

# **Magic Hypercube approach applied to twenty economies between 1992 and 2018: the effectiveness of Brazilian macroeconomic policy using DEA-Malmquist**

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**Abstract:** Based on the magic hypercube approach, this paper uses a tool called DEA-Malmquist to numerically measure the relative effectiveness of twenty selected economies according to the four variables of Kaldor: current account balance (CAB), GDP growth, inflation and unemployment. We start from the Kaldor's (1971) view, which led to the construction of a diagram known as the "Magic Square" (MS). This seminal idea offers a contextual perspective. Then we turn to the "Magic Hypercube" (MH), a broader panorama where through the Data Envelopment Analysis (DEA) procedure and Malmquist index we quantify its geometry. To the application of it, we extracted data from the World Bank's Open Access Indicators System, dividing it in two periods: 1992 to 2007 and 2010 to 2018. As an outcome, we reached numerical results that capture the global performance of the sample. The encountered relative effectiveness to these twenty nations allows a macroeconomic analysis, in which we focus on how the variable unemployment is being tackled by Brazil compared with other nations.

**Keywords:** Brazil; DEA-Malmquist; Magic Hypercube; Effectiveness; Unemployment.

## **Abordagem do Hipercubo Mágico aplicada à vinte economias entre 1992 e 2018: a eficácia da política macroeconômica brasileira usando o DEA-Malmquist**

**Resumo:** Com base na abordagem do hipercubo mágico, este artigo usa uma ferramenta chamada DEA-Malmquist para medir numericamente a eficácia relativa de vinte economias selecionadas de acordo com as quatro variáveis de Kaldor: balanço de pagamentos, crescimento do PIB, inflação e desemprego. Começamos da visão de Kaldor (1971), que levou à construção de um diagrama conhecido como "quadrado mágico". Essa ideia seminal oferece uma perspectiva contextual. Em seguida, nos voltamos para o "hipercubo mágico", um panorama mais amplo em que, através do procedimento DEA e do índice de Malmquist, quantificamos sua geometria. Para sua aplicação, extraímos dados do Banco Mundial, dividindo-os em dois períodos: 1992 a 2007 e 2010 a 2018. Como resultado, alcançamos resultados numéricos que capturam o desempenho global da amostra. A eficácia relativa encontrada nesses vinte países permite uma análise macroeconômica, na qual focamos em como a variável desemprego está sendo conduzida pelo Brasil em comparação com outros países.

**Palavras-chave:** Brasil; DEA-Malmquist; Hipercubo Mágico; Eficácia; Desemprego.

## **Área 4 - Macroeconomia, Economia Monetária e Finanças**

**JEL Classification:** C15; E24; O57.

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## 1. Introduction

The dynamics of the world of work is closely related to the behaviour of a series of macroeconomic variables, both internal and external to a given country. Brazil, as an intermediate economic power that aspires to the status of a so-called advanced nation, presents a myriad of diversities and complexities that must be considered for a better articulation and socioeconomic insertion in global terms in general and with regard to the labour market in particular.

In this sense, the content of the seminal paper of Kaldor (1971) – called *Conflicts in National Economic Objectives* – is a relevant academic reference. Going beyond the scope of United Kingdom (UK), Kaldor's main purpose was to present a logical and empirical reconsideration of a basic macroeconomic framework of necessary relations to achieve some desirable targets, or economic policy objectives. Assuming analogous proposal, we aim to shed light specifically on the macroeconomic analysis of the Brazilian labour market, adopting for the purposes of analysis the four *kaldorian* variables: CAB, GDP growth, inflation and unemployment. We choose to investigate the labour category – focusing on the results obtained for the variable unemployment. In fact, any other *kaldorian* variable could be inquired in a similar perspective. Nonetheless, we consider that employment should not be analysed as an isolated variable, so the importance of observing it interacting with three other macroeconomic variables.

Kaldor's (1971) pioneering analysis did not benefit from quantitative nor graphical instruments. However, through a series of other studies it has unfolded into the construction of a diagram known as the "Magic Square" (MS), which further in time was continuously improved until achieve the format of the "Magic Hypercube" (MH) (SAAVEDRA-RIVANO & TEIXEIRA, 2017). We embrace the last structure in this paper as a beacon.

Our proposal consists of integrating this framework to the Data Envelopment Analysis (DEA) procedure and Malmquist index to quantify the geometry of the MH. It allows investigating quantitatively the relative effectiveness and frontiers of a selection of twenty economies. Thus, we reach numerical results that intend to capture the global performance of the sample, which allows a macroeconomic analysis focusing on how the variable unemployment is being tackled by Brazil compared with other nations. We extracted data from the World Bank's Open Access Indicators System (WB, 2020) concerning the four variables of Kaldor, dividing it in two periods: 1992-2007 and 2010-2018. Doing so, this study aims to contribute in an original way to the economic analysis.

In this paper, after this introduction, the second section brings forward an historical view of the configuration of the MH according to the *kaldorian* variables. Section three uncovers the methodology, exposing the production possibility set, the frontier of best practices and its shift, followed by the explanation regarding the source and treatment of data. Section four present and analyse the results for both periods and their Malmquist index, thus allowing the interpretation of changes in effectiveness and frontier. Section five summarizes the main conclusions of the research, accompanied by a macroeconomic analysis focusing on the Brazilian labour market.

## 2. History of the Magic Hypercube: rethinking the model

Aiming to contribute with a measurement by means of indicators that are multi-varied, the use of the Magic Square of *Kaldorian* inspiration appears as an appropriate analytical tool, since it allows the comparison of results between countries of different levels of development. It permits the study of several variables simultaneously and a more direct comparative performance, evolving socioeconomic issues.

The idea of evaluating and comparing the accomplishment of countries first appeared in a seminal paper of Kaldor (1971), an evaluation of the UK macroeconomic performance. The analysis was performed using four variables (inflation, balance of payments, unemployment and income), without mathematics or diagrams. With this in mind, Schiller (1973) introduced a graphical representation of the ideas of Kaldor. Such geometric figure became known as "Magic Square".

In 1987, economists at Organization for Economic Co-operation and Development (OECD) began using this tool with a minor modification (GDP growth was used instead of income), preserving the fundamental Kaldor's idea<sup>4</sup>. Bernard *et al.* (1988) indicated a basic illustration of this approach to the MS. Such view has been conceived in a way that its four directions (north, south, east and west) were aligned with growth, inflation, trade and unemployment indexes, respectively. All four directions were at different scales, even though expressed in percentages. The state of the economy were related to the size "area" of the resulting quadrangle.

Medrano-B and Teixeira (2013) considered that the area of such quadrangle cannot be properly calculated, because of the non-uniform scales of the axes. To solve it they re-defined the figure to make the axes uniform. In order to accomplish this task rebounding the initial conditions, they introduced the notion of an ideal country (wonderland macroeconomic configuration). This concept was constructed by means of intervals that establish values of maximum and minimum for each variable. The larger the area of the country in the MS, the better will be the results obtained and the closer the country will be in relation to the optimal performance. However, it is very difficult for a nation to achieve a good result in all the parameters present in the geometric figure. In this way, the definition of an ideal country, even being a stimulating quantitative criterion of comparison, is virtually impossible to be achieved.

The use of the MS as an analytical instrument is not limited to the macroeconomic performance. The index may favour different interpretations, depending on the chosen variables to compose it. This arrangement was tested in the article of Teixeira, Pinheiro & Vilasboas (2015), where it was used to compare the performance of China and the United States of America (USA) in relation to their respective paths of socioeconomic development. The variables were CO<sub>2</sub> emissions, access to basic sanitation, human development index and the percentage of renewable resources in energy matrix. In this new geometric form (with only three variables), the results of the index were indifferent to its ordination, once it provides always a single value for the indicator.

In a recent work of Saavedra-Rivano & Teixeira (2017), they observed a problem in the ordering the variables, showing that different results may be generated for the index. Since the geometric figure chosen has more than three dimensions, its graphical representation becomes problematic. This is not an inconsequential oversight and, in order to avoid this problem, they proposed a solution called "Magic Hypercube", which was a normalization of the four used variables: CAB, GDP growth, inflation and unemployment. The use of this new approach involves a hypercube diagram, which produces the same index, regardless the ordering of the variables. An empirical application of the concept was offered using economic data from Brazil and Chile. Essentially, except in the case of particular symmetry, the ordering of the variables around the square is fundamental to avoid multiple values for an index composed by more than three variables.

In the present work, we replicate the approach of Saavedra-Rivano & Teixeira (2017) to calculate the index of the MH. In our geometric diagram, we chose the same four *kaldorian* variables. The novelty is that we rethink their model, analysing twenty selected economies between 1992 and 2108 through Malmquist-DEA method, as explained in the next section.

### 3. Methodology

Two subsections compose this section. The first will present – through concepts, figures and equations – the production possibility set, the frontier of best practices and its shift. The second will explain the database source and how it will be treated in this research.

