On measurement and continuity in neoclassical economics: The Pareto-Cassel controversy, 1899-1902

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Abstract

This paper explores the controversy over the use of mathematics in economics happened between 1899 and 1902 involving Vilfredo Pareto, Gustav Cassel, Knut Wicksell, and Gaetano Scorza, with Maffeo Pantaleoni playing the role of intermediary. It begins by recapping the content of Pareto's early articles and his book *Cours d'Économie Politique* (1896-97), where he lays out his method of successive approximations, the mutual interdependence of economic phenomena, and Leibniz's principle of continuity. After that, Cassel's criticism of formalization in economics is presented, as firstly set forth in his *Grundrisse einer elementaren preislehre* (1899) and further developed in later works. Cassel's own simplified system of equations for determining prices through the concept of scarcity is introduced. The next section covers a set of unpublished letters between Pareto, Pantaleoni, and Cassel sitting at the National Library of Sweden. These documents reveal significant aspects of academic life in Europe at the time, as well as the correspondents' interests on pure theory. The last section reviews the reception of Cassel's *Grundriss* by Wicksell, Scorza, and Pareto. A few appointments on the history of mathematics are included to indicate how the rift among the mentioned economists echoed the influence of French and German traditions in infinitesimal analysis and algebra.

Key words: utility, measurement, continuity, scarcity, general equilibrium

JEL: B13, B16, B31

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Introduction

Is it valid to construct a theory assigning a prominent role to variables that are impossible to measure? And if so, how should conclusions reached through such a procedure be set against reality? As early as 1889-91, a fierce controversy emerged between Marie-Esprit Léon Walras, Ladislau von Bortkiewicz, and Francis Y. Edgeworth over the extent of formalization in economics, the meaning of free competition and whether the idea of general equilibrium could represent the actual operation of markets.[1] Essentially that engagement was between two different approaches to equilibrium, that is, the continental one placing weight on general interdependence and Walras's notion of price-setting through *tâtonnement*, and the British one

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[1] Roberto Marchionatti, "On the application of mathematics to political economy. The Edgeworth–Walras–Bortkievicz controversy, 1889–1891", *Cambridge Journal of Economics* 31 (2007): 291-307.

associated with Edgeworth and Alfred Marshall, which privileged more realistic representations of the economic world.[2]

The next great controversy, which is the subject of this paper, developed from 1899 to 1902 among the continental followers of the Walrasian approach. In those years, Vilfredo Pareto was teaching political economy at the University of Lausanne, where he held the chair formerly occupied by Walras. He had already published several articles on pure economic theory,[3] as well as his two-volume *Cours d'Économie Politique* (1896-97), along with other writings on socialism and politics.[4] Gustav Cassel, after being awarded a degree in mathematics in 1894, had geared his mind towards economic thinking and visited Germany and England to become acquainted with the subject. Over this period, he was also searching for a post from which he could teach political economy. In 1901 he missed out on the chair at the University of Lund, which went to Knut Wicksell. Cassel would eventually take up the chair of national economy and finance at the University of Stockholm in 1904, where he remained until his retirement in 1933.[5]

What is most relevant for the purposes of this study is that Cassel, in his famous 1899 article *Grundriss einer elementaren preislehre* (Outline of an elementary price theory) was severely critical of the concept of marginal utility and the alleged continuity of the demand function adopted by many economists of the time. He proposed instead a singular and much more simplified theory of prices based on the notion of scarcity (Cassel 1899). Cassel tried to convince Pareto, and later Maffeo Pantaleoni, to review his *Grundriss*, but without success. Eventually, the Italian mathematician Gaetano Scorza accepted the task and published, in 1902, his assessment of Cassel's work in the *Giornale degli Economisti*. Wicksell would also publish a full review of the *Grundriss* in 1900, with Pareto taking the opportunity to comment on Cassel's concerns in a 1902 entry to the *Encyklopädie der Mathematischen Wissenschaften*.

In what follows, we shall submit an assessment of the episode by examining the views of the aforementioned economists on the formalization of economics, the measurement of utility and the role of continuity in pure analysis.[6] We not only refer to published works and the correspondence between Pareto and Pantaleoni, but also to a series of unpublished letters

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[2] Michel de Vroey, "Marshall and Walras: Incompatible bedfellows?", *The European Journal of the History of Economic Thought* 19 (2012): 765-83; Franco Donzelli, "Marshall vs. Walras on equilibrium and disequilibrium", *History of Economics Review* 48 (2008): 1-38.

[3] John S. Chipman (ed.) "Special issue: Articles by Vilfredo Pareto"*, Giornale degli Economisti* 67 (2008): 273-566.

[4] Fiorenzo Mornati, *Vilfredo Pareto: An Intellectual Biography* (London: Palgrave Macmillan 2018), vol. II, pp. 2-14, 93-124.

[5] Hans-Michael Treutwein, "Gustav Cassel (1866-1945)", in Gilberto Faccarello and Heinz Kurz (eds.) *Handobook on the History of Economic Analysis, Volume I.* (Cheltenham: Edward Elgar, 2016), pp. 404-9; Lars Magnusson, "Gustav Cassel, popularizer and enigmatic Walrasian", in Bo Sandelin (ed.) *The History of Swedish Economic Thought* (New York: Routledge 1991), pp.122-40.

[6] Alain Alcouffe and Donald A. Walker have covered partially the same ground, but without making these epistemological issues the main object of their expositions, centered instead on a comparison between Walras's, Pareto's and Cassel's formal conceptions of general equilibrium. See Alain Alcouffe "Walras, Cassel, the German connection revisited," in Volker Caspari, (ed.) *The Evolution of Economic Theory. Essays in Honour of Bertram Schefold* (New York: Routledge 2011), pp. 113-31; Donald A. Walker, "Early general equilibrium economics: Walras, Pareto and Cassel", in Warren Samuels, Jeff E. Biddle and John B. Davis (eds.) *A Companion to the History of Economic Thought* (Oxford: Blackwell 2007), pp. 278-93.

between Cassel and these two Italian economists from the period under scrutiny stored within the National Library of Sweden. This material throws some light not only on academic life in Europe, but also on the theoretical interests of the involved parties at the time. Abiding by this plan, the second section presents an account of Pareto's personal understanding of how science should be done, his position on the use of mathematics in pure theory, and its connection with the principle of continuity. The third section contains an overview of Cassel's concerns from 1899 over the unwarranted use of mathematics in economics, the limited soundness of marginal utility, along with his own system of general equilibrium. The fourth section commences with some context on Pantaleoni as a preamble to the correspondence between him, Pareto, and Cassel at the time of the controversy. The responses to Cassel's criticisms from Wicksell, Scorza, and Pareto are covered in the fifth section. The study ends with some final reflections on the analytical implications of formalization in economic theory and their connection with distinct traditions of mathematical thinking.

Pareto on mathematics, economic theory, and continuity

The basic building blocks in Pareto's method of economic analysis are set out in the opening section of his *Cours*. The first foundation for his theoretical disquisitions is the idea of successive approximations, which he borrowed from the physical sciences. As any concrete phenomenon results from the confluence of several factors, any explanation encompassing all forces in play would simply be impossible. Pure theory is not, therefore, an ideal representation of the world. Rather, it is a representation of the general or average form of the phenomena it seeks to represent. Secondary aspects of this object of inquiry then become the focus of applied study, which considers those elements that have been excluded by abstract analysis. Now, the second foundation of Pareto's analytical method lies in his embracing of Walras' conception of general equilibrium. Such rather complex outlook allows the Italian economist to frame his theories of consumption, production, and capital formation, respectively, as conforming coexisting systems of equations. From the simultaneous interaction of these integrated systems, a set of equilibrium values can be found for all traded quantities for a given array of equilibrium prices. Under free competition and assuming the expenditure of all earned incomes, the price increases from excess demand and, conversely, the price reductions from excess supply provide concrete grounds for the associated presumption that a specific price set will emerge that clears markets. Moreover, the resulting equilibrium would result in society attaining an economic maximum, in terms of ophelimity,[7] for some initial endowment of capital goods and skills.[8]

[7] The word *ophelimité* is Greek and Pareto introduced it to distinguish the strict subjective satisfaction each individual experiences by consuming certain goods from the more general meaning of utility, a term supposed to convey a sense of the material, moral and scientific progress of society. Vilfredo Pareto, *Cours d'Économie Politique* (Lausanne: F. Rouger 1896-97), vol. I pp. 4-14. In an article from 1894, Pareto firstly proved that, under free competition, social ophelimity or the total of every individual's ophelimity would be maximum from the most efficient use of productive services. In the second part of the article, though, warned by Enrico Barone and Maffeo Pantaleoni as to the difficulties surrounding a common measure of utility, Pareto changed his method of calculation by establishing a correspondence between individual ophelimities and the amount of some specific good taken as equivalent to the remaining others according to their respective relative prices. Vilfredo Pareto, "The maximum of utility given by free competition", *Giornale degli Economisti and Annali di Economia*, Nuova Serie 67 (2008 [1894]): 387-403; see also Michael McLure, "The Pareto-Scorza polemic on collective economic welfare", *Australian Economic Papers* 29 (2000): 347-71.

