Cartel Screening in the Brazilian Fuel Retail Market

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Abstract

This article is aimed at contributing to the challenges and pitfalls faced by the antitrust authorities in the identification process of anti-competitive market behavior. We propose two econometric models to select possible cases to be investigated: (i) The Markov-Switching GARCH (MSGARCH) Models; (ii) The Local Gaussian Correlation (LGC) approach. We compare both models. Our results indicate that the LGC model, based on the correlation of the resale price margin and the variability of prices, may provide a biased estimation of the likelihood that a market is practicing cartel. The MSGARCH model, based only on the log deviation of the gasoline sale price, showed a better accuracy in cartel detection.

Keywords: Cartel filter, price dynamics, fuel retail market
JEL classification: L41 · L95 · C22 · C63

Resumo

Esse artigo busca contribuir com as autoridades antitruste na identificação de comportamento anticompetitivo no mercado de varejo de gasolina. Dois modelos econométricos são propostos: (i) Markov-Switching GARCH (MS-GARCH); (ii) Correlação Gaussiana Local (CGL). Comparando-se os dois modelos, há indícios de que o CGL, baseado na correlação da margem de preço de revenda e na variabilidade de preços, pode viesar a probabilidade de um mercado estar cartelizado. O modelo MS-GARCH, baseado apenas no log-desvio do preço de venda da gasolina no varejo, mostrou-se melhor na detecção de cartéis.

Palavras-chave: Filtros de Cartel, dinâmica de preços, mercado de gasolina
Área 8 - MICROECONOMIA, MÉTODOS QUANTITATIVOS E FINANÇAS

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1. Introduction

Frequently, gasoline markets are accused of cartel formation. Given the persistence of anti-competitive behavior in the industry, the best way to investigate and identify them are relevant issues. In an environment where the number of complaints and suspicions of cartels is increasing and, given the restriction of resources available to initiate an investigation, the econometric methods of screening and filtering are useful tool. As we can see in [16], these problems in cartel identification have led to a prominent new field of research in the last decades, but there is still no universal method in Economics that allows us to infer about the existence of cartel in a certain market. In this sense, the economic evidence is insufficient, maintaining the need for a criminal investigation. However, the econometric filters may reduce the authorities' search effort by ordering the most likely candidates for the cartel.

One of the key variables used to chart the behaviour of cartels, capable of transmitting information about how the market works and its strategic decisions, is the retail price. There are several works following this literature[1]. In light

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[1]Please see the discussion shown in [2, 16, 11, 24, 22, 23].

of these contributions, this paper is aimed at collaborating with the antitrust authorities in the analysis of behavioural patterns of cartelized markets, as well as in the task of distinguishing them from competitive behaviour.

In order to achieve our objective, we apply both the MSGARCH and the LGC models for screening and filtering markets with greater indications of cartel. As far as we are concerned, this is the first paper that empirically evaluates the performance of a MSGARCH model in the Brazilian gasoline retail market. Specifically, our objective is to analyze the market behaviour of the following cities: (i) Brasília (DF), the capital of Brazil; (ii) Goiânia (GO), a city in the vicinity of Brasília; (iii) Rio de Janeiro (RJ) and (iv) São Paulo (SP). We used continuous weekly price data provided by ANP[2] between 2014 and 2017. The choice of these cities is justified by their relevance in the market. In addition, the city of Brasília was recently judged and condemned by CADE[3] for cartel practice. Thus, we have a good way of evaluating the accuracy of the proposed models in identifying cartels.

In this regard, our research is related to cartel detection through dynamic pricing. As reported in [22, 43], economists widely apply this methodology in an attempt to generate patterns of collusive behaviour and, from then on, to formulate and validate a specific hypotheses, in order to distinguish a competitive pattern from the collusive one. The seminal contribution in this field of research came from [29], where the authors provide a game theoretic foundations for the classic kinked demand curve equilibrium and Edgeworth cycle. The analysis is based on a model in which firms take turns choosing prices. By using the Markov perfect equilibrium (MPE) concept, they conclude that a firm's move in any period depends only on the other firm's current price.

Using a Markov-switching regression model to estimate both prevalence and structural characteristics of the pricing patterns, [31] analyzes dynamic pricing in 19 Canadian cities over 574 weeks. The author finds that sticky-pricing (cycles) is more prevalent when there are few (many) small firms. [42] studies oligopoly firms' dynamic pricing strategies in the Australian gasoline market before and after the introduction of a unique law that constrains firms to set price simultaneously and only once per day. The observed pricing behaviour, both before and after law implementation, is well captured by the Edgeworth price cycle equilibrium in the Maskin and Tirole dynamic oligopoly model. Thus, the results highlight the importance of price commitment in tacit collusion.

As shown in [26], the role of price leadership in coordinating price increases in cycling gasoline retail markets in the U.S. The author concludes that the first price increases tend to stem from retail chains that operate a large number of stations. Following this approach, [12] used court documents from a gasoline cartel in Canada to characterize the strategies played by heterogeneous firms to collude and highlight the role of transfers based on adjustment delays during price changes. The cartel leaders systematically allowed the most efficient firms to move last during price-increase episodes in order to compensate. Furthermore, [13] use weekly station-level price data from before and after the cartel's collapse to compare pricing behaviour in stations affected and unaffected by the investigation. The results indicate that collusion is associated with asymmetric price adjustments and high margins.