#### 3.1. The production possibility set, the frontier of best practices and its shift

In evaluating performance, the concepts of productivity, efficiency and effectiveness are used. Productivity can be defined as the relationship between the quantity of goods and services produced (objectives) and the quantity of resources used (means). Efficiency is about achieving the objectives of the production unit in an optimized way, with maximum productivity. Effectiveness refers to doing the

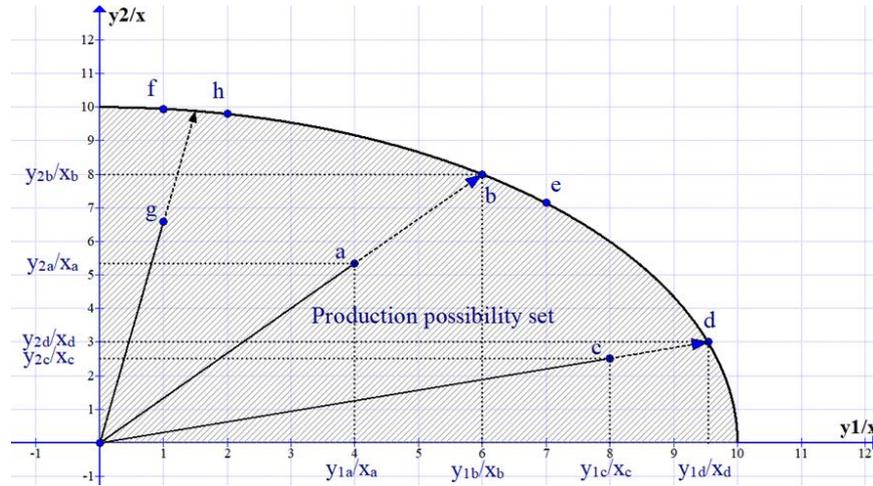
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<sup>4</sup> That is why we assume in this study the expressions "kaldorian variables" or "variables of Kaldor".

right task, achieving the intended objectives and disregarding the resources used. In the literature, these definitions are used to evaluate firms, organizations, public policies and others (AVKIRAN & ROWLANDS, 2008).

The quantification of productivity, efficiency and effectiveness often uses the productive frontier methods. These methods start from the definition of the production possibility set (PPS) (or frontier, PPF) and its properties. Figure 1 illustrate these concepts. PPS frontier is represented for as a production process that uses a given amount of inputs ( $x$ ) to generate two products ( $y_1$  and  $y_2$ ). The shaded area is the PPS and the curve is the boundary of this set, which represents the maximum possible combinations with a given level of input and the existing technology at a given point in time. The points **a**, **b**, **c**, **d**, **e**, **f**, **g** and **h** represent eight production units, an observed subset of the PPS.

**Figure 1 – Production possibility set (PPS)**



**Source:** own elaboration.

The coordination of these points (vectors) indicate the partial productivity ( $y_1/x$  and  $y_2/x$ ) of the respective units. These productivities can be measured at different scales, which makes comparison difficult. We can only compare the length (norm and therefore the magnitude of the productivities) of the vectors if their directions are the same. Note that **a** and **b** are collinear (have the same direction) and the length of vector **b** is bigger than the one of the vector **a**. Therefore, in terms of productivity, point **b** exceeds the performance of **a**. Similarly, **c** is surpassed by **d**.

However, if the vectors have different directions, the production mix is different and if we do not have weights or market prices for the evaluated products, it is impossible to compare the productivity of the points. For example, the productivity of the set with mix of **a** and **b** ( $\frac{y_{2a}}{y_{1a}} = \frac{y_{2b}}{y_{1b}}$ ) is not comparable to the performance of units **c** and **d** that have a different mix ( $\frac{y_{2c}}{y_{1c}} = \frac{y_{2d}}{y_{1d}}$ ). In this case, there being free will in the choice of “what and how much to produce”, nothing authorizing us to assume that **b**  $\succ$  **d** or **c**  $\succ$  **b** regarding productivity. Nevertheless, when prices or the relative importance of products are available, we reach the total factor productivity (PTF):

$$PTF = \frac{y_1 v_1 + \dots + y_n v_n}{x_1 u_1 + \dots + x_m u_m} \quad (3.1)$$

where  $v_i$ , for  $i = 1, 2, \dots, n$ , are the weights (or prices) of  $n$  products and  $u_j$ , for  $j = 1, 2, \dots, m$ , are the weights (or prices) of  $m$  inputs.

For Farrell (1957), the reference in quantification of efficiency is productivity. The empirical demarcation of the PPS can be built with the productive units that maximize production for a given level of input, achieved when it is impossible to increase the level of one product without worsening that of another (Pareto’s optimal points). Convex linear combinations of the  $k$  efficient units (equation 3.2) result in a linear boundary per part that mimics the representation of the figure 1. The points **b**, **d**, **e**, **f** and **h** form the boundary, representing the efficient units and points **a**, **c** and **g** are the inefficient ones.

$$\lambda_1 \begin{bmatrix} y_{11}/x_1 \\ y_{12}/x_1 \end{bmatrix} + \dots + \lambda_k \begin{bmatrix} y_{1k}/x_k \\ y_{2k}/x_k \end{bmatrix} \text{ (in which } \sum \lambda_r = 1, \lambda_r \geq 0, r = 1, \dots, k \text{)} \quad (3.2)$$

With the PPS demarcated, the level of efficiency can be estimated by the shortest distance that separates each unit from the border. The smallest relative distance from the frontier determines a value called by Farrell (1957) index of relative technical efficiency (IE) that must be greater than or equal to one: IE = 1 for efficient units and IE > 1 for inefficient units, in such a way that the farther from the border the unit is evaluated, the more inefficiency must be evident. For example, for the point **a** this distance is estimated by dividing the length of the vector **b** ( $y_{1b}/x_b, y_{2b}/x_b$ )' by the length of the vector **a** ( $y_{1a}/x_a, y_{2a}/x_a$ )', that is,  $IE_a = \overline{0b}/\overline{0a}$ .

This (scalar) value, being greater than one, indicates the projection of **a** at the border,  $IE_a * \begin{bmatrix} y_{1a}/x_a \\ y_{2a}/x_a \end{bmatrix}$ . Similarly, the relative inefficiency index of point **c** ( $IE_c$ ) takes as reference the point of border **d**, the closest target and with the same production mix (that is, the same technological profile). This results in the relationship  $\overline{0d}/\overline{0c} = IE_c$  which is used to project the vector **c** on the efficient frontier,  $IE_c * \begin{bmatrix} y_{1c}/x_c \\ y_{2c}/x_c \end{bmatrix}$  and estimate how much both products should be increased (with the same inputs) to make **c** efficient.

Note that there may be some inefficient units without real benchmarks at the border. See the case of point **g** in figure 1. For these cases, a virtual reference can be estimated, formed by the linear combination of the nearest optimal points. Thus, the virtual reference of **g** must be defined with the vectors **f** and **h** (reference set) and denoted by (3.3), where  $\lambda_f$  and  $\lambda_h$  indicate the proportion in which **f** and **h** respectively contribute to determine the projection point of **g** on the efficient frontier. Therefore, the projection point at the border is equivalent to the weighted sum coordinated by the units of the reference set:

$$\lambda_f \begin{bmatrix} y_{1f}/x_f \\ y_{2f}/x_f \end{bmatrix} + \lambda_h \begin{bmatrix} y_{1h}/x_h \\ y_{2h}/x_h \end{bmatrix} \quad (3.3)$$

In order to calculate efficiency, the performance of each evaluated unit is compared with the best technologically closest practices, contemplating the possibility of each unit emphasize its individual competitive advantages. Hence, we can conclude that if productivity is how much is produced in relation to the inputs used, efficiency is how much is produced in relation to how much could be produced.

Using Farrell's (1957) concepts and measures as a reference, Charnes, Cooper & Rhodes (1978) modelled a linear programming problems (LPP) to estimate the relative efficiency indexes and the weights (or relative importance) of the multiple inputs and products considered in the analysis of production units. This type of estimation was categorized as Data Envelopment Analysis (DEA).

The choice of the DEA method in this work is justified by its popularity in the literature and by its advantages over the other frontier methods, the so-called stochastic frontier analysis. The DEA determines the relative efficiency indexes and the weights of multiple inputs and outputs for each unit evaluated, eliminating the need to define *a priori* a functional relation of the aggregated product and inputs. However, like all methods, it has limitations. As it is a deterministic (non-statistical) approach, it disregards the random effects of the production process and is very sensitive to extreme values with measurement errors (COELLI, PRASADA & BATTESE, 1998).

