Do banks have right incentives to finance innovations? *

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Abstract

We present a model of bank credit in which the risk profile of a project affects a bank's lending decision and the loan's interest rate. Entrepreneurs seek to finance risky and safe projects by borrowing from a monopolistic bank, or entrepreneurs can obtain financing from sophisticated investors. Good risky projects advance while bad risky projects default. The bank cannot observe the type of the borrower, but decides whether to lend and at what interest rate. Three possible equilibria can emerge: *(i)* no loans are made, and interest rates are so high all types default; *(ii)* interest rates extract full surplus from good risky projects, and all other types default; *(iii)* interest rates extract full surplus from safe projects, and only bad risky projects default. When considering the resulting comparative statics in the context of innovation, our results imply that good innovative projects might be abandoned due to adverse selection issues in bank financing.

Keywords: debt, bank, innovation, project financing.

Resumo

Apresentamos um modelo de crédito bancário em que o perfil de risco de um projeto afeta a decisão do banco em conceder ou não um empréstimo e a taxa de juros. Empreendedores buscam financiar projetos, arriscados ou seguros, tomando empréstimos de um banco monopolista ou obtendo financiamento de investidores sofisticados. Projetos arriscados bons são levados adiante enquanto projetos arriscados ruins não seguem adiante. O banco não pode observar o tipo do tomador, mas tem que decidir se concede o empréstimo e a taxa de juros. Três possíveis equilíbrios podem emergir: (i) nenhum empréstimo é concedido e as taxas são tão altas que todos os tipos de projetos não seguem adiante; (ii) as taxas de juros extraem todo o excedente dos projetos arriscados bons e os demais tipos não seguem adiante; (iii) as taxas de juros extraem todo o excedente dos projetos seguros e somente projetos arriscados ruins não seguem adiante. Quando consideramos os desdobramentos da estática comparativa no contexto de inovação, nossos resultados implicam que projetos inovativos bons poderiam ser abandonados devido a problemas de seleção adversa no financiamento bancário.

Palavras-chave: dívida, bancos, inovação, financiamento de projetos.

Área ANPEC: Microeconomia, Métodos Quantitativos e Finanças (Área 8)

Classificação JEL: C73, D21, D82, G14, G21.

*The views expressed in the paper are those of the authors, and do not necessarily reflect those of the Central Bank of Brazil. All remaining errors are the authors sole responsibility

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1 Introduction

Innovation and credit markets are deeply related. As a matter of fact, Laeven et al. (2015) argue that a condition necessary for product innovation to be sustained in the long run is that financiers keep improving their screening methods through financial innovation. Separating good projects from bad ones is vital for firms that operate in credit markets. Banks tend to choose safe projects over risky ones because limited liability implies that only the downside risk matters for creditors. This feature of the credit market tends to negatively affect R&D projects, which are selected out by banks' screening processes.

We provide a model of the credit market that replicates this feature of credit markets. In our model, a monopolistic bank decides whether or not to give credit to entrepreneurs looking for financing for their projects. Banks and entrepreneurs play a sequential game of complete but imperfect information. Projects are of two types: risky and safe, and the projects' types are the entrepreneurs' private information. Before applying for a loan, an entrepreneur receives a signal that perfectly reveals to him whether the project will succeed. This signal is also the entrepreneur's private information. The banker chooses whether to finance a project and at what interest rate without observing the project's type or success prospects. Before applying for a loan with the monopolistic bank, the entrepreneurs may encounter a venture capitalist willing to finance the project. Because of the projects' heterogeneity, different interest rates attracts different types of projects, so that the banker may choose an interest rate that services only projects of a certain type. There is adverse selection, so that bad risky projects will always be serviced, followed by a default. Very high interest rates only attract entrepreneurs with this type of project. Moderate interest rates attract both good and bad risky projects, but not the safe project. Low interest rates service all types of projects. These three different scenarios are all possible equilibrium outcomes that occur if the combination of parameters is consistent with that outcome.

Our model shows that good risky projects might not find an investor nor be accepted to receive a loan from the bank. We also show that the *ex ante* probability that a project is risky, the probability that a risky project is good, and the availability of a sophisticated investor affect the loan decisions of the bank and the interest rate charged in equilibrium. The impact of an increase in the proportion of risky projects is ambiguous, and the minimum project return required to obtain a loan can either rise or fall as a result, depending on parameter values. The impact of having more risky projects around on the interest rate is less ambiguous. Having more risky projects may raise, but never lower, the equilibrium interest rate. A higher rate of success in risky projects reduces the project return required to obtain a loan, regardless of whether the project is risky or safe. Changes in the availability of sophisticated investors is fundamental to the bank's decision of financing debt to innovative projects entrepreneurs, but does not affect the equilibrium interest rate.

Extant literature states that banks should be more likely to lend if the project is good, if the borrower can offer a valuable collateral, or if the entrepreneur contributes with a large enough share of the initial investment (Hart and Moore, 1990). Our model includes collaterals and beliefs about the type and success prospects of a project before financing. Moreover, recent work of Laeven et al. (2015) has modelled financial innovation as a process of improving the screening of technological entrepreneurs. One of their results is that screening methods become inadequate as the technological frontier advances. In our framework, the availability of sophisticated investors plays a similar role as the dynamics of financial innovation in Laeven et al. (2015). In fact, we can interpret these two hypotheses as being the same. Financial innovations increase investors' ability to find innovative projects to invest in. Higher investors' ability to assess innovative projects imply a higher availability of private equity funds to finance entrepreneurs.

The importance of the role of banks in financing innovative projects has been documented in several empirical studies. Robb and Robinson (2014) showed, using an eight-year panel based on confidential, restricted-access data from the Kauffman Firm Survey, that newly founded firms rely heavily on owner-backed bank loans, business bank loans, and business credit lines. Berger and Udell (1998) state that, in 1993, commercial banks provided 18.75% of total finance to US small businesses. These works used small and young firms as proxies for innovative firms. Observing innovative outcome, Nanda and Nicholas (2014) measured patent novelty, using data from the Great Depression, and the results suggest that firms were more conservative in innovation and R&D after bank distress. Innovation can also benefit indirectly from other sources of financing such as retained earnings, non-bank financial institutions, or individuals. Besides providing financing directly, these investors increase the credit market's capacity to provide funding because, when they have a lot of resources available, borrowing conditions improve (Amore et al., 2013).

Researchers have also identified many sources of interaction between credit markets and innovation. For example, an increase in banks' market and bargaining powers can allow banks to extract a large share of the future cash flows from the firm, leading banks to finance projects of lower upfront returns (Petersen and Rajan, 1995). Manso (2011) shows that a managers' contract that optimally motivates innovation is one that exhibits tolerance for early failure and reward for long-term success. Firms with good innovative projects should seek private financing because the presence of non-sophisticated investors and liquidity traders on the market imply that the stock market never pays the innovator the full value of its project (Ferreira et al., 2012). On the other hand, an innovative firm that chooses to remain private is exposed to higher sensitivity to local banking conditions than it would otherwise be if instead it went public, as evidence provided by Cornaggia et al. (2015) suggests. Hsu et al. (2014) tested the effects of financial development, in equity and credit markets, and found that credit markets discourage innovation in industries that are more dependent on external finance or in high-tech industries.

From the point of view of banks, the relevant risk is default risk, and a good assessment of a borrower's default risk depends on having good information about the borrower's project, which banks seldom have. Because of this to information asymmetry, banks have to rely on a variety of screening devices to uncover the real nature of the project to be financed. One such device is the interest rate. Stiglitz and Weiss (1981) showed that adverse selection and moral hazard effects can lower loanable funds to a level below that of a "Walrasian equilibrium", the one at which demand equals supply, and thus credit rationing may occur in equilibrium. In this scenario, banks may not have incentives to increase the supply of loans by raising interest rates, even in the presence of excess demand. The reasons are that higher interest rates i) attract riskier projects and discourage safer projects, and ii) induce firms to undertake projects with lower probability of success but higher payoffs when successful. The first is an adverse selection effect, while the latter is a moral hazard effect. When credit markets are characterized by rationing, these information asymmetry problems cause reduction in loan profits.

The second screening device is the collateral requirement. Stiglitz and Weiss (1981) point out that a lender's decision to increase collateral requirements may decrease the returns to the bank by inducing individual investors to undertake riskier projects. Riskier borrowers may more often pledge collateral, implying that secured loans can be riskier than unsecured ones (Berger and Udell, 1990). However, if we assume that low-risk borrowers do not face a binding constraint on the amount of collateral they can provide, then collateral requirements can work as signalling devices. Contracts with collateral requirements may then allow banks to sort borrowers into risk classes, eliminating credit rationing with this signalling mechanism (Bester, 1985; Besanko and Thakor, 1987).

