_t}{P_{t-1}} \right]^{1-\epsilon} \quad (13)$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. Notice that in the zero-inflation steady state $P^*_t = P_{t-1} = P_t$. Equation 13 describes the inflation dynamics in the model.

Let us now turn attention to the firm’s problem. As mentioned before, firms seek to maximize their current market value of expected profits by choosing its prices. Thus their problem can be expressed as:

$$\max_{P_t} \sum_{k=0}^{\infty} \theta^k E_t [Q_{t,t+k}(P_t Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))] \quad (14)$$

subject to

$$Y_{t+k|t} = \left[ \frac{P^*_t}{P_{t+k}} \right]^{-\epsilon} C_{t+k} \quad (15)$$

for $k \geq 0$ and where $Q_{t,t+k} = \beta^k \left[ \frac{C_{t+k}}{C_t} \right]^{-\sigma} \left[ \frac{P_t}{P_{t+k}} \right]$ is the discount factor for nominal payoffs, $Y_{t+k|t}$ represents output in $t+k$ for a firm that reset its price at $t$, and $\Psi(\cdot)$ is the cost function. After some mathematical transformations, one may transform the problem above...
∞ \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}Y_{t+k,t}(P^*_t + \bar{M}\Psi_{t+k|t}^t)] = 0

where \( \Psi_{t+k|t} = \Psi_{t+k}(Y_{t+k|t}) \) is the nominal marginal cost at \( t + k \) for a firm which reset its price at period \( t \). \( \bar{M} \) represents the desired mark-up by firms in the absence of frictions on the frequency of price adjustment. The above equation, as mentioned before, follows closely Gali (2008). Let us do further transformations in it. Diving it by \( P_{t-1} \) and making \( \Pi_{t,t+k} = P_{t+k}/P_t \) gives:

\[ \sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}Y_{t+k,t}\left(\frac{P^*_t}{P_{t-1}} + \bar{M}\Pi_{t-1,t+k}MC_{t+k|t}\right)] = 0 \]

where \( MC_{t+k|t} = \frac{\Psi_{t+k|t}^t}{P_{t+k}} \) is the real marginal cost at \( t + k \) of a firm which reset its price at \( t \).

Finally, it is possible to rewrite the equation above after some transformations as:

\[ \frac{P^*_t}{P_{t-1}} = (1 - \beta\theta) \sum_{k=0}^{\infty} \theta^k \beta^k E_t[MC_{t+k|t}\left(\frac{P_{t+k}}{P_{t-1}}\right)] \] (16)

The equation above rules how firms will readjust their prices. Together with equation (13) it is possible to arrive at the price dynamics for this economy.

### 2.3 Monetary Policy

Monetary policy follows a rule that is meant to describe the way it works in Brazil. Since 1999, Brazil has had an inflation targeting regime in which the central bank pursues a target for inflation over the course of a year. The target is defined two years before, to allow for forward-guidance. In this model, the monetary authority is allowed to try to stabilize public debt trajectory under fiscal dominance contexts. Under usual circumstances (i.e., monetary dominance), the central bank worries only about inflation and output volatility.

Following a similar rule to that of Castro et al. (2011), and adding public debt to the rule as in Kumhof, Nunes, and Yakadina (2007) and Furtado (2017), the central bank policy becomes:

\[ R_t = R_{t-1}^{\Gamma_B} \left( \frac{1}{\beta} E_t[\Pi_{t+1}]^{\Gamma_Y} \left( \frac{Y_t}{Y} \right)^{\Gamma_Y} \left( \frac{B^y_t}{B^y} \right)^{-\Gamma_{Byy}} \right)^{1-\Gamma R} Z_t^R \] (17)

where \( R_t \) is the nominal interest rate set by the monetary authority, \( \Pi_t \) is the inflation rate at time \( t \), \( Y_t \) is the economy’s product at period \( t \), \( Y \) is the economy’s steady state product, \( B^y_t \) is public debt at time \( t \) as a portion of current product, \( B^y \) is the steady state public debt as a portion of the steady state product, \( \Gamma_B \in (0,1) \) is an interest rate smoothing parameter, \( \Gamma_{\Pi} \geq 0 \) is the monetary authority’s reaction to inflation, \( \Gamma_Y \geq 0 \) is the monetary authority’s reaction to output gap, \( \Gamma_{By} \geq 0 \) is the monetary authority’s reaction to public debt’s deviation from the steady state, and \( Z_t^R \) is a monetary policy shock. In the log-linearized version of the model, it is assumed that the target for inflation is zero.

Following Kumhof et al. (2007), monetary dominance contexts are given by \( \Gamma_{\Pi} \geq 1 \) and \( \Gamma_{By} = 0 \) while fiscal dominance regimes are defined by \( \Gamma_{\Pi} \geq 0 \) and \( \Gamma_{By} > 0 \). In words,
in a monetary dominance regime the central bank does not worry about the trajectory of public debt and fights inflation often following the Taylor Principle\(^6\). This resembles the rules established by Leeper (1991), according to which the passive authority is the one responsible for accommodating public debt. When a fiscal dominance context is established, the central bank incorporates public debt dynamics in his policy rule — taking into account the impact of interest rates in the country’s debt to decide on its policy.