[8] Pareto, *Cours*, vol. I pp. 16-18, 38-46.

The synthesis devised by Pareto in the *Cours* affords a helpful insight into a major, but often neglected, facet of his thought: the adoption of Gottfried Leibniz's principle of continuity. Building upon Kepler's studies on conic sections describing how ellipses evolve into parabolas, the Saxon philosopher defined continuity as ontological density, with every being developing itself through small and imperceptible steps, and also formally, indicating the possibility of inserting a point between any two other points in a time or space continuum. When the principle of continuity is taken as the backdrop of knowledge, what initially appear to the eye as distinct states of matter should be construed instead as limiting cases of the more general laws of motion. A position of rest, for instance, could be interpreted as the limiting condition of slowness. It is immediately evident then that, to Leibniz, abstraction and generality are the bases for any type of comprehensive reasoning. The corollary of this perspective consists in the epistemological rule that theories alien to the principle of continuity should be discarded as lacking proper logical foundation. Leibniz himself framed his thoughts on the subject in the following manner:

> Nothing takes place suddenly, and it is one of my great and best confirmed maxims that nature never makes leaps. I called this the Law of Continuity when I discussed it formerly in the *Nouvelles de la republique des lettres* ['Letter on a general principle useful in explaining the laws of nature']. There is much work for this law to do in natural science. It implies that any change from small to large, or vice versa, passes through something which is, in respect of degrees as well as of parts, in between; and that no motion ever springs immediately from a state of rest, or passes into one except through a lesser motion.[9]

Confining the matter to the mathematical realm, as the Calculus progressed over the eighteenth and, particularly, the nineteenth century, the notion of continuity came to be associated with two diverging views, one as physical displacement, which was in line with Isaac Newton's concept of fluxions, the other as a sequence of infinitely small (*infiniment petits*) variations, which seemed closer to Leibniz's understanding of differentials.[10] Eventually, both approaches would coalesce and prove themselves extremely valuable to the advance of the Calculus, even though the notion of limits as devised by Augustin-Louis Cauchy (1789-1857) would eventually dominate the field. To Cauchy, continuity was a philosophical principle assuring that all laws of mathematical analysis kept their validity in the transition of finite to infinite quantities.[11] Looking also at the institutional context in France over the nineteenth century,

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[9] Gottfried W. Leibniz, *New Essays on Human Understanding*, Peter Remnant and Jonathan Bennett (eds.), (Cambridge: Cambridge University Press 1996 [1704]), §55; see also Larry M. Jorgensen, "The principle of continuity and Leibniz's theory of consciousness", *Journal of the History of Philosophy* 47 (2009): 223-48; Timothy Crockett, "Continuity in Leibniz's mature metaphysics", *Philosophical Studies* 94 (1999): 119-38; Benson Mates, *The Philosophy of Leibniz* (New York: Oxford University Press 1996), pp. 146-49.

[10] Niccolò Guicciardini, *The Development of Newtonian Calculus in Britain 1700-1800* (Cambridge: Cambridge University Press 1989), pp. 9-88; Charles H. Edwards Jr, *The Historical Development of the Calculus* (New York: Springer Verlag 1979), pp. 169-267; Carl B. Boyer, *The History of the Calculus* (New York: Dover 1949), pp. 147-223.

[11] In his landmark books summarizing his lessons at the *École Polytechnique*, Cauchy firstly defines continuity, differentiation, and integration for a finite interval, and later investigates if those operations remain valid when both edges of functions are extended towards infinity. In his *Cours d'Analyse*, as well in later works, a function is qualified as continuous between points x_0 and X if, for any intermediary value x within that interval, infinitely smaller increases α are added to x for which the difference $f(x+\alpha)-f(x)$ diminishes along with α. Interestingly, the examples of continuity provided by Cauchy are of everywhere differentiable functions such as $a+x$, $\sin x$, and A^x, among others. He presents only a few cases of pointwise discontinuity such as $1/x$ at $x=0$. Augustin L. Cauchy, *Cours d'Analyse de L'École Royale Polytechnique* (Paris: Libraires du Roi 1821), pp. 34-7; Augustin L. Cauchy, *Resumé des Leçons Donnés a L'École Royale Polytechnique sur le Calcul Infinitésimal.*

within the walls of the leading and influential *École Royale Polytechnique*, the Calculus never strayed from its close connection with Mechanics. This strong linkage implied that the continuity of functions, however defined, remained commonly accepted as a condition for differentiability and, therefore, for all fundamental theorems established from this key assumption. Anomalous cases were simply discarded as oddities lacking both analytical and practical interest.[12]

Although German mathematicians would introduce deeper algebraical rigor into the Calculus after mid-nineteenth century, debunking thus the tacit continuity-differentiability connection, their effort took some time to reach widespread currency in Europe, with the French tradition being ultimately informing Pareto's view of science, in general, and the use of the mathematical approach to economic theory, in particular.[13] Leibniz's principle of continuity is readily evident in several instances of Pareto's work. To begin with, the Lausanne professor assumed at the very outset of the *Cours* that economic variables, when defined in their aggregate values within society, could be handled as continuous entities. "The science of quantities teaches us how," notes Pareto, "when working with large numbers and averages such as the ones considered by political economy, one can suppose that the quantities grow by imperceptible and continuous degrees". Also, he claimed there being no clear distinction between the alleged different species of capitals or even, for that matter, of labor, all of which constituted just diverse manifestations of the larger genus "productive services" mobilized by entrepreneurs to obtain delayed and increased ophelimity.[14]

Pareto's acceptance of the principle of continuity is equally conspicuous in his first systematic contributions to pure economic theory outlined in a series of five articles published in the *Giornale degli Economisti* in 1892 and 1893 under the common title *Considerazioni sui principi fondamentali dell'economia pura.* In those articles, Pareto distinguished between the

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Oeuvres Complétes d'Augustin Cauchy, IIe Série, Tome IV (Paris: Gauthier-Villars 1899 [1829]), pp. 122-194, 305-39; see also Gert Schubring, *Conflicts between Generalization, Rigor, and Intuition. Number Concepts Underlying the Development of Analysis in 17–19th Century France and Germany* (New York: Springer 2005), pp. 147-166; Umberto Bottazzini, *The Higher Calculus: A History of Real and Complex Analysis from Euler to Weierstrass* (New York: Springer Verlag 1986), pp. 103-108; Philip E. B. Jourdain, "The origin of Cauchy's conceptions of a definite integral and of the continuity of a function", *Isis* 1 (1913): 661-703.

[12] Schubring, *Conflicts*, pp. 381, 410, 460-2, 607; Jesper Lützen, "The foundation of analysis in the 19th century", in Hans N. Jahnke (ed.) *A History of Analysis*. (Providence: American Mathematical Society 2003), pp. 155-212. The German mathematician Karl T.W. Weierstrass (1815-1897), in an article published in 1872, proved that the function $f(x) = \sum_{n=1}^{\infty} b^n \cos(a^n x)$ is everywhere continuous but nowhere differentiable for $ab > 1 + 3\pi/2$. Other pathological functions were similarly created afterwards. Ernst Hairer and Gottfried Wanner, *Analysis by Its History* (New York: Springer 2008), pp. 273-4. Famous French mathematicians, however, dismissed such examples as irrelevant. Charles R. Méray (1835-1911), for instance, wrote that "The discontinuous functions, with no derivatives, no integrals etc. are just found in metaphysical dissertations; it is useless to worry about such cases" Charles Méray, C. 1894. *Leçons Nouvelles sur l'Analyse Infinitesimale et ses Aplications Géométriques*. (Paris: Gauthier-Villars 1894), Tome 1 vol. I p. XIII. The distinguished *Polytechnique* professor Charles Hermite (1822-1901), whose book on analysis is mentioned approvingly by Pareto, raised a similar point in a letter from 1893 to his Dutch friend Thomas Stieltjes (1856-1894), when he commented: "I run away with fright and terror from such regrettable plague of continuous functions without any derivatives", in Benjamin Baillaud and Henry Bourget, *Correspondance d'Hermite et de Stieltjes* (1889-1904) (Paris: Gauthier-Villars 1905), vol. II p. 318; see also Vilfredo Pareto, *Considerations on the Fundamental Principles of Political Economy*, Roberto Marchionatti and Fiorenzo Mornati (eds.) (New York: Routledge 2007 [1892, 1893]), p. 6.