In honour of their creators (CHARNES, COOPER & RHODES, 1978), the basic DEA models are called CCR-OO (output-oriented) or CCR-IO (input-oriented). The CCR-OO estimates the efficiency oriented to maximize the outputs with the given inputs and the CCR-IO calculates the efficiency oriented to minimize the inputs with the fixed products. These two models assume technologies with a constant return of scale (CRE). Note that there are others possible scales, but they are outside the scope of this study (see, for example, COOPER, SEIFORD & TONE, 2000).

The CCR-OO LPP is given by the system (3.4) for any unit **O** (from a group of **S** organizations evaluated) that produces the vector  $y_{no}$ , using the input vector  $x_{mo}$  and a technology with constant scale return. The solution for this programming involves: i) obtaining a minimum  $IE_o$  value that, multiplied by the vector  $y_{no}$ , project this on the border; ii) the calculation of the vectors  $u_j$  and  $v_i$ , which represent the weights (relative importance or shadow price) of the inputs and outputs respectively. This formulation (also called the multiplicative model) is calculated for each of the units evaluated, in such a way that the weights of the inputs and outputs can be different in each unit depending on the mix of inputs and productions.

$$Min IE_o = \sum_{j=1}^m u_j x_{jo} \quad (3.4)$$

subject to:

$$\sum_{i=1}^n v_i y_{io} = 1$$

$$\sum_{i=1}^n v_i y_{ir} - \sum_{j=1}^m u_j x_{jr} \leq 0, \quad r = 1, \dots, k, \dots, o, \dots, S$$

$$u_j \text{ e } v_i \geq 0$$

The dual LPP of the system 3.4 determines the same IE, but adds another important information: the vector  $\lambda$  (the coefficient of the linear combination determining the boundary) determines the point of the virtual benchmark at which  $y_{no}$  must project to make the unit **O** efficient. Therefore, this LPP is known in the literature as an envelope model. If **O** is efficient,  $IE_o = 1$  and  $\lambda_s = 0$ , except if  $\lambda_o = 1$ . If **O** is inefficient,  $IE_o > 1$  and the  $\lambda_s > 0$ , it indicates that the reference units of **O** must mirror to be efficient.

Intrinsically linked to efficiency is the concept of effectiveness. As seen, effectiveness is related to the main objectives of organizations and assesses the extent to which these objectives were achieved, regardless of the resources used and the way in which the results were obtained<sup>5</sup>. In this way, the closer a unit gets to the desired goal or best practices, the more effective it is.

The measurement of relative effectiveness is intricate when there are multiple objectives to be achieved simultaneously. It requires weighing the objectives, defining the relative importance of each one. If a single hierarchy of objectives is inadequate, if it is recognized that each production unit may have different priorities or should take greater advantage of its competitive advantages and seek an objective method to weigh these criteria, the frontier analysis can be used as part of the demarcation of the boundary between the obtained and the possible results.

Therefore, to quantify the relative effectiveness, an adaptation of the CCR-OO envelope model is used. In this adaptation, inputs are disregarded, estimating the frontier with previously defined objectives or with the organizations with the highest outputs. The envelope model used to calculate the relative effectiveness for an **O** unit is given by (3.5):

$$Max EC_o \quad (3.5)$$

subjected to:

$$\sum_{r=1}^S y_{ir} \lambda_r \geq EC_o y_{io} \quad r = 1, \dots, k, \dots, o, \dots, S$$

$$\sum_{r=1}^S \lambda_r = 1$$

$$EC_o \geq 1, \quad \lambda_r \geq 0, \quad \forall r$$

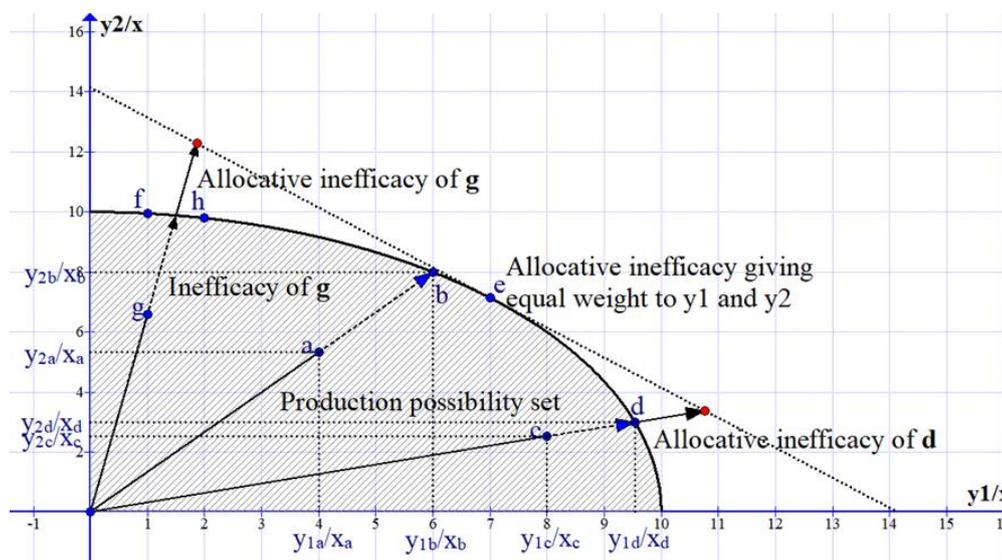
<sup>5</sup> In the context of economic policy, effectiveness can refer to the measure of the success of governmental actions related to the issue of economic development and the social welfare of the country.

This formulation estimates the relative distance of unit **O** from the border, considered as the  $EC_o$  effectiveness index and determines the projection point at the border, resulting from the weighted sum of the coordinates of the units of unit **O** reference set. If the unit evaluated is below from the border it will have an  $EC > 1$  that will indicate how much all outputs must be increased simultaneously to become a good practice. In addition, the inverse of the effectiveness index shows the performance related to best practices. Thus, if  $EC = 1.25$ , the evaluated unit must simultaneously increase its objectives by 25%. Using the inverse ( $1/1.25 = 0.8$ ) it can be said that the performance of the evaluated unit is 80% of what it could be, that is, of the indicated by the best practices.

From the dual LPP of (3.5) derives the multiplicative model that determines the same  $EC$  effectiveness index and additionally calculates the weights ( $v_i$ ) of the products that evaluated unit inputs. Being called the shadow price, the vector  $v_i$  plays an important role in the sensitivity analysis of the effectiveness index (COOPER, SEIFORD & TONE, 2000) and can be used to define the most similar groups of units. In addition, the dual LPP (3.5) can be reformulated by giving the same weight to the outputs considered in the research. This is interesting in determining which units perform best on all variables.

To make the idea clear, consider figure 2, which represents the frontier of best practices of a sample of production units, considering two objectives ( $y_1$  and  $y_2$ ) achieved regardless of the resources used and the way in which the results were obtained. It is observed that unit **d** prioritizes the objective  $y_1$  instead of the objective  $y_2$ . The unit **f** behaves differently, giving greater weight to objective  $y_2$ . If we give the same weight to both objectives we can find an isorevenue curve (dotted line) that touches the border at point **e**. Hence, **e** is the unit that maximizes both objectives when they have the same weight. Based on these weights, it is also possible to calculate the allocative effectiveness, using the distance that separates the border points from the isorevenue. Note the allocative inefficiency of **d** and **g**.

Figure 2 – Frontier of best practices



Source: own elaboration.

It should also be observed that there is great research potential with regard to the effectiveness of socioeconomic policies. The analysis of the effectiveness indices developed so far was static, since variables are used and units are compared in a given period. The introduction of observations done sequentially over time allows the creation of a dynamic model. It shifts the central question to other very important problems, such as: i) the evolution of the effectiveness of each unit in relation to the evolution of the set of units evaluated; ii) the decomposition of the dynamic effectiveness index in terms of changes in the frontier and changes in effectiveness.

