PUBLIC DEBT SUSTAINABILITY AND FISCAL CYCLICALITY IN BRAZIL: FACING THE FRACTIONAL INTEGRATION APPROACH

Área 4: Macroeconomia, Economia Monetária e Finanças

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Abstract

This article contributed to the literature on fiscal rules applied to Brazil, as we found empirical evidence regarded as robust to long-memory in time series, in contrast with the most part of the published studies, which have adopted conventional unit root tests. We found evidence of a pro-cyclical fiscal rule in Brazil, mainly after 2014, so that it diverged from the public debt sustainability principle developed by Bohn (1995; 2005). Moreover, using GMM regressions and an interaction term for the period marked by a higher fiscal stress, we also obtained evidence of expected inflationary effects as a result of lower fiscal cyclicality in Brazil.

Keywords: fiscal rules; public debt; long-memory; Brazil.

JEL Classification: E31; H30; H63.

Resumo

Este artigo contribuiu para a literatura sobre regras fiscais aplicadas ao Brasil, à medida que encontramos evidências consideradas robustas à memória-longa em séries temporais, em contraste com a maior parte dos estudos publicados, os quais têm adotado testes de raiz unitária convencionais. Encontramos evidências de regra fiscal pró-cíclica no Brasil, principalmente após 2014, de modo que ela divergiu do princípio de sustentabilidade da dívida pública desenvolvido por Bohn (1995; 2005). Ademais, usando regressões por GMM e um termo de interação para o período marcado por maior estresse fiscal, também obtivemos evidências de efeitos inflacionários esperados como resultado de menor ciclicidade fiscal no Brasil.

Palavras-chave: regras fiscais; dívida pública; memória longa; Brasil.

1. Introduction

Since the contribution of Bohn (1995), empirical works on fiscal sustainability have been developed for both international and national cases. Basically, such works have reported estimates on coefficients which measure the sensitivity of the primary surplus-to-GDP ratio to the lagged debt-to-GDP ratio. Despite the several differences in terms of data and methodological strategies each work adopts, the conclusions are generally straightforward: if those coefficients are positive and statistically significant then these studies suggest the existence of fiscal sustainability.

For Brazil in particular, Pires and Andrade (2009), Luporini (2015), Moreira (2017) and Campos and Cysne (2019) are some examples of efforts to test for fiscal rules or fiscal sustainability. Specifically, Luporini (2015), Moreira (2017) and Campos and Cysne (2019) pointed to the deterioration of the

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fiscal dynamics in Brazil over the last years of their samples, thereby capturing fiscal facts which are nowadays stylized and associated with Dilma Rousseff's presidential mandate (2011-2016), especially the increasing path of the public debt/GDP ratio.

However, all the results extracted from these studies may be conditional to a type of statistical fragility, that is, *the lack of appropriate treatments for long-memory in data*. The statistical error of disregarding long-memory is that, for instance, when a time series is assumed as stationary (or as having unit root) by the conventional unit root tests, but it actually presents long-memory, the resulting auto-regressive inference commonly misestimates coefficients. One way to deal appropriately with long-memory in data is by the *fractional integration approach* (Geweke and Porter-Hudak, 1983; Baillie et al. 1996; Reisen et al., 2001; Reisen et al., 2003; Wei, 2006).

Our initial contribution was then to estimate fiscal rules for Brazil, *but only after controlling for long-memory.* The particular case of Brazil is an important one as some authors have previously identified pro-cyclical fiscal rules (Moreira, 2017; Campos and Cysne, 2019). However, these studies did not deal with potential long-memory in the adopted time series, thereby possibly estimating the associated fiscal cyclicality in a biased way. Such a fiscal cyclicality degree is represented by the coefficient relating primary surplus responses to lagged debt-to-GDP ratio changes.

The second contribution, in turn, was to identify possible effects of changes in the fiscal cyclicality degree on expected inflation rates. A worse fiscal cyclicality can lead an economy to several disruptions, such as increasing interest rates and risk premia, as well as lower output growth and a rise in inflationary indicators (Reinhart and Rogoff, 2010). Our hypothesis was that a worse fiscal cyclicality – meaning a weaker response of primary surpluses to public debt/GDP ratio – was followed by a context of higher expected inflation rates, thereby causing a deterioration in monetary policy's effectiveness. These estimates were conducted by *Generalized Method of Moments* (GMM) regressions and controlling for the effects of a structural break in the public debt path.

The article is structured as follows: in section 2 we described the fiscal rules theory underlying our subject; section 3 made a brief empirical literature review focusing on evidence for Brazil; section 4, in turn, presented the statistical methodology to deal with long-memory in our data; section 5 presented the dataset, while section 6 analyzed the results. At last, we brought concluding remarks and the list of references.

2. A brief overview: fiscal rules theory and macroeconomic effects

The fiscal rule theory have developed in different directions since the seminal contribution of Bohn (1995), but our main interest is focused on the primary surpluses responses to public debt changes over time. Equation (1) describes the relevant relationship in such a case:

$$(ps/y)_t = \alpha(D/y)_{t-1} + \rho_t, \qquad (1)$$

where $(ps/y)_t$ stands for the primary surpluses (ps) to DGP (y) ratio in period t; and $(D/y)_{t-1}$ represents the public debt (D) to GDP ratio in lagged period $t-1$. Moreover, α measures the fiscal cyclicality, that is, the magnitude of reaction of the primary surpluses to the debt/GDP ratio. We consider a countercyclical fiscal policy as presenting a positive parameter $\alpha > 0$, meaning that when the public debt increases, the fiscal authorities perform a rise in primary surpluses, thereby avoiding an indefinite uptrend in public financial liabilities and so maintaining fiscal solvency in long-run.