The current study contributes to the discussion Stiglitz and Weiss (1981) started by analyzing how bank loan decisions made by banks affect innovative projects. We use some of the features of Ferreira et al. (2012) to obtain a model in which the most innovative projects are selected out of financing because they are too risky. This theoretical prediction is consistent with empirical evidence relating innovation and financing that found that sample selection problems are present in innovation data (Lewbel, 1997).

Several empirical studies point out that the effects of finance on technological progress can be biased due to sample selection problems. In order to overcome this problem, many studies used banking deregulations passed by state legislatures in different U.S. states as exogenous shocks in credit supply. Results from these studies indicate that deregulation had impacted the economy's innovative activity, in terms of quantity (number of patents) and quality (risk associated to originality and generality of patents). Banks' bargaining power (Chava et al., 2013) and firms' financing constraints (Amore et al., 2013; Cornaggia et al., 2015) were affected by changes in competition in the banking industry due to deregulation, and the increased bank competition that resulted is associated with more and better innovation. These studies provide evidence linking bank competition, credit availability to firms and innovative activity. Additionally, firms more dependent on lending relationships were more affected by the shocks (Hombert and Matray, 2017). However, these studies do not evaluate the effects of innovative activities on loan markets.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 discusses the equilibrium and the effects on the bank's loan supply of: *(i)* a project's riskiness (probability that a particular project is risky); *(ii)* a project's quality (probability that a risky project is good); and *(iii)* the availability of private equity financing (by a sophisticated investor). Section 5 shows how innovative entrepreneurship affects the bank loans market. Section 6 concludes with a brief review and issues for further research.

2 Model setup

There are two risk-neutral players, an entrepreneur (E) and a monopolistic bank (B). Private financing is provided by a monopolistic bank and sophisticated investors (SI). As in Ferreira et al. (2012), sophisticated investors know the true value of the project they might invest in.

There are two types of project: safe (C) and risky (I). In contrast to entrepreneurs and sophisticated investors, the bank cannot observe the type nor the return of the project. The return of the project can assume three possible values, $R_\pi \in \{R_C, R_I, 0\}$, and it depends on the type of the project and the nature of a risky project (good or bad). A safe project yields a strictly positive return of R_C with certainty. Before financing the project, a risky project entrepreneur receives a signal that perfectly reveals the quality of their project. This signal's outcomes are success (S) and failure ($\overline{S}$). A success outcome reveals that the project is good, while a failure outcome reveals that the project is bad. The return of a good (successful) risky project is $R_I > R_C$, and the return of a bad (unsuccessful) risky project is zero.

Sophisticated investors may be available (A) or unavailable ($\overline{A}$). If available to finance the entrepreneur's project, sophisticated investors compete Bertrand-style for the project so that they end up paying the entrepreneur the project's true value for their share in the project.

A state of nature consists of the type of project, $\pi \in \{C, I\}$, the signal revealing the risky project's type (good or bad), $\tau \in \{S, \overline{S}\}$, and the availability of a sophisticated investor to finance the project, $\alpha \in \{A, \overline{A}\}$. There is a joint probability distribution determining the probability of occurrence of a particular state of nature. Let $\mu \equiv Pr(I)$ denote the marginal probability that a project is risky, $p \equiv Pr(S)$ the marginal probability that the risky project is good, and $e \equiv Pr(A)$ the marginal probability that a sophisticated investor is available. Given the true state of nature, the entrepreneur first searches for a sophisticated investor willing to invest in his project.

The investment needed to fund the project is normalized to unity. If the bank chooses to grant the loan, it will finance the project in its entirety. The bank grants a loan at a strictly positive interest rate r, and requires the entrepreneur to pledge a non-negative collateral γ, which we assume worth no more than the value of the loan, that is, $0 \leq \gamma < 1$. [1] If the gain in defaulting, the difference between the principal and the collateral value, $1 - \gamma$, is greater than the gain in moving forward with the project and repaying the loan and interest to the bank, the difference between return of the project and loan's interest rate, $R_\pi - r$, then the borrower will choose to default. As in Allen (1983), there are no other costs in defaulting, such as criminal or other legal penalties. The game is played once, and thus, there are no penalties involving future exclusion from the loan market. We assume the tie-breaking rule that when entrepreneurs are indifferent between to default and not to default, they choose the latter. For simplicity, there are no costs in the execution of the collateral.

The game starts at date 1. At this time, Nature (N) chooses π and τ. Then, given π and τ, the entrepreneur decides whether to search for a sophisticated investor. If he does search for a sophisticated investor, then nature chooses α, which reveals whether sophisticated investors are available to finance the entrepreneur's project. If available, the sophisticated investors compete Bertrand-style for the project, making a take-it-or-leave it offer to the entrepreneur for their share in the project if its value is strictly positive. Let $\delta \in \{D, \overline{D}\}$ represent the entrepreneur's decision on whether to accept the sophisticated investor's offer. If a deal is reached and the entrepreneur accepts the offer (D), the entrepreneur moves forward with the project, the game ends and payoffs are realized.

If unable to obtain financing from a sophisticated investor, the entrepreneur then applies for a bank loan to finance his project. There are three scenarios in which the entrepreneur applies for a bank loan: *(i)* the entrepreneur rejects the offer ($\overline{D}$), *(ii)* sophisticated investors are not available ($\overline{A}$), or *(iii)* the entrepreneur does not receive a financing offer from a sophisticated investor (which occurs in case the project is risky but bad and, thus, has zero value).

Still at date 1, given that an entrepreneur has requested a loan, the bank decides whether to lend the requested amount and the interest rate r to be applied to the loan, if the loan is granted. Let $\lambda \in \{L, \overline{L}\}$ be the bank's lending decision, where L represents a decision to lend the money to the entrepreneur, and $\overline{L}$ represents the decision of not lending the money to the entrepreneur. The bank does not know the type of the project he is asked to finance, but decides about lending and interest rate based on its beliefs. If the bank decides not to lend, the game ends, and the payoff is zero for both players.

At date 2, given that the bank has granted a loan, the entrepreneur decides whether to honor the debt, a decision we denote by $\eta \in \{H, \overline{H}\}$, where H represents the decision of honoring the debt, and $\overline{H}$ represents the decision to default on the debt. This decision depends on the type of the project that the entrepreneur is financing. In the case the entrepreneur decides to default on the debt, he surrenders assets worth γ to the bank, and keeps the remaining amount $1 - \gamma$. Otherwise, the entrepreneur moves forward with the project, repays his loan, and obtains the returns of the project. In either case, the game

[1] Collaterals can be used as part of an incentive mechanism to solve the moral hazard problem that arises in loan repayment (Bester, 1987), but can only solve the moral hazard problem completely if the collateral value is at least as high as the debt balance. Therefore, the assumption that the collateral value is lower than the principal keeps the moral hazard problem relevant in the model, as the borrower may choose not to repay the loan and surrender his collateral.

ends and payoffs are realized. We assume that players' strategies are sequentially rational, and look for sets of strategies and beliefs that constitute pure strategy Perfect Bayesian Equilibria. Figure 1 shows the timeline of the game. Figure 2 shows the extensive form representation of the game.