In contrast, following the arguments presented in [39], we use price series rather than margin[4] because there is an ambiguity[5] in the reasons behind the behaviour that sustains the cartels. For example, if the profit margin decreases, how can you be sure that this is not a period of punishment after some firm has cheated a cartel agreement? So, the decrease in the profit margin should not be seen merely as an indicator of competition, since firms may be punishing those who deviated. To sum up, in one hand, cartel can lead to an increase in the profit margin. On the other hand, it may have a punishment phase with lower profits, but this will still be an anti-competitive behaviour. We can say that this aspect is a limitation in the methodology of the antitrust authority if the data is restricted in a short period of time.

Other behavioural issues of economic agents may affect the price variance in the market under analysis. Firms in collusion can practice parallel price behaviour, that is, firms adjust their prices identically and simultaneously, for some common factor[6] of knowledge between them. Such conduct would lead to a similar trajectory of prices among

[2]National Agency of Petroleum, Natural Gas and Bio-fuels: http://www.anp.gov.br.

[3]The Brazilian antitrust authority - Administrative Council for Economic Defense.

[4] [5] propose a regime-switching model based on mean-reverting and local volatility processes to comprise the market structure of the French fuel retail market. By analyzing the volatility of prices and margins, the authors provided a better understanding of the behaviour of oligopolies. In this same market, [34] found evidence of tacit collusion from the margin analysis, but they emphasize that, the collusive behavior in the gasoline market is still an open question.

[5]Theoretically, the stability of the cartel depends on the ability of firms to detect and punish the defectors. To implement the punishment mechanism, cartelized firms can reduce price and, consequently, profit margin. However, this behaviour of reduction of the retail price is compatible with that expected in a market environment in which there is an effective competition - without collusion of the firms.

[6]Such as identical mark-ups, price levels.

firms, resulting in a low variance[7]. Related to the structural changes in the price series over time, [7, 24] argue that when firms have a low discount rate on future earnings, collusion equilibrium is given with equal prices.

Besides that, when firms exercise some market power, they can also act asymmetrically in the relation between product pricing and cost structure. In this way, the greater the cartel's interference in price formation, the lower the price-to-cost ratio. There is an extensive literature[8] dealing with the problem of asymmetry in collusive markets, with a reasonable consensus on the non-linearity of the relationship between price variations and cost adjustments in collusive markets. Another remarkable feature of collusive markets, according to [33], refers to coefficients of price variation, which may be relatively different in non competitive markets.

In order to reach our purpose and develop the discussion shown in this introduction, the remainder of this paper is organized as follows. Section 2 describes the methodology and the database. Section 3 brings the results of the empirical analysis. Section 4 shows our conclusions. Section 5 shows the appendix, in which it is possible to find detailed tables with information about the estimation of the models, as well as the validation of some asymptotic properties.

2. Methodology

In this section, we begin with a brief description of the Brazilian fuel market and the database provided by ANP. Then, we introduce the methodology used to model the behavioural pattern of the gasoline sales market between January 2014 and December 2017 in Brasília (DF), Goiânia (GO), Rio de Janeiro (RJ) and São Paulo (SP). Next, we will describe the Markov-Switching Model, in line with [19, 5]. Then, we will explain the Local Gaussian Correlation approach, as proposed by [37].

2.1. Characteristics of the Brazilian Gasoline Market

According to the exposed in [15], the composition of the price of gasoline in the Brazilian market went through many changes over the years[9]. Until the middle of 1990, the Brazilian State interfered in distribution and resale of automotive fuels, controlling prices, margins of sale and freight. However, the price liberation process was initiated throughout the oil production chain. Thus, in 1996, the prices of automotive gasoline were released in the wholesale and retail trade units, as well as the sales margins of the resellers and distributors in the South, Southeast and Northeast regions. In 1997, the Petroleum Law[10] was published, which created the Brazilian National Agency of Petroleum (ANP), whose function is to regularize the Brazilian fuel market. In addition, it allowed for a new model to enter into force, leaving behind the monopoly of the oil sector, hitherto exercised by Petrobras S.A, and opening up the fuel market in Brazil. Finally, the most recent change that is still in force today was the one that occurred in 2002. The main objective of the opening was to create a competitive market, avoiding cartel situations in the sector. Thus, since this period, gasoline imports have been released and the price has been defined by the market.

2.2. The ANP Sampling of the Fuel Retail Price

One of the roles of the ANP is to preserve the competitiveness of the fuel sector. Thus, there are several efforts to defend competition and consumer welfare in the pursuit of economic efficiency. It is also the ANP that provides the database[11] to CADE to be analyzed at the administrative level of complaints about cartel practices in the fuel retail market. With this set of information, it is possible to have a preliminary investigation signalling whether there is any cartel behaviour or not in a certain city. Therefore, the database used in this paper is the same as that used by CADE.