In contrast, when $\alpha < 0$ we find a pro-cyclical fiscal policy: fiscal reactions are not consistent with the public debt stabilization or sustainability in long-run. In turn, ρ_t stands for other factors explaining the primary surplus-to-DGP ratio, such as inflation rates or economic activity, which also can calibrate fiscal decisions over time.

In our context, we can define α as a measure for fiscal cyclicality. The higher the fiscal cyclicality, the higher the α coefficient. It is important to note that fiscal decisions are subject to several rigidities, either because of the associated legislative channel in democracies, or due to delays in data processing and instrument adjustments by the fiscal authorities. As a consequence, Taylor (2000) argued for constraining discretionary decisions to long-run goals, such as solvency maintenance and public efficiency, while the short-term output cycle should have be faced by the classical automatic stabilizers.

A related contribution to this subject was made by the *Fiscal Theory of the Price Level* (Woodford, 2001; Cochrane, 2001; Leeper, 1999). Basically, a worse fiscal cyclicality can decrease the monetary policy`s effectiveness by means of potential effects on expected and effective prices dynamics. Even a Central Bank committed to inflation targets would not be enough to avoid inflationary deviations if fiscal authorities did not react adequately to public debt trajectory over time. In other words, a fiscal deterioration has a potential nominal effect (inflationary effect), despite the existence of an autonomous and independent Central Bank.

Equation (2) represents the government's intertemporal budget constraint. It equals the real public debt $(\frac{D_t}{P_t})$ to the present value of current and expected future primary surpluses (sp_t) and seigniorage revenue (m_t) over time, from t to $t+n$. The main correction mechanism in Equation (2) is the general prices level (P). Under inflation targeting regimes, Central Banks are committed to an inflation target, so that monetary policy is conducted autonomously and so seigniorage revenue does not adjust to satisfy the budget constraint.

$$\frac{D_t}{P_t} = sp_t + \sum_{i=1}^{n} sp_{t+i} + m_t + \sum_{i=1}^{n} m_{t+i}. \qquad (2)$$

Therefore, when there exists a positive impulse to the public debt, thereby increasing real public financial liabilities, fiscal authorities are called to react by means of a rise in primary surpluses, or public savings. If it occurs appropriately (that is, if $\alpha > 0$ in Equation (1)) the budget constraint will continue to be satisfied and thus we will not observe impacts on the prices level. In contrast, if fiscal authorities do not adjust to the rise of the real public debt (that is, if $\alpha < 0$ in Equation (1)), Equation (2) will present a divergence, with the left side being higher than the right one. Although such a situation can be verified in short-run, it does not hold in long-term. The continuity of a real public debt which is not covered by current and expected future primary surpluses eventually will stimulate a substitution process, in which pubic bonds will be replaced by consumption (Woodford, 2001). In turn, it will contribute to the decrease of the nominal value of public bonds (*D*) and also to the rise in services and goods prices. In sum, an inflationary pressure emerges from the fiscal disequilibrium and, at the same time, serves as a correction mechanism, so that the budget constraint gap can be closed in long-term.

3. Empirical works on debt sustainability and fiscal rules

Since Bohn (1995), the literature on tests for fiscal sustainability has developed through two alternative empirical methods: on the one hand, the older method based on unit root or stationary tests on public debt time series; on the other hand, the new method based on testing for primary surpluses responses to public debt changes. Applying the latter, Bohn and Inman (1996) used a panel of 47 US states from 1970 to 1991 and found asymmetric results. Basically, they demonstrated evidence of fiscal sustainability in states with end-of-the-year balance requirements enforced as constitutional constraints and with an independently elected state supreme court; in contrast, statutory end-of-the-year requirements or constitutional end-of-the-year requirements enforced by a politically appointed court did not present statistically significance in constraining deficits behavior.

Moreover, Bohn (2005), using aggregate data for the US, found robust evidence for fiscal sustainability over the period 1792-2003. Mahdavi (2014) also took into account the Bohn's approach in order to test for the primary surplus responses to lagged public debt covering the post-Subprime crisis period. With a panel of 48 states, from 1961 to 2008, Mahdavi (op. cit.) found robust evidence in favor of fiscal sustainability, besides a higher adjustment of taxes into primary surpluses changes in comparison to the spending adjustment.

Small et al. (2020) focused on 53 developing countries and examined the response of the primary balance to lagged debt-to-GDP ratio. The authors found evidence for positive relationships between the debt and primary surpluses. Furthermore, the results showed that countries adjusted both revenues and expenditure margins at roughly the same rate.

Especially for Brazil, Pires and Andrade (2009) found evidence of an important influence of the public debt composition (regarding the types of public bonds carried by the market) on the strength and duration of the macroeconomic responses to different kind of shocks. In particular, they found that the higher the proportion of the public bonds indexed by the basic interest rate, the higher the inflationary inertia as a response to demand shocks. In other words, a worse public debt composition was a source of monetary policy ineffectiveness.

Luporini (2015) estimated a time-varying parameter for Brazil's fiscal rule over the period from 1991 to 2011, and its empirical findings suggested a weaker fiscal policy since 2006 in comparison with the previous sub-sample, meaning a reduction in the sensitivity of the primary surplus to the public debt/GDP ratio. Such a result was interpreted as a potential damage to a fiscal consolidation plan and also to the monetary policy's available choices, in a context of increasing inflationary expectations and low output growth.