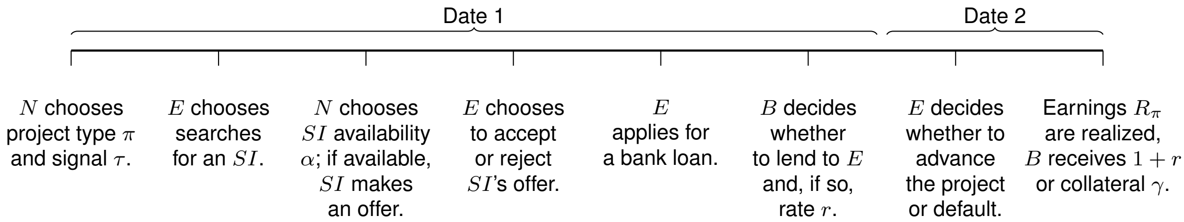

Figure 1: Timeline of the game

3 Credit market equilibrium

The characterization of equilibrium is done by backward induction. However, before examining the last stage, the first thing to notice is the entrepreneur's decision on whether to accept a deal offer from the sophisticated investor. An entrepreneur with a risky project of signal $\overline{S}$ is not faced with such a decision to make, given that the project has zero value, and thus no offer is made by sophisticated investors. There also are a probability $1-e$ of unavailability of a sophisticated investor to finance the project. However, if a sophisticated investor is available, an entrepreneur with a risky project with signal S is faced with an offer of R_I, the true value of the project. If the entrepreneur rejects this offer, the maximum project's gain is $R_I - r$ by applying for a bank loan. This occurs because the risky project with signal S is the most profitable project and $R_I - r < 1-\gamma$ implies that this entrepreneur will want to default on the loan if he is granted one. Thus the bank will not be willing to make any loans, as the Bank would incur a loss of $1-\gamma$, and in this case the entrepreneur obtains zero. If, on the other hand, $R_I - r \geq 1-\gamma$, and the entrepreneur has an incentive to repay the loan if granted one, then the entrepreneur obtains $R_I - r$. One way or another, the entrepreneur obtains a higher payoff by accepting the sophisticated investor's deal, which means entrepreneur's action is always D. In other words, the probability $Pr(\overline{D} \mid I, S, A)$ of a successful risky project's entrepreneur to reject an investor's offer is zero. Consequently, the node in bank's information set where the state of nature is (I, S, A) and the entrepreneur chooses $\overline{D}$ is never reached, provided that rejecting the sophisticated investor's offer is strictly dominated in this case. The bank attributes probability zero to this type of entrepreneur.

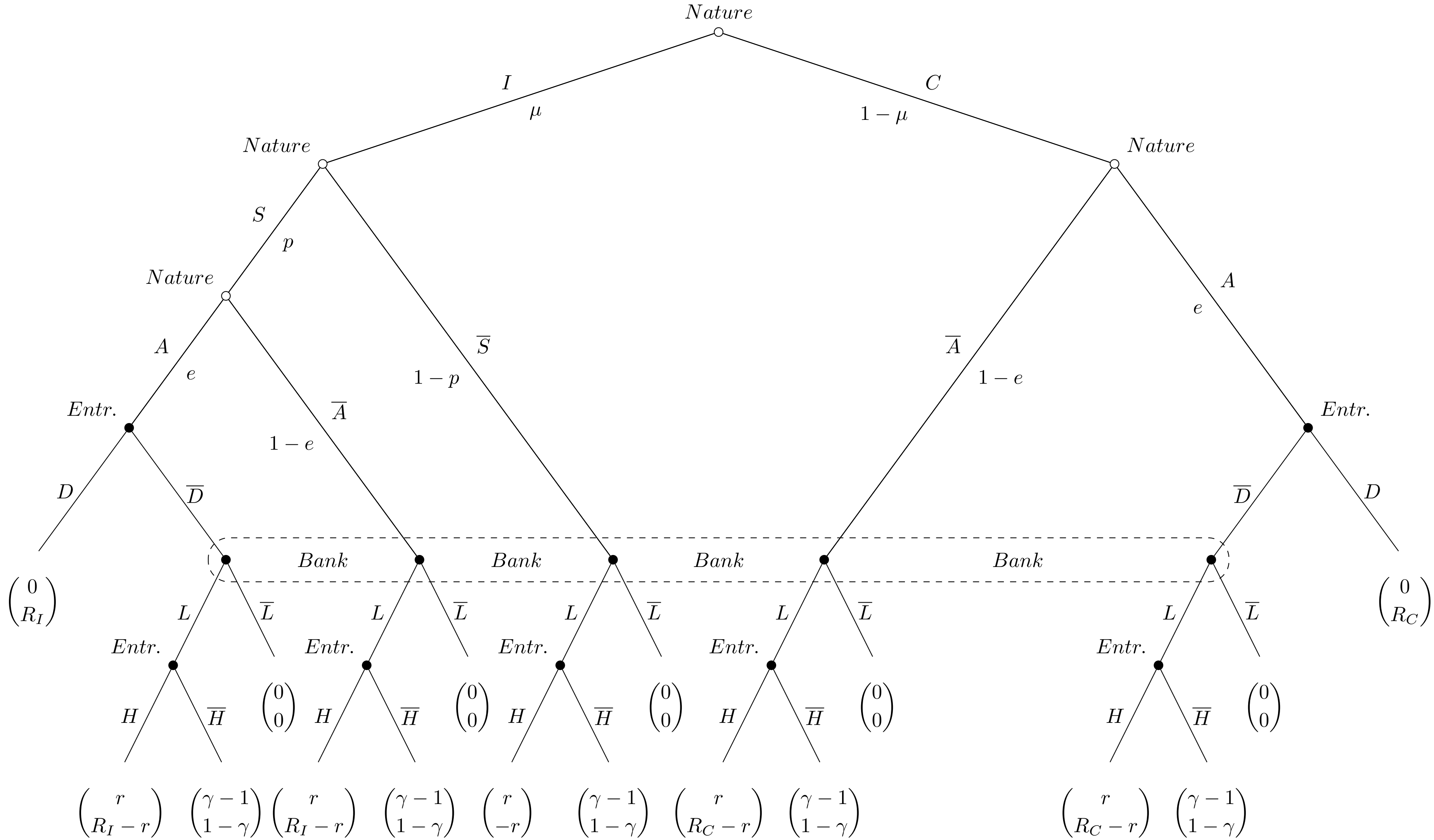

Figure 2: Extensive-form representation of the game

Consider now the situation in which a sophisticated investor makes an offer to an entrepreneur with a safe project. This entrepreneur obtains value R_C by accepting the sophisticated investor's deal, and obtains the maximum between $R_C - r$ and $1-\gamma$ if rejecting the offer and applying for a bank loan. If $R_C \geq 1-\gamma$, the entrepreneur has no incentive to reject the sophisticated investor's deal. However, if $1-\gamma > R_C$, the entrepreneur may be willing to apply for a bank loan, and the probability $P(\overline{D} \mid C, A)$ of a conventional project entrepreneur rejecting a sophisticated investor's offer may assume any value (zero or one in pure strategy equilibria). It depends on bank's decision of granting or not a loan in next stage of the game:

$$Pr(\overline{D} \mid C, A) = \begin{cases} 1 & \text{, if } 1-\gamma > R_C \text{ and } \lambda = L \\ 0 & \text{, otherwise} \end{cases} \tag{1}$$

If the entrepreneur believes that bank will not grant the loan, it is better to accept sophisticated investor's offer. We assume that between the option of accepting investor's offer and the option of going to the bank and defaulting, in case payoffs are the same, entrepreneur's choice is the former.

We can also make use of the fact that the five separate actions that Nature makes give us five different states of nature. Three of them take the entrepreneur directly to the bank and two of them take the entrepreneur to a sophisticated investor. In the latter case, a bank node will be reached only if a sophisticated investor is unavailable or the entrepreneur refuses a sophisticated investor offer. Then, there are five nodes in Bank's information set which are reached according to the situation of each entrepreneur that may be applying for a bank loan (one of these nodes, as we have seen before, has probability zero, but it is described to simplify the equilibrium characterization). Figure 3 shows these five nodes, referred to as E_0, E_1, E_2, E_3, and E_4. Node E_0 represents a good risky project entrepreneur who found an available sophisticated investor but applied for a loan (this is the node where bank's belief puts probability zero). Node E_1 represents the state in which a good risky project entrepreneur who did not find an available sophisticated investor applied for a loan. Node E_2 represents the state in which a bad risky project entrepreneur applied for a loan. Node E_3 represents the state in which a safe project entrepreneur who did not find an available sophisticated investor applied for a loan. Node E_4 represents a safe project entrepreneur who found an available sophisticated investor but rejected its offer and applied for a loan.

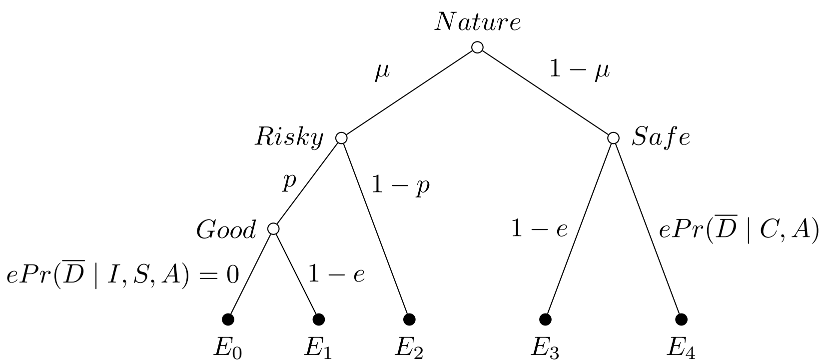

Figure 3: Tree diagram of conditional probabilities of an entrepreneur going to the bank

In order to analyze the interaction between the bank and the entrepreneur, we can look at a simplified version of the Game Tree that uses these features of the game. This is shown in Figure 4. There the five branches stemming out of the initial node correspond to the five nodes described in the previous paragraph. Using this simplified game extensive form representation greatly simplifies the analysis, so we make use of it in the analysis that follows.