### 2.4 Fiscal Policy

Fiscal policy in Brazil has had a rule of annual targets for the non-financial public sector primary surplus as a proportion of GDP. According to Castro et al. (2011), the government’s ultimate goal is to stabilize public sector debt to GDP ratio, from which one concludes the primary budget surplus is an intermediate target and the real objective is to stabilize debt trajectory. Fiscal policy modeling accompanies closely the one proposed by the authors of the SAMBA model, with minor changes. Notice that we do not consider the “spending ceiling” (Teto de Gastos) approved in late 2016 because it did not have serious effects in any variables for the period of our analysis.

Firstly, an equation is defined where the actual primary surplus responds to the announced targets:

\[
S_y^t = \bar{S}_y + \phi_S (S_y^{t-1} - \bar{S}_y) + \phi_S (\bar{S}_y - S_y^t)
\]

where \(\phi_S\) belongs to the interval \([0,1]\) and represents the inertia of the primary surplus as a proportion of GDP, \(S_y^t = \bar{S}_y \frac{P_t}{Y_t}\) is the actual primary budget surplus as a proportion of GDP (where \(y\) represents this proportionality), \(\bar{S}_y\) is the primary surplus to GDP ratio at the steady state, \(\bar{S}_y^t\) is the adjustable target for the primary surplus as a ratio of GDP, and \(\phi_S > 0\) represents the weight of the adjustable primary surplus target as a deviation from the steady state.

It is worth mentioning that the tax rate is exogenous in this model, which implies that the fiscal policy instrument is government spending. This implies that any deviations of the primary surplus from target are corrected via government consumption. The primary surplus target follows:

\[
\bar{S}_y^t = \bar{S}_y + \rho_S (\bar{S}_y - \bar{S}_y^t) + \phi_B (B_y^{t-1} - B_y^t)
\]

where \(\rho_S \in [0,1]\) is the smoothing parameter for the primary surplus deviation from steady state, \(B_y^t = \frac{B_t}{P_t Y_t}\) is government debt as a proportion of GDP, \(B_y^t\) is government debt as proportion of GDP at the steady state, and \(\phi_B\) is the parameter that captures changes in the primary surplus target due to deviations of public debt from its steady state value.

As for government aggregate consumption, it is assumed that government demands the same variety of goods as households. Then for a continuum of goods indexed by \(i \in [0,1]\):

\[
G_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} G_t
\]

where \(G_t = \left(\int_0^1 G_t(i)^{1-\epsilon} di\right)^{-\frac{1}{1-\epsilon}}\). This follows the same consumption logic as in the households’ problem. It basically means government consumes goods the same way house-

\(^6\)According to which the percentage change of interest rates set by the central bank in response to a percentage change in the inflation rate is higher than 1.
holds do. This assumption simplifies the aggregation procedures conducted in the next section.

The nominal primary surplus is given by the difference between non-interest government revenues and expenses. It is assumed revenues are proportional to nominal output and an AR(1) stationary process is defined for the difference between the average tax rate and its steady state value. Similar to Furtado (2017), the rules follow:

$$S_t^n = \tau_t(P_tY_t) - P_tG_t$$  \hspace{1cm} (21)

$$\tau_t = \tau_{ss} + \rho_t(\tau_{t-1} - \tau_{ss}) + \varepsilon_t$$  \hspace{1cm} (22)

where $\tau_t = \frac{T^n_t}{P_tY_t}$ is the average tax rate, $T^n_t$ represents nominal lump-sum revenues, $G_t$ is real government expenditures (consumption), and $\tau_{ss}$ is the average tax rate steady state value. Rewriting equation 21 for $G_t$, it is possible to arrive at:

$$G_t = Y_t(\tau_t - S^n_t)$$  \hspace{1cm} (23)

To complete the model, it is important to have in mind the law of motion of government debt. The government finances its expenditures by tax revenues and one-period non-contingent bonds issuance at the rate of return $R_t$. Those revenues are used both for consumption and for paying debt. Hence, the government budget constraint is given by:

$$P_tG_t + B_{t-1} - 1 = B_tR_t + \tau_t(P_tY_t)$$  \hspace{1cm} (24)

And using current nominal GDP and 23 it is possible to arrive at the following law of motion:

$$B_t^\psi = R_t[\frac{B_{t-1}^\psi Y_{t-1}}{\Pi_t} - S^n_t]$$  \hspace{1cm} (25)

Let us notice that this last equation sheds light on the relationship between monetary and fiscal policies. This is the equation which links them in the model. Once the monetary authority decides to raise interest rates to fight inflation, for example, the fiscal authority will have to increase the primary surplus target, and as a consequence reduce government consumption.

Additionally, under monetary dominance one has $\phi_B > 0$ and $\phi_{\psi} > 0$, which means that higher government debt will result in higher primary surplus targets and an increasingly higher actual primary surplus. In other words, fiscal policy seeks to stabilize public debt to GDP ratio. This is the final objective of the fiscal authority in this model, following the rules established in Brazil.