In turn, Moreira (2017) analyzed the nature of the fiscal rule in Brazil, during the period from January 2005 to April 2015, and by using a *Vector Error Correction* approach. The main findings indicated that Brazil's fiscal policy was a pro-cyclical one, thus contributing to understand why such a policy was not enough to impose a sustained downturn movement of the public debt/GDP ratio over the entire sample. Moreover, it meant that fiscal reactions were not consistent with the inflation-targeting regime in Brazil.

More recently, Campos and Cysne (2019) evaluated the sustainability of public debt in Brazil using monthly data from January 2003 to June 2016, and based on the estimation of fiscal reaction functions with time-varying coefficients. Three estimation methods were applied: Kalman filter, penalized spline smoothing and time-varying cointegration. All these methods and their findings indicated that the Brazilian public debt reached an unsustainable trajectory in the last years of the sample.

Specifically for the Brazilian case, these studies brought relevant findings which pointed to a lack of fiscal consolidation consistency over time, while some of them stressing the pro-cyclical bias of the fiscal rule regarding the last years (Moreira, 2017; Campos and Cysne, 2019). However, there exist a weak point of such a literature: the lack of adequate treatments for long-memory in data, so that their main evidence could be a result of potential over or underestimation for the fiscal cyclicality coefficient.

4. Statistical methodology

The analysis of the Brazilian fiscal dynamics was divided into two steps. First, we performed an univariate analysis, based on the method of Geweke and Porter-Hudak (GPH) (Geweke and Porter-Hudak, 1983). Such an analysis allowed testing for the existence of long-memory in time series (Reisen, 1994; Reisen et al. 2001; Wei, 2006).

Second, after testing for the fractionally integrated process of data and filtering eventual long-memory behavior of all variables, we developed a multivariate analysis based on the estimation of *Generalized Method of Moments* (GMM) regressions, in order to estimate fiscal rules for Brazil. The GMM method was applied for purposes of robustness of the estimates, as it is regarded useful to overcome potential problems of heteroskedasticity, autocorrelation and endogeneity (Hansen, 1982). In turn, the instrumental variables set was based on the i-period lag of the time series, which thus satisfied the hypothesis of exogeneity of the instruments (Johnston, 1984). Finally, to assure the appropriate specification of the instrumental variables, an analysis of *overidentification* was performed by the J-test (Gragg, 1983; Hansen, 1982).

It is also worth noting that before applying the multivariate analysis we filtered the long-memory component into the series with $\hat{d}_i$ (fractional integrated parameter), by means of the filter $(1 - B)^{\hat{d}_i} X_{t,i}$, where i representing each times series X_t and B is the backward shift operator. The parameter d wad estimated based by the GPH method aforementioned. In contrast, the most part of the empirical studies regarding fiscal rules in Brazil used typical Dickey-Fuller tests, which have low power against the fractional alternative (Diebold and Rudebush, 1991; Hasler and Wolters, 1994). Besides, Lee and Amsler (1997) found that the KPSS test cannot consistently distinguish between a process with nonstationary long-memory $(0.5 \leq d < 1)$ and a unit root process $(d = 1)$. In this perspective, our current effort of using times series based on the fractional integration approach, as well as filtering possible long-memory process in data, can be regarded as an innovation to applied fiscal studies.

Furthermore, the paper estimated some macroeconomic effects as a consequence of fiscal cyclicality changes over the time sample, specifically by means of inferring the expected inflationary effects as a result of the degree of primary surpluses responses to the public debt path. To do so, we especially defined a sub-sample in which Brazil clearly presented a deterioration in its gross public debt-to-GDP ratio, particularly the sub-period from October 2014 to April 2019. The moderating effect (Aguinis, 2004; Jaccard and Turrisi, 2003) of this dummy variable – meaning the effect of the latter on a relationship between two other variables – was useful in identifying both the pro-cyclical fiscal bias and its expected inflationary damages during such a period.

5. Dataset

The analysis comprised the period from January 2003 to April 2019, with monthly frequency. The time series were the following ones and their graphical behaviour and descriptive statistics are presented in Figure 1 and Table 1, respectively.

PS: the public sector primary surplus as a proportion of GDP (%);

DEBT: the general government gross debt as a proportion of GDP (%);

P: the 12-month cumulative inflation by IPCA (*Broad Consumer Prices Index*) (%);