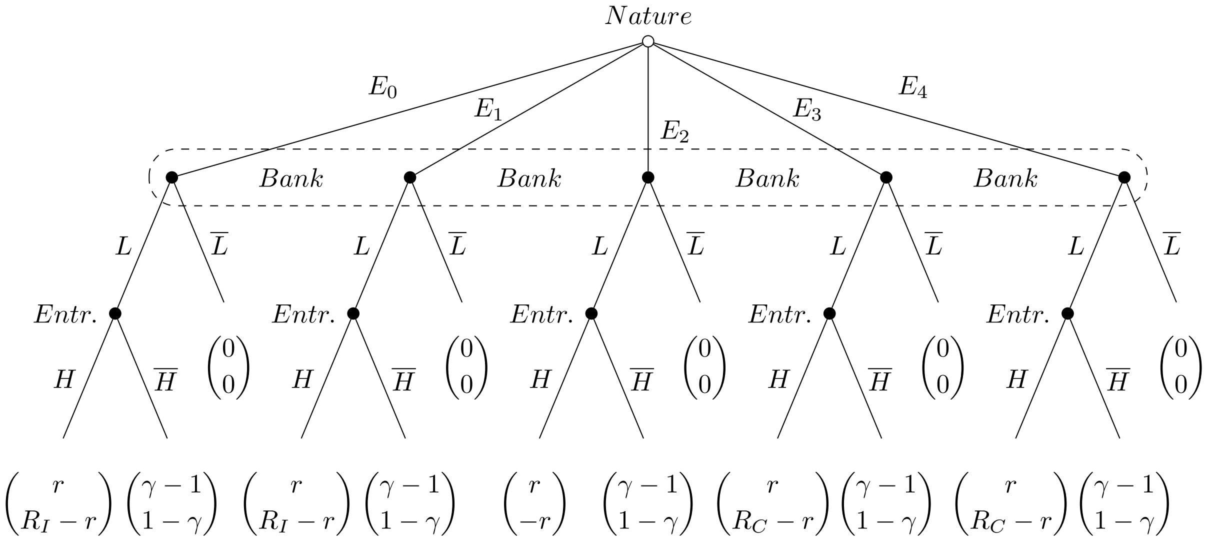

Figure 4: Simplified Extensive-form representation of the game

At date 2, the entrepreneur compares the gains of carrying out the project ($R_\pi - r$) with defaulting the project($1-\gamma$), and chooses the action η^* that gives him the higher payoff. At date 1, the bank compares the expected payoff of lending against the payoff of not lending. The bank chooses whether to grant a loan or not, λ^*, and the loan's interest rate, r^*, to maximize its expected profits given its beliefs about entrepreneur's type conditional on the fact that the entrepreneur has requested a loan from the bank.

An entrepreneur requests a bank loan with a joint probability $Pr(B) = \mu p(1-e) + \mu(1-p) + (1-\mu)(1-e) + (1-\mu)ePr(\overline{D} \mid C, A)$, where $B \equiv \{E_0 \cup E_1 \cup E_2 \cup E_3 \cup E_4\}$. We take the last two terms and define the probability of a safe project entrepreneur to go to the bank as $d \equiv (1-e) + ePr(\overline{D} \mid C, A)$. Then, we can rewrite the joint probability:

$$Pr(B) = \mu p(1-e) + \mu(1-p) + (1-\mu)d$$
$$\text{where } d = \begin{cases} 1 & \text{, if } 1-\gamma > R_C \text{ and } \lambda = L \\ 1-e & \text{, otherwise} \end{cases} \tag{2}$$

The probability d emphasizes that all safe projects will only go to the bank if the gain of defaulting is higher than the project's return and if the entrepreneur believes that the bank will grant a loan, otherwise a fraction e will prefer to accept sophisticated investor's offer and the game finishes. Considering that the number of entrepreneurs is fixed, we can interpret probability $Pr(B)$ as the demand for bank loans. Given that node E_4 is conditioned to bank's decision of lending, demand is also conditional to the same bank's choice.

3.1 The entrepreneur's behavior

We can enumerate three scenarios at date 2, when entrepreneur has to decide to honor the loan granted by the monopolistic bank. In the first scenario the payoff of defaulting is higher than the payoff of borrowing and moving forward with the project, for any kind of project. This scenario occurs when $1-\gamma > R_I - r^*$, where r^* is the optimal bank's interest rate. Because of the assumption that $R_I > R_C$, regardless of the type of project, the entrepreneur will always default on the bank loan. Thus, the probability of repayment is zero, $Pr(H \mid r^*) = 0$, and entrepreneur's best response in this scenario is $\eta^* = \overline{H}$.

In the second scenario, the payoff of defaulting is not higher than the payoff of borrowing and moving forward with a good risky project, but is higher than the payoff of borrowing and moving forward with a conventional project, $R_I - r^* \geq 1-\gamma > R_C - r^*$. In this case, only an entrepreneur with a good risky project will repay his debt. So, the entrepreneur will repay the loan with probability $Pr\left(E_0, E_1 \mid r^*\right)$, and will default with probability $Pr\left(E_2, E_3, E_4 \mid r^*\right)$ yielding $Pr(H \mid r^*) = \frac{\mu p(1-e)}{\mu p(1-e)+\mu(1-p)+(1-\mu)d}$. Since this scenario assumes $\lambda^* = L$, we can obtain d by applying this decision to equation 2:

$$Pr(H \mid r^*) = \begin{cases} \frac{\mu p(1-e)}{\mu p(1-e)+\mu(1-p)+(1-\mu)(1-e)} & \text{, if } R_I - r^* \geq 1-\gamma > R_C \\ \frac{\mu p(1-e)}{\mu p(1-e)+\mu(1-p)+(1-\mu)} & \text{, if } R_C \geq 1-\gamma > R_C - r^* \end{cases}. \tag{3}$$

Consequently, this scenario can be divided into two subscenarios, which differ in beliefs about E_4: the probability of an entrepreneur who asks for a bank loan to be this type is zero in the latter subscenario. In any subscenario, entrepreneur's best response is $\eta^* = H$, if $\{E_0, E_1\}$, or $\eta^* = \overline{H}$, otherwise.

The third scenario occurs when the payoff of defaulting is not higher than the payoff of borrowing and moving forward with a safe project, $R_C - r^* \geq 1-\gamma$. Because $R_C \geq 1-\gamma$, we have that $d = 1-e$ and $Pr(E_4 \mid r^*) = 0$. In this scenario, only the entrepreneur of a bad risky project defaults. So, the entrepreneur will repay his debt with probability $Pr\left(E_0, E_1, E_3, E_4 \mid r^*\right)$, and will default with probability $Pr\left(E_2 \mid r^*\right)$, yielding $Pr(H \mid r^*) = \frac{\mu p(1-e)+(1-\mu)(1-e)}{\mu p(1-e)+\mu(1-p)+(1-\mu)(1-e)}$. So, entrepreneur's best response is $\eta^* = \overline{H}$ when E_2, and $\eta^* = H$ when $\{E_0, E_1, E_3, E_4\}$.

To conclude the description of entrepreneur behavior, we make some additional remarks. First, note that the action of honoring the debt is strictly dominated for an entrepreneur of a bad risky project, because $-r^* < 1-\gamma$, and thus it is not part of any equilibrium, which is quite intuitive. Second, the complete description of the entrepreneur's strategy must include his response to a sophisticated investor's offer, if available, when nature draws a project of any value (either a good risky project or a safe one). Then, for both entrepreneurs E_0 and E_4 the optimal strategy is a pair (δ^*, η^*).