On the other hand, in a fiscal dominance context one should have $\phi_{\psi} = 0$. In this theoretical case, the actual primary surplus equation 18 will not depend on a primary surplus target anymore. The central bank then tries to control inflation at the same time as it seeks to stabilize public debt trajectory, according to the established rule $\Gamma_{By} > 0$ for fiscal dominance regimes in the previous section. It is also assumed government spending is partially financed by public debt issuance when debt deviates from its steady state value.
2.5 Aggregation and Equilibrium

Households have the same utility and face the same budget constraint. Therefore, all of them arrive at the same solution for their control variables. Aggregation follows from the representative agent as:

\[
\int_0^1 C_{j,t}dj = C_t \quad \int_0^1 N_{j,t}dj = N_t \quad \int_0^1 B_{j,t}dj = B_t
\]  \hspace{1cm} (26)

Where \( C_t \) is aggregate consumption, \( N_t \) represents labor, and \( B_t \) is aggregate demand for one-period government bonds. From the firm’s problem, labor market clearing requires:

\[
N_t = \int_0^1 N_t(i)di
\]  \hspace{1cm} (27)

Using the representative firm’s production function it is possible to rewrite the above as:

\[
N_t = \int_0^1 \left[ \frac{Y_t(i)}{A_t} \right]^\frac{1}{1-\alpha} di
\]  \hspace{1cm} (28)

\[
N_t = \left( \frac{Y_t(i)}{A_t} \right)^\frac{1}{1-\alpha} \int_0^1 \left[ \frac{P_t(i)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} di
\]  \hspace{1cm} (29)

This aggregation is less obvious due to the term \( \int_0^1 \left[ \frac{P_t(i)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} di \), which represents a measure of price dispersion across the economy. It is worth noting that the steady state price dispersion is close to 0, so that when a first-order Taylor expansion is conducted, price dispersion can be ignored at the steady state.

As mentioned in the previous section, government consumption is given by \( G_t \) - following the same pattern as household consumption for final goods. Additionally, goods market clearing requires:

\[
Y_t = C_t + G_t
\]  \hspace{1cm} (30)

The economy’s resource constraint henceforth is respected and all the resources used are either consumed or saved in the form of government bonds. As a consequence, all markets clear.

2.6 The log-linearized model

We log-linearize all equations described in the sections above and find the linear model, shown fully below. All variables are treated as deviations from the steady state (zero for all variables). Notice that lower case letters in the equations below represent the respective log-linearized variables from the equations described in the above sections with capital case letters.

\[
w_t - p_t = \sigma c_t + \phi n_t + z_t^C
\]  \hspace{1cm} (31)

\[
c_t = E_t(c_{t+1}) - \frac{1}{\sigma} E_t[r_t - E_t(\pi_{t+1})] + \frac{1}{\sigma} [z_t^C - z_{t+1}^C]
\]  \hspace{1cm} (32)
\[ \pi_t = (1 - \theta)(p_t^* - p_{t-1}) \]  
(33)

\[ y_t = s_C c_t + s_G g_t \]  
(34)

\[ \pi_t = \beta E_t[\pi_{t+1}] + \lambda \hat{mc}_t \]  
(35)

\[ \hat{mc}_t = \left[ \frac{\sigma}{s_C} + \frac{1 + \phi}{1 - \alpha} - 1 \right] (y_t - y_t^n) \]  
(36)

\[ y_t^n = \psi_a a_t - \psi_g g_t + v_y^n \]  
(37)

\[ r_t = \Gamma_R * r_{t-1} + (1 - \Gamma_R) [\Gamma \pi_t + \Gamma_y * (y_t - y_t^n) - \Gamma_{By} * b_t^y] + z_t^R \]  
(38)

\[ b_t^y = r_t + R(b_{t-1}^y - \pi_t + y_{t-1} - y_t) - (R - 1)s_t^y \]  
(39)

\[ s_t^y = \phi_s s_{t-1}^y + \phi_{\bar{s}} \bar{s}_t^y \]  
(40)

\[ \bar{s}_t^y = \rho_s \bar{s}_{t-1}^y + \phi_B R \frac{R}{R - 1} b_{t-1}^y \]  
(41)

\[ g_t = y_t + \frac{1}{s_G} (\tau_{ss} \tau_t - S^Y s_t^y) \]  
(42)

\[ \tau = \rho \tau_{t-1} + \frac{1}{\tau_{ss}} \varepsilon_t^\tau \]  
(43)

\[ a_t = \rho_a a_{t-1} + \varepsilon_a \]  
(44)

\[ z_t^R = \rho_z z_{t-1}^R + \varepsilon_z^R \]  
(45)

\[ z_t^C = \rho_z z_{t-1}^C + \varepsilon_z^C \]  
(46)

### 2.7 Behavioral Transformation

We apply Gabaix’s behavioral transformation for the Euler equation [32] and Phillips curve [35] given above to add inattention (bounded rationality) to the model. The behavioral biases are placed based on the “sparsity” model cited in the introduction section. We apply Gabaix (2020)’s lemmas 1 and 2, which are fully presented in his paper — briefly, both lemmas transform the rational expectations equations into behavioral ones. We describe them below:

**Lemma 1** (Cognitive Discounting of all Variables) For any variable \( z(X_t) \) with \( z(0) = 0 \) the
beliefs of the behavioral agent satisfies, for all k:

\[ E_t^{BR}[z(X_{t+k})] = \tilde{m}^k E_t[z(X_{t+k})] \quad (47) \]

where \( E_t^{BR} \) is the subjective expectation operator ("BR" stands for "bounded rationality") and \( E_t \) is the rational expectation operator. This makes it clear the behavioral agent (with \( \tilde{m} < 1 \)) does not discount inter-temporally the same way the rational expectations agent does. The former places less weight onto future events, mainly due to her inability to process information.