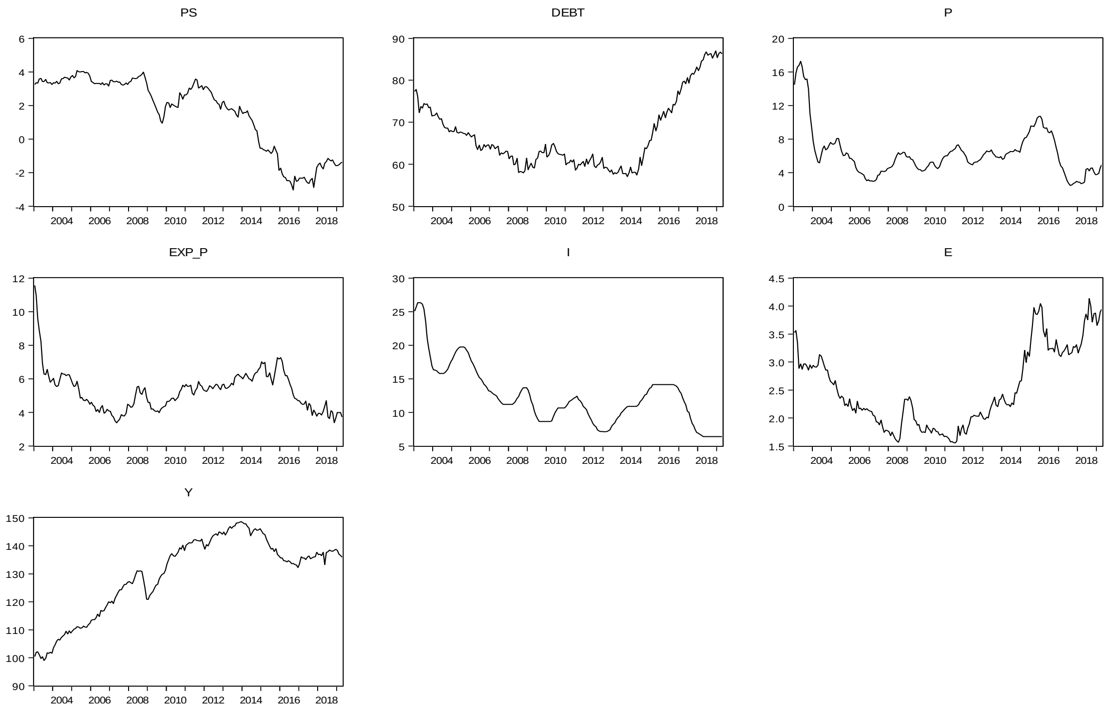
EXP_P: the cumulative expected inflation for 12 months ahead by the IPCA (*Broad Consumer Prices Index*) (%);

I: the annualized basic interest rate (Selic rate) (%);

E: the nominal exchange rate (R$/US$);

Y: the *Index of Economic Activity of the Central Bank* (IBC-Br), seasonally adjusted.

Figure 1: Graphical behaviour



Source: drawn from the survey data.

Table 1: Descriptive statistics

Variable	PS	DEBT	P	EXP_P	I	E	Y
Mean	1.59	67.04	6.28	5.26	12.46	2.52	129.81
Median	2.32	64.21	5.88	5.34	11.70	2.31	135.14
Maximum	4.08	86.97	17.24	11.56	26.32	4.13	148.66
Minimum	-3.04	57.03	2.46	3.37	6.40	1.56	99.03
Std. Dev.	2.19	8.33	2.85	1.21	4.43	0.71	13.94
Skewness	-0.77	0.96	1.84	1.58	1.05	0.53	-0.68
Kurtosis	2.09	2.88	7.11	8.52	4.25	2.10	2.27
Jarque-Bera	26.34	30.23	248.75	330.71	48.34	15.81	19.58
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: drawn from the survey data.

6. Results

6.1. Univariate analysis for the gross public debt (DEBT)

The autoregressive moving average (ARMA(p, q)) process is often referred to as a short memory process since the autocorrelation function (ACF), $\rho(h)$, decreases rapidly as $h \to \infty$ (lags tend to infinity) (Brockwell and Davis, 2006; Brockwell and Davis, 2016). Stationary processes whose ACF converges to zero rapidly, for instance ARMA processes, are said to have "short memory". In contrast, stationary processes with much more slowly decreasing autocorrelation function are called fractionally integrated ARMA processes, ARFIMA(p, d, q). ARFIMA processes are said to exhibit "long memory", or "long-range positive dependence".

According to Reisen et al. (2003), the ARFIMA(p, d, q) process can be characterized as follows: i) $d = 0$, short memory; ii) $0 < d < 0{,}5$, stationary with long memory and mean reversion; iii) $0{,}5 \leq d < 1$, nonstationary with long memory, but still shows mean reverting; and, iv) if $d \geq 1$, nonstationary and does not present mean reversion[3]. In the latter case, if $d = 1$, the process has a unit root.

The performance of traditional unit root tests in the presence of long memory has received much attention. As described by Diebold and Rudebush (1991) and Hasler and Wolters (1994), Dickey-Fuller type tests present low power against the fractional alternative. In addition, Lee and Amsler (1997) demonstrated that KPSS statistics present problems to distinguish consistently between a process with nonstationary long memory ($0{,}5 \leq d < 1$) and a unit root process ($d = 1$).

It is noteworthy that an ARFIMA process presents several characteristics that are similar to those of a nonstationary process in finite samples (Wei, 2006). For instance, the ACF of an ARFIMA(p, d, q) model decays very slowly, that is, similarly to the sample ACF of a nonstationary time series (ARIMA(p, d, q)). In addition, the ARFIMA process and the nonstationary ARIMA present periodograms[4] that diverge at the zero frequency. These similarities often lead to model misspecification. For instance, a stationary ARFIMA model could be misspecified as a nonstationary ARIMA model. This overdifferencing may lead to some undesirable effects on parameter estimation and forecasting.