To estimate the evolution of the relative effectiveness, the adapted Malmquist index (IM) is useful, following one of the most popular IM developments, the one carried out by Färe, Grosskopf, Norris & Zhang (1994). Suppose that the best practice boundary, shown in figure 2, does not change between the period  $t$  and  $t + 1$  and that the point **a** represents the performance of a country ( $\alpha$ ) in the period  $t$  and **d** the performance of the same country in  $t + 1$ . Thus, the change in relative effectiveness ( $\Delta Ec_\alpha$ ) over time is given by (3.6):

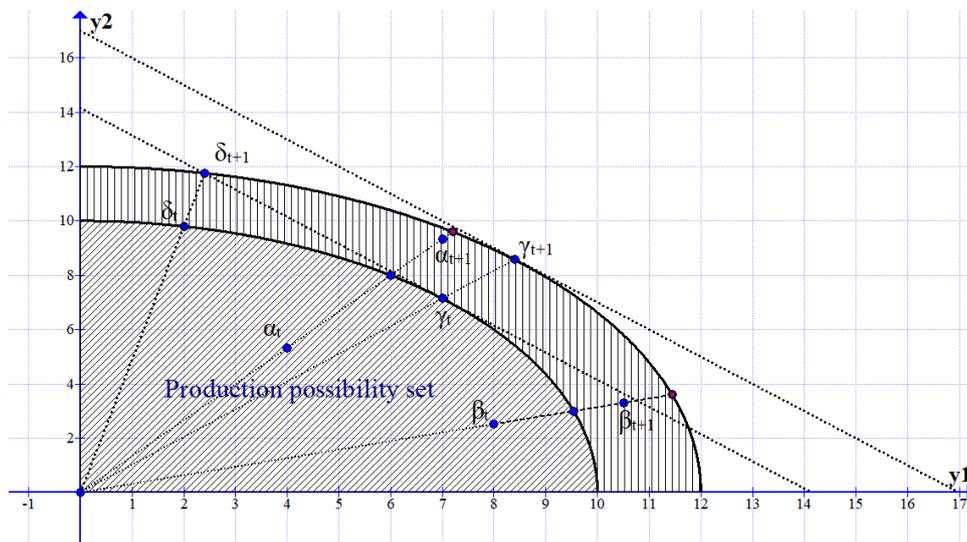
$$\Delta Ec_\alpha = \frac{Ec_{\alpha,t+1}}{Ec_{\alpha,t}} = \frac{Ec_b}{Ec_a} \quad (3.6)$$

In this case,  $\Delta Ec_\alpha > 1$ , once  $Ec_{\alpha,t+1} > Ec_{\alpha,t}$ . It means that the distance of the boundary is reduced, so there is an improvement in the effectiveness on the given period. The opposite would happen if  $\Delta Ec < 1$ , showing a decrease in effectiveness.  $\Delta Ec = 1$  indicates no change in relative effectiveness.

If the analysis considers a much longer period, a shift in the best practice boundary is likely, upwards or downwards. It makes possible two processes. The first captures the change in effectiveness related to the evolution of the set of units evaluated in the given period (the catching-up effect), for which less effective production units tend to grow faster than best practices, since they imitate and copying is always easier than innovating. The second expresses the displacement of the border that requires learning and innovation.

These two processes are related in the figure 3. In the first period, the points  $\gamma_t$  and  $\delta_t$  are on the  $t$  border, representing the best practices. The points  $\alpha_t$  and  $\beta_t$  show ineffectiveness, with  $\alpha_t$  being farther from the boundary than  $\beta_t$ . In the second period, with the displacement of the boundary ( $t + 1$ ), the point  $\alpha_{t+1}$  is closer to that than  $\beta_{t+1}$ . Therefore, the variation in the relative effectiveness of the first period is bigger than in the second. The displacement of the border was performed by the points  $\gamma_{t+1}$  and  $\delta_{t+1}$ . If the products receive the same weights, the relative allocative effectiveness of  $\gamma_{t+1}$  is greater than that shown by  $\delta_{t+1}$ .

**Figure 3 – Shifting the frontier of best practices**



**Source:** own elaboration.

The measurement of these two processes follows the development of Färe, Grosskopf, Norris & Zhang (1994) of the IM. Equation (3.7) expresses how to quantify the dynamic index of relative effectiveness (Malmquist index – IDEc) of the  $\alpha$  unit, separated in terms of changes in effectiveness (MEc) and frontiers changes (MF). The first element on the right (in parentheses) expresses the change in relative effectiveness in the period studied; the second (in brackets) expresses the border changes.

$$IDEc = \left( \frac{Ec_{\alpha,t}^t}{Ec_{\alpha,t+1}^{t+1}} \right) \left[ \frac{Ec_{\alpha,t+1}^{t+1} Ec_{\alpha,t}^{t+1}}{Ec_{\alpha,t+1}^t Ec_{\alpha,t}^t} \right]^{0,5} \quad (3.7)$$

The calculation of DEA requires the: 1) relative effectiveness of the unit evaluated in period  $t$  in relation to the boundary  $t + 1$  ( $Ec_{\alpha,t}^t$ ); 2) relative effectiveness of the unit evaluated in period  $t + 1$  in relation to the border  $t + 1$  ( $Ec_{\alpha,t+1}^{t+1}$ ); 3) relative effectiveness of the unit evaluated in period  $t$  in relation to the border  $t + 1$  ( $Ec_{\alpha,t}^{t+1}$ ); 4) relative effectiveness of the unit evaluated in the period  $t + 1$  in relation to the boundary  $t$  ( $Ec_{\alpha,t1}^t$ ).

In this case, the MEc, MF and IDEc of the evaluated unit can take different directions with positive values greater than, less than or equal to one. When  $MEc > 1$  the distance from the border is reduced, meaning that there is an improvement in relative effectiveness in the given period. If  $MEc < 1$  means a decrease in relative effectiveness.  $MEc = 1$  indicates no change in effectiveness or relative ineffectiveness.  $MF > 1$  means that the displacement of the border was upward.  $MF < 0$  indicates shift down;  $MF = 1$  means border immobility. Thus, MEc and MF can evolve in opposite directions when in the same period, meaning that an improvement in one and a worsening in another occurred.

As IDEc is the product of the two previous changes,  $IDEc > 0$  indicates a positive evolution of the unit's effectiveness in relation to the evolution of the set of units evaluated. If  $IDEc < 1$  means that the balance of the two processes indicates a decrease in relative effectiveness, while  $IDEc = 1$  show that there are zero sum gains and losses.

### 3.2. Source and treatment of data

The extraction of the data comes from the World Bank's Open Access Indicators System (WB, 2020). We selected twenty economies in America, Europe, Africa, Asia and Oceania. Due to shortages in the data sequence, we exclude some countries (e.g. Argentina) from the sample. To "run" all the data extracted from World Bank we use *MaxDEA 8 Basic Free DEA software* (MaxDEA, 2019).

The four variables from the MH model of Saavedra-Rivano & Teixeira (2017) were applied: GDP growth (annual %), inflation – consumer prices (annual %), unemployment – total (% of total labour force) (modelled ILO estimate) and current account balance (CAB, % of GDP). We grouped the historical series of these variables into two periods. The first considers the time series from 1992 to 2007 and the second from 2010 to 2018. Then, we sought to eliminate the influence of the international financial crisis of 2008-2009 and observe whether the effectiveness of economic policies changes in the period before and after the crisis. In this sense, we follow Bernard *et al.* (1988), who compares the pre-crisis economic performance in the 1960-1973 period and during the crisis of 1974-1980 in France, Japan and England, using the same four variables of the Kaldor model (also substituting income for inflation).

Like Medrano & Teixeira (2012) and Teixeira, Pinheiro & Ferreira (2014), the variables were normalized to mitigate possible biases due to their different orders of magnitude. This process started from calculating the median of each variable for each period. Subsequently, from the median, all variables were transformed in the same order of magnitude within the range of 0 and 1. The normalization of GDP growth and CAB were using the formula (3.8) and for unemployment and inflation the formula (3.9). In this way, the same direction is obtained (the bigger, the better) and the maximum value one (1) that allows representing an ideal country (wonderland macroeconomic configuration), which would have its indicators optimized.

$$\frac{x_{median} - x_{minimum}}{x_{maximum} - x_{minimum}} \quad (3.8)$$

$$\frac{x_{median} - x_{maximum}}{x_{minimum} - x_{maximum}} \quad (3.9)$$

Given the above, table 1 shows the initial results of the normalized medians for the variables of Kaldor, according to the selected economies in the relevant periods for the proposed analysis.

**Table 1 – Initial results of the four variables of Kaldor (normalized medians)**

Country	Country Code	Period 1 (1992-2007)				Period 2 (2010-2018)			
		CAB	GDP Growth	Inflation	Unempl.	CAB	GDP Growth	Inflation	Unempl.
Australia	AUS	0.205	0.641	0.998	0.802	0.237	0.595	0.998	0.853
Brazil	BRA	0.300	0.620	0.996	0.716	0.249	0.544	0.996	0.764
Canada	CAN	0.335	0.604	0.998	0.768	0.248	0.586	0.999	0.794
China	CHN	0.375	0.855	0.998	0.948	0.377	0.759	0.998	0.898
France	FRA	0.340	0.582	0.999	0.661	0.301	0.544	0.999	0.694
Germany	DEU	0.306	0.568	0.999	0.720	0.497	0.580	0.999	0.878
India	IND	0.304	0.752	0.996	0.973	0.262	0.754	0.996	0.973
Indonesia	IDN	0.359	0.688	0.995	0.832	0.276	0.685	0.997	0.905
Italy	ITA	0.329	0.560	0.998	0.673	0.346	0.535	0.999	0.619
Japan	JPN	0.390	0.549	0.999	0.915	0.391	0.548	0.999	0.935
Korea, Rep.	KOR	0.335	0.716	0.997	0.944	0.438	0.607	0.999	0.935
Mexico	MEX	0.278	0.615	0.995	0.937	0.279	0.607	0.997	0.885
Netherlands	NLD	0.437	0.611	0.998	0.897	0.532	0.559	0.999	0.843
Russian Fed.	RUS	0.515	0.676	0.989	0.745	0.423	0.568	0.996	0.858
Saudi Arabia	SAU	0.390	0.583	0.999	0.850	0.563	0.632	0.998	0.850
Spain	ESP	0.233	0.625	0.998	0.477	0.344	0.553	0.999	0.196
Switzerland	CHE	0.536	0.557	0.999	0.932	0.555	0.564	0.999	0.886
Turkey	TUR	0.287	0.732	0.970	0.730	0.194	0.717	0.995	0.659
UK	GBR	0.272	0.599	0.998	0.844	0.226	0.567	0.998	0.831
United States	USA	0.239	0.628	0.998	0.866	0.262	0.583	0.999	0.829

**Source:** own elaboration based on World Bank's Open Access Indicators System (2020).

In this sense, the next section present and discuss the mathematical results derived from the selected data, allowing then a critical macroeconomic analysis on how the variable unemployment is being tackled by Brazil compared with other nations.