Lemma 2 (Optimal Price for a Behavioral Firm Resetting its Price) A behavioral firm resetting its price at time \( t \) will set it to a value \( p^*_t \) following the same logic as the equation below (which comes from the log-linearization of Gali’s Phillips Curve), though with cognitive discount factors as given below:

\[ p^*_t - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\theta \beta \tilde{m})^k E_t[m^f_{\pi}(\pi_{t+1} + \ldots + \pi_{t+k}) - m^f_x \mu_{t+k}] \quad (48) \]

where \( m^f_{\pi} \) and \( m^f_x \) parametrize attention to inflation and macro disturbances, respectively, and \( \tilde{m} \) is the overall cognitive discounting factor.

Next, the results above are used to derive the behavioral Euler equation and Phillips curve. Equation 32 transformed by Lemma 1 results in:

\[ c_t = ME_t[c_{t+1}] - \frac{1}{\sigma} [r_t - E_t[\pi_{t+1}]] + \frac{1}{\sigma} [z^C_t - z^C_{t+1}] \quad (49) \]

where \( \tilde{m}^k \) becomes \( M \), a macro parameter of inattention to future consumption. Equation 35 transformed by Lemma 2 is given by:

\[ \pi_t = M^f \beta E_t[\pi_{t+1}] + \lambda m \hat{c}_t \quad (50) \]

where \( M^f \) is the cognitive discounting factor for future prices — it is a function of \( m^f_{\pi} \) — and \( \lambda = m^f_x \lambda \), where \( m^f_x \) is the cognitive discounting factor for marginal costs. It is assumed \( m^f_x = 1 \), in order to simplify the model. Even so, bounded rationality is still very well captured by the \( M \) and \( M^f \) parameters — which are objects of our estimation.

The interesting lemma proofs can be found in Gabaix (2020). Interestingly, one can write the rational expectations model as a particular case of the above model, in which \( M = M^f = m^f_x = 1 \).

3 Empirical Evidence

We then proceed to estimating the model using Bayesian methods. We use Brazilian quarterly data from 2000Q1 to 2017Q4. There are four observable variables in the model (thus accounting for four exogenous shocks — given by equations 43 through 46). Data was obtained in the official websites of the Instituto Brasileiro de Geografia e Estatística (IBGE) and of the Banco Central do Brasil (BCB).

Data received a similar treatment described in Castro et al. (2011) for the SAMBA model. Quarterly GDP and government consumption series underwent the application of
Table 1: Data Used in the Estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product ($Y_t$)</td>
<td>GDP - seasonally adjusted (s.a.) (1995 values)</td>
<td>IBGE</td>
</tr>
<tr>
<td>Government Consumption ($G_t$)</td>
<td>Government Spending - s.a. (1995 values)</td>
<td>IBGE</td>
</tr>
<tr>
<td>Inflation ($\Pi_t$)</td>
<td>Quarterly IPCA-E (12 months)</td>
<td>IBGE</td>
</tr>
<tr>
<td>Interest Rates ($R_t$)</td>
<td>Quarterly SELIC (12 months)</td>
<td>BCB</td>
</tr>
</tbody>
</table>

the first-log difference to transform them in stationary series. The resulting series was demeaned to eliminate differences in the trends across growing variables. Following Castro et al. (2011), the inflation target was subtracted from the actual quarterly IPCA-E series. The resulting series was then demeaned. For the interest-rates series, SELIC, the sample average was subtracted from the actual series. These are the same procedures used by Castro et al. (2011) and the resulting series became stationary with mean 0.

3.1 Calibrated Parameters

There are nine calibrated parameters, following closely Castro et al. (2011). Aggregate shares and steady state values which appear as parameters in the linear model are calibrated. The table below summarizes the values:

Table 2: Calibrated Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Time discount factor</td>
<td>0.989</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Frisch elasticity of labor supply</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in the product</td>
<td>0.448</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Demand elasticity</td>
<td>6.0</td>
</tr>
<tr>
<td>$\Gamma_B$</td>
<td>Central Bank reaction to debt</td>
<td>0</td>
</tr>
<tr>
<td>$s_G$</td>
<td>Government spending/GDP at the steady state</td>
<td>0.2</td>
</tr>
<tr>
<td>$s_Y$</td>
<td>Primary Surplus/GDP at the steady state</td>
<td>0.022</td>
</tr>
<tr>
<td>$B^y$</td>
<td>Net public debt/GDP at the steady state</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>Tax revenues/GDP at the steady state</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Castro et. al (2011) calculate the GDP shares based on the sample average from the national accounts. They calibrate $\beta$ and $\phi$ based on existing literature and match the average capital income share in GDP with $\alpha$. Demand elasticity $\epsilon$ is taken from Gali (2008), and also is a common value found in the literature. Finally, $\Gamma_B$ is initially calibrated to be 0, simulating a situation in which the monetary authority does not intend to influence debt trajectory with its interest rate policy. In a fiscal dominance context, when the fiscal authority ceases to follow a rule compatible with a sustainable debt trajectory, $\Gamma_B > 0$ meaning the central bank will take into account such trajectory when deciding interest rates.