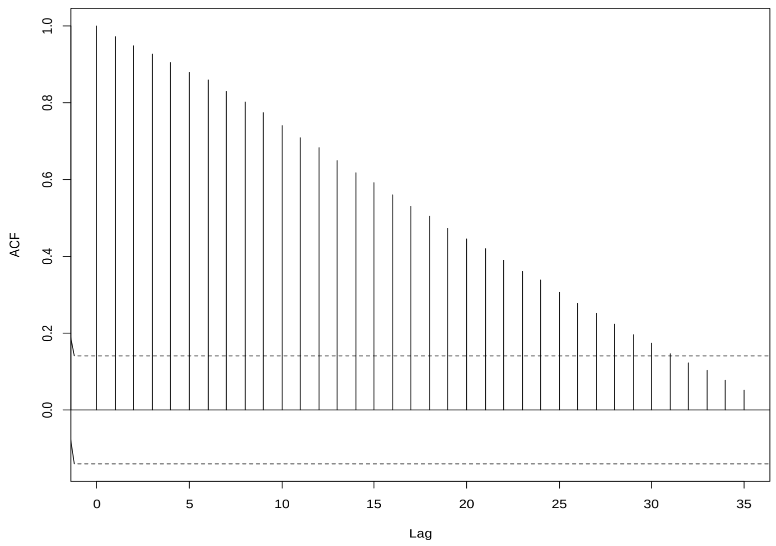
Empirical results reveal that the long memory process occurs frequently in several research areas such as hydrology and economics (see Hurst, 1956; Lawrence and Kottegoda, 1977; Granger, 1980; Baillie, et al., 1996; Hassler and Wolter, 1995; Baum et al., 1999; Reisen, Cribari-Neto and Jensen, 2003; among others). In economics, time series analyses are mostly based on typical tests to verify the stationarity of the former, such as: Augmented Dickey-Fuller – ADF (Dickey and Fuller, 1981); Phillips-Perron – PP (Phillips and Perron, 1988); and, Kwiatkowski-Phillips-Schmidt-Shin – KPSS (Kwiatkowski et al., 1992). In turn, one of the problems of such tests is that it can easily overestimate (or underestimate) the integration order of a specific time series, since they do not deal with the long-memory in data.

Figure 2 shows the ACF for DEBT variable (the general government gross debt as a proportion of GDP), considering 35 lags. As the sample autocorrelations decay slowly, the series seems to be nonstationary and possibly characterized by long memory behavior.

[3] The intermediate memory of an ARFIMA model is defined when d is in [-0,5; 0).

[4] The periodogram function describes how the total variability of the series is partitioned over the various components relating to each of the Fourier frequencies.

Figure 2: Sample autocorrelation function for DEBT



Source: drawn from the survey data.

Before checking if DEBT series presents features of a fractionally integrated process, the ADF, PP and KPSS tests were performed. The results in Table 2 reveal that DEBT has a unit root (so not stationary at level). Then, if DEBT is characterized by a long-range dependence, its first difference could result in an overdifferencing (to $0 < d < 1$) or even in a subdifferentiation ($d > 1$) of the series, when compared with a fractionally integrated process (ARFIMA).

Table 2: Unit root test for DEBT in level

Variable	ADF	K	PP	K	KPSS	K
DEBT	-0.9122 ^{ns}	6	-0.9581^{ns}	1	0.3848***	11

Source: drawn from the survey data.

Note: 1) *** Significant at 1%, ^{ns} Non-significant at 10%; 2) K = number of lags; 3) Tests performed with constant and/or trend. However, tests without constant and/or without constant and trend showed similar results.

Concerning this possible problem, this paper adopted the Geweke and Porter-Hudak (GPH) method (Geweke and Porter-Hudak, 1983) to test the presence of a fractionally integrated process in DEBT. Estimates of the fractionally integrated parameter (d) were made considering different bandwidths, $g(n) = n^{\alpha}$, with n equal to the number of observations and $0 < \alpha < 1$. Firstly, the parameter d was estimated for the whole period (Jan/2003 to Apr/2019; Table 3). In sequence, as the DEBT variable presents a period of downtrend (Jan/2003 to Dec/2013) and a period with an uptrend (Jan/2014 to Apr/2019), the estimates of d were performed for each of these sub-periods; Tables 4 and 5, respectively. According to the results, regardless of the choice of $g(n)$, in all cases, the DEBT series presented a high degree of persistence, since the $\hat{d}$ values ranged between 0.8925 and 1.6436 and the $I(1)$ hypothesis cannot be rejected (95% confidence interval). It is noteworthy that in many cases the parameter $\hat{d}$ was greater than one, thereby signaling for potentially explosive paths over the sample.

Table 3: Estimated GPH for different bandwidths for DEBT (Jan/2003 to Apr/2019)

α	Bandwidths ($g(n)$)	GPH ($\hat{d}$)	Standard-Error
0.4	8	1.6436	0.1990
0.5	14	1.5768	0.1505
0.6	24	1.3469	0.1264
0.7	40	1.3011	0.1017

Source: drawn from the survey data.

Table 4: Estimated GPH for different bandwidths for DEBT (Jan/2003 to Dec/2013)

α	Bandwidths ($g(n)$)	GPH ($\hat{d}$)	Standard-Error
0.4	7	0.8939	0.1588
0.5	11	0.9359	0.1049
0.6	19	0.9506	0.0988
0.7	31	0.8925	0.0772

Source: drawn from the survey data.