3.2 Bank behavior

The bank's payoff when granting a loan is $E[R_{B,i} \mid B] = rPr(H \mid r) + (\gamma-1)Pr(\overline{H} \mid r)$, where i indicates a scenario described in previous subsection. For each scenario, there is a set of bank's strategies (λ_i^*, r_i^*) that maximizes $E[R_{B,i} \mid B]$. The bank's strategy in first scenario is trivial. Because the expected return of the bank from lending is $\gamma - 1$, strictly negative, and from not lending is 0, whatever the project type, the best response is not unique: the bank never lends ($\lambda_1 = \overline{L}$), sets any interest rate $r_1 > 0$, and obtains payoff $R_{B,1} = 0$ for sure. We call this the rationing scenario, in which entrepreneurs are willing to borrow at sufficiently low interest rates but the bank prefers not to lend because it expects a certain default. In scenarios 2 and 3, the optimal interest rates are the upper limits of conditions that define those scenarios, respectively, $r_2 = R_I - 1 + \gamma$ and $r_3 = R_C - 1 + \gamma$, since expected bank's returns from lending are increasing in r. As $r_2 > r_3$, we call scenario 2 the high-interest-rate scenario (or, simply, the high scenario), and scenario 3 a low-interest-rate scenario (or, simply, the low scenario). Obviously, bank's best response in the last two scenarios also includes $\lambda_2 = \lambda_3 = L$.

The bank's problem is to find:

$$(\lambda^*, r^*) = \underset{(\lambda,r)\in\{(\lambda_i,r_i)\}_{i\in\{1,2,3\}}}{\arg\max} \left\{rPr(H \mid r) + (\gamma-1)Pr(\overline{H} \mid r)\right\}. \tag{4}$$

To characterize which interest rate should be chosen, we have to compare: *(i)* each expected return from lending versus zero (the return from not lending), and *(ii)* if the expected returns from lending is greater or equal than zero, which of them is the highest. The bank lends only in the low and high scenarios. Then, to be optimal, these scenarios must have expected returns greater than zero. The minimal expected bank's return condition when playing the low scenario is showed in the following lemma.

Lemma 1 *The bank's payoff in the low scenario is strictly positive if, and only if, the safe project's return is greater than a critical value $\underline{R}_C$, that is, $R_C > \underline{R}_C$, where*

$$\underline{R}_C \equiv (1-\gamma)\left(1+\frac{\mu(1-p)}{\mu p(1-e)+(1-\mu)(1-e)}\right) \tag{5}$$

To be optimal to lend at the high scenario interest rate, the following lemma must be satisfied.

Lemma 2 *The bank's payoff in the high scenario is strictly positive if, and only if, the good risky project's return is greater than a critical value, that is, $R_I > \underline{R}_I$, where*

$$\underline{R}_I = \begin{cases} \underline{R}_I' \equiv (1-\gamma)\left(1+\frac{\mu(1-p)+(1-\mu)}{\mu p(1-e)}\right) & \text{, if } R_C < 1-\gamma \\ \underline{R}_I'' \equiv (1-\gamma)\left(1+\frac{\mu(1-p)+(1-\mu)(1-e)}{\mu p(1-e)}\right) & \text{, if } R_C \geq 1-\gamma \end{cases} \tag{6}$$

Note that the minimal values $\underline{R}_I$ and $\underline{R}_C$ for project returns in scenarios low and high are both greater than the payoff of the entrepreneur in defaulting, $1-\gamma$.

Now, we need to compare scenarios low and high to evaluate which of them yields the highest expected return. Initially, we analyze the case where $R_C < 1-\gamma$. This implies that $R_C < \underline{R}_C$, which means that the bank's payoff in the low scenario is not positive because of lemma 1. Consequently, if R_I fulfils lemma 2, then the high scenario is the best choice to the bank. A general solution depends on how high the return of a good risky project is related to the return of a safe project. The following lemma describes the rule to rank those scenarios.

Lemma 3 *The expected return to the bank from choosing the low scenario's interest rate is higher than the expected return from choosing the high scenario interest rate if, and only if, the ratio between the returns of a good risky project and a safe project lies below a certain threshold β, that is, $\frac{R_I}{R_C} < \beta$, where*

$$\beta \equiv 1+\frac{1-\mu}{\mu p} \tag{7}$$

We have that $\beta > 1$, in accordance to our assumption that $R_I > R_C$. Lemma 3 states that the difference between those returns is also fundamental to the bank's choice regarding the loan's interest rate. When $\frac{R_I}{R_C} > \beta$, the high scenario's interest rate is a better option for the bank than the low scenario's.

3.3 Equilibrium

The equilibrium is characterized by $\lambda^*(R_C, R_I, \mu, p, e, \gamma)$ and $r^*(R_C, R_I, \mu, p)$. To obtain an intuitive view of the equilibrium, we can make use of figure 5 to identify which scenario the bank must choose to maximize his payoff. The gray area in the graph represents all feasible combinations of R_C and R_I. The set of feasible combinations of R_C and R_I is limited by the diagonal $R_I = R_C$ and by the vertical axis because we have assumed that $R_I > R_C > 0$. This assumption also implies that the boundary $R_I = R_C$ does not belong, as long as the points on the R_I axis, to the set of feasible combinations of R_C and R_I. Formally, we define the set of all possible ordered pairs (R_C, R_I) in this game as:

$$\mathbb{V} \equiv \left\{(R_C, R_I) \in \mathbb{R}_+^2 : R_I > R_C\right\} \tag{8}$$

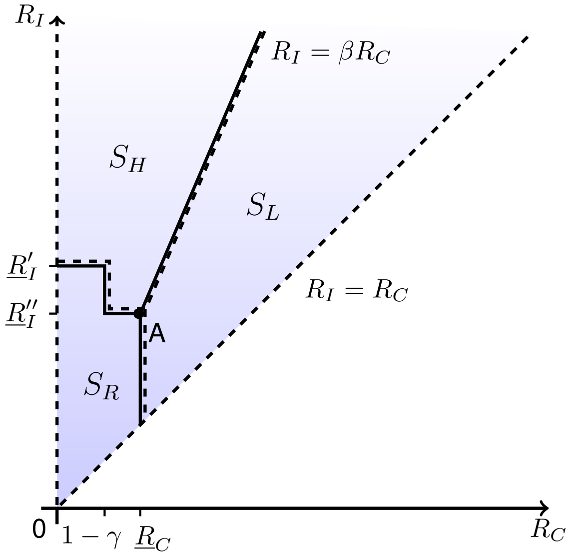

Figure 5: Aggregating in equilibrium subsets

The subsets S_R, S_H, and S_L of $\mathbb{V}$ showed in Figure 5 are functions of parameters μ, p, e, and γ. The subset S_R corresponds to the rationing scenario mentioned in the previous section. It contains all pairs (R_C, R_I) for which the bank's best response is not to lend at any interest rate. The high scenario corresponds to the subset S_H where the bank's best response is to lend at $r^* = R_I - 1 + \gamma$. Finally, subset S_L contains the low scenario combinations of R_C and R_I) in which the bank's best response is to lend at $r^* = R_C - 1 + \gamma$.

Without loss of generality, we assume that the tie-breaking rule for the bank's decision between lending or not is not to lend when the bank's expected return is zero, and to choose the high scenario interest rate, when indifferent between the low and high scenarios. The boundary between S_R and S_H is part of S_R. This boundary is composed of two horizontal lines with minimal returns of R_I and a vertical line in $1-\gamma$, and it is a function of μ, p, e, and γ. The boundary between subsets S_R and S_L is also part of S_R. This boundary is vertical in the minimal return R_C, and depends on μ, p, e, and γ. The line that limits regions of scenarios low and high is part of S_H, and has a slope decreasing in μ and p. Lemma 4 characterizes the point A where the three subset's boundary lines intercept.

Lemma 4 *The boundary lines separating subsets S_R, S_L and S_H from one another intercept at point $A = (\underline{R}_C, \underline{R}_I'')$, for all values of parameters μ, p, and e.*

A corollary of lemma 4 is that point $A = (\underline{R}_C, \beta\underline{R}_C)$ is the bottom limit of the boundary between S_H and S_L, for all values of parameters μ, p, and e. We can then characterize the subsets of $\mathbb{V}$ as:

$$S_R \equiv \{(R_C, R_I) \in \mathbb{V} : R_C \leq \underline{R}_C \text{ and } R_I \leq \underline{R}_I\} \tag{9}$$

$$S_H \equiv \left\{(R_C, R_I) \in \mathbb{V} : R_I > \underline{R}_I \text{ and } R_C \leq \beta^{-1} R_I\right\} \tag{10}$$

$$S_L \equiv \{(R_C, R_I) \in \mathbb{V} : R_C > \underline{R}_C \text{ and } R_I < \beta R_C\} \tag{11}$$

To proceed to the equilibrium, we rewrite conditions of probability d in terms of parameters. The conditions necessary for $d = 1$ are $1 - \gamma > R_C$ and $\lambda = L$. A bank will lend only in scenarios low and high. However, $R_C - r \geq 1 - \gamma$ in the low scenario, which is not consistent with the other condition of $d = 1$. On the other hand, $R_I - r \geq 1 - \gamma > R_C - r$ in the high scenario. Then, replacing condition $\lambda = L$ for lemma 2's condition we can rewrite the probability d as

$$d = \begin{cases} 1 & \text{, if } 1-\gamma > R_C \text{ and } R_I > R_I' \\ 1-e & \text{, otherwise} \end{cases} \tag{12}$$

Now, we describe the equilibrium in proposition 1, using all that was developed so far.