With this in mind, the development of the ANP sampling is described as follows. The price collection service, as stated in [32, 17], is developed through the structuring and execution of the following steps: (i) weekly collection of sales prices to the final consumer and the corresponding acquisition prices by the economic agents selected to integrate

[7]We highlight the following literature: [27, 36, 35].
[8]Please see [13, 14, 30].
[9]See `www.anp.gov.br` for more details.
[10]The law 9.478/97, which is available at `http://www.planalto.gov.br/ccivil_03/leis/L9478.htm`.
[11]The database is composed of, among other pieces of information, the evolution of fuel prices (buying and selling), the evolution of the resale margin and the coefficient of variation between prices.

the sample defined by the ANP; (ii) quality control of the information; (iii) data entry into the system; (iv) creation of a database containing the information specified through contracts; and (v) forwarding the results to the ANP.

Field planning within each municipality is based on a geographical identification (plotting) of the resale points within the sample. The determination of the weekly collection routes in each of the municipalities is carried out based on the registration data of resellers to be forwarded by the ANP and the quantity defined in the sample design. The main objective of the distribution of the points of sale is to optimize the geographical representation of each of them. Considering the number of resale points in the total sample determined for each municipality and the criteria listed above, a random sample selection is made by the company conducting the survey to be collected weekly. In the selection procedures, however, we must observe the geographic coverage of the municipality as well as guarantee the randomness.

2.3. *Price Volatility with Markov-Switching GARCH Models*

The MSGARCH parameters vary dynamically according to an unobservable variable. In this way, we can use it to identify changes in model behaviour. Another good property of these models, according to [28, 3], is their fast adaptation to the observed perturbations in unconditional volatility. Therefore, a better statistical inference process is possible as well as a better prediction of the variable of interest. In order to verify how well the model is able to capture the characteristics of the markets used in this article, a broad empirical analysis was carried out.

In order to better evaluate the impact of the estimation methods on the performance of the MSGARCH[12] model, both Bayesian[13] estimation techniques and the traditional maximum likelihood[14] (ML) estimation will be used. In many situations, the Bayesian approach has been shown to be more advantageous, since the Markov Chain Monte Carlo (MCMC) procedure is able to explore the joint posterior distribution of the model parameters.

2.4. *MSGARCH Model Specification*

Let p_t denote the gasoline sales price (per liter) in the retail market at time t. Then, the price variation between t and $t-1$, which is our variable of interest, z_t, is given by

$$z_t = \frac{p_t - p_{t-1}}{p_{t-1}} \approx log(p_t) - log(p_{t-1}).$$

Thus, we assume the following conditions: $E[z_t] = 0$ and $E[z_t z_{t-\ell}] = 0$ for $\ell \neq 0$ and all $t > 0$. The Markov-Switching GARCH specification is given by the following expression:

$$z_t | (m_t = k, \Im_{t-1}) \sim \mathfrak{D}(0, h_{k,t}, \xi_k), \tag{1}$$

In (1), we denote by $\Im_{t-1}$ the information set observed up to time $t-1$, that is, $\Im_{t-1} \equiv \{z_{t-i}, i > 0\}$. The continuous distribution $\mathfrak{D}(0, h_{k,t}, \xi_k)$ has zero mean and time-varying variance $h_{k,t}$. The vector ξ_k[15] assembles additional shape parameters. The stochastic variable, m_t, which is an integer number and takes discrete values between $1, ..., K$, characterizes the MSGARCH model. Standardized innovations are defined as

$$\lambda_{k,t} \equiv \frac{z_t}{\sqrt{h_{k,t}}} \overset{iid}{\sim} \mathfrak{D}(0, 1, \xi_k).$$

2.4.1. *The Dynamics of the State Variable*

Following [5], two procedures are implemented in order to capture the dynamics of the state variable. As shown in [19], a first-order ergodic homogeneous Markov chain is assumed to distinguish the MSGARCH model. Departing from a multinomial distribution and assuming the hypothesis of independent draws, we typify the mixture of

[12] R package `MSGARCH` developed by [5].
[13] See [4, 9, 10].
[14] See [19, 28, 8].
[15] See [6] for details.

GARCH models as [18]. The first-order Markov-chain, m_t, evolves in accordance to an unobserved first-order ergodic homogeneous Markov chain with $K \times K$ transition probability matrix $\mathbf{P}$:

$$\mathbf{P} \equiv \begin{bmatrix} p_{1,1} & \cdots & p_{1,K} \\ \vdots & \ddots & \vdots \\ p_{K,1} & \cdots & p_{K,K} \end{bmatrix},$$

in which the term $p_{i,j} \equiv \mathsf{P}[m_t = j | m_{t-1} = i]$ corresponds to the transition probability from the state $m_{t-1} = i$ to the state $m_t = j$. By intuition, we see that the constraints $0 < p_{i,j} < 1$ $\forall i, j \in \{1, ..., K\}$, and $\sum_{j=1}^{K} p_{i,j} = 1$, $\forall i \in \{1, ..., K\}$ is sustained. Given the parametrization of $\mathfrak{D}(\cdot)$, we have $E[z_t^2 | m_t = k, \mathfrak{I}_{t-1}] = h_{k,t}$. In other words, $h_{k,t}$ represents the variance of z_t conditional on $m_t = k$.