Table 5: Estimated GPH for different bandwidths for DEBT (Jan/2014 to Apr/2019)

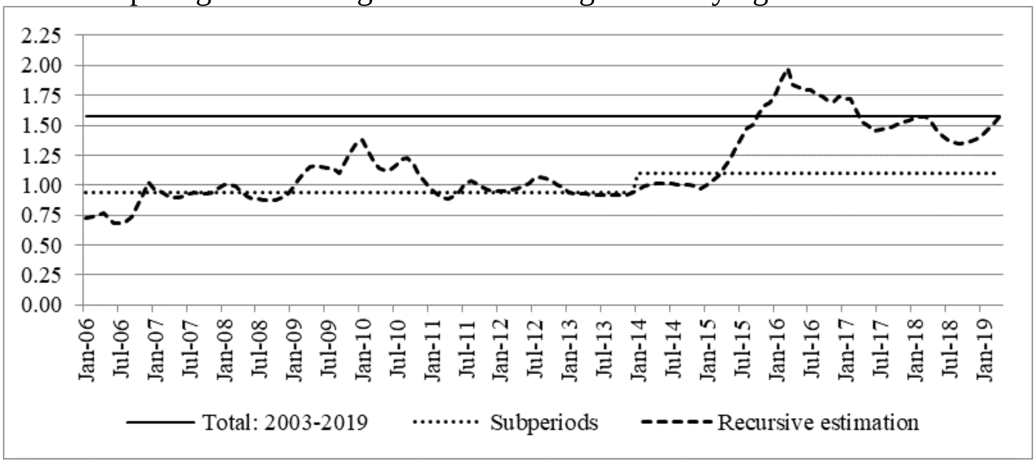
α	Bandwidths ($g(n)$)	GPH ($\hat{d}$)	Standard-Error
0.4	5	1.2663	0.0667
0.5	8	1.1019	0.0759
0.6	12	1.0890	0.0465
0.7	18	1.0765	0.0369

Source: drawn from the survey data.

The disadvantage of looking at longer time horizons and even at subsamples is that such an analysis provides only a very general picture of the fractionally integrated parameter ($\hat{d}$). Since the estimates for the latter may vary over time, it was useful to perform a recursive estimation technique in order to capture the time-varying $\hat{d}$. In the estimates of d it was considered $g(n) = n^{0,5}$ (see Reisen, 1994). To perform the recursive estimation, we took a 36-month regression window, starting from January 2003 to January 2006. Then, the starting date was kept fixed and only the end date was moved forward by one month at each time. Recursive methods have some shortcomings, but such a technique provides a good first proxy of the time-varying $\hat{d}$.

Figure 2 shows the 36-month recursive estimation showed values ranging between 0.68 (at the beginning of the period) and 1.98. Furthermore, the estimated coefficient ($\hat{d}$) generally was greater than 1.0, thereby corroborating the aforementioned high persistence of DEBT. We can clearly observe an increase of $\hat{d}$ especially over the second subsample (Jan/2014 to Apr/2019).

Figure 2: Comparing methodologies for estimating time-varying fractional coefficient (d)



Source: drawn from the survey data.

6.2. Estimated GPH for the dataset

The main goals of this paper require dealing with long-memory in all the dataset, not only in DEBT. Thus, we performed the GPH method to yield the semiparametric estimates for their $\hat{d}$, which are shown in Table 6 (including for DEBT)[5]. According to the results, our time series should be classified

[5] In all estimates of d we considered $g(n) = n^{0,5} = 14$ (see Reisen, 1994).

as follows: i) stationary with long memory and mean reversion (EXP_P); ii) nonstationary with long memory, but with mean reversion (P and I); and, nonstationary (PS, DEBT, E and Y). To extract the corresponding short-memory time series, the filter $(1-B)^{\hat{d}_i}X_{t,i}$ was then adopted, where i representing each times series X_t and B is the backward shift operator.

Table 6: Estimated GPH ($\hat{d}$) for the time series

Variable	GPH ($\hat{d}$)	Standard Error
PS	1.0983	0.1562
DEBT	1.5768	0.1505
P	0.5218	0.1532
EXP_P	0.4897	0.1471
I	0.6475	0.1096
E	1.1232	0.2397
Y	1.0187	0.1131

Source: drawn from the survey data.

6.3. Estimates for fiscal rules in Brazil

We started our multivariate analysis by estimating a fiscal rule of the following general type:

$$PS_t = a_1 PS_{t-1} + a_2 DEBT_{t-1} + \sum_{i=1}^{n} \beta_i V_{i,t-1} + \rho_t. \qquad (3)$$

The fiscal rule is a response function of the public sector primary surplus (%GDP) to the public debt (%GDP), so that a_2 represents the main coefficient in our analysis. A countercyclical and sustainable fiscal rule is expected to yield $a_2 > 0$. The higher the a_2 the higher the fiscal cyclicality degree. In contrast, if $a_2 \leq 0$ we then have a pro-cyclical fiscal rule (Moreira, 2017) in which public sector solvency is threatened. The other parameters and time series work as control variables in our fiscal rule estimates. So, we included an auto-regressive component (a_1), due to the common persistence in fiscal variables. In turn, $V_{i,t-1}$ represents a vector of i macroeconomic control variables, e.g. inflation rate (P) and economic activity (Y). At last, ρ_t stands for a white noise fiscal disturbance. The GMM estimates for fiscal rules in Brazil are presented in Table 7.

As a way to confirm the apparent structural break in DEBT persistence, which was signalized in our univariate analysis, we also included a dummy variable into the GMM regression. So, BURDEN stands for an interaction term or a moderating effect (Aguinis, 2004; Jaccard and Turrisi, 2003). It controls for the period from October 2014 to April 2019 (thus assuming 1.0 over this subsample), which encompasses the increasing fiscal stress in Brazil, amid a strong uptrend and a rising persistence of the public sector's gross debt-to-GDP ratio. The purpose was to apply BURDEN to the response of PS to $DEBT$, so that we could test for a structural break on the fiscal cyclicality degree. A possible specification is then the following one:

$$PS_t = a_1 PS_{t-1} + a_2 DEBT_{t-1} + \beta_1 (BURDEN_t) DEBT_{t-1} + \beta_2 P_{t-1} + \varepsilon_t. \quad (4)$$

We expect $a_2 + \beta_1 < a_2$, that is, the time sample represented by the BURDEN dummy brought a worse cyclicality degree ($a_2 + \beta_1$) in comparison with such a degree over the previous period (a_2). We regressed five equations and the first three represent the estimation of simple fiscal rules (Table 7), or without the interaction term (BURDEN). In all of them (Eq.1-Eq.3), the fiscal response coefficient (or the fiscal cyclicality degree) presents a negative and statistically significant signal at 1%. It means that we had a pro-cyclical fiscal policy over the entire sample: a reduction of the primary surplus as a response to a rise in public debt-to-GDP ratio. Such a result corroborates the findings obtained in Moreira (2017) and Campos and Cysne (2019).