[13] Pareto, *Considerations*, p. 6 and Mornati, *Vilfredo Pareto*, vol. II pp. 34-39.

[14] Pareto, *Cours*, vol. I pp. 9, 40-2.

existence of total utility, which he left as an open question, and utility (by 1892 he had not yet coined his neologism ophelimity, which firstly appears in the *Cours*) that can be contemplated by individuals, which concerns the utility relating to very small quantity of a good. It is individual's general awareness of their 'final degree of utility' for economic goods, and not the existence of a utility function per se, that emboldened Pareto to treat very small variations in utility as if the utility function existed and was continuous. "Let us assume that economic goods are indefinitely divisible. The consideration of non-divisibility beyond a certain limit does not create major difficulties with the general principles of sciences and quantities."[15]

When the final degree of utility of a good depends on the quantity of two or more other goods, then, in a formal sense, the total utility function depends upon its integrability.[16] Pareto was generally aware of this condition, even at this early stage of his career as a theoretical economist. But he did not attempt to provide such a demonstration, as these issues are not of the first order of importance for the relationship between utility and exchange, production and capital formation, Pareto simply accepted the decreasing final degree of utility from the consumption of a good as an "an experimental fact" and regarded issues associated with integrability as the subject of study in individual psychology rather than political economy.

> The case where total utility exists corresponds perfectly to the cases considered by the science of mechanics, where there exists a function of the forces. Almost all authors who have written about the mathematical theories of political economy assume as evident *a priori* the existence of a function representing total utility [with the final degree of utility given by the partial derivatives of the total utility function]. But this existence is not demonstrated, at least in the most general cases.[17]

Of course, Pareto did acknowledge that, in many cases, the quantities consumed could not change in small amounts in order to satisfy the maximum condition that the ratios between the final degrees of utility and the price of each good must be equal at the margin of expenditure. But even that did not stop him from employing mathematical ideas grounded in the principle of continuity to advance the development of economic theory. He listed several reasons for this approach, but three of them shall be enough here for our purposes. First, large quantities of consumption goods can, in Pareto's assessment, be considered as varying in small amounts when the demand for each good or service is defined in its totality, encompassing all members of society. Second, statistical tools and data may be developed in the future that have the potential to render utility measurable to a reasonable degree of accuracy. Lastly, but perhaps the most important argument of all, Pareto envisaged significant gains for economic theory in the circumstances where continuous functions present a good approximation to discontinuous quantities, with the employment of the advanced tools of differential analysis enhancing scope for simplification, which will contribute to the development of pure theory:

> To remove these difficulties [associated with discontinuity], the method that is usually adopted consists in making the discontinuous functions continuous, ensuring that the error generated by so doing is negligible. There are several reasons for this, one of the main ones being the fact that

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[15] Pareto, *Considerations*, pp. 15-16.

[16] Surprisingly, at no stage in his scholarly career did Pareto correctly establish the conditions for the integrability of the utility function with three or more independent variables.

[17] Pareto, *Considerations*, p. 105.

> discontinuous functions are much more difficult to deal with than continuous functions, which is also the case when using mathematics.[18]

Nevertheless, the Italian economist was cognoscente of the potential dangers of employing formal reasoning in economics. Most notably, as a devout positivist, Pareto believed in the evolution of economics through mathematical tools just as a first step towards a better knowledge of truth, never as a goal in itself.[19] As a result, he warned of the possibility of theoretical misrepresentation, especially by the choice of incorrect assumptions about the world, and cautioned against the unmediated transition from pure theory to reality without the due introduction of additional assumptions. Pareto also voiced his apprehension about an excess of refinement in theoretical explorations, a danger that tends to obscure the meaning of the formulae even to expert readers.[20] Nonetheless, in a speech delivered at the University of Lausanne students' Union *Societé Stella* in 1898, Pareto insisted that the question of the existence of a continuous and diminishing ophelimity function was a completely distinct one from the premises about the measurability of that variable. The definition and the measurement of any concept consisted in two quite separate issues. Astronomers were beset by similar difficulties when attempting to calculate the distance between the stars, he observed, but this long-standing measurement problem did not impair the steady progress of celestial mechanics.[21]

Cassel and the theory of prices

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[18] Ibid., p 36. For Pareto's reasons on the continuity of the utility function, pp. 11, 36, 41, 58, 60, 74, 92, 117, 128 passim.

[19] While Pareto followed Walras in general equilibrium, there was indeed a serious rift between the two masters of Lausanne, with Pareto being scornful of Walras's rational approach, preferring instead to adopt the experimental method, in which observation informs both the development of theory and judgements about its scientific standing. For instance, in 1909, he wrote to Pantaleoni that: "Walras is now living in the clouds with his metaphysics, and he is meeting the same fate as the astronomer who fell into the well", Pareto to Pantaleoni, June 17 1909, *apud* Erich Schneider, "Vilfredo Pareto. The economist in the light of his letters to Maffeo Pantaleoni", *PSL Quarterly Review*, 14 (2014 [1961]): 247-95. On Pareto's assessment of Walras's overall work, see Pascal Bridel and Fiorenzo Mornati, "De l'équilibre général comme 'branche de la métaphysique': Ou de l'opinion de Pareto sur le projet walrasien", *Revue Économique*, 60 (2009): 869-90, as well as Roberto Marchionatti and Enrico Gambino, "Pareto and political economy as a science: Methodological revolution and analytical advances in economic theory in the 1890s", *Journal of Political Economy* 105 (1997): 1322-48. For most of his *Elements of Pure Economics*, which firstly appeared in 1874, Walras worked with continuous functions, but acknowledging the difficulty of measuring utility. He managed to derive the demand curves in a two-party exchange by assuming a declining *rareté* [marginal utility] as traded quantities increased. He even studied how equilibrium is achieved under a discontinuous curve of supply by one of the parties. Léon M. E. Walras, *Elements of Pure Economics*. (London: George Allen and Unwin 1965 [translation of the 4th ed. 1900]), pp. 115-31.

[20] Pareto, *Considerations*, pp. 35, 124-5. As he remarked once about the hardships of thinking about general equilibrium through mathematical expressions: "Certainly the reasoning does not flow smoothly and rapidly; and even in this case which I have tried to reduce to the simplest terms, when I see the formulae stretch out like snakes, and even more when I contemplate those that are still to be written — and I see no means of shortening them — I am assailed by the doubt as to whether I shall be fortunate enough to find even a single reader with the patience to follow me up to this point and the even greater patience to persist (Pareto 1892b: 335). Vilfredo Pareto, "The theory of prices of Messrs. Auspitz and Lieben and Professor Walras's observations", *Giornale degli Economisti* and *Annali di Economia*, Nuova Serie, 67 (2008 [1892]): 321-53, 335.

[21] Vilfredo Pareto, "Comment se pose le problème de l'économie pure: Mémoire présenté, en décembre 1898, à la Société Stella, *Cahiers Vilfredo Pareto,* 1 (1963 [1898]): 121-30.

Early in his career as an economist, Cassel set himself the "gigantic undertaking" of reconstructing the entire science he thought built upon vain endeavors on metaphysical speculations. The first outcome of this quite ambitious goal came out in his *Grundriss*, published in 1899 and offering an outline of his future works on economic theory, many with lasting contributions to the science.[22] After acknowledging Walras's notion of general equilibrium as a source of inspiration, Cassel goes on to regret the excessive mathematization of the subject by the French economist. In part, this regret was due to mathematical formalism blocking access to that line of thought to the public. To make general equilibrium more accessible, a more consequential approach to economic theory would be required to achieve a drastic simplification of methods and tools. To improve its scientific grounding, equilibrium economics should be made more succinct by eliminating artificial concepts that are devoid of any means of measurement and, therefore, completely divorced from reality. In short, Cassel advocates for a simplified general equilibrium approach way less abstract than the Walrasian one. The main target of his attack is the fundamental proposition in neoclassical theory that the individual distributes their expenditures in such a way that the ratio between marginal utility and price is equivalent for all purchased goods during an interval. To demonstrate the weakness of that assertion, Cassel disputed the validity of two key assumptions subjacent to it: first, that consumer goods are infinitely divisible and, second, that the utility function is continuous.