The Mixture of GARCH established by [18] introduces a specification related to the independent states. By doing so, the random sampling process is done independently over time, in addition to considering a Multinomial distribution of dimension $\{1, ..., K\}$. The term $\mathsf{P}[m_t = k] = \omega_k$ is the probability vector given by $\omega = (\omega_1, ..., \omega_K)^\mathsf{T}$. Thus, the same parametric formulation of (1) with K several GARCH-type models is defined for each element of the mixture[16].

2.4.2. *Conditional Variance Dynamics*

As in [19, 6], the conditional variance of the variable of interest, z_t, follows a GARCH-type specification. Therefore, $h_{k,t}$ depends on the regime $m_t = k$ and corresponds to a function of the past observation, z_{t-1}, past variance, $h_{k,t-1}$, and the Θ_k[17]:

$$h_{k,t} \equiv h(z_{t-1}, h_{k,t-1}, \Theta_k), \tag{2}$$

where the term $h(\cdot)$ is a function dependent on the information set available at time $t-1$, given by $\mathfrak{I}_{t-1}$. It also defines the appropriate filter for conditional variance and guarantees its positivity. In the MSGARCH model proposed by [5], the variance is estimated recursively and its start values, that is, $h_{k,1} (k = 1, ..., K)$, are assumed to be equal to the unconditional variance in regime k. Depending on the parameters observed in $(\cdot)$, we obtain different specifications for the variance of the error term. It is useful to reduce the model complexity.

2.5. *Conditional Distribution*

The normal distribution, also known as the Gaussian or standard normal distribution, is the probability distribution that plots all of its values in a symmetrical fashion, and most of the results are situated around the probability's mean. Values are equally likely to plot either above or below the mean. Grouping takes place at values close to the mean and then tails off symmetrically away from the mean. The probability density function (PDF) of the standard Normal distribution is given by:

$$f_N(\lambda) \equiv \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\lambda^2}, \lambda \in \mathbb{R}.$$

The PDF of standardized Student-t distribution is given by:

$$f_s(\lambda; \nu) \equiv \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(\nu-2)\pi}\Gamma(\frac{\nu}{2})} \Big(1 + \frac{\lambda^2}{(\nu-2)}\Big)^{-\frac{\nu+1}{2}}, \lambda \in \mathbb{R}.$$

where $\Gamma(\cdot)$ is the Gamma function. In order to guarantee the existence of the second order moment, the constraint $\nu > 2$ is imposed. The kurtosis increases when the term ν decreases[18].

The PDF of the standardized generalized error distribution (GED[19]) is given by:

$$f_{GED}(\lambda; \nu) \equiv \frac{\nu e^{-\frac{1}{2}|\lambda/\phi|^\nu}}{\phi 2^{(1+1/\nu)}\Gamma(1/\nu)}, \lambda \equiv \Big(\frac{\Gamma(1/\nu)}{4^{1/\nu}\Gamma(3/\nu)}\Big)^{1/2}, \lambda \in \mathbb{R},$$

where $\nu > 0$ is the shape parameter. By considering $\nu = 1$ ($\nu = 2$), the Laplace (Normal) distribution is obtained. If $\nu \to \infty$, then the Uniform distribution is obtained.

[16] According to what is exposed in [18, 6] the Mixture of the GARCH model contemplates the interactions between the mixture component variances.

[17] It contains a additional regime-dependent vector of parameters.

[18] For the Student-t distribution to be equivalent to the Normal distribution, it is necessary that $\nu = \infty$.

[19] For a complete specification of the PDF of the conditional distributions see [38].

2.6. *MSGARCH Model Estimation*

The MSGARCH models can be estimated either by ML or by MCMC (Bayesian) techniques. For this, it is necessary to evaluate the likelihood function.. By doing so, let $z \equiv (z_1, ..., z_t)^\mathsf{T}$ be the vector of T observations and let $\psi \equiv (\Theta_1, \xi_1, ..., \Theta_K, \xi_K, P)$ be the vector of the model parameters. The likelihood function is:

$$\mathfrak{L}_{(MSGARCH)}(\psi|\Im) \equiv \prod_{t=1}^{T} f(z_t|\psi, \Im_{t-1}), \tag{3}$$

in which $f(z_t|\psi, \Im_{t-1})$ corresponds to the density of z_t given past observations, $\Im_{t-1}$, and the model parameters ψ. The conditional density of z_t for the MSGARCH is given by:

$$f(z_t|\psi, \Im_{t-1}) \equiv \sum_{i=1}^{K}\sum_{j=1}^{K} p_{i,j} z_{i,t-1} f_{\mathfrak{D}}(z_t|m_t = j, \psi, \Im_{t-1}), \tag{4}$$

and the filtered probability of the state i observed in $t-1$ is represented by $z_{i,t-1} \equiv \mathsf{P}[_{t-1} = i|\psi, \Im_{t-1}]$. It is obtained via Hamilton's filter [20]. Analogously, the conditional density of z_t for the MSGARCH is:

$$f(z_t|\psi, \Im_{t-1}) \equiv \sum_{j=1}^{K} \omega_j f_{\mathfrak{D}}(z_t|m_t = j, \psi, \Im_{t-1}). \tag{5}$$