We follow Castro et al. (2011) in establishing the priors and their distributions, with a few modifications in some parameters. Table 3 below summarizes the priors, their distributions, their means and standard deviations:
Table 3: Priors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma)</td>
<td>Intertemporal elasticity of substitution</td>
<td>Normal</td>
<td>1.3</td>
<td>0.05</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Calvo parameter</td>
<td>Beta</td>
<td>0.65</td>
<td>0.1</td>
</tr>
<tr>
<td>(M_f)</td>
<td>Firm’s inattention parameter</td>
<td>Beta</td>
<td>0.8</td>
<td>0.05</td>
</tr>
<tr>
<td>(M)</td>
<td>Consumer’s inattention parameter</td>
<td>Beta</td>
<td>0.8</td>
<td>0.05</td>
</tr>
<tr>
<td>(\Gamma_R)</td>
<td>Monetary policy smoothing parameter</td>
<td>Beta</td>
<td>0.75</td>
<td>0.05</td>
</tr>
<tr>
<td>(\Gamma_{II})</td>
<td>Monetary policy response to inflation</td>
<td>Normal</td>
<td>2.0</td>
<td>0.35</td>
</tr>
<tr>
<td>(\Gamma_y)</td>
<td>Monetary policy response to output gap</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td>(\phi_S)</td>
<td>Primary surplus inertia</td>
<td>Beta</td>
<td>0.4</td>
<td>0.05</td>
</tr>
<tr>
<td>(\phi_{S\bar{S}})</td>
<td>Primary surplus reaction to target deviation</td>
<td>Normal</td>
<td>0.35</td>
<td>0.05</td>
</tr>
<tr>
<td>(\rho_S)</td>
<td>Primary surplus target inertia</td>
<td>Beta</td>
<td>0.5</td>
<td>0.05</td>
</tr>
<tr>
<td>(\phi_B)</td>
<td>Primary surplus reaction to public debt</td>
<td>Inv-Gamma</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>(\rho_{\nu})</td>
<td>Monetary policy autocorrelation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>(\rho_a)</td>
<td>Technology shock autocorrelation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>Tax rate shock autocorrelation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>(\rho_c)</td>
<td>Consumption shock autocorrelation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>(\epsilon_{\nu})</td>
<td>Monetary policy shock variance</td>
<td>Inv-Gamma</td>
<td>1.0</td>
<td>inf</td>
</tr>
<tr>
<td>(\epsilon_a)</td>
<td>Technology shock variance</td>
<td>Inv-Gamma</td>
<td>1.0</td>
<td>inf</td>
</tr>
<tr>
<td>(\epsilon_r)</td>
<td>Tax rate shock variance</td>
<td>Inv-Gamma</td>
<td>1.0</td>
<td>inf</td>
</tr>
<tr>
<td>(\epsilon_c)</td>
<td>Consumption shock variance</td>
<td>Inv-Gamma</td>
<td>1.0</td>
<td>inf</td>
</tr>
</tbody>
</table>

In the table above, the mean and standard deviation values of \(\sigma\) and \(\theta\) are the same as in the SAMBA model. \(M_f\) and \(M\), the inattention parameters, have their means and standard deviations close to the ones used by Gabaix (2018) — a working paper version of Gabaix (2020) which presents a preliminary Bayesian estimation of a simple behavioral New Keynesian model using USA data. The monetary policy parameters \(\Gamma_{II}\) and \(\Gamma_y\) follow the same values of Castro et al. (2011). The fiscal parameters \(\phi_S\), \(\phi_{S\bar{S}}\) and \(\phi_B\) follow the SAMBA model exactly, as do all remaining autocorrelation parameters and shocks. The parameter \(\rho_S\) has the same prior but not the same standard deviation, which is lower in this model. The only parameter that differs from the priors established in Castro et al. (2011) is \(\Gamma_R\), whose value is inspired in the posterior distribution found in the Bayesian estimation of the SAMBA model.

4 Estimation Results

The estimation uses Bayesian methods to arrive at the posterior means and distributions of the selected parameters. It is conducted in MATLAB using the Dynare extension. The results are summarized in the table below:
Table 4: A Posteriori Means And Distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Lower Bound</th>
<th>Higher Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.3823</td>
<td>1.3608</td>
<td>1.3998</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.7093</td>
<td>0.6788</td>
<td>0.7541</td>
</tr>
<tr>
<td>$M^f$</td>
<td>0.8649</td>
<td>0.8310</td>
<td>0.9126</td>
</tr>
<tr>
<td>$M$</td>
<td>0.7580</td>
<td>0.7410</td>
<td>0.7790</td>
</tr>
<tr>
<td>$\Gamma_R$</td>
<td>0.7105</td>
<td>0.7010</td>
<td>0.7208</td>
</tr>
<tr>
<td>$\Gamma_R$</td>
<td>2.3555</td>
<td>2.2555</td>
<td>2.4191</td>
</tr>
<tr>
<td>$\Gamma_y$</td>
<td>0.3965</td>
<td>0.3549</td>
<td>0.4371</td>
</tr>
<tr>
<td>$\phi_S$</td>
<td>0.4739</td>
<td>0.4440</td>
<td>0.5086</td>
</tr>
<tr>
<td>$\phi_S$</td>
<td>0.4802</td>
<td>0.4480</td>
<td>0.5197</td>
</tr>
<tr>
<td>$\rho_S$</td>
<td>0.5540</td>
<td>0.5280</td>
<td>0.5874</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>0.0304</td>
<td>0.0179</td>
<td>0.0448</td>
</tr>
<tr>
<td>$\rho_\nu$</td>
<td>0.2532</td>
<td>0.2019</td>
<td>0.3008</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.5471</td>
<td>0.4472</td>
<td>0.6722</td>
</tr>
<tr>
<td>$\rho_\tau$</td>
<td>0.6047</td>
<td>0.4737</td>
<td>0.7505</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.5361</td>
<td>0.5246</td>
<td>0.5531</td>
</tr>
<tr>
<td>$\epsilon_\nu$</td>
<td>0.1225</td>
<td>0.1176</td>
<td>0.1293</td>
</tr>
<tr>
<td>$\epsilon_\sigma$</td>
<td>0.1779</td>
<td>0.1340</td>
<td>0.2193</td>
</tr>
<tr>
<td>$\epsilon_\sigma$</td>
<td>0.1215</td>
<td>0.1176</td>
<td>0.1265</td>
</tr>
<tr>
<td>$\epsilon_\sigma$</td>
<td>0.1256</td>
<td>0.1146</td>
<td>0.1347</td>
</tr>
</tbody>
</table>