As to the first assumption, Cassel lists quite a few examples of goods that cannot be divided at will. For instance, he mentions travel tickets, newspapers, meals and renting rooms. In cases like these, Cassel concludes that the marginal utility would be greater than the price in question, otherwise the individual would not even consider buying the ticket or renting the room. While recognizing that certain goods, such as sugar, are well suited to traditional economic analysis, Cassel contended that such example is not representative for supporting a general theory. When one considers articles of consumption in their totality, the supposed continuity of the utility function is not only compromised by the indivisibility of most goods, but also, by the circumstance that in daily situations individuals hardly redo their expenditures when faced with small variations in price. Even if one talks about the whole demand for a single good by society, some might have indeed their marginal utility equal to its respective price, but many would be rather willing to accommodate a rise in price within their current income, making thus the demand function discontinuous. To leave no doubt about the seriousness of the issue for economic science, Cassel pushes the point further by indicating that the idea of utility, and inference drawn from that, is nothing but pure metaphysical conjecture because of the utter impossibility to attribute any meaningful number to individual assessments of utility. Therefore, interpersonal comparisons of satisfaction and related conclusions on social welfare are all empty of significance.[23] We should briefly observe, at this point, that Cassel's extremely critical position on the use of utility in economic theory appears to derive, in part, from the German influence on his formative studies in mathematics.[24]

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[22] Hans Brems, "Gustav Cassel revisited", *History of Political Economy* 21 (1989): 165-78 and Eric Englund, "Gustav Cassel's Autobiography", *The Quarterly Journal of Economics* 57 (1943): 466-93.

[23] Gustav Cassel, "Grundriss einer elementaren preislehre", *Zeitschrift für die Gesamte Staatswissenschaft* 3 (1899): 395-458, 415-20.

[24] In German states over the nineteenth century, mathematicians worked away from applied sciences faculties, a condition that allowed the development of an algebraic approach to the Calculus towards strengthening its foundations by the rigorous axiomatic definitions of numbers, limits, and integrals. Functions, for instance, were no longer associated with an analytical expression, but conceived as any kind of association between values. This

To the Swedish economist, assuming goods as infinitely divisible so that economic theory can be constructed to benefit from the use of mathematical tools in economic analysis is akin to the tail wagging the dog: it is the reverse of the correct approach to developing science. Any theory, explains Cassel, ought to incorporate the properties of the specific object it is inquiring upon, instead of trying to squeeze reality into some available mathematical apparatus. When commenting on Marshall's marginalist treatment of consumption decisions based on marginal utility, Cassel bitterly criticizes the British economist for his disposition to hold on to an account of human behavior based on complex mind operations detached from the observations of everyday life.

> That this violence to reality reaches such a degree is, first, probably explained by the fact that one wants to press it into a mathematical form at all costs. Then there was the strange idea that "a mathematical law is in theory always continuous" (as Jevons puts it). This view must be rejected. The scientific form of any quantitative description is surely mathematical, but this form can and must depend on the content - not the other way around.[25]

If utility is then completely removed from the picture, what would be now the chief factors driving consumers' choices? For Cassel, economic analysis must be firmly anchored on real and measurable entities. And for him the essential component of economic life was the monetary price of every transacted good, a well-known and measurable phenomenon.[26] There was no place, therefore, for ethereal digressions on value stemming from labor or utility and just resulting in a massive waste of room on the shelves of economic literature. Instead, all reasoning pertaining to how consumers make their choices are to be framed in reference to prices, with the price of each good being the key unit of analysis. In addition, a correct study of the economy's working mechanism can only be achieved through the acknowledgment of the general interdependency of markets within the national economy. Thus, the demand for any good would depend indeed on objective factors, not the subjective notion of utility. First, as an individual's real income grows, his consumption basket changes, whatever the constellation of prices in vigor. Second, the demand of any good by an individual depends not only on its respective price, but also on the prices of every remaining goods, tying together all strands of analysis into one single coherent whole. In a nutshell, general equilibrium should be expressed by a system of simultaneous equations which could be easily grasped by any reader with average instruction. "The mathematics really demanded by the popular economy is elementary

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trend culminated in the formulation of the Set Theory by Georg Cantor (1845-1918) and the famous Dedekind Cut by Richard Dedekind (1831-1916) in 1872, fundamental concepts in establishing the continuity of the real domain. Richard Dedekind, *Essays in the Theory of Numbers* (Chicago: Open Court 1924 [translation from the 1875 ed.]), pp. 1-27; Schubring, *Conflicts*, pp. 481-565; Roger Bunn, "Developments in the Foundations of Mathematics", in Ivor Grattan-Guinness (ed.) *From the Calculus to Set Theory 1630-1910* (Princeton: Princeton University Press 2000), pp. 220-55; Bottazzini, *Higher Calculus*, pp. 257-294; José Ferreirós, "On the relations between Georg Cantor and Richard Dedekind", *Historia Mathematica* 20 (1993): 343-63. Cassel, in his *Fundamental Thoughts in Economics*, published in 1925, showed himself conversant with these analytical advances: "In modern higher mathematics the arithmetical foundation of the elementary series of whole numbers is looked upon as the most essential thing. Economics cannot do without a corresponding carefulness in defining the units in which it proposes to measure the quantities it is contemplating." Gustav Cassel, *Fundamental Thoughts in Economics* (London: Ernst Benn 1925), p. 43.

[25] Cassel, *Grundriss*, p. 418.

[26] Hans Brems has contended that Cassel was the pioneer of the revealed preference approach to consumers' decisions, a contention fiercely opposed by Paul Samuelson, who is generally considered the first economist to fully formalize the idea. Hans Brems, "Gustav Cassel", 1989 and Paul Samuelson, "Gustav Cassel's scientific innovations: claims and realities", *History of Political Economy*, 25 (1993): 515-27.

in nature and does not go beyond the domain of the common systems of simultaneous equations that everyone learned at school."[27]

In fact, the structure of Cassel's price theory in the *Grundriss* is quite straightforward. Let p_i stands for the price of each finished good ($i = 1, 2, \ldots n$), with a demand function given by $F_i(p_1, p_2, \ldots, p_n)$. If the respective supply of each good within the system is taken as fixed and represented by A_i, then the problem can be depicted as the determination of all prices by simply establishing the equality between supply and demand in each market:

$$F_1(p_1, p_2, \ldots, p_n) = A_1 \quad (1)$$

$$F_2(p_1, p_2, \ldots, p_n) = A_2 \quad (2)$$

$$\ldots \quad \ldots$$

$$F_n(p_1, p_2, \ldots, p_n) = A_n \quad (3)$$

Here, the demand by every individual depends on the whole array of prices through three channels, namely: the direct competition of substitute goods; the effect of prices on the individual's real income and consequent consumption decisions, and, lastly, on the amount of personal inventory brought to the market, since every buyer is supposed also to be a seller of some commodity at the previous stage of production. Similar method would apply to the prices q_j of the given supply of raw materials R_j ($j = 1, 2, \ldots r$), including labor, where firms are supposed to operate with fixed coefficients of production a_{ij} ($i = 1, 2, .. n$). Once the prices p_i of final goods are found, so are prices q_j, determined alongside the distribution of productive resources satisfying a second group of r equations $R_j = a_{1j}A_1 + a_{2j}A_2 + \cdots + a_{nj}A_n$.[28]

The above system would be later fully developed by Cassel in his *Theory of the Social Economy*, where he extends his model of general equilibrium to dynamic conditions under a constant rate of growth and, further on, to a situation of long-term changing parameters. The whole exposition has as its pillars the principles of scarcity, indicating the limitation of productive factors to attend an infinity of wants, and the principle of minimal means, assuring the most efficient use of finite resources towards a specific end. When discussing prices, Cassel's wording is even reminiscent of Dedekind's Cut, for their role is expressed as splitting demand into two distinct segments, namely, one comprising the consumers willing to pay for it, and the other clumping together all remaining individuals who feel the prevailing price as too expensive.[29] "The demand for a commodity", writes Cassel, "must thus be cut down somewhere by means of its price".[30] The fiction of marginal utility is deemed entirely

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[27] Cassel, *Grundriss*, pp. 396-7, 405-15.

[28] Ibid., pp. 422-4, 438-58.