#### 4. Analysis of results

This section is divided into three subsections. The first and second have static character and in them will be analysed: i) the main results, general score, projections, benchmarks (with its respective *lambdas*) and weights that each country assumes for each variable; ii) the general score of the twenty countries that make up the sample when the *kaldorian* variables assume the same weights. In these cases, the MH forms the geometric figure that summarizes the general argument of both subsections.

The analysis of the third section is dynamic (comparative), consisting of considerations on the effectiveness and changes in the frontier based on the results found for the Malmquist index and its decomposition. Lastly, we discuss the MH comparing the possibility frontier with the frontier achieved in the first and in the second periods, besides the performances of Brazil in the same time frame.

##### 4.1. Analysis of the results of the first period (1992 a 2007)

With regard to the main results of the first period (tables 2 and 3), let us start with the general score. It is observed that Switzerland, China, India, Japan and the Russian Federation have the maximum score (one), conforming the countries of the existing frontier of best practices between 1992 and 2007. Brazil is seventeenth in this regard (0.996906), ahead of Mexico (0.996783), Indonesia (0.99619) and Turkey (0.971867)<sup>6</sup>. These indicate that countries with low performance should increase the value of the variables in those registered in the table as projections. Note that the difference between the first and the last country's score is less than 0.5 (similarly to table 5). Once we use a deterministic method, it is just a matter of scale, which means that the calculation is relative (not dimensional).

<sup>6</sup> Being currently the main world economy, it is interesting to note that (along this paper) the USA always appear in intermediate positions according to the numbers revealed by the DEA-Malmquist method. Unlike China, the second economic power, which appears ahead in most of the numerical results.

Considering the free will in determining the priorities and weights of the Kaldorian variables, the countries that form the border are those with the best performance in these variables (in red): in the CAB was Switzerland (0.536497), in GDP growth was China (0.855437), in inflation was Japan (0.999313) and in unemployment was India (0.972844). It should be noted that in the specific case of inflation, the values are very close to each other for all countries (in both periods), possibly due to the adoption of similar monetary policies for this parameter. It is also relevant that, according to the data, the Russian Federation appears with a score of one due to the good performance in one of the variables (CAB), being a major exporter of natural resources (mainly oil and gas).

Table 3<sup>7</sup> also reveals this contour, once identical values (weight) for the same parameters among nations suggest the same profile. Brazil presents zero (0) for CAB and unemployment, 0.003534548 for GDP growth and 0.998747137 for inflation. Thus, it reinforces the same profile of Japan and China, among others. Relevant is the fact that the five countries that have score one shows four distinct weights composition. It seems that better performances give different weights to one or the another variable, being able to occupy this position precisely by “betting” on (at least) one of them. Therefore, the weight of each variable is determined by the country’s priority profile.

**Table 3 – Weights of Kaldorian variables in determining the effectiveness index (2)**

Country	Weight Dual Price (CAB)	Weight Dual Price (GDP Growth)	Weight Dual Price (Inflation)	Weight Dual Price (Unemployment)
Switzerland	1.86394233	0	0	0
China	0	0.003534548	0.998747137	0
India	0	0	0.936788416	0.068435688
Japan	0	0.003534548	0.998747137	0
Russian Fed.	1.568985307	0.283852655	0	0
Saudi Arabia	0.002415906	0.003645994	0.997743998	0
Netherlands	0.002415906	0.003645994	0.997743998	0
France	0	0.003534548	0.998747137	0
Germany	0	0.003534548	0.998747137	0
Canada	0	0.003534548	0.998747137	0
UK	0	0.003534548	0.998747137	0
Australia	0	0.003534548	0.998747137	0
Korea Rep.	0	0	0.961307192	0.042616034
United States	0	0.003534548	0.998747137	0
Italy	0	0.003534548	0.998747137	0
Spain	0	0.003534548	0.998747137	0
Brazil	0	0.003534548	0.998747137	0
Mexico	0	0	0.961307192	0.042616034
Indonesia	0	0.003534548	0.998747137	0
Turkey	0	0.003534548	0.998747137	0

**Source:** own elaboration based on World Bank’s Open Access Indicators System (2020).

Analysing the benchmarks and the  $\lambda_s$  (proportional importance of a country as a beacon) (table 2), we can see that only three countries among those that obtained the maximum score serve as a reference for best practices for another countries. They are, according to their respective frequencies: China (15), Japan (13) and Switzerland (4). For the specific case of Brazil, it indicates that the best strategy to reach the existing frontier is to assume a linear combination between Japan ( $\lambda_{JPN} = 0.7613926589$ ) and China ( $\lambda_{CHN} = 0.2386073421$ ).

<sup>7</sup> We changed proposedly the orders of the tables 2 and 3 only to better fit in this paper, without prejudice of its content.

**Table 2 – Main results for the first period (1)**

Country	Score	Projection (CAB)	Projection (GDP Growth)	Projection (Inflation)	Projection (Unemployment)	Benchmark ( $\lambda_s = \text{Lambdas}$ )	Times as a benchmark for another country
Switzerland	1	0.536497285	0.557485157	0.998924862	0.93219247	CHE(1.000000000)	4
China	1	0.375025074	0.855437451	0.998227057	0.947933134	CHN(1.000000000)	15
India	1	0.303736315	0.75220602	0.996407251	0.972843721	IND(1.000000000)	0
Japan	1	0.389630368	0.548600456	0.999312948	0.914663569	JPN(1.000000000)	13
Russian Fed.	1	0.515133745	0.675571357	0.989312886	0.745383857	RUS(1.000000000)	0
Saudi Arabia	0.999899	0.389504917	0.583168127	0.999085903	0.8495591839	CHE(0.0106050711); CHN(0.1125426193); JPN(0.8768523096)	0
Netherlands	0.999339	0.437194135	0.611256886	0.998308858	0.897197042	CHE(0.3431749825); CHN(0.1942641324); JPN(0.4625608851)	0
France	0.999309	0.340456451	0.582848124	0.998501707	0.660525661	CHN(0.1116151848); JPN(0.8883848162)	0
Germany	0.9993	0.306431187	0.567562489	0.998544712	0.720182162	CHN(0.0630945685); JPN(0.9369054324)	0
Canada	0.99928	0.335410785	0.604451956	0.998394530	0.768111119	CHN(0.1834424583); JPN(0.8165575427)	0
UK	0.999159	0.271881280	0.598886883	0.998292673	0.843861767	CHN(0.1655298064); JPN(0.8344701936)	0
Australia	0.999147	0.205090214	0.641007934	0.998131800	0.802087661	CHN(0.3029450744); JPN(0.6970549256)	0
Korea Rep.	0.999107	0.334964912	0.715513140	0.997482306	0.943774437	CHE(0.2106010618); CHN(0.7893989382)	0
United States	0.998937	0.238579423	0.627515893	0.997969822	0.866131576	CHN(0.2593653685); JPN(0.7406346315)	0
Italy	0.998849	0.328523373	0.559825805	0.998121120	0.672731431	CHN(0.0386858800); JPN(0.9613141210)	0
Spain	0.998646	0.232750195	0.624761073	0.997688113	0.476732102	CHN(0.2509718343); JPN(0.7490281657)	0
Brazil	0.996906	0.300101995	0.619890000	0.995962576	0.716210608	CHN(0.2386073421); JPN(0.7613926589)	0
Mexico	0.996783	0.278444680	0.614634527	0.995364075	0.937037347	CHE(0.5001086421); CHN(0.4998913579)	0
Indonesia	0.99619	0.358885438	0.688292496	0.995004242	0.832342182	CHN(0.4638428905); JPN(0.5361571095)	0
Turkey	0.971867	0.286983270	0.732421438	0.970493813	0.729892718	CHN(0.6681818053); JPN(0.3318181947)	0