In turn, Eq.4 and Eq.5 applied the interaction term so as to test for a potential moderating effect of the particular period of higher fiscal burden. When we control for it, the cyclicality fiscal coefficient shows an important structural break between the two subsamples. On the one hand, before 2014, we indeed had a countercyclical fiscal policy, as a_2 becomes positive and statistically significant; on the other hand, from 2014 to 2019 the fiscal policy became pro-cyclical as $a_2 + \beta_1 < a_2$. Thus, we can suggest that Brazil really experienced a deterioration of its fiscal cyclicality starting from 2014, thereby transforming a countercyclical fiscal policy into a pro-cyclical one. Besides, all regressions presented a valid instrument list as the J-stat confirmed the exogeneity condition and so the overidentifying hypothesis (J-stat prob. > 0.1).

Table 7: Fiscal rules estimates using GMM regressions (May/2003-Apr/2019).
Dependent variable: PS

	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5
PS(-1)	0.534***	0.137		0.147	0.898***
	(0.074)	(0.108)		(0.107)	(0.054)
DEBT(-1)	-0.031***	-0.039***	-0.050***	0.070***	0.077***
	(0.003)	(0.004)	(0.002)	(0.012)	(0.015)
P(-1)	-0.066***	-0.065***	-0.070***	-0.028***	
	(0.006)	(0.007)	(0.004)	(0.006)	
Y(-1)		-0.053***	-0.046***		0.060***
		(0.012)	(0.009)		(0.006)
(BURDEN)DEBT(-1)				-0.163***	-0.206***
				(0.025)	(0.038)
DW stat	2.77	2.31	2.125	2.479	2.902
S.E. regression	0.268	0.252	0.250	0.267	0.359
Inst. Rank	22	22	22	22	22
J-stat	17.691	13.998	14.558	13.063	17.758
Prob(J-stat)	0.543	0.729	0.750	0.787	0.471

Notes: Estimation weighting matrix by HAC with AIC for lag definition; Instrument list: P(-2), DEBT(-2), PS(-2), I(-1), E(-1), Y(-2), EXP_P(-1), P(-3), DEBT(-3), PS(-3), I(-2), E(-2), Y(-3), EXP_P(-2), P(-4), DEBT(-4), PS(-4), I(-3), E(-3), Y(-4), EXP_P(-3); *** for 1% of significance; ( ) for Std. Error.
Source: drawn from the survey data.

6.4. Macroeconomic effects of changes in fiscal cyclicality

A worse fiscal cyclicality can lead an economy to several disruptions, such as increasing interest rates and risk premia, as well as lower output growth and a rise in inflationary indicators (Reinhart and Rogoff, 2010). Specifically in terms of the Brazilian economy, and over our time sample, we aimed at testing potential effects on expected inflation rates. Our hypothesis was that a worse fiscal cyclicality – meaning a weaker response of primary surpluses to public debt/GDP ratio – is followed by a context of higher expected inflation rates, thereby causing a damage to monetary policy's effectiveness.

The main goal of the Central Bank of Brazil, under its inflation targeting regime, is to control inflationary expectations in order to make the latter convergent to the inflation target over relevant time horizons. In turn, if a period of losses in fiscal cyclicality is accompanied by an increase of inflationary expectations, then the Central Bank needs to adjust interest rates in a more aggressive way so as to react to eventual inflation deviations. It means that a worse fiscal regime implies a higher sacrifice ratio.

Therefore, we estimated GMM regressions taking EXP_P (expected inflation rate) as dependent variable and using the conventional controls for such a rate, based on a type of expected *Phillips Curve*. Thus, P, Y and E were included as control variables, while BURDEN performed the role of testing for our hypothesis. We expected that the statistical significance for a positive coefficient of BURDEN would validate our intuition. In order words, when the fiscal rule in Brazil becomes pro-cyclical – represented by the BURDEN period (see Eq. 4 and Eq. 5 in Table 7) – the expected inflation for 12 months ahead increases, thereby imposing obstacles to inflation stabilization in Brazil.

$$EXP\_P_t = d_1 P_{t-1} + d_2 Y_{t-1} + d_3 E_{t-1} + d_4 BURDEN_t + \mu_t. \qquad (5)$$

Another way of testing for these macroeconomic effects is by adding a moderating effect of the BURDEN period on the relationship between Y and EXP_P. In normal circumstances, a rise in economic activity is followed by an increase of the inflationary expectations, due to an uptrend in wages and production costs, so that $d_2 > 0$. However, based on the *Fiscal Theory of the Price Level* (Woodford, 2001; Cochrane, 2001; Leeper, 1999), if there exists a pro-cyclical fiscal policy, meaning that primary surpluses do not correct for public debt changes, the underlying relation between Y and EXP_P can be inverted[6], so that even under an economic activity reduction we could observe increasing expected inflation rates. It would corroborate the hypothesis according to which there exists a loss of the monetary policy`s effectiveness as a consequence of a worse fiscal cyclicality over time. We then also estimated the following form:

$$EXP\_P_t = d_0 BURDEN_t + d_1 P_{t-1} + d_2 Y_{t-1} + d_3 E_{t-1} + d_4 (BURDEN_t) Y_{t-1} + \mu_t. \ (6)$$