[29] Look at how Dedekind defines a cut in the one-dimensional line R produced by a rational point, which could sit at either one the contiguous extremes of both segments resulting from this procedure: "If now any separation of the system R into two classes A_1, A_2 is given which possesses only *this* characteristic property that every number a_1 in A_1 is less than every number a_2 in A_2, than for brevity we shall call such separation a *cut* [*Schnitt*] and designate it by (A_1, A_2)" (Dedekind 1924 [1872]: 12-13,emphasis in the original). The memoir than proceeds to show that cuts can be effected also by an infinity of irrational numbers, making R a densely ordered line infinitely continuous in both directions. Every real number is thus defined by its respective cut on R.

[30] Gustav Cassel, *The Theory of Social Economy* (New York: Augustus M. Kelley 1967 [translated from the 1st ed. 1918]), p. 83.

dismissible since prices provide all necessary information to the man of science. Or again, as Cassel himself posits the problem:

> But the complete absence of arithmetical basis in this theory [based on pure exchanges] though it was so often expressed in arithmetical forms and mathematical formulae, prevented it from having that internal solidity which is necessary in a scientific theory, and showed at the same time where the essential fault of the theory lay. It was the rejection of the standard of valuation actually used in economic life, the exclusion of money from the whole study of the economics of exchange.[31]

In *On Quantitative Thinking on Economics*, a book published amidst the Great Depression in 1935, one finds Cassel naturally more concerned about the causes of economic fluctuations and monetary disturbances associated with the violent price dislocations after World War I and the dismantling of the interwar gold standard in the 1930s.[32] More relevant for our purposes though is Cassel's detailing of his general equilibrium approach and comprising three basic steps. The first one consists in assuming a monetary unit and its corresponding supply as fixed from the very beginning, avoiding hence the usual dichotomy between the real and monetary domains of analysis, the source of what he considers most of the errors committed by economic science. Once relative prices are known, so are absolute prices as well. Second, the fixing of prices in a competitive environment can be interpreted as being carried out by an auctioneer guiding all markets at the same time. Otherwise, it would be impossible for any individual to calculate his demand for each good without knowing the whole set of prices and, therefore, his own real income, a process reminiscent of Walras' *tâtonnement* process presented in his final edition of his *Elements*.[33] The last and most significant aspect of Cassel's approach to general equilibrium lies in his refusal to concede the existence of a supply curve of every productive factor connected to its respective price. For him, it was impossible to calculate the marginal productivity of any element of production without the previous knowledge of all other costs. "The system of equations which I use for representing economic equilibrium contains only 'full-size' quantities and no increments of them. Thus the equilibrium theory of prices is shown to be entirely independent of the concept of margins."[34]

Pareto, Pantaleoni, and Cassel's exchanges

Maffeo Pantaleoni was a prominent figure in the advancement of marginalist economics in Italy in the years following the peninsula unification. After graduating in Law at the Potsdam Gymnasium in 1877, he started a prolific academic career which took him, among other institutions, to the universities of Naples (1895-1897), of Geneva (1897-1990) during his voluntary political exile, afterwards to Pavia (1900-1901), and finally to the University of Rome (1902-1924), where he retained the chair of political economy until his death. Two aspects of

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[31] Ibid., p. 48.

[32] On Cassel's formulation of the purchasing power parity doctrine, his quest for monetary reform and controversy with John Maynard Keynes during the interwar years, see Rogério Arthmar and Michael McLure, "John Maynard Keynes and the Royal Swedish Academy", *History of European Ideas* 44 (2018): 605-22 and Denis Kadochnikov, "Gustav Cassel's purchasing power parity doctrine in the context of his views on international economic policy coordination", *European Journal of the History of Economic Thought* 20 (2013): 1101-21.

[33] Walras, *Elements*, pp. 164-81, see also Pascal Bridel and Elizabeth Huck, "Yet another look at Walras's *tâtonnement*", *European Journal of the History of Economic Thought* 9 (2002): 513-40.

[34] Gustav Cassel, *On Quantitative Thinking in Economics* (Oxford: Clarendon 1935), p. 165; for Cassel's general equilibrium method, see pp. 164-81.

Pantaleoni's life are of relevance here. First, his longtime association with Pareto, both in the fields of science and politics, and second, his purchase, along with Antonio de Viti de Marco and Ugo Mazzola, of the *Giornali degli Economisti* in 1890, of which he remained one of the leading editors onwards.[35] Besides many works on several fields of applied, historical and abstract economics, Pantaleoni published in 1889 his masterpiece, *Principii di Economia Pura*, which was translated into English in 1898.[36]

A true believer in a minimal state and freedom of economic activity, Pantaleoni met a kindred soul in Pareto in 1890. A lifelong friendship started based on common values. Both were active liberals without fear of speaking out against the protectionist policies of the successive Italian cabinets, an attitude that kept them in constant conflict with political authorities. In 1891, Pantaleoni introduced Pareto to Walras, and three years later, in 1893, with the support of both economists, Pareto was appointed as an extraordinary professor of the Law Faculty at the University of Lausanne. After a one-year trial period, he eventually assumed the chair of Political Economy formerly occupied by Walras himself. In 1901, Pareto settled down for life in a country house in Céligny, on Lac Genève, some thirty miles away from Lausanne.[37]

In 1900, Cassel first contacted Pareto to introduce him to a friend who would be coming to Lausanne in search of economic information. Pareto, in reply, took the opportunity to forward a copy of some articles on mathematical economics he had just published in the *Giornale*.[38] After that, Cassel sent Pareto a copy of two of his works in economics, one the *Grundriss* and the other his book on labor economics *Das Recht auf den vollen Arbeitsertrag*, that had been published in the previous year. Pareto responded he would not be able to read the material because of his poor command of the German language, but that he would nonetheless seek the views of someone in Italy able to do that. Significantly, though, Pareto did not notice in Cassel's

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[35] Marco Dardi, "Maffeo Pantaleoni (1857-1924)", in Gustavo Faccarello and Heinz Kurz (eds.) *Handobook on the History of Economic Analysis, Volume I* (Cheltenham: Edward Elgar 2016), pp. 367-73; Piero Bini, "When Economics Talked to Society: The Life, Thought and Works", in Mario Baldissera (ed.) *Maffeo Pantaleoni: At the Origin of the Italian School of Economics and Finance* (London: Macmillan 1997), pp. 11-46; Piero Sraffa,"Obituary of Maffeo Pantaleoni" (from *The Economic Journal* 1924), in Baldissera, *Maffeo Pantaleoni*, pp. 5-10.

[36] In his book, Pantaleoni ties together hedonism and natural selection. Man, as any other species, he says, lives within a natural environment and the search for pleasant sensations with lesser effort is not only a psychological phenomenon, but also the way natural selection manifests itself among mankind. The genetic origin of hedonism would come from the interaction of the individual with their surroundings, and those most able to get pleasure in life-sustaining activities would outlast those unable to feel pain. Maffeo Pantaleoni, *Pure Economics*. (London: Macmillan 1898 [translated by T. Boston Bruce from the 1889 ed.]), pp. 3-57.

[37] Mornati, *Vilfredo Pareto*, vol. II, pp. 1-39; Louis Chauvel and Jean-Paul Fitoussi, "Pareto and Pantaleoni: Parallel Lives and Secant Lives", in Baldissera, *Maffeo Pantaleoni*, pp. 155-76. There are three letters from Pareto to Cassel, along with eight letters between Pantaleoni (five) and Cassel (three), sitting at the Handwritten Section of the National Library of Sweden, the correspondence starting in 1900 and finishing most likely in 1903. Cassel's annotations to Pantaleoni are sketches riddled with stricken and added words and just one of those pieces is dated. Gustav Cassel (1866–1945), Brevsamling 13–1, Ke-Kisch, Handwritten Unit, National Library of Sweden.

[38] Pareto to Cassel, July 14, 1900, National Library of Sweden. These articles are almost certainly the following ones: Vilfredo Pareto, "Sunto di alcuni capitoli di un nuovo trattato di economia pura del Prof. Pareto", *Giornale degli Economisti*, Serie Seconda, 20 (Marzo 1900): 216-35; Vilfredo Pareto, "Sunto … (continuazione and fine)", *Giornale degli Economisti* (Giugno 1900): 511-49; see also Michael McLure, "A note on Pareto's 'Sunto'", *Journal of the History of Economic Thought*, 27 (2005): 399-403.

works enough signs of a serious mathematical approach to economic theory, which he declared to be of his uppermost interest at the occasion. The corresponding letter is reproduced below.