Proposition 1 *The equilibrium of this game consists of beliefs and strategies such that:*

1. *at date 1, bank's beliefs are that the applicant for a loan is:*
   - E_0 *(good risky project entrepreneur with investor's offer) with probability* 0*;*
   - E_1 *(good risky project entrepreneur without investor's offer) with probability* $\frac{\mu p(1-e)}{Pr(B)}$*;*
   - E_2 *(bad risky project entrepreneur) with probability* $\frac{\mu(1-p)}{Pr(B)}$*;*
   - E_3 *(safe project entrepreneur without investor's offer) with probability* $\frac{(1-\mu)(1-e)}{Pr(B)}$*;*
   - E_4 *(safe project entrepreneur with investor's offer) with probability* $\frac{(1-\mu)(d-1+e)}{Pr(B)}$*,*
2. *the strategies that maximize payoffs of entrepreneurs* E_0, E_1, E_2, E_3 *and* E_4*, and of the banker, are:*
   (a) *case* $1-\gamma > R_I > R_C$*:* $\{(D,\overline{H}); \overline{H}; \overline{H}; \overline{H}; (D,\overline{H}); (\overline{L}, r>0)\}$
   (b) *case* $R_I \geq 1-\gamma > R_C$*:*
      i. *case* $1-\gamma > R_I - r$*:* $\{(D,\overline{H}); \overline{H}; \overline{H}; \overline{H}; (D,\overline{H}); (\overline{L}, r > R_I - 1 + \gamma)\}$
      ii. *case* $1-\gamma \leq R_I - r$*:*
         A. *case* $rPr(E_0, E_1 \mid B) \geq (1-\gamma)Pr(E_2, E_3, E_4 \mid B)$*:*
            $\{(D,H); H; \overline{H}; \overline{H}; (D,\overline{H}); (L, R_I - 1 + \gamma)\}$
         B. *otherwise:* $\{(D,H); H; \overline{H}; \overline{H}; (D,\overline{H}); (\overline{L}, 0 < r \leq R_I - 1 + \gamma)\}$
   (c) *case* $R_I > R_C \geq 1-\gamma$*:*
      i. *case* $R_I > R_C \geq 1-\gamma > R_C - r$*:*
         A. *case* $R_I - r \geq 1-\gamma$*:*
            - *case* $rPr(E_0, E_1 \mid B) \geq (1-\gamma)Pr(E_2, E_3, E_4 \mid B)$*:*
              $\{(D,H); H; \overline{H}; \overline{H}; (D,\overline{H}); (L, R_I - 1 + \gamma)\}$
            - *otherwise:* $\{(D,H); H; \overline{H}; \overline{H}; (D,\overline{H}); (\overline{L}, 0 < r \leq R_I - 1 + \gamma)\}$
         B. *case* $R_I - r < 1-\gamma$*:* $\{(D,\overline{H}); \overline{H}; \overline{H}; \overline{H}; (D,\overline{H}); (\overline{L}, r > R_I - 1 + \gamma)\}$
      ii. $R_I > R_C > R_C - r \geq 1-\gamma$*:*
         A. *case* $rPr(E_0, E_1, E_3, E_4 \mid B) \geq (1-\gamma)Pr(E_2 \mid B)$*:*
            $\{(D,H); H; \overline{H}; H; (D,H); (L, R_C - 1 + \gamma)\}$
         B. *otherwise:* $\{(D,H); H; \overline{H}; H; (D,H); (\overline{L}, 0 < r \leq R_C - 1 + \gamma)\}$

In the low scenario, because the equilibrium interest rate is $r^* = R_C - 1 + \gamma$, only entrepreneurs E_1 and E_3 choose to move forward with their projects and honor their debt. Entrepreneurs E_0 and E_4 do not apply for bank loans while entrepreneur E_2 defaults. The payoff is $R_I - R_C > 0$ for E_1 and is zero for E_3. In the high scenario, the equilibrium interest rate is $r^* = R_I - 1 + \gamma$ and only entrepreneur E_1 chooses to move forward with the project and honor his debt. Entrepreneur E_0 does not apply for a bank loan and entrepreneurs E_2 and E_3 default. Entrepreneur E_4 does not appy for a bank loan, in some situations, and will default, in other. The payoff is zero for E_1. In any scenario and type of entrepreneur, the payoff of defaulting is $1 - \gamma$. In the rationing scenario, no project is financed, including the good ones. So, given R_I, R_C and γ, even good projects might be refused by banks in equilibrium. This result is in line with the incentive effect of Stiglitz and Weiss (1981), where a raise in interest rates changes the behavior of the borrower. Even if banks grant loans, given r, entrepreneurs of conventional projects might prefer to default rather then to move forward with their project.

Note that the bank's beliefs are determined by Bayes' rule provided that they are conditional not only to the joint probability distribution of the states of nature, but also to the projects' returns R_C and R_I and to the collateral γ. Particularly, the collateral can be used as a screening device by banks or a signal by entrepreneurs. In section 5, we consider a case in which the value of a collateral is correlated to the project characteristics in order to analize the financing of innovative projects in our model.

4 Effects on credit supply

Changes in μ, p, and e modify the areas covered by the sets S_R, S_H, and S_L of $\mathbb{V}$, which can affect the bank's lending decisions and the equilibrium interest rate. The following propositions allow us to analyze the effects of riskier entrepreneurship variables on credit supply.

Proposition 2 *An increase in parameters μ or p reduces the slope, β, of the boundary between S_H and S_L, whereas changes in parameter e do not have any effect on the same boundary.*

Proposition 3 *The boundaries of S_R, $\underline{R}_C$ and $\underline{R}_I$, are decreasing in p and increasing in e. An increase in parameter μ raises $\underline{R}_C$ and lowers $\underline{R}_I$.*

Let us take two distinct values for the parameter μ, say μ' and μ'', where $\mu'' > \mu'$, and denote $\{S_i'\}_{i\in\{R,L,H\}}$ the subsets that result from μ' and $\{S_i''\}_{i\in\{R,L,H\}}$ the subsets that result from μ''. Using propositions 2 and 3, we can verify that: *(i)* $S_H'' \supset S_H'$, $S_H'' \cap S_R' \neq \emptyset$, and $S_H'' \cap S_L' \neq \emptyset$; *(ii)* $S_L'' \subset S_L'$, $S_R'' \cap S_L' \neq \emptyset$, and $S_H'' \cap S_L' \neq \emptyset$. Figure 6 shows these effects. The first effect of a rise in μ is the incorporation of elements of subsets S_R and S_L by S_H, which increases the possibilities of the bank start lending at a high interest rate, if it is not currently lending, or raises the interest rate, if the bank is lending at a lower interest rate. The second effect is a migration of elements from subset S_L to subsets S_R and S_H, which increases the required minimum rate of return that the project must have to receive loan, at a lower interest rate in favour of a reduction in the risky project's required rate of return to receive a loan (and an increase in the safe project's required rate of return to receive a loan), or that the bank will only lend at a higher interest rate. Riskier projects can change the bank's decision of lending, denying credit to a few entrepreneurs with safe projects but giving access to credit to a few entrepreneurs with risky projects, and correspondingly raising the interest rate for the projects whose decision changes as a consequence of the change in μ.