The lower and higher bounds fall within the 90% confidence interval level. The acceptance ratios were 33.673% and 34.083% for the Metropolis Hasting algorithm, which was replicated 100,000 times in two blocks. This falls within the 20%-40% interval that is well accepted by the literature. All parameters converged. The results are close to those of the SAMBA model constructed by Castro et al (2011), which is reasonable since the data used and the model specification are similar - though much simpler in this model.

Some interesting estimations are found. It is the first time a Bayesian estimation for the cognitive factors $M$ and $M^f$ is conducted for the Brazilian economy. Their posterior means, 0.7580 and 0.8648, respectively, make theoretical sense once they are lower than 1 but still allow for enough rationality. In addition, firms have a higher cognitive discount factor meaning they pay more attention to macro variables — this too makes theoretical sense since firms can hire consultants and information services to back their decisions.

The parameters that guide monetary policy are also sound, given Brazil’s policy regime in the period our analysis. Parameters $\Gamma_R > 1$ and $\Gamma_y < 1$ account for the inflation-targeting regime, in which the central bank has a strong reaction function towards inflation — following the Taylor principle. As for fiscal policy, parameters $\phi_S > 0$, $\rho_S > 0$, and $\phi_B > 0$ indicate a rule in which the primary surplus target matters for fiscal policy decisions and so does public debt trajectory. This certainly leads one to conclude that over the past years Brazil has generally operated under a monetary dominance regime.

5 Simulations

We run the model to simulate monetary dominance situations and fiscal dominance contexts (a hypothetical that still uses most of the values for the parameters estimated in
the last section while changing the values of other parameters). Since the objective is to analyze monetary and fiscal interactions with a behavioral model, the variances of output and inflation are used as proxies for social welfare — that is, the monetary and fiscal authorities’ goals include pursuing their targets while keeping inflation and product variances low. In other words, higher variance values for output and inflation lead to lower social welfare. Under such assumptions, the model and its variations are analyzed next.

First, let us look at the results under monetary dominance — which can be understood as the country’s current context, at least under the rules and laws that apply today. Last section’s estimation suggests the country has lived under such a regime over the past twenty years. The model is run considering a one standard deviation monetary policy shock and different responses the central bank can exert when fighting inflation — parameter $\Gamma_\Pi$ — and different responses the fiscal authority can weigh on public debt trajectory — parameter $\phi_B$. We simulate output and inflation variances by changing those parameters while keeping other parameters constant.

Table 5 below summarizes the variances of output $y$ and inflation $\pi$ given a one standard deviation monetary policy shock and different responses by the central bank and the fiscal authority according to the behavioral case — using $M$ and $M^f$ from our Bayesian estimation:

<table>
<thead>
<tr>
<th>$\phi_B$</th>
<th>$\Gamma_\Pi$</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0442</td>
<td>0.0028</td>
<td>0.0379</td>
<td>0.0021</td>
<td>0.0333</td>
</tr>
<tr>
<td>0.02</td>
<td>0.0444</td>
<td>0.0035</td>
<td>0.0371</td>
<td>0.0026</td>
<td>0.0321</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0455</td>
<td>0.0053</td>
<td>0.0360</td>
<td>0.0038</td>
<td>0.0298</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0549</td>
<td>0.0087</td>
<td>0.0407</td>
<td>0.0063</td>
<td>0.0317</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1574</td>
<td>0.0280</td>
<td>0.0912</td>
<td>0.0160</td>
<td>0.0617</td>
</tr>
</tbody>
</table>

Notice that when the central bank fights inflation fiercely, the variance of inflation lowers for all cases. In addition, notice that for certain values of $\phi_B$, especially for those that belong to the [0.01, 0.05] interval, product variance can be lowered in most cases (for instance, when $2 \leq \Gamma_\Pi \leq 3$). This means the fiscal authority can pursue its objective (e.g., debt stability) with care, otherwise it could generate too much product volatility.