> I have just received your two works for which I thank you very much. It would be a pleasure to review them in some Italian journal, but I scarcely read German for such a task. I shall send your works to someone among my friends who knows the language well and who could evaluate them better than me.
>
> Your study on the theory of prices will be interesting, but I doubt that you could establish this theory in a general form without making use of mathematics. At this moment, I am busy with my work on mathematical economics, but it will take a long time before this treatise is published.
>
> I had got a lot to do before moving away. I have just settled down in Céligny, a small canton between Lausanne and Geneve. I have a villa on the lake. If you ever come to Switzerland, I shall be happy to receive you in my modest house.[39]

Later that year, Cassel contacted Pareto again to follow up on the promised review of his works and to inquiry on how he could get a paper published in an Italian journal. Pareto replied he had already forwarded the material to Pantaleoni, who, however, had scanty time to go over it because of his constant travels between Geneve and Rome. Pareto also informed Cassel that he had just finished the manuscript of his two-volume book *Les Systémes Socialistes*, to be soon published in Paris, and expressed regret over Cassel's lack of familiarity with the Italian language, since he was sending to him another article on the foundations of the new mathematical methods in economics.[40] In the end, Pareto suggested to his Swedish colleague a direct contact with Pantaleoni and wrote down the University of Rome address.[41]

Pareto's reference to his work on socialism affords us another instance of an application of the principle of continuity, this time around its being heavily influenced by Herbert Spencer's and Gustave de Molinari's ideas.[42] In the second volume of the *Cours*, Pareto embarks upon a journey through social dynamics predicated on natural selection as a better approximation of economics to reality. Evolution, meaning the progressive and continuous diversification of society, although not always treading a uniform path, could proceed either through an automatic process, following the natural laws regulating commerce and liberty, or it may be imposed by coercive external forces, as planned by the enthusiasts of socialist systems. The latter course, however, advises Pareto, would go against what had already been proved successful over

[39] Pareto to Cassel, February 5, 1901, National Library of Sweden.

[40] Probably Vilfredo Pareto, "Le nuove teorie economiche: Appunti", *Giornale degli Economisti*, Serie Seconda 23 (Settembre 1901): 235-52, which ended with an appendix outlining the equations for dynamic equilibrium.

[41] Pareto to Cassel, October 10, 1901, National Library of Sweden.

[42] In his *First Principles*, published in 1862, Spencer portrays all phenomena as conforming a continuous process encompassing the different states of motion, in which every object or living thing contains two forces in permanent conflict: integration or accumulation of matter and disintegration or decaying. Evolution constitutes the progressive change of a less coherent homogenous mass into a more complex, integrated and heterogenous aggregate, be that of a natural or social kind (Spencer 1867: 180-195, 278-396). Herbert Spencer, *First Principles* (London: Williams and Norgate, 2nd ed. 1867), pp. 180-95, 278-396. The Belgian economist Molinari, for his turn, in his *Les Lois Naturelles*, from 1887, claims that economic activity is subject to natural laws, particularly the creation of maximum value through minimum effort, an unavoidable human behavior designed to increase man's vital strength (*puissance vitale*). Free competition, for Molinari, constitutes the channel through which progress in all productive spheres comes about, promoting alongside the continuous expansion of trade among nations. Gustave de Molinari, G. de. 1887. *Les Lois Naturelles de l'Économie Politique* (Paris: Guillaumin 1887), pp. 1-32, 78-138.

centuries of mankind's material and moral advancement.[43] In *Les Systèmes Socialistes*, Pareto denounces the egalitarian doctrines as forging imaginary worlds whereas rejecting the objective evidence that people are different whatever the social arrangement in place. Any scheme of income distribution based on the supposed merits of individuals or even on their effort would deploy an ill-defined standard without paying attention to the objective circumstance that everybody has their own preferences, skills, vices, and other attributes. Above all else, a State-imposed homogeneity of incomes would subvert the continuous progress of diversification reached through evolution based on competition and specialization, thereby harming the production of wealth and, consequently, the welfare of society.[44]

Coming now to the correspondence between Pantaleoni and Cassel, the Italian professor first explained to his Swedish correspondent in 1901 the impossibility of himself writing a review of the works he had just received because of his almost "gypsy" lifestyle, with a lot of traveling between locations, kids living scattered in different cities and his wife's mental ailment.[45] Six months later, when Pantaleoni was about to move to Rome for good, he wrote an extensive letter detailing his attempt to present his Swedish acquaintance as a possible name to replace him at the University of Geneva, in view of what Pantaleoni saw as Cassel's able handling of economic theory and simultaneous equations. The effort, however, turned out unsuccessful owing to the university manager's reluctance to hire a foreign professor not fluent in French.

> You have requested that I give you my conclusive judgment on your writings. I can do this very simply and succinctly by now telling you what I would probably never have told you otherwise: Namely that when I gave up [the Chair at] Geneva and was asked to select any successors or, more correctly, to propose their designation, I suggested *you* to the Dean, Mr Naville, along with a few others. I pointed out to Mr Naville that you were definitely one of the very best newer buds that our tree would have and because the Genevans seemed to care - at least the faculty was concerned, not so much the current cantonal government - that in Geneva, just as in Lausanne, pure or abstract economics would have a representative, I told Mr Naville that you were one of the few candidates who could represent this subject.
>
> It was doubted though that you would be able to lecture fluently in French - a purely external condition, but to which the Genevans strongly adhere, because many German students come to Geneva for the presentation rather than the content. I noticed that a Frenchman had been chosen and did not insist further.
>
> The reasons that led me to propose your name to my friend Naville can be summarized briefly by saying: 1. That you have a very clear idea of what a system of conditional equations is and consequently understand the incompleteness or one-sidedness of the so-called Austrian School as well as of the production cost theorists ; 2. That your attention to discontinuity is a perfectly true correction of the first approximation carried out by those who have worked with continuous functions; 3. That your writings contain abundant ingenious remarks, some of which are entirely original, I mean objectively original, while others have already been made by different economists without you being aware of that.[46]

In November 1902, Pantaleoni reached out again to Cassel by sending him a recent issue of the *Giornale* in which Gaetano Scorza's review of Cassel's *Grundriss*, prepared at Pantaleoni's request, had been published. We shall deal with it in the next section. Pantaleoni also invited

[43] Pareto, *Cours*, vol. II pp. 1-71.

[44] Vilfredo Pareto, Pareto, *Les Systèmes Socialistes* (Paris: V. Giard & E. Brière 1902), vol. II pp. 129-251.

[45] Pantaleoni to Cassel, November 1, 1901, National Library of Sweden.

[46] Pantaleoni to Cassel, May 12, 1902, National Library of Sweden, emphasis in the original.

Cassel to write an article for the *Giornale*, which by then had become the main outlet for Paretian economics in Italy. He advised Cassel there being no need to avoid any kind of formal expressions since the readers and contributors to the journal were already familiar with the mathematical approach to economics. He even offered to translate the paper to Italian.[47] A few days later, Cassel wrote back to Pantaleoni thanking him for the journal issue but declining the invitation due to his current commitments. Cassel announced still the imminent publication of a book in English, which he hoped Pantaleoni could write a review about after its publication. The book hit the bookshelves in 1903 under the title *The Nature and Necessity of Interest*:

> It is very gracious of you to offer me a place in your journal and provide translation services. Unfortunately, I cannot presently handle this opportunity because I am already very engaged with other works. My *Grundriss* must be accompanied by two other pieces at the *Zeitschrift für die Gesamte Staatswissenschaft* of which I have sent you the last issue recently. I place the main weight [of my theory] on the right identification of the cause behind the price-setting mechanism and on seeing how the problem is to be attacked.
>
> I believe thereby that to defend my views, it is best that I can prove the prolificacy of my starting point. This is what I want to do by focusing on the central problem of price theory, which is the theory of interest. This work I have now finished; the M.S. is in London, but I have not yet found a publisher. Before it is published, I shall write a bit more about the general price theory. I shall be pleased if I will be able to make good on your invitation afterwards.
>
> I hope you will also find time to review my book in your journal.[48]

Directed to English speaking readers, Cassel's book is essentially Marshallian in tone, containing just sparse mentions to the simultaneous determination of economic variables. In a rather long first chapter on previous theories of the interest rate, Cassel praises Anne-Robert J. Turgot and Jean-Baptiste Say for their practical treatment of the issue by looking at the conditions of supply and demand for money, while he disparages David Ricardo for having embarked on a sophistic path by analysing the interest rate as a mere deduction on profits. Harshly criticized as well is the proposition, which had been advanced by Eugen von Böhm-Bawerk in his *Positive Theory of Capital*, that capital goods requiring larger "average periods of production" yielded increased productivity.[49] This hypothetical average, says Cassel, was an entirely vague concept and hence analytically worthless due to the impracticality of its measurement with any degree of precision.[50]

Waiting, as defined by Marshall[51], was then adopted by Cassel as an elementary factor of production whose amount would be calculated by the product of a certain monetary value multiplied by the time interval this same sum is invested before it being ready to produce

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[47] Pantaleoni to Cassel, November 30, 1902, National Library of Sweden.