In equations (4) and (5), for a certain ψ and $\Im_{t-1}$, the conditional density of z_t in state $m_t = k$ is defined as $f_{\mathfrak{D}}(z_t|m_t = k, \psi, \Im_{t-1})$. Through the maximization of equation (3), the ML estimator, $\tilde{\psi}$, is obtained. Following [3] for the MCMC estimation, the likelihood function is combined with a truncated prior $f(\psi)$. In sequence, the kernel of the posterior distribution $f(\psi|y)$ is generated. As the posterior is of an unknown shape, simulation techniques are required. For both ML and MCMC estimations, positivity and covariance-stationarity constraints of the conditional variance in each regime are ensured during the estimation and are stronger than those derived by [19]. Nevertheless, it allows for an insertion of a wider range of conditional variance specifications and distributional assumptions in our model. In the next subsection, we present the Local Gauss Correlation approach. The method is able to suggest the periods where the economic collusion agreement is more likely to have occurred.

2.7. *The Local Gaussian Correlation*

In essence, cartels are unstable - since their participants, at any instant of time, may come to break the agreement, which reduces their lifetime. This way, the observed effect from the profit margin ratio and the resale price variability can occur in short-time intervals. In practice, an evaluation that considers only aggregate (or global) measure may despise observations of a collusive behaviour for a short period of time, since the whole series was dominated by a pattern of economic competition.

Thus, according to [1], a local correlation measure may be used to identify not only whether there are indications of cartelization, but also periods when the cartel may have occurred. The authors, following [37], proposed a local Gaussian correlation using local maximum likelihood to evaluate the Brazilian gasoline retail market. Specifically, they propose to consider any random variables (X_1, X_2) with arbitrary bivariate density defined by $f(\mathbf{X})$ and locally set in a neighborhood of $\mathbf{x} = (x_1, x_2)$ following a bivariate normal distribution for each observed point so as to minimize the distance, in the same way as shown in [25]:

$$\int K_{\mathbf{b}}(\upsilon - \mathbf{x})\{\Psi(\upsilon, \Theta(\mathbf{x})) - log[\Psi(\upsilon, \Theta(\mathbf{x}))]f(\mathbf{x})\}^2 d\upsilon \tag{6}$$

Equation (6) is the distance of Kullback-Leibler between f and $\Psi(\nu, \Theta(\mathbf{x}))$ locally weighted. The term $K_{\mathbf{b}}(\nu - \mathbf{x}) = (b_1 b_2)^{-1} K[b_1^{-1}(\nu_1 - x_1)] K[b_2^{-1}(\nu_2 - x_2)]$ is the product of kernels with bandwidth $\mathbf{b} = (b_1, b_2)$, and $\Psi(\nu, \Theta(\mathbf{x}))$ is a bivariate normal distribution with local density in the form:

$$\Psi(\upsilon, \Theta(\mathbf{x})) = \frac{1}{2\pi\sigma_1(\mathbf{x})\sigma_2(\mathbf{x})\sqrt{1-\rho(\mathbf{x})^2}} exp\left[-\frac{z(\mathbf{x})}{2(1-\rho(\mathbf{x})^2)}\right], \tag{7}$$

[20] See [20, 21].

with:

$$\Theta(\mathbf{x}) = (\mu_1(\mathbf{x}), \mu_2(\mathbf{x}), \sigma_1(\mathbf{x}), \sigma_2(\mathbf{x}), \rho(\mathbf{x})), \upsilon = (\upsilon_1, \upsilon_2);$$

and

$$z(\mathbf{x}) = \frac{(\upsilon_1 - \mu_1(\mathbf{x}))^2}{\sigma_1(\mathbf{x})^2} - \frac{2\rho(\mathbf{x})(\upsilon_1 - \mu_1(\mathbf{x}))(\upsilon_2 - \mu_2(\mathbf{x}))}{\sigma_1(\mathbf{x})\sigma_2(\mathbf{x})} + \frac{(\upsilon_2 - \mu_2(\mathbf{x}))^2}{\sigma_2(\mathbf{x})^2} \tag{8}$$

By maximizing the local log-likelihood function, for bivariate sample jointly independent and identically distributed $X_i = (X_{1,i}, X_{2,i})$ and bandwidth $\mathbf{b} = (b_1, b_2)$, as presented in [37], we have the following equation for $\mathfrak{L}_{(LGC)}(X_1, ..., X_n; \Theta_{\mathbf{b}}(\mathbf{x}))$:

$$\mathfrak{L}_{(LGC)}(\cdot) \equiv n^{-1} \sum_{i=1}^{n} K_{\mathbf{b}}(X_i - \mathbf{x}) log\Psi(X_i, \Theta_{\mathbf{b}}(\mathbf{x})) - \int K_{\mathbf{b}}(\nu - \mathbf{x})\Psi(\nu, \Theta(\mathbf{x}))d\nu. \tag{9}$$