Next, the same is done for the rational expectations case — in which we use $M = M^f = 1$. Table 6 below summarizes the results given a one standard deviation monetary policy shock:

Table 6: Behavioral Case ($M = 0.7580$, $M^f = 0.8649$)

<table>
<thead>
<tr>
<th>$\phi_B$</th>
<th>$\Gamma_\Pi$</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0442</td>
<td>0.0028</td>
<td>0.0379</td>
<td>0.0021</td>
<td>0.0333</td>
</tr>
<tr>
<td>0.02</td>
<td>0.0444</td>
<td>0.0035</td>
<td>0.0371</td>
<td>0.0026</td>
<td>0.0321</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0455</td>
<td>0.0053</td>
<td>0.0360</td>
<td>0.0038</td>
<td>0.0298</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0549</td>
<td>0.0087</td>
<td>0.0407</td>
<td>0.0063</td>
<td>0.0317</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1574</td>
<td>0.0280</td>
<td>0.0912</td>
<td>0.0160</td>
<td>0.0617</td>
</tr>
</tbody>
</table>

Notice that when the central bank fights inflation fiercely, the variance of inflation lowers for all cases. In addition, notice that for certain values of $\phi_B$, especially for those that belong to the [0.01, 0.05] interval, product variance can be lowered in most cases (for instance, when $2 \leq \Gamma_\Pi \leq 3$). This means the fiscal authority can pursue its objective (e.g., debt stability) with care, otherwise it could generate too much product volatility.

Next, the same is done for the rational expectations case — in which we use $M = M^f = 1$. Table 6 below summarizes the results given a one standard deviation monetary policy shock:

---

We are inspired by the simulations of Furtado (2017), which is a benchmark for this section.
Table 6: Rational Expectations Case ($M = 1, M_f = 1$)

<table>
<thead>
<tr>
<th>$\phi_B$</th>
<th>$\pi$</th>
<th>$\pi$</th>
<th>$\pi$</th>
<th>$\pi$</th>
<th>$\pi$</th>
<th>$\pi$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0532</td>
<td>0.0027</td>
<td>0.0457</td>
<td>0.0022</td>
<td>0.0401</td>
<td>0.0019</td>
<td>0.0356</td>
</tr>
<tr>
<td>0.02</td>
<td>0.0467</td>
<td>0.0029</td>
<td>0.0401</td>
<td>0.0022</td>
<td>0.0352</td>
<td>0.0018</td>
<td>0.0314</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0325</td>
<td>0.0047</td>
<td>0.0266</td>
<td>0.0031</td>
<td>0.0231</td>
<td>0.0022</td>
<td>0.0206</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0267</td>
<td>0.0097</td>
<td>0.0197</td>
<td>0.0067</td>
<td>0.0156</td>
<td>0.0049</td>
<td>0.0132</td>
</tr>
<tr>
<td>0.15</td>
<td>0.2567</td>
<td>0.0566</td>
<td>0.1470</td>
<td>0.0304</td>
<td>0.1052</td>
<td>0.0202</td>
<td>0.0852</td>
</tr>
</tbody>
</table>

The results are similar to those in Table 5, though one can notice the variance values for both output and inflation vary more in this case. Additionally, note that the interval in which the fiscal authority improves its performance is between $[0.01, 0.10]$, wider than in the behavioral case. This is interesting, since more product volatility does not necessarily translate into higher variances for inflation.

Above all, one can conclude that under monetary dominance the central bank should fiercely pursue its goals — while the fiscal authority should carefully adjust not to cause higher volatility in the variances of product and inflation. This is a result in both specifications, considering behavioral and fully rational agents. One main difference between both is that variances are smoothed when behavioral agents are considered — which makes theoretical sense, once inattentive agents do not pay attention to all prices.

Next, the results from fiscal dominance situations are shown. Let us first emphasize that some changes are made in the original model to make it fit for such conditions. First, it is assumed $\phi_S = 0$, implying the primary surplus target is not relevant for the fiscal authority. That means the actual primary surplus is not influenced by a target anymore, depending only on its past values. This implies that fiscal policy does not place any weights on debt trajectory.

Note that the model does not allow for both monetary and fiscal policies to be dominant — it results in an explosive situation in which the Blanchard-Kahn conditions are not usually satisfied. Such situation resembles a context similar to the one treated by Loyo (1999) for explaining hyperinflation in Brazil during the 1970s and 1980s. In addition, the central bank becomes responsible for debt trajectory and thus $\Gamma_b > 0$. Also, since monetary policy is not dominant, parameter $\Gamma_\Pi$ is run for values less than 1 — meaning the central bank does not follow a Taylor-rule in these cases, it just adjusts for fiscal policy. Furthermore, simulations are also made for $\Gamma_\Pi \geq 1$, which often results in explosive situations. Finally, some simulations are made for hypothetical contexts in which the monetary authority wishes to fight inflation at the same time as it pursues debt stability.

Again, variance values for output and inflation are analyzed as proxies for social welfare. Simulations are run for different $\Gamma_\Pi$ and $\Gamma_b$ values. These values were chosen based on the fact that monetary policy is not dominant and on the equilibrium conditions for the model. Note that $\Gamma_b$ values are run for certain values close to $\phi_B$, and that is done on purpose: the attention given to debt by the fiscal authority under monetary dominance could be close to the attention given to debt by the monetary authority under fiscal dominance. The parameter $\Gamma_b$ is also tested with higher values. Table 7 below summarizes the results for the behavioral case — using $M$ and $M_f$ from our Bayesian estimations — when a one standard deviation monetary policy shock takes place:
Table 7: Behavioral Case \((M = 0.7580, \ M^f = 0.8649)\)