[48] Cassel to Pantaleoni, December 8, 1902, National Library of Sweden.

[49] Eugen von Böhm-Bawerk, *The Positive Theory of Capital* (New York: G.E. Stechert 1930 [translated by William Smart from the 1888 ed.]), pp. 87-91.

[50] Gustav Cassel, *The Necessity and Nature of Interest* (London: Macmillan 1903), pp. 1-37, 55-67.

[51] "Since, however, the term [abstinence] is liable to be misunderstood, we may with advantage avoid its use, and say that the accumulation of wealth is generally the result of a postponement of enjoyment, or of a *waiting* for it. Or, in other words again, it is dependent on man's *prospectiveness;* that is, his faculty of realizing the future". Alfred Marshall, *Principles of Political Economy* (London: Macmillan, 1920 8th ed., p. 233 emphasis in the original, see also pp. 230-6.

consumable commodities.[52] The main determinants of the demand for waiting are then explained as the growth of population, the higher standard of living of the lower classes, the internationalization of production and new inventions. As to the supply of waiting, according to Cassel, it primarily depends on the surplus production over the basic necessities of society, as well as on the improved social stance and longevity of people that tend to make families more prudent toward the future. Moreover, Cassel indicates that a lower interest rate would be followed by an almost unlimited demand for waiting, there never being anything like an excess of savings or insufficiency of investment since that situation would obviously violate the general principle of scarcity regulating all prices.[53]

Finally, in a letter to Pantaleoni, possibly from 1903, Cassel shows how distinguished was the Italian economist within academic circles across Europe by asking for a personal testimony about his skills as an economist in order to boost his candidacy for a chair of political economy in Sweden: "In Stockholm, a professorship of economics is now free and I have good prospects to be appointed. It will enhance my case in the highest degree if you will send me your opinion about my scientific proficiency".[54] Pantaleoni's help might have aided Cassel somehow, for the latter would ultimately be appointed by the University of Stockholm in 1904.

Responses to Cassel's *Grundriss*

Despite their common nationality and solid background in mathematics, Knut Wicksell and Gustav Cassel did not enjoy each other's work. Wicksell thought Cassel snobbish and averse to concede that his ideas were hardly original, whereas Cassel regarded Wicksell only as a blind follower of Böhm-Bawerk.[55] Already in 1900, Wicksell published in the same journal a reply to Cassel's *Grundriss* and entitled *Zur Verteidigung der Grenznutzenlehre* (later translated into English as "In defence of marginal utility"). Striking directly at Cassel's central argument, namely, the general indivisibility of consumption goods, Wicksell reaffirms the soundness and, most remarkably, the continuity of the law of marginal utility by recalling that within any large society, those same goods listed by Cassel as indivisible are available in different sizes and gradations. For instance, renting rooms in Berlin could be found at different prices, locations, and spaces, the same applying for meals, newspapers and almost any other item of consumption.

When addressing Cassel's own theory of prices, Wicksell regrets the lack of recognition of Walras' fundamental contribution to economics and points out that a complete treatment of general equilibrium under a static situation would have to include a missing equation expressing the condition that the income received by the owners of productive factors must be equal to the value spent by them on consumption goods. The system would not become overdetermined since, once this correction is introduced, one of Cassel's equations becomes redundant given

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[52] Cassel was to raise the same charge against Böhm-Bawerk in *Quantitative Thinking*. The Oxford professor Henry E. Phelps-Brown, in his book review, questioned exactly Cassels' definition of waiting, which involved a time interval equally hard to be estimated. "[W]e may ask whether he [Cassel] does not sometimes apply a sterner test of quantitative precision to the doctrines of others than to his own." Ernst H. Phelps-Brown, "On Quantitative Thinking in Economics (Book review)", *The Economic Journal* 46 (1936): 104-105; Cassel, *Quantitative Thinking*, pp. 13-14.

[53] Cassel, *Nature of Interest*, 96-157, see also Cassel, *Social Economy*, 649-52.

[54] Cassel to Pantaleoni, undated, National Library of Sweden.

[55] Brems, "Gustav Cassel", 1989.

that equilibrium in all markets but one necessarily meant that this very remaining one would be also in equilibrium.[56] Or yet, in simpler words, what is to be spent by the people should be earned through their participation in the productive process. Finally, even if the coefficients of production are taken as fixed, as assumed by Cassel, there could be some maneuvering room for marginal adjustments through the displacement of productive factors among alternative branches of production.[57]

Let us now move back to Italy. Born in 1876 in Morano Calabro, Gaetano Scorza obtained his degree in mathematics in 1898 from the University of Pisa.[58] After several years teaching in technical institutes, he won a professorship at the University of Cagliari in 1912, moving to other Italian institutions over the years until obtaining a chair at the University of Rome in 1934. Scorza main fields of research were algebraic geometry, Abelian functions, matrixes, and the theory of groups. He was also a member of several scientific associations, as the distinguished *Accademia dei Lincei*, and along with numerous memoirs, he wrote the book *Corpi Numerici e Algebre*, published in 1921. He would experience a brief but honorable stint in economics by discussing issues related to marginalist analysis, general equilibrium, and welfare during the first years of the twentieth century.[59]

By invitation from Pantaleoni and having already been acquainted with Walras's theory of prices, the young Scorza agreed to review the *Grundriss* in late 1902. In his rather brief but very objective appointments, the Italian mathematician express his agreement with Cassel's criticism of the association between pure competition and maximum utility. Such common ground, however, goes on Scorza, does not imply that the Swedish economist had opened a more constructive route by breaking up with the already well-established Walras' and Pareto's tradition of analysis, which started from utility functions to reach equilibrium prices and quantities. To Scorza, Cassel had fallen into the trap of scientific ingenuity by flatly denying the continuity of economic functions, since such a drastic stance meant an outright refusal to

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[56] A property of general equilibrium known as Walras' Law. See Walras, *Elements*, pp. 166-7.

[57] Knut Wicksell, "In Defence of the Marginal Utility Theory", in Bo Sandelin (ed.) *Knut Wicksell, Selected Essays in Economics* (London: Routledge 1997 [1900]), pp. 3-14. Years later, in a review in the *Ekonomisk Tidskrift* of the first German edition of the *Theory of Social Economy*, Wicksell regarded Cassel's principle of scarcity as quite poorly defined, such wording being indeed just a smart way of avoiding to acknowledge that wants were nothing else but the manifestation of marginal utility, and that equilibrium prices were the result of the interplay of demand and supply. What Cassel lacked the most, added Wicksell with a touch of contempt, was "sober scholarship" and respect toward the achievements of great past economists such as Walras and Böhm-Bawerk. Knut Wicksell, "Professor Cassel System of Economics", in Lionel Robbins (ed.) *Lectures on Political Economy*, *Vol. I. General Theory* (Fairfield: Augustus M. Kelley 1977 [1918]), pp. 219-57.

[58] Mathematics in Italy experienced a quick development after unification, especially in the fields of analysis, mathematical physics and algebraic geometry, as illustrated by scholars of international recognition such as Giuseppe Peano (1859-1932), Tullio Levi-Civitá (1873-1941) and Luigi Cremona (1830-1903). Angelo Guerragio and Pietro Nastasio, *Italian Mathematics Between the Two World Wars* (Basel: Birkhäuser Verlag 2006), pp. 1-39.

[59] Ibid., pp. 120-22; Francesco Severi, "L'opera scientifica di Gaetano Scorza", *Annali di Matematica*, Serie VI, XX (1941): 1-20; Gaetano Scorza, "Osservazioni sulla teoria del baratto secondo il Prof. Walras", *Giornale degli Economisti*, Serie Seconda, 25 (Aprile 1902a): 282-302; Gaetano Scorza, "Antonio Graziadei. *Intorno alla legge del godimento decrescente ed al principio del grado finale di utilità*. Valparaiso, Imprenta del Universo, 1901", *Rivista Internazionale di Scienze Sociali e Discipline Ausiliarie*, 28 (Aprile 1902b): 644-7; Gaetano Scorza, "Dr. G. Cassel. *Grundriss einer elementaren preislehre*", *Giornale degli Economisti*, Serie Seconda 25 (Agosto 1902c): 191-4; Gaetano Scorza, "A proposito del massimo de ofelimità dato dala libera concorrenza", *Giornale degli Economisti*, Serie Seconda 26 (1903): 41-62.

benefit from all major advances in mathematics that could be easily put at the service of economic theory. And, more importantly, Cassel's rudimentary system of equations had ditched not only the powerful tools of infinitesimal calculus, but also the elementary properties of algebra, ignoring thus the fundamental unity of mathematical analysis. The Swedish economist's simplified price-setting system might very well be proven insoluble owing to the possible nonexistence of a common intersection among discontinuous equations.