On this matter, through equation (9), for $K = \{1, 2\}$, each point observed in the sample has local average ($\mu_k(\mathbf{x})$), local variance ($\sigma_k(\mathbf{x})$) and local correlation ($\rho(\mathbf{x})$). Thus, the choice of the optimum bandwidth is made by restricting the variance of the parameters so as to be sized by $(nb_1^3 b_2^3)^{-1}$, and the square of bias is constrained by $(b_1^2 + b_2^2)^2$. One of the advantages of this estimator is its flexibility in dealing with models whose correlation is nonlinear. Another advantage is the fact that it does not require a normal distribution of the vector $X_i = (X_{1i}, X_{2i})$. Thus, it is possible to apply the local Gaussian correlation approach to detect periods where the correlation between the resale price margin series and the variability of fuel price in the market may be signalling the presence of collusion.

3. Results

To generate the results shown in this section, our sample period is formed by the log deviation of weekly gasoline sales prices in DF, GO, RJ and SP, provided by the Brazilian National Agency of Petroleum, Natural Gas and Bio-fuels (ANP), from January 5, 2014 to December 24, 2017, for a total of 207 observations. The elaboration of the sample plan is important to guarantee that the statistical inference process is credible. As pointed out by [40], to carry out a statistical study by means of sampling, it is necessary to know the concept of probabilistic sampling plans, in which all units of the population have a nonzero probability of belonging to the sample[21], and that probability is calculable.

3.1. The Best Fitted MSGARCH Model

From Figure (1) it is possible to make a first exploratory and descriptive analysis of the data. Note that the time interval selected to adjust the MSGARCH model suggests two different regimes. The first is characterized by a low volatility of the data. The second regime is identified as a period of greater dispersion and variability of the data. This suggests that conditional variance is time-varying according to a regime-switching specification.

3.1.1. Brasília

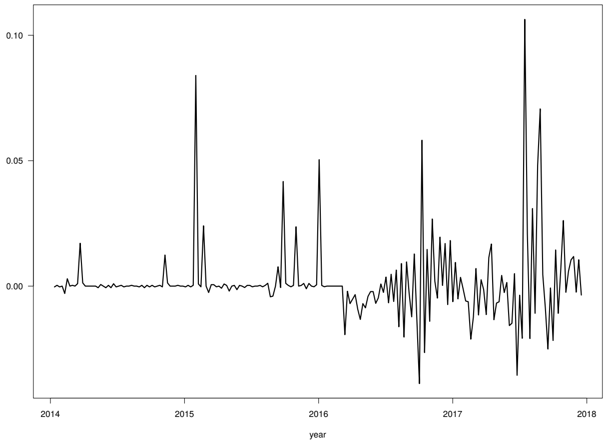
In order to capture this effect, we exhaustively performed many estimations to fit the best GARCH model to describe the behaviour observed in the data. In this way, the model that identified more precisely the two regimes in Brasília is the one with heterogeneous regime, i.e., each regime is characterized by a different conditional volatility. Thus, a GARCH variance specification with a skewed[22] Standardized t- Student ($S.t{-}Student$) distribution is assumed in the first regime and an ARCH variance specification with a S.t-Student distribution is assumed in the second regime. The model is given by the equation (10):

$$\begin{aligned} z_t|(m_t = k, \Im_{t-1}) &\sim S.t - Student(0, h_{k,t}, \nu, \xi), \\ h_{k=1,t} &\equiv \alpha_{0,1} + \alpha_{1,1} z_{t-1}^2 + \beta_1 h_{1,t-1}, \\ h_{k=2,t} &\equiv \alpha_{0,2} + \alpha_{1,2} z_{t-1}^2. \end{aligned} \tag{10}$$

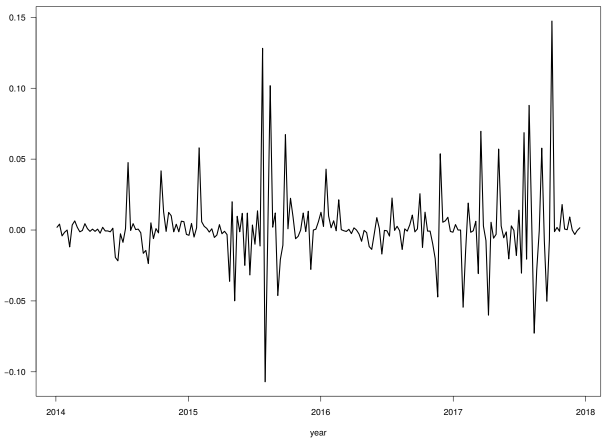
where $k \in 1, 2$.

[21] By doing so, the information obtained from the data can be generalized to the population, since the random selection guarantees its representativeness and allows us to control possible sample errors. In this sense, it is necessary to choose the best sampling plan for the case studied, as this will have a great influence on the results obtained. However, since there is no precise description of the sample selection process adopted by the agency responsible for data collection, we carefully inform the reader that we are aware of the possible limitations of the statistical estimations made in this study.