If our hypothesis is correct so $d_2 > 0$ and $d_2 + d_4 < 0$. In such a case, the BURDEN period implies an inverse relationship between economic activity and expected inflation in Brazil, in contrast with the previous period of a better fiscal cyclicality. The estimates showed in Table 8 confirmed such a hypothesis. The BURDEN period implied both a direct effect and a moderating one on expected inflationary rates in Brazil. While Eq. 1 and Eq.2 test only for the direct effect (with d_0 positive and statistically significant at 1%), Eq. 3 to Eq. 5 also included the interaction term.

Table 8: Expected inflation using GMM regressions (May/2003-Apr/2019).
Dependent variable: EXP_P

	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5
P(-1)	0.392***	0.381***	0.299***	0.312***	0.299***
	(0.036)	(0.043)	(0.054)	(0.040)	(0.051)
Y(-1)	0.282***	0.257***	0.310***	0.382***	0.365***
	(0.047)	(0.048)	(0.080)	(0.072)	(0.090)
E(-1)	0.242*			0.431***	
	(0.141)			(0.118)	
BURDEN	0.212***	0.224***	0.187**		
	(0.072)	(0.067)	(0.062)		
(BURDEN)*Y(-1)			-0.382**	-0.414***	-0.450***
			(0.153)	(0.119)	(0.129)
DW stat	1.345	1.249	1.019	1.23	1.155
S.E. regression	0.505	0.493	0.463	0.483	0.477
Inst. Rank	25	25	25	25	25
J-stat	24.004	24.030	23.105	22.791	22.948
Prob(J-stat)	0.292	0.345	0.338	0.355	0.404

[6] Although our purpose was not to explore all the potential channels by which such an inversion of the relation between Y and EXP_P may occur, a possible way is to regard the worse fiscal period as a type of positive cost shock to firms. A related exchange rate undervaluation, as a consequence of rising country-risk, can be complimentary to this hypothesis.

Notes: Estimation weighting matrix by HAC with AIC for lag definition; Instrument list: I(-1), E(-1), I(-2), E(-2), I(-3), E(-3), I(-4), E(-4), EXP_P(-2), DEBT(-2), P(-2), Y(-2), EXP_P(-3), DEBT(-3), P(-3), Y(-3), EXP_P(-4), DEBT(-4), P(-4), Y(-4), EXP_P(-5), DEBT(-5), P(-5) and Y(-5); *** for 1%, ** 5%, * 10%; ( ) for Std. Error.
Source: drawn from the survey data.

Doing so, there existed an inversion of the classical relationship between Y and EXP_P. Before 2014, the estimated correlation between them was positive ($d_2 > 0$), as predicted by a new-keynesian Phillips Curve approach. In contrast, from 2014 to 2019 such a correlation was inverted into a negative one: $d_2 + d_4 < 0$. It suggests that under the higher fiscal burden period even a tighter monetary policy, so implying a reduction of the economic activity, would be accompanied by a rising expected inflation rate. Such an evidence can be interpreted as an indirect loss of monetary policy's effectiveness amid a period of worse fiscal cyclicality. In turn, effective inflation rate (P) and nominal exchange rate (E) performed their expected positive lagged effects on inflationary expectations. All equations showed a validation of their instrument list as the correspondent J-stat satisfied the exogeneity condition and thus the overidentifying hypothesis (J-stat prob. > 0.1).

7. Concluding remarks

Our empirical work analyzed the behavior of the fiscal rule in Brazil by means of a fractional integration approach. Doing so, we did not applied conventional unit root tests to our time series, differing the estimates from the most part of the related literature, so that we appropriately dealt with the potential existence of long-memory in the dataset.

Initially we performed an univariate analysis on the Brazilian gross public debt-to-GDP ratio. The findings suggested a high persistence of such a variable over the time sample, with a particular increase from 2014 to 2019, thereby confirming a recent stylized fiscal fact, according to which there existed a fiscal deterioration over the last years of Dilma Rousseff`s presidential mandate.

In turn, we applied GMM regressions so as to estimate different fiscal rules for Brazil, in order to measure the sensitivity of the primary surplus (%GDP) to the gross public debt (%GDP), that is, the fiscal cyclicality degree. An interaction term was also adopted to controlling for a potential moderating effect of a higher fiscal burden on the fiscal cyclicality coefficient. We found evidence of a pro-cyclical bias specifically over the subsample 2014-2019. Such a fiscal policy was not convergent to the Bohn`s principle of public debt sustainability.

Furthermore, we also tested for expected inflationary effects as a consequence of a higher fiscal stress in Brazil. To do so, we also applied a moderating effect to GMM regressions based on an expected Phillips curve. Our findings demonstrated that the subperiod from 2014 to 2019 was marked by an inversion of the common relationship between economic activity and expected inflation. It means that, under a fiscal deterioration or a pro-cyclical fiscal policy, even a tighter monetary policy was followed by a rise in expected inflation rates in Brazil. We can interpret it as a loss of monetary policy`s effectiveness amid a higher fiscal burden.

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