> He [Cassel] states that whoever wants to read his article needs just to be familiar with the basic elements of a system of simultaneous equations which any educated person has learned in school. He could have stated as well which theory of simultaneous equations dispenses with the hypotheses of continuity and also which mathematical theorem authorizes him to conclude that a problem, translated into *n* equations with *n* unknowns and equaled to *n* constants, is determined if the corresponding functions are discontinuous? [60]

Returning to Pareto, in the *Cours*, the issue on the measurement of utility comes about when he discusses the difference between accepting the existence of a quantifiable entity and its actual numerical expression. In §21, for instance, Pareto invokes Francis Y. Edgeworth's conception of "unnumerical" mathematics to reinforce his belief that relevant analytical headway could be achieved by a formal theory of consumption, even with no direct estimation of the variables involved. Furthermore, in two extensive footnotes (84 and 114), Pareto introduces a graphical depiction of Edgeworth's indifference curves showing how an individual can replace some quantity of one good for another while keeping his satisfaction or ophelimity unchanged, there being thus no need to attribute any number to these curves besides some purely ordinal sequencing.[61] Edgeworth's remarks on the subject in the opening chapter of his *Mathematical Psychics* are worth being reproduced here: "The criterion of a maximum turns, not upon an *amount*, but upon the *sign* of a certain quantity. We are continually concerned with the ascertainment of a certain loose quantitative relation, the *decrease-of-rate-of-increase* of a quantity. Now, this is the very quantitative relation which it is proposed to employ in mathematical sociology".[62]

A few years later in his *Sunto* (see footnote 37), Pareto developed a full system of choice theory where the equilibrium point is established without regard to ophelimity as a quantity, but to the fact of choice based on preference ordering. This should have been acceptable to Cassel since the continuity of ophelimity is not required for indicating an increase or decrease in utility. Pareto even sent the article to him, but it was in Italian and he could not read it. It is true that Cassel had access to similar ideas from the French edition of the *Manual*, but they are set out less clearly (and with an element of confused exposition) because Pareto addresses there two complex and interrelated ideas – choice theory when ophelimity exists (integrability conditions hold such that ophelimity is treated in closed cycles) and when ophelimity does not exist (integrability conditions are not satisfied such that ophelimity is treated in open cycles). Had Cassel been able to read the simpler and clearer *Sunto*, he may well have been satisfied with Pareto's choice framing of economic equilibrium.

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[60] Scorza, "Dr. G. Cassel", 1902c: 194.

[61] Pareto, *Cours*, v. I pp. 9, 35, 66.

[62] Francis Y. Edgeworth, *Mathematical Psychics* (London: Keegan and Paul 1881), p. 6, emphasis in the original. See also Michael McLure and Aldo Montesano, "Thinking Outside the Box: Edgeworth, Pareto and the Early History of the Box Diagram', *Economic Record* 95 (2019): 301-11.

Pareto would also directly address Cassel's concerns in an entry to the *Encyklopädie der Mathematischen Wissenschaften*, published in 1902. It was translated in 1906 and published in the *Giornale* as *Applicazioni della mathematica all'economie politique.* Cassel, says Pareto, would be correct in accusing economics of treating discontinuous functions as if they were indeed continuous only if the corresponding analysis focused solely on the behavior of a single individual. But as the science deals with large numbers of people, the error implied by such method would be minimal. This practice, in addition, was common to many scientific fields such as physics, probability, and statistics. For instance, to explain the dynamics of a population N of a certain country over time t one would have to denote it by a continuous function $N = f(t)$ even though the codomain N of $f(t)$ could only, in actual fact, take integer values.[63]

A more elaborate response to Cassel's arguments though would come up at the French edition of Pareto's *Manuel of Political Economy*, from 1909, a revised version of the *Manual* published in Italian three years earlier. There, Pareto insists again on the analytical advantages of continuous functions as a suitable representation of situations involving a multitude of individuals. Ample use of Edgeworth's indifference curves is made in the *Manuel* and Pareto even calls the attempts at quantifying ophelimity as entirely metaphysical. This seemingly radical change in position from the *Cours* to the *Manuel*, however, does not represent a full reversal of perspective about a previous equivocal position. To the contrary, as explained by the Italian economist, it only mirrors the natural advance of science. If the assumption of measurable utility could be left aside and even criticized now, the same attitude was absolutely inappropriate in the past when the concept had just come to light and hence allowed a huge leap forward in economic theory through a more systematic use of mathematics.

> The old theories of economics were necessary to the development of new ones; and these, though still very imperfect, will help us to develop better theories, and so on. But improving theory is something quite different from trying to destroy it by silly, pedantic quibbling. The first approach is sensible and useful; the second is foolish and futile, and anyone with no time to waste would do better not to bother about it.[64]

Pareto, when talking about the properties of the general equilibrium, conceded yet that it would be easier to start off his analysis from well behaved utility functions but recognizes that they were just no longer needed. The whole theory developed in his book is built upon the ordinal character of indifference curves, extracted from experience according to him and, therefore, dispensing with any abstract idea of quantifiable ophelimity or utility for the study of equilibrium (although not for the study of welfare consequences of movements within the commodity space, not even in his mature works, as such studies typically represent ophelimity as a quantity). Such objective approach permitted thus a closer proximity of pure economics to rational mechanics, as he always professed to be his primary scientific goal.

Concluding remarks

As the Calculus and Mechanics evolved at a remarkably fast pace over the nineteenth century, specially at the *École Royal Polytechnique* where Leibniz's ideas on continuity were more

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[63] Vilfredo Pareto, "Applicazione dela matemática all'economia politica", *Giornale degli Economisti*, Serie Seconda 33 (1906 [translated by Guido Sensini from the 1902 version]): 424-53.

[64] Vilfredo Pareto, *Manual of Political Economy. A Critical and Variorum Edition*, Aldo Montesano, Alberto Zanni, Luigino Bruno, John S. Chipman, Michael McLure (eds.) (Oxford: Oxford University Press 2014 [translated from the 1906 and 1909 eds.]), p. 77.

prevalent, the obvious advantages of having such powerful assortment of tools applied to other quantitative sciences would not fail to reach the province of economics. As the bond between analysis and physics was the strongest in France, the approach there prevailing, based on continuous analytical functions, naturally appeared as the more convenient to be replicated elsewhere. By so doing, any field of inquiry involving numbers could take full advantage of the theorems for maximization of functions and the framing of problems through systems of equations. That was the line of thought more attractive to Walras, Wicksell and Pareto, who devised economic truths as mathematical laws, even though some instances of discontinuity could generate an undesired margin of error. For them, however, the benefits of mathematical analysis applied to pure economic theory far surpassed the troubles of eventual misrepresentations in some instances of human behavior. Even when a variable was hard to be measured, much could be inferred about its behavior once continuity is accepted, as sharply observed by Edgeworth and Pareto.

Cassel, who had studied the work of German mathematicians of the late nineteenth century, rejected the notion of continuity and strived to place economics upon a more rigorous foundation regarding arithmetic and quantities. His effort, in the end, proved itself too restrictive to prosper, and Scorza did not let the inconsistency of Cassel's price system pass unnoticed. Anyhow, the Swedish economist most likely was hardly fully aware that the quest for measurement was not especially attractive to German mathematicians of the nineteenth century. Differently from their French colleagues, most of them wished indeed to free mathematics from any connection with other branches of knowledge, transforming it into a pure science on its own. Thus, in the new algebra of axioms and groups being developed in the late nineteenth century, a variable did not necessarily mean a precise quantity nor even a real entity.[65] Curiously, Dedekind wrote about this very matter in his pathbreaking memoir on real numbers: "And if we knew for certain that space was discontinuous there would be nothing to prevent us, in case we so desired, from filling up its gaps, in thought, and thus making it continuous."[66] That was a prophetical advice about the path economists were about to trail in pure theory soon afterwards.

[65] Victor J. Katz and Karen H. Parshall, *Taming the Unknown. A History of Algebra from Antiquity to the Early Twentieth Century* (Princeton: Princeton University Press 2014), pp. 558-62; Schubring, *Conflicts*, pp. 540-61.

[66] Dedekind, *Essays*, p. 12.