**Source:** own elaboration based on World Bank's Open Access Indicators System (2020).

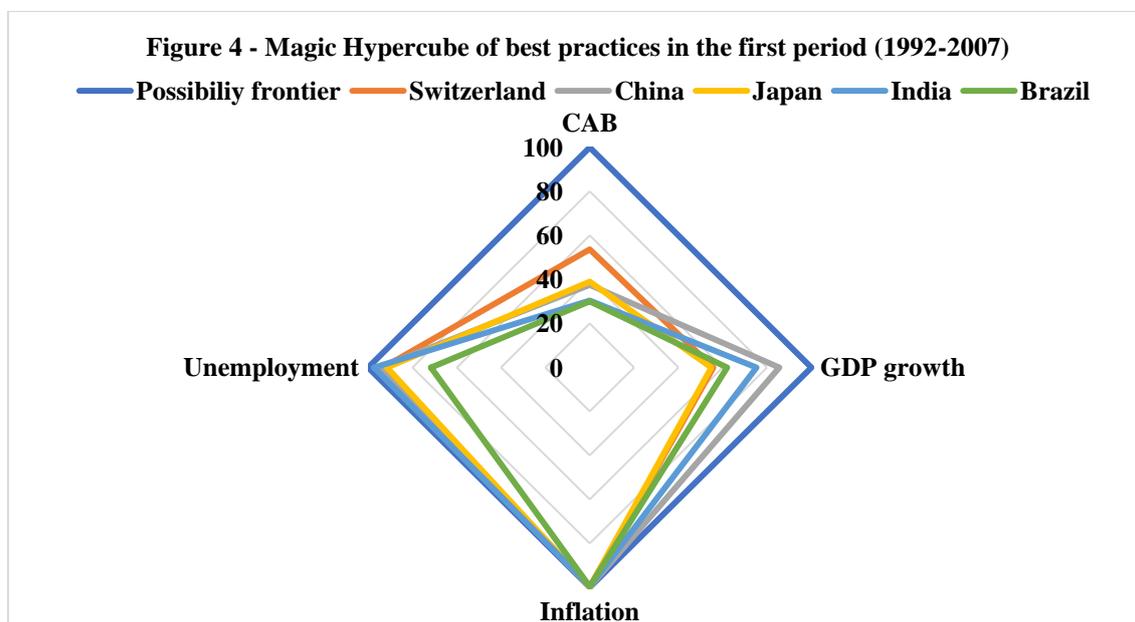
Table 4 shows the general score of the twenty countries when the four *kaldorian* variables assume the same weight (0.25 each). China is the country with the best performance (score one), followed by India (0.952330062), Switzerland (0.952300618), Korea Republic (0.941797331) and Netherlands (0.926538824). Brazil (0.82860491) occupies the sixteenth position, ahead only of mature economies – all members of the European Union – such as Germany, France, Italy and Spain, which is undesirable for a developing country that aims to catch-up in relation to the advanced countries.

**Table 4 – General score of the first period with the same weight for the four variables**

Ranking	Country	Score	Ranking	Country	Score
1	China	1	11	United States	0.859465211
2	India	0.952330062	12	Turkey	0.856189571
3	Switzerland	0.952300618	13	UK	0.854027326
4	Korea, Rep.	0.941797331	14	Canada	0.851964062
5	Netherlands	0.926538824	15	Australia	0.33060091
6	Russian Fed.	0.920915735	16	Brazil	0.82860491
7	Indonesia	0.904899516	17	Germany	0.816187752
8	Japan	0.897874125	18	France	0.81279071
9	Mexico	0.889460563	19	Italy	0.805636035
10	Saudi Arabia	0.88815021	20	Spain	0.734091421

**Source:** own elaboration based on World Bank’s Open Access Indicators System (2020).

Concerning the analysis of the MH for the first period (figure 4), it geometrically denotes the limits of the frontier of possibility (wonderland macroeconomic configuration, in blue = 100 %), the results of the countries that stood out in each of the variables respectively and the Brazilian performance in relation to it.



**Source:** own elaboration based on World Bank’s Open Access Indicators System (2020).

The magic hypercube makes evident the distance of Brazil (except in the case of inflation, for reasons described before), both in relation to the possibility frontier and the best practices of each leading country in a specific variable. From this perspective, the worst performance of the country is in CAB, followed by GDP growth (also the biggest gap – in relation to China, which has the best performance on it) and unemployment.

## 4.2. Analysis of the results of the second period (2010 to 2018)

Observing the main results (tables 5 and 6<sup>8</sup>), the country (in red) that stood out the most numerically in CAB was Saudi Arabia (0.562863551), in GDP growth was China (0.758995973), in inflation it was Switzerland (0.999331704) and unemployment was India (0.973301174). Korea Republic and Japan were also countries that got maximum score (one), despite not isolated in the leadership in any parameter. The case of Saudi Arabia can be interpreted as the general increase in the prices of the oil commodity in the period. Note that India was the country that most emphasized the issue of unemployment in both periods compared to any other nation.

Concerning the benchmarks and the  $\lambda_s$  (table 5), Switzerland (14 times) and China (9 times) are among those that obtained the maximum score and serve as a reference for best practices for other countries. For the specific case of Brazil in this period, it indicates that the best strategy is to integrally assume the Swiss practices as a benchmark ( $\lambda_{CHE} = 1$ ). This means that in order to improve effectiveness, Brazil must fully mimic Switzerland and increase the values of the variables indicated in the projections in table 5, which are consistent with the original values of Switzerland.

It is interesting to verify that Brazil and Switzerland do not give the same weight to the variables as indicated in table 6. This means that Brazil is setting priorities that are very different from those adopted by the best practices. Nevertheless, the most similar benchmark to Brazil is Switzerland.

**Table 6 – Weights of Kaldorian variables in determining the effectiveness index (2)**

Country	Weight Dual Price (CAB)	Weight Dual Price (GDP Growth)	Weight Dual Price (Inflation)	Weight Dual Price (Unemployment)
China	0	0.008622	0.9708	0.02709
India	0	0.008622	0.9708	0.02709
Japan	0	0	0.934	0.07137
Korea Rep.	0.0002423	0.0087	0.9703	0.02745
Saudi Arabia	0.01188	0.01592	0.9851	0
Switzerland	0	0.004801	0.9935	0.005015
Australia	0	0.005145	0.9978	0
Canada	0	0.005145	0.9978	0
France	0	0	1.001	0
Germany	0	0.005145	0.9978	0
Italy	0	0	1.001	0
Netherlands	0	0	1.001	0
Spain	0	0	1.001	0
UK	0	0.005145	0.9978	0
United States	0	0.005145	0.9978	0
Indonesia	0	0.004801	0.9935	0.005015
Mexico	0	0.005145	0.9978	0
Brazil	0	0	1.001	0
Russian Fed.	0	0.005145	0.9978	0
Turkey	0	0.005145	0.9978	0

**Source:** own elaboration based on World Bank's Open Access Indicators System (2020).

<sup>8 8</sup> We changed proposedly the orders of the tables 5 and 6 only to better fit in this paper, without prejudice of its content.

**Table 5 – Main results for the second period (1)**

Country	Score	Projection (CAB)	Projection (GDP Growth)	Projection (Inflation)	Projection (Unemployment)	Benchmark ( $\lambda_s = \text{Lambdas}$ )	Times as a benchmark for another country
China	1	0.377301274	0.758995973	0.998326539	0.898028781	CHN(1.000)	9
India	1	0.261675269	0.75441952	0.99626693	0.973301174	IND(1.000)	0
Japan	1	0.391164897	0.547727622	0.99915856	0.935457048	JPN(1.000)	1
Korea Rep.	1	0.438166917	0.607054385	0.998614861	0.935457048	KOR(1.000)	0
Saudi Arabia	1	0.562863551	0.632197466	0.998138239	0.850203775	SAU(1.000)	0
Switzerland	1	0.555194335	0.564064452	0.999331704	0.885511102	CHE(1.000)	14
Australia	0.999	0.236940871	0.594517289	0.998387263	0.852532654	CHN(0.159); CHE(0.841)	0
Canada	0.999	0.247613149	0.586224109	0.998556601	0.794102968	CHN(0.116); CHE(0.884)	0
France	0.999	0.300626659	0.54390289	0.998828398	0.694252699	CHE(1.000)	0
Germany	0.999	0.497181502	0.580207054	0.998600968	0.878025456	CHN(0.085); CHE(0.915)	0
Italy	0.999	0.346028627	0.535049263	0.998738035	0.618938683	CHE(1.000)	0
Netherlands	0.999	0.53240761	0.5591407	0.998660306	0.843092405	CHE(1.000)	0
Spain	0.999	0.343511525	0.553188775	0.998518964	0.195625074	CHE(1.000)	0
UK	0.999	0.225923399	0.567417001	0.99822157	0.831073772	CHN(0.020); CHE(0.980)	0
United States	0.999	0.261932497	0.58342084	0.998535824	0.828661725	CHN(0.101); CHE(0.899)	0
Indonesia	0.998	0.276216924	0.685002008	0.996853748	0.904849044	CHN(0.648); JPN(0.257); CHE(0.095)	0
Mexico	0.998	0.279444592	0.606787036	0.997390788	0.88517841	CHN(0.225); CHE(0.775)	0
Brazil	0.997	0.249045382	0.544065145	0.99633859	0.763869259	CHE(1.000)	0
Russian Fed.	0.997	0.423368178	0.567791242	0.996028062	0.858188467	CHN(0.029); CHE(0.971)	0
Turkey	0.997	0.193502342	0.716800965	0.99520148	0.659485983	CHN(0.796); CHE(0.204)	0

**Source:** own elaboration based on World Bank's Open Access Indicators System (2020).

Forward in the analysis is the general score of the second period with the same weight for the four variables (table 7). Saudi Arabia is the country with the best performance (score one), followed by China, Switzerland, India and Korea Republic. Brazil (0.838968205) occupies the seventeenth position.