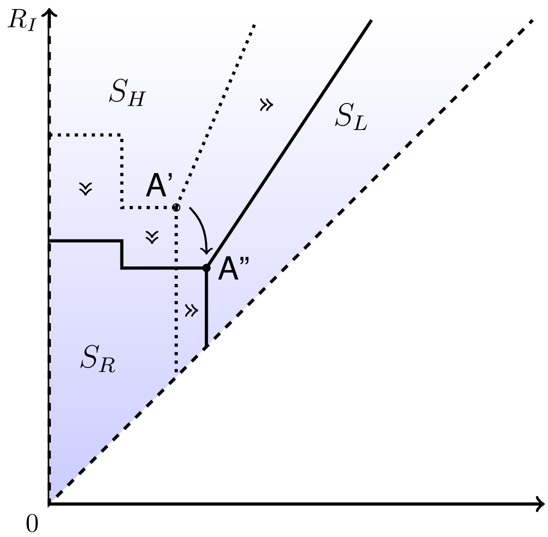

Figure 6: Effects on subsets of an increase in μ

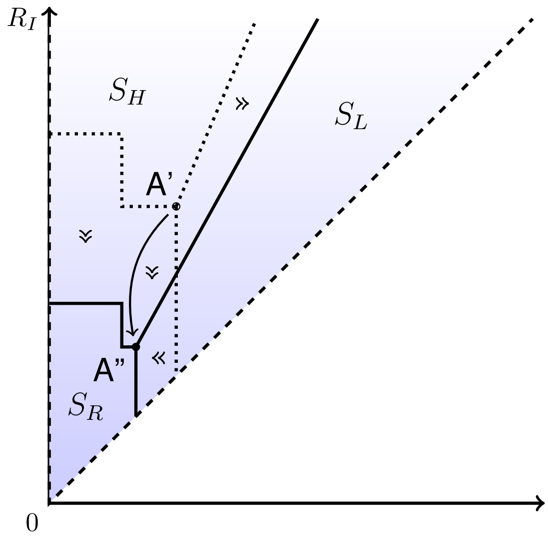

Figure 7: Effects on subsets of an increase in p

The effects on subsets of an increase in p, from p' to $p'' > p'$, are: *(i)* $S''_R \subset S'_R$, $S''_H \cap S'_R \neq \emptyset$, and $S''_L \cap S'_R \neq \emptyset$; *(ii)* $S''_H \supset S'_H$, $S''_H \cap S'_R \neq \emptyset$, and $S''_H \cap S'_L \neq \emptyset$. The first effect means that an increase in the probability that a risky project is successful increases the number of projects receiving loans through a reduction in the minimum project return, not only to this type of project, but also to E_3 entrepreneurs with conventional projects. The second effect, similarly to an increase in μ on S_H, increases the loan's interest rate for projects with intermediate returns. All these effects can be seen in figure 7. Increase in the probability of success of a risky project can lead the bank to offer more loans, including to safe projects, but can raise interest rate.

A higher e, moving, say, from e' to $e'' > e'$, yields a larger subset S_R where, following the notation: *(i)* $S''_R \supset S'_R$, $S''_R \cap S'_H \neq \emptyset$, and $S''_R \cap S'_L \neq \emptyset$; *(ii)* $S''_H \cap S'_L = \emptyset$ and $S''_L \cap S'_H = \emptyset$. We describe the first effect as a decrease in the number of projects receiving bank loans through an increase in the required minimum rate of return, and the second effect is that there is no change in the loan's interest rate if bank still lends at e''. The easier it is for entrepreneurs to find sophisticated investors, the more probability the bank attributes to the entrepreneur being of type E_3. These entrepreneurs default for sure, and always demand

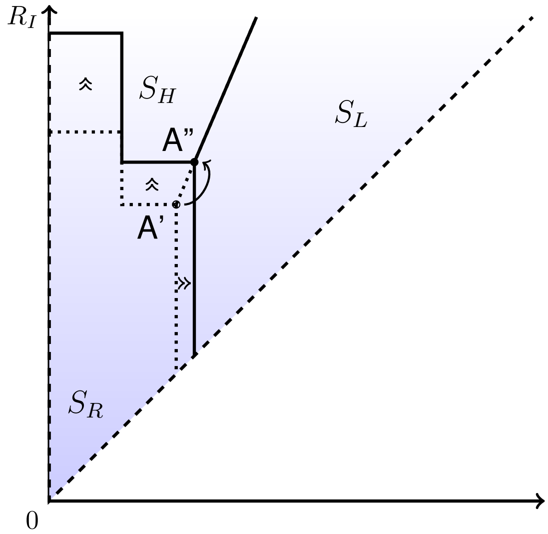

Figure 8: Effects on subsets of an increase in e

debt financing. Because of the larger probability of default, the bank only grants the loan for higher (successful) project returns R_C, R_I, or for a higher collateral value γ. Higher projects' returns give the bank higher margin that good risky and safe projects will repay the loan. A higher collateral requirement makes default less attractive for the entrepreneur. Figure 8 illustrates these effects. An increase in the availability of a sophisticated investor increases the required project return for the bank to offer a loan. However, if banks keep lending, interest rates do not change.

5 Innovation

Holmstrom (1989) argues that innovative activities are, by their very nature, riskier than non-innovative activities. An increase in the number of risky projects, as shown in propositions 2 and 3, reduces the area where loans are offered to both innovative and conservative projects, and increases the area where either loans are offered only to innovative projects or to no project at all. One way or another, the payoff of entrepreneurs with conservative projects is reduced after an increase in the presence of innovative projects. A larger presence of safe projects, on the other hand, does not necessarily imply in a reduction in innovative entrepreneurs' payoffs, because they can obtain loans at a lower rate when the bank serves both types of entrepreneurs.

Schumpeter (1912) has conjectured that small firms at the fringes of society that are responsible for most of the innovative activity, creating a product that makes its predecessor, made by an established firm, obsolete. The established firm is then driven out of the market, and the innovator takes its place, becoming the target of the next technological revolution. This process has come to be known as creative destruction.

If creative destruction describes the innovative process well, then innovative firms tend to be small and, thus, then there is a negative correlation between the probability of a project being innovative (the parameter μ in our framework) and the value of its physical assets. In most cases, only physical assets can be collateralized, and thus there is a negative correlation between μ and γ. A consequence of this feature of the market for bank loans is that most innovative project entrepreneurs will not have the required assets to apply for a bank loans. It follows that the bank collateral requirement causes banks to deny credit to innovative projects. The higher the collateral requirement, the more innovative projects are dropped.

The assumption of Schumpeter's conjecture in the model allows the bank to learn the entrepreneur's innovativeness. To illustrate this, we turn collateral into an endogenous variable, perfectly negatively correlated with the probability of a project to be innovative. We also assume that entrepreneur always offers all available assets he owns as collateral when he goes to the bank. Thus, at date 1, the entrepreneur who applies for a bank loan offers an amount of collateral γ to the bank, revealing the true value of μ, the probability of his project being innovative, to the bank.

A perfect negative correlation between entrepreneur's innovativeness and entrepreneur's assets implies that $\mu = 1 - \gamma$. We can rewrite equations 5, 6, and 7 to obtain:

$$\underline{R}_C \equiv (1-\gamma)\left(1 + \frac{(1-\gamma)(1-p)}{(1-\gamma)p(1-e)+\gamma(1-e)}\right) \tag{13}$$

$$\underline{R}_I = \begin{cases} \underline{R}_I' \equiv (1-\gamma) + \frac{(1-\gamma)(1-p)+\gamma}{p(1-e)} & \text{, if } R_C < 1-\gamma \\ \underline{R}_I'' \equiv (1-\gamma) + \frac{(1-\gamma)(1-p)+\gamma(1-e)}{p(1-e)} & \text{, if } R_C \geq 1-\gamma \end{cases} \tag{14}$$

$$\beta \equiv 1 + \frac{\gamma}{(1-\gamma)p} \tag{15}$$

The following proposition allow us to analyze the effects of innovativeness in bank incentives to lend.

Proposition 4 *When innovativeness of an entrepreneur is perfect negative correlated with his assets, an increase in parameter γ raises the slope β of the boundary between S_H and S_L, lowers $\underline{R}_C$, and can affect $\underline{R}_I$ increasing it, if $\left(R_C < 1-\gamma\right) \cup \left(R_C > 1-\gamma \cap p < \frac{e}{2-e}\right)$, or decreasing it, otherwise.*

In Figure 9, we can see the effects of a change in collateral when it is perfectly negative correlated with innovativeness. In contrast to the model with exogenous collateral, not all

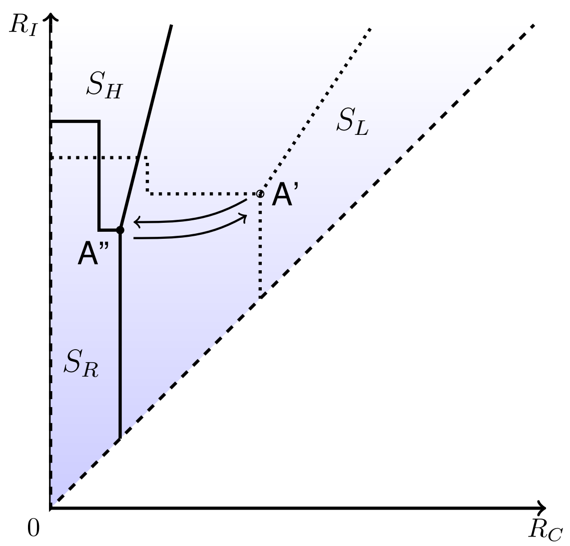

Figure 9: Changes in collateral when it is perfect negative correlated with innovativeness. Low collateral's subsets are limited by dotted lines and high collateral's subsets are limited by solid lines.

innovative projects that would receive a bank's loan in the high scenario will receive after an increase in μ (decrease in collateral γ). This occurs because a higher proportion of innovative firms implies a higher proportion of small firms with few assets that can be offered as collateral. While a higher probability of a project to be innovative encourages banks to increase lending because more good innovative projects will repay the loans, a lower collateral acts in the opposite direction. Hence, creative destruction exacerbates the adverse selection effect common in bank financing. This effect might lead to an increase in innovation activity to further reduce the credit available to such projects.