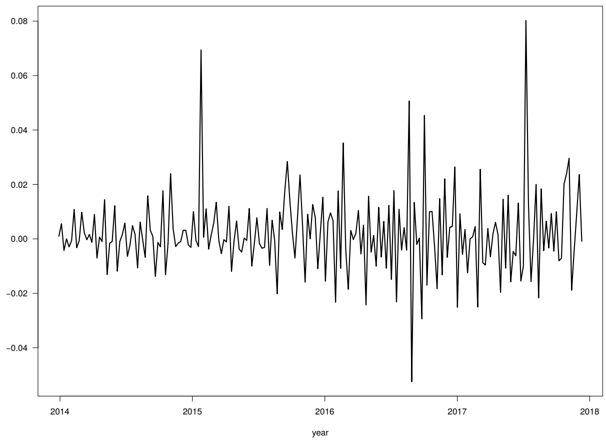
[22] Please see [38] for details.



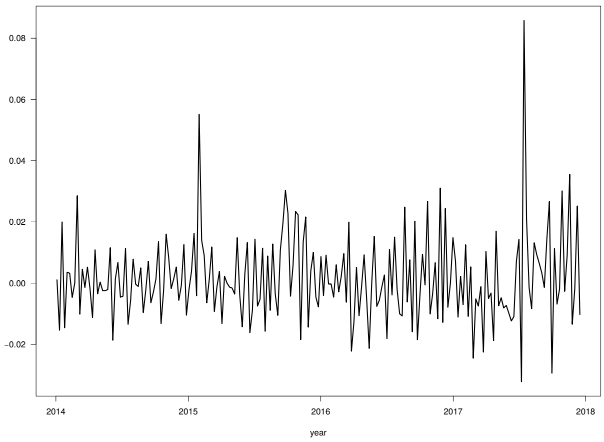
(a) (DF)



(b) (GO)



(c) (RJ)



(d) (SP)

Figure 1: Percentage weekly log-deviation of the gasoline sales price in the cities of Brasília (a), Goiânia (b), Rio de Janeiro (c) and São Paulo (d) for a period ranging from January 05, 2014, to December 24, 2017, for a total of 207 observations.

3.1.2. Goiânia

The best extension of the MSGARCH model to describe the market behaviour in Goiânia is represented in equation (11). We adjust an EGARCH model with skewed Generalized Error Distribution (SGED) for the first regime ($k = 1$), i.e., the one that suggests a collusive behaviour. The second regime ($k = 2$), which may indicate a competitive behaviour, follows an ARCH model with skewed normal distribution (SNORM):

$$\begin{gathered} z_t|(m_t = 1, \Im_{t-1}) \sim SGED(0, h_{k=1,t}, \nu, \xi), \\ ln(h_{k=1,t}) \equiv \alpha_{0,1} + \alpha_{1,1}(|\lambda_{1,t-1}| - E[|\lambda_{1,t-1}|]) + \alpha_{2,1} z_{t-1} + \beta_1 ln(h_{1,t-1}), \\ z_t|(m_t = 2, \Im_{t-1}) \sim SNORM(0, h_{k=2,t}, \nu, \xi), \\ h_{k=2,t} \equiv \alpha_{0,2} + \alpha_{1,2} z_{t-1}^2. \end{gathered} \tag{11}$$

3.1.3. Rio de Janeiro

A GJR GARCH variance specification with a skewed $S.t - Student$ distribution is assumed in each regime for Rio de Janeiro. The model may be written as:

$$\begin{gathered} z_t|(m_t = k, \Im_{t-1}) \sim S.t - Student(0, h_{k,t}, \nu, \xi), \\ h_{k,t} \equiv \alpha_{0,k} + (\alpha_{1,k} + \alpha_{2,k}\mathbb{I}_{\{z_{t-1}<0\}}) z_{t-1}^2 + \beta_k h_{k,t-1}, \end{gathered} \tag{12}$$

where $k \in 1, 2$.

3.1.4. São Paulo

The best extension of the MSGARCH model to describe the market behaviour in São Paulo is represented in the equation (13). We adjust an ARCH model for the first regime ($k = 1$), i.e., the one that suggests a collusive behaviour. The second regime ($k = 2$) follows an EGARCH model. A skewed Generalized Error Distribution (SGED) is assumed in each regime:

$$\begin{gathered} z_t|(m_t = k, \Im_{t-1}) \sim SGED(0, h_{k,t}, \nu), \\ h_{k=1,t} \equiv \alpha_{0,1} + \alpha_{1,1} z_{t-1}^2. \\ ln(h_{k=2,t}) \equiv \alpha_{0,1} + \alpha_{1,1}(|\lambda_{2,t-1}| - E[|\lambda_{2,t-1}|]) + \alpha_{2,2} z_{t-1} + \beta_2 ln(h_{2,t-1}), \end{gathered} \tag{13}$$

where $k \in 1, 2$.