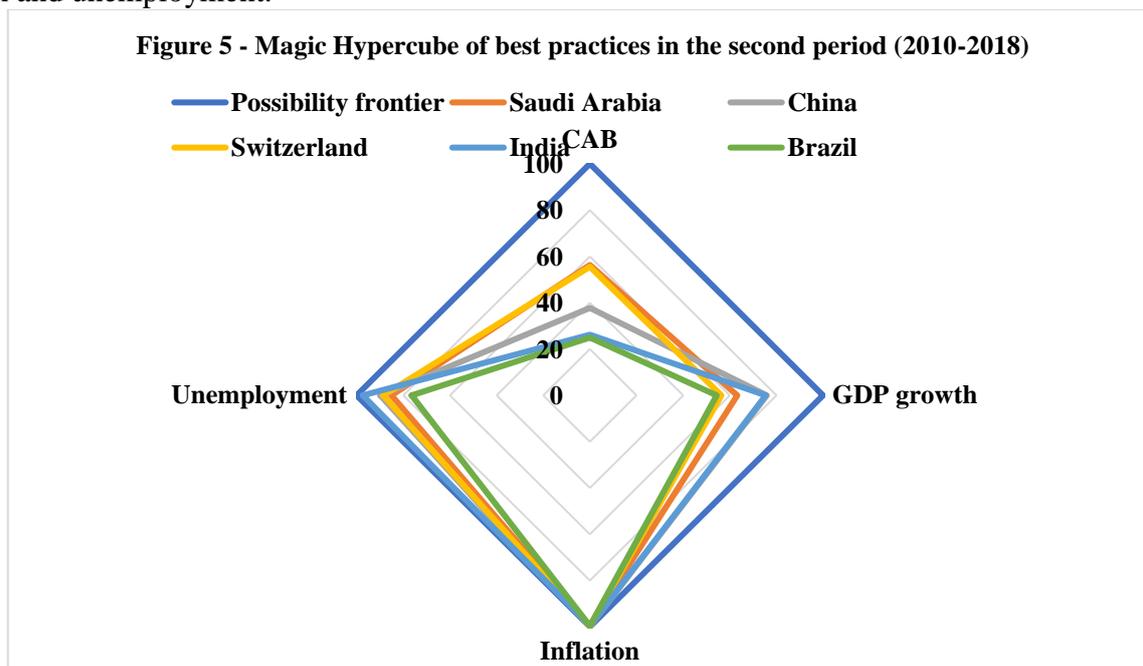
**Table 7 – General score of the second period with the same weight for the four variables**

Ranking	Country	Score	Ranking	Country	Score
1	Saudi Arabia	1	11	Mexico	0.909771331
2	China	0.996467618	12	Australia	0.881374583
3	Switzerland	0.987086351	13	United States	0.878145569
4	India	0.981027772	14	Canada	0.863013147
5	Korea, Rep.	0.978934824	15	UK	0.861744474
6	Germany	0.970628915	16	Turkey	0.842803514
7	Netherlands	0.963822731	17	Brazil	0.838968205
8	Japan	0.944176009	18	France	0.833806966
9	Indonesia	0.940697533	19	Italy	0.821039666
10	Russian Fed.	0.934932351	20	Spain	0.687008693

**Source:** own elaboration based on World Bank’s Open Access Indicators System (2020).

Concerning the analysis of the MH for the second period (figure 5), equally to the figure 4 it geometrically denotes the limits of a frontier of possibility (in blue = 100), the results of the countries that stood out in each of the four variables respectively and the Brazilian performance in relation to it.

Once again, the MH makes evident the distance of Brazil (except in the case of inflation) both in relation to the possibility frontier and the best practices of each leading country relating to a specific variable. From this perspective, the worst performance of the country is in CAB, followed by GDP growth and unemployment.



**Source:** own elaboration based on World Bank’s Open Access Indicators System (2020).

### 4.3. Malmquist index and changes on effectiveness and frontier

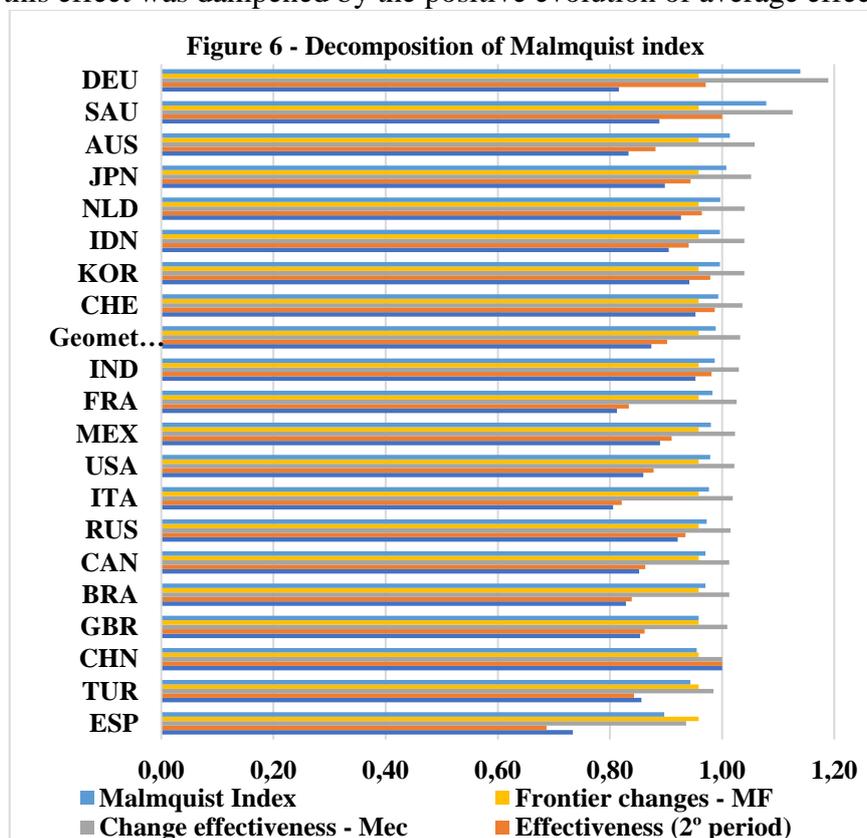
From equation (3.7) and data obtained, the dynamic index of relative effectiveness (Malmquist index – IDEc) of the selected countries was determined, separated in terms of change of effectiveness (MEc) and frontier changes (MF) of two periods. These results were obtained by giving the same weight in the four kaldorian variables and are recorded in table 8. Remember that: i) values below the unit here indicate the fall, that is, the negative variation; ii) those greater than one correspond to growth; iii) those equal to one show that there were no changes.

**Table 8 – Dynamic index of relative effectiveness (IDEc), changes in effectiveness (MEc) and frontier changes (MF)**

Country	Change effectiveness	Frontier changes	Malmquist index
Australia	1.06	0.96	1.01
Brazil	1.01	0.96	0.97
Canada	1.01	0.96	0.97
Switzerland	1.04	0.96	0.99
China	1.00	0.96	0.95
Germany	1.19	0.96	1.14
Spain	0.94	0.96	0.90
France	1.03	0.96	0.98
UK	1.01	0.96	0.97
Indonesia	1.04	0.96	1.00
India	1.03	0.96	0.99
Italy	1.02	0.96	0.98
Japan	1.05	0.96	1.01
Korea	1.04	0.96	1.00
Mexico	1.02	0.96	0.98
Netherlands	1.04	0.96	1.00
Russian Fed.	1.02	0.96	0.97
Saudi Arabia	1.13	0.96	1.08
Turkey	0.98	0.96	0.94
United states	1.02	0.96	0.98
<b>Geometric mean</b>	<b>1.03</b>	<b>0.96</b>	<b>0.99</b>

Source: own elaboration based on World Bank’s Open Access Indicators System (2020).