6 Final remarks

This paper presented a model of debt project financing in which the development of risky activities affects a monopolistic bank's decision of lending and the corresponding interest rate in a market with imperfect information. Some risky projects entrepreneurs eventually apply for a bank loan because of two reasons: *(i)* there are no sophisticated investors available to invest in a good project; or *(ii)* because the project is bad and they plan to default in order to get the loan principal net of collateral. The monopolistic bank chooses the optimal loan's interest rate but, as long as it does not observe the real type of the borrower, this can result in negative incentive effects for entrepreneurs whose project's return net of interest isn't sufficiently high. The proportion of risky projects, the probability of a project being good and the availability of sophisticated investors are conditions that affect the number of loans offered and the interest rate charged on these loans.

We use this framework to evaluate the effects of Schumpeter's creative destruction effect on the adverse selection effect of bank loans described above. In particular, we consider the case in which there is a perfect negative correlation between a firm's assets and the the innovativeness of its projects. This feature exacerbates adverse selection because the collateral requirement will select out the innovative projects in favor of conservative ones.

This model can be extended in several ways. First, the entrepreneurs' capacity to offer collaterals can be used as a sort of signalling mechanism. The bank can update its belief and make a better decision if the probability of an entrepreneur to have a riskier project is higher when it offers a low level of collateral. This has implications for firms that choose collateral requirements in their loan contracts.

A second extension is to model imperfect competition in credit market. It is expected that, if banks compete Bertrand-style, then the bank's behavior will be different and changes in innovative activities development conditions can yield distinct effects on the interest rate and in whether an entrepreneur will be able to obtain a loan.

Finnaly, some of this model's implications could be empirically tested. Several studies have tested effects on innovation of financial development and exogenous shocks in credit market (Chava et al., 2013; Amore et al., 2013; Hsu et al., 2014; Cornaggia et al., 2015; Hombert and Matray, 2017). However, according to our results, we should also see the effects of innovative activities on credit markets. For instance, the probability that a project is innovative can be seen as a fraction of high-tech companies production over GPD. We would expect that aggregate investments in R&D and education improve the probability that a risky project will be successful. The volume of investments made by venture capital and private equity seems to be a proxy for the availability of sophisticated investors. Those proxies could be tested against credit indicators, particularly the volume of loans and the mean interest rate.

Appendix: proofs

Lemma 1. $E[R_{B,3} \mid B] > 0 \iff r_3 Pr(E_0, E_1, E_3, E_4 \mid B) + (\gamma - 1)Pr(E_2 \mid B) > 0.$

Substituting the optimal low scenario's interest rate and reorganizing the terms in the right side we find $R_C > (1-\gamma)\left(1 + \frac{Pr(E_2)}{Pr(E_0,E_1,E_3,E_4)}\right)$. Replacing the probabilities with parameter μ, p, and e, yields $R_C > (1-\gamma)\left(1 + \frac{\mu(1-p)}{\mu p(1-e)+(1-\mu)(1-e)}\right)$. ■

Lemma 2. $E[R_{B,2} \mid B] > 0 \iff r_2 Pr(E_0, E_1 \mid B) + (\gamma-1)Pr(E_2, E_3, E_4 \mid B) > 0$. Substituting the optimal high scenario's interest rate and reorganizing the terms in the right side inequation we find $R_I > (1-\gamma)\left(1 + \frac{Pr(E_2,E_3,E_4)}{Pr(E_0,E_1)}\right)$. Remember that, in the high scenario, $Pr(H \mid r^*)$ is given by equation 3, which means that $Pr(E_4 \mid B)$ equals to $(1-\mu)$ or to $(1-\mu)(1-e)$ according to relation between R_C and γ. Considering that and replacing the probabilities with parameter μ, p, and e, yields $R_I > \begin{cases} (1-\gamma)\left(1 + \frac{\mu(1-p)+(1-\mu)}{\mu p(1-e)}\right) & \text{, if } R_C < 1-\gamma \\ (1-\gamma)\left(1 + \frac{\mu(1-p)+(1-\mu)(1-e)}{\mu p(1-e)}\right) & \text{, if } R_C \geq 1-\gamma \end{cases}$. ■

Lemma 3. $E[R_{B,3} \mid B] > E[R_{B,2} \mid B] \iff r_3^* Pr(E_1, E_3 \mid r_3^*) + (\gamma-1)Pr(E_2 \mid r_3^*) > r_2^* Pr(E_1 \mid r_2^*) + (\gamma-1)Pr(E_2, E_3, E_4 \mid r_2^*)$. Because Lemma 1, $R_C > (1-\gamma)$, which implies $Pr(E_1 \mid r_2^*) = \frac{\mu p(1-e)}{1-e[1-\mu(1-p)]} = Pr(E_1 \mid r_3^*)$ and $Pr(E_2, E_3, E_4 \mid r_2^*) = \frac{\mu p(1-e)}{1-e[1-\mu(1-p)]} = Pr(E_2, E_3 \mid r_3^*)$. Substituting the optimal interest rates in each scenario and reorganizing the terms in the inequation's right side we find the condition $R_C < R_I \frac{Pr(E_1,E_3)}{Pr(E_1)} + (1-\gamma)\frac{Pr(E_4)}{Pr(E_1)}$. Because $(1-\gamma)\frac{Pr(E_4|B)}{Pr(E_1|B)} \geq 0$, then stricter condition $R_C < R_I \frac{Pr(E_1,E_3)}{Pr(E_1)}$ must be satisfied. Replacing the probabilities with parameter μ, p, and e, yields $\frac{R_I}{R_C} < 1 + \frac{1-\mu}{\mu p}$. ■

Lemma 4. The ratio of $\underline{R}_I$ (boundary between S_R and S_H) and $\underline{R}_C$ (boundary between S_R and S_L), calculated by using the definitions in equations 6 and 5, always yields β (boundary between S_H and S_L), whatever the values of parameters μ, p, and e. ■

Proposition 1. This proposition results from the construction developed in subsections 3.1 and 3.2. ■

Proposition 2. It is straightforward to calculate derivatives of β using equation 7 to find that $\frac{\partial\beta}{\partial\mu} < 0$, $\frac{\partial\beta}{\partial p} < 0$, and $\frac{\partial\beta}{\partial e} = 0$. ■

Proposition 3. It is straightforward to calculate derivatives of $\underline{R}_C$ and $\underline{R}_I$ using definitions 5 and 6 to find that $\frac{\partial \underline{R}_C}{\partial\mu} > 0$, $\frac{\partial \underline{R}_C}{\partial p} < 0$, $\frac{\partial \underline{R}_C}{\partial e} > 0$, $\frac{\partial \underline{R}_I}{\partial\mu} < 0$, $\frac{\partial \underline{R}_I}{\partial p} < 0$, and $\frac{\partial \underline{R}_I}{\partial e} > 0$. ■

Proposition 4. It is straightforward to calculate derivatives of $\underline{R}_C$, $\underline{R}_I$, and β, using equations 13, 14, and 15, to find that $\frac{\partial \underline{R}_C}{\partial\gamma} < 0$, $\frac{\partial \underline{R}'_I}{\partial\gamma} > 0$, $\frac{\partial \underline{R}''_I}{\partial\gamma} \begin{cases} > 0 & \text{, case } p < \frac{e}{2-e} \\ < 0 & \text{, case } p > \frac{e}{2-e} \\ = 0 & \text{, case } p = \frac{e}{2-e} \end{cases}$, and $\frac{\partial\beta}{\partial\gamma} > 0$.

■

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