3.1.5. Comparison Between ML and MCMC Estimations

The ML and MCMC estimations for the cities selected in this article are available in the Appendix. In Table (1), we summarize the information of the Transition Matrix and stable probabilities for ML and MCMC estimations. Estimates suggest that the volatility behaviour is not homogeneous over the two regimes. In fact, from Figures (8) and (9) in the Appendix, it is possible to see the heterogeneity in the conditional volatility of the two regimes. The persistence of volatility also shows a different behaviour between the regimes. Thus, in accordance with equations (10) - (13), the first regime has low unconditional volatility and low persistence of the volatility process. In a different way, the second regime presents high unconditional volatility and high persistence of the volatility process. In this sense, regime one may be perceived as a cartel behaviour, while regime two may be perceived as a competitive behaviour.

Transition Matrix - ML Estimation								
	Brasília		**Goiânia**		**Rio de Janeiro**		**São Paulo**	
	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2
t|k=1	0.9181	0.0819	0.9805	0.0195	0.9705	0.0295	0.9866	0.0134
t|k=2	0.0729	0.9271	0.1995	0.8005	0.7658	0.2342	0.3915	0.6085
Stable Probabilities								
State 1:	0.471		0.911		0.9629		0.9669	
State 2:	0.529		0.089		0.0371		0.0331	
Transition Matrix - MCMC Estimation								
	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2	t+1|k=1	t+1|k=2
t|k=1	0.9085	0.0915	0.8647	0.1353	0.9617	0.0383	0.9881	0.0119
t|k=2	0.0978	0.9022	0.8620	0.1380	0.6203	0.3797	0.5246	0.4754
Stable Probabilities								
State 1:	0.5169		0.8643		0.9419		0.9778	
State 2:	0.4831		0.1357		0.0581		0.0222	

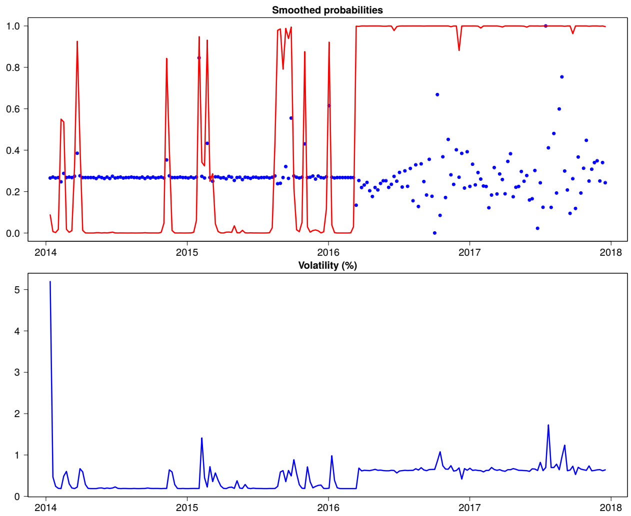
Table 1: Transition Matrix and stable probabilities for ML and MCMC estimation.

As reported in [3], ML estimation may not be an easy task for MSGARCH-type models. The MCMC estimate circumvents some potential problems of the ML method, since the exploratory analysis of the joint posterior distribution of the model parameters avoids the convergence to a local maximum - which is a common problem in ML approach. The Bayesian framework has many advantages, such as the lower cost to obtain the exact distributions of non-linear functions of the model parameters by simulations from the joint posterior distribution. For the Bayesian estimation of the MSGARCH models, the MCMC adaptive sampler proposed by [41] was applied. In other words, this method is a random-walk Metropolis-Hastings algorithm with coerced acceptance rate. A satisfactory performance was observed. The outcomes are presented in Table (1). The smoothed probabilities of being in the second regime ($k = 2$), given by $\mathbf{P}[m_t = 2|\Im_T]$ for $t = 1, ..., T$, superimposed on the log-deviation of the gasoline sales price (top graph) is displayed in Figure (2). The filtered (annualized) conditional volatilities of the overall process corresponds to the bottom graph.

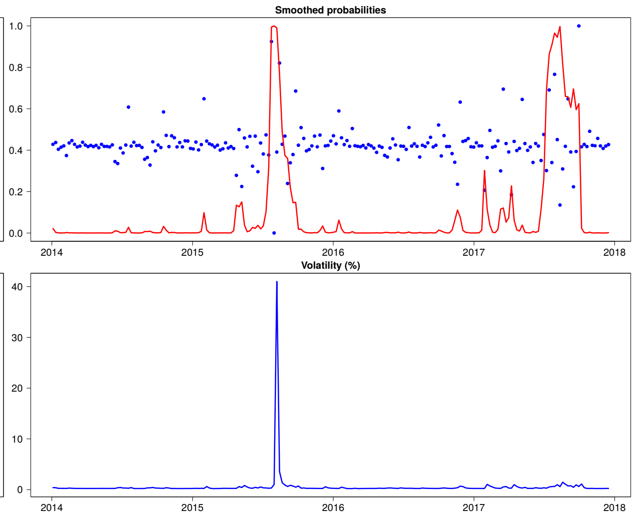
As expected, when the smoothed probability of being in a more competitive environment (regime 2) is approximately equal to one, the filtered volatility of the process increases quite noticeably. It is also possible to observe that the Markov chain[23] evolves persistently over time and that, at the limit, as reported in Table (1), the probabilities of

[23]The stability and stationarity test of the Markov chain is available in Figure (10) of the appendix.