In table 8 and figure 6 it is observed that the geometric mean of the countries’ Malmquist index indicates a reduction of 1% in the effectiveness of macroeconomic policies expressed by the four *kaldorian* variables. This retraction was due to the 4% shift down the border in the second period, motivated by the slow recovery after the 2008-2009 financial crisis. It justifies the elimination of these two years of the time series, removing the influence of this specific event and observe whether the effectiveness of economic policies changes in the period before and after it. However, the geometric mean shows that this effect was dampened by the positive evolution of average effectiveness (3%).



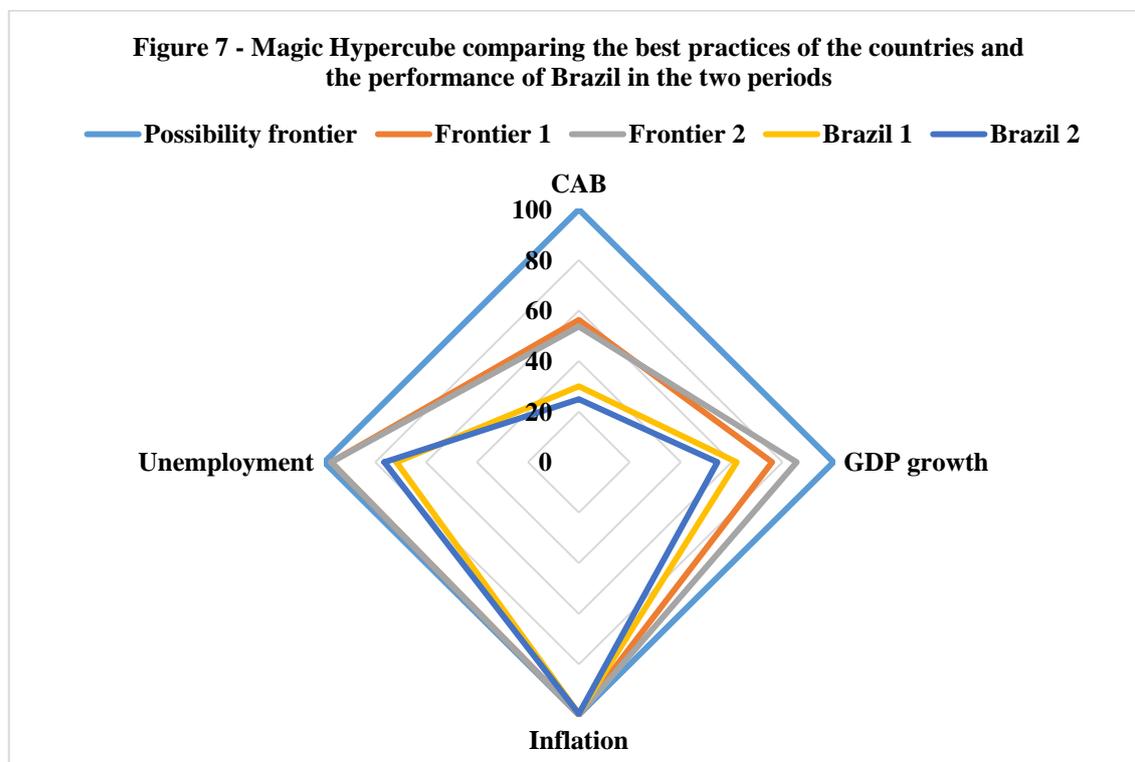
Source: own elaboration based on World Bank’s Open Access Indicators System (2020).

In what concerns the MH (table 9 and figure 7), here we compare the possibility frontier with the frontier achieved in the first and in the second periods, besides the performances of Brazil in the same period. In the first case, the frontiers are almost the same for inflation and unemployment, while the second frontier shrinks for CAB and expand considerably in GDP growth (most responsible for the boundary change). For Brazil, we observe that in the second period CAB and GDP growth retract considerably, inflation slightly expand and unemployment expand reasonably.

**Table 9 – Comparison of the best practices of the countries and the performance of Brazil in the two periods**

Variable	Frontier 1	Frontier 2	Brazil 1	Brazil 2
<b>CAB</b>	56.28635508	53.64972853	30.01019952	24.90453822
<b>GDP growth</b>	75.89959733	85.5437451	61.98900001	54.4065145
<b>Inflation</b>	99.93317039	99.93129477	99.59625757	99.63385901
<b>Unemployment</b>	97.33011742	97.2843721	71.62106084	76.38692593

Source: own elaboration based on World Bank’s Open Access Indicators System (2020).



Source: own elaboration based on World Bank’s Open Access Indicators System (2020).

In this scenario, Brazil indicates a certain resilience to this shift from the border downwards. The change in Brazilian effectiveness was positive ( $IDEc = 1.01$ ), indicating that it approached the frontier of best practices (figure 6). In this context, China registered a significant drop in the effectiveness of its macroeconomic policy, but, as indicated by  $MEc = 1$  and figure 7, the country remained on the border in both periods.

The (majority of the) other countries have adapted better to changing borders, specially Germany, Saudi Arabia, Japan and Australia. This gain in relative effectiveness can be interpreted as an indication of the “catching-up” effect of countries that are below the border and of the greater vulnerability of the countries that compose it. If this indication becomes evidence, we can talk about a process of convergence.

## 5. Concluding remarks

In this paper, the DEA-Malmquist enabled to measure the relative effectiveness and frontiers of twenty selected economies between 1992 and 2018. This was proceeded according to the MH approach, using the four variables of Kaldor: CAB, GDP growth, inflation and unemployment. Given the presentation and analysis undertaken along the study, we reached numerical results to capture the global performance of these nations.

In the first period (1992-2007), the MH makes evident how far Brazil is (except for inflation) both in relation to the possibility frontier (ideal country) and the best practices of each leading country with reference to a specific variable. Thus, the worst performance of the country was in CAB, followed by GDP growth and unemployment. In the second period (2010-2018), Brazil's results shows that its CAB and GDP growth retracted considerably, inflation slightly expanded and unemployment expanded reasonably. Thus, whatever the criteria discussed along this research, there is ample room for improvement of Brazil regarding the frontier of possibilities, since it is far from the leading nations.

For the general changes of effectiveness and frontier, it is observed that the geometric mean of the countries' Malmquist index indicates a reduction of 1% in the effectiveness of macroeconomic policies expressed by the *kaldorian* variables – due to the 4% shifting down the border in the second period. In this perspective, Brazil indicated resilience to this shift from the border downwards, changing its effectiveness positively (approaching the best practices – the so-called “catching-up” effect).

Considering now that the MH compared the possibility frontier with the frontiers achieved in the first and in the second periods, besides the performances of Brazil in the same periods. In the first case, the frontiers were almost the same for inflation and unemployment, while the second frontier shrunk for CAB and expands considerably in GDP growth. The case of Brazil was a simple (but revealing) juxtaposition of the results of the first and second periods.

These outcomes allows a macroeconomic analysis specifically on how the variable unemployment is being tackled by Brazil compared with other nations, shedding light on some points that may be of interest for reflection and action with regard to the development of the country in general and its world of work in particular. Nevertheless, it should be clarified that this numerical exercise do not intend to explain this variable only by a single (or a set) of indicator(s) derived from one quantitative tool, once it is not enough to capture all the aspects of a diverse and complex Brazilian labour market. Besides, we have to be aware that with distinct or additional variables interacting with unemployment we possibly could reveal a different panorama, even more realistic.

The numerical results found revealed that the unemployment situation in Brazil is consistent with the observable empirical reality. In fact, the movements of its labour market shows great volatility resulting from the different internal and external movements in relation to the other three Kaldor variables. Suffice it to mention that since mid-2014 the unemployment rate has remained at around 12% (with more than 40% of employees performing informal jobs), with a high rate of inflation (above the central target stipulated by monetary policy), low GDP growth and exports basically of low value added commodities (following as dependent on imported technology). It is true that it was not that different since the 1990s, although with some favourable nuance here and there. This pleads the question: are we on the right path for the country's development?

The fundamental consideration to be highlighted is that in Brazil, labour market are poorly compatible with the desired promotion of business competitiveness and increased investment, generating jobs in unsatisfactory quantity and quality. Essentially, the country has labour market institutions that are not yet fully adequate to promote the effective and sustainable reduction of historically existing unemployment, even though cyclical movements have contributed in specific periods of its history in this aspect. Furthermore, the need for institutional improvements is challenging in order to provide the Brazilian labour market with an arrangement that favours its production to compete in more equitable conditions in the international markets and, at the same time, guarantees the protection of workers in national scope. In this sense, proposals such as the universal minimum income and the state as an employer of last resort are new and promising.

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