<table>
<thead>
<tr>
<th>(\Gamma_{\Pi})</th>
<th>0.00</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_b)</td>
<td>0.00 0.0686</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.01</td>
<td>0.1240 0.0303</td>
<td>0.0298 0.1296</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.05</td>
<td>0.1016 0.0437</td>
<td>0.0299 0.1190</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0349 0.0260</td>
<td>0.0289 0.0543</td>
<td>0.0258 0.2118</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0196 0.0144</td>
<td>0.0128 0.0029</td>
<td>0.0162 0.0036</td>
<td>0.0223 0.0063</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0130 0.0024</td>
<td>0.0144 0.0029</td>
<td>0.0162 0.0036</td>
<td>0.0223 0.0063</td>
</tr>
</tbody>
</table>

Results with “-” mean the Blanchard-Kahn conditions were not met and there is no stable equilibrium for those parameter values. First, let us notice that given a certain value of \(\Gamma_b\), when the central bank responds with higher intensity to inflation, the outcome is increasing inflation and output variances. Take for instance \(\Gamma_b = 1.0\). For higher values of \(\Gamma_{\Pi}\), one can observe increasing variances for both output and inflation. Additionally, for a given value of \(\Gamma_{\Pi}\), the more the monetary authority responds to debt trajectory (the higher \(\Gamma_b\)), the lower the product and inflation variances become. This suggests the monetary authority should take into account public debt trajectory when faced with fiscal dominance contexts and not fight inflation fiercely. If it pursues the latter, it could cause price instability. In fact, what this model specification suggests is that the monetary authority ought to adjust for fiscal policy and accommodate for public debt. By doing this, it keeps inflation and product variances low and possibly generate less welfare losses.

Next, a similar simulation is conducted for the rational expectations case, in which we use \(M = M^f = 1\). Table 8 below summarizes the results:

Table 8: Rational Expectations Case \((M = 1, M^f = 1)\)

<table>
<thead>
<tr>
<th>(\Gamma_{\Pi})</th>
<th>0.00</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_b)</td>
<td>0.00 0.0460</td>
<td>0.0298 0.1296</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0303 0.0437</td>
<td>0.0299 0.1190</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0264 0.0260</td>
<td>0.0289 0.0543</td>
<td>0.0258 0.2118</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0222 0.0159</td>
<td>0.0252 0.0281</td>
<td>0.0274 0.0674</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0185 0.0105</td>
<td>0.0211 0.0165</td>
<td>0.0241 0.0312</td>
<td>0.0138 0.8259</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0130 0.0024</td>
<td>0.0144 0.0029</td>
<td>0.0162 0.0036</td>
<td>0.0223 0.0063</td>
</tr>
</tbody>
</table>

Generally, the results are in line with the behavioral case from Table 7. For instance, for a given value of \(\Gamma_b\), the more the central bank responds to inflation, the higher the variances for product and inflation becomes. Let us consider again \(\Gamma_b = 1\). Then, for higher values of \(\Gamma_{\Pi}\), product and inflation variances increase. Additionally, for a given value of \(\Gamma_{\Pi}\), higher responses of the monetary authority to public debt (a higher \(\Gamma_b\)) generate lower product and inflation variances.

This leads one to conclude the same as before: under a fiscal dominance context, the central bank should not fiercely pursue its inflation target. In particular, when the central bank’s response to public debt and inflation is high enough, a strong instability can be
generated — as is the case for \( \Gamma_b = 1.5 \) and \( \Gamma_{II} = 2 \), which results in an inflation variance of 0.8259.

Comparing the behavioral and rational cases is useful for checking how the theoretical results differ. Placing a cognitive discount factor for economic agents generates smoother paths for the resulting variables under normal conditions (e.g., monetary dominance). This has happened in the comparisons above. For instance, one arrives at similar conclusions for both the behavioral and rational cases under monetary dominance, though the amount of weight the fiscal authority places on debt trajectory should be taken into account with care. The resulting variances, and thus social welfare, can differ significantly due to higher or lower weights placed by the fiscal authority on public debt trajectory. Under a fiscal dominance context, the best the central bank can do about inflation is almost nothing — just accommodating for fiscal policy and adjusting for debt trajectory. Interestingly, in the rational expectations case, fiscal dominance is not so costly in terms of output — this could suggest fully rational agents are able to understand inflation as a monetary phenomena, not having too much impact on real variables, as mentioned — while the opposite could be true for behavioral agents.

6 Conclusions

This paper builds a behavioral New Keynesian model with Brazil’s monetary and fiscal rules and estimates it using Bayesian methods and Brazilian data. Monetary and fiscal dominance situations are then simulated and the resulting variances of output and inflation are analyzed.

For the first time, the cognitive discount factors of Gabaix are calculated for Brazil. We estimate \( M \) and \( M^f \) to be 0.7580 and 0.8649 for consumers and firms, respectively. Our simulations then show that the central bank should fiercely pursue its inflation target under a monetary dominance context, while the fiscal authority should carefully choose the amount of effort it places on controlling debt trajectory under the penalty of generating high output variance. The behavioral model demonstrates a tenuous threshold for the fiscal authority, thus suggesting it should be vigilant in such contexts.

In addition, under fiscal dominance the monetary authority should only accommodate for fiscal policy, controlling for the risk of generating high inflation variance. This is more obvious in the behavioral model than in the rational one. The former generated higher variance results for both inflation and product as the central bank places higher weights in fighting inflation and debt trajectory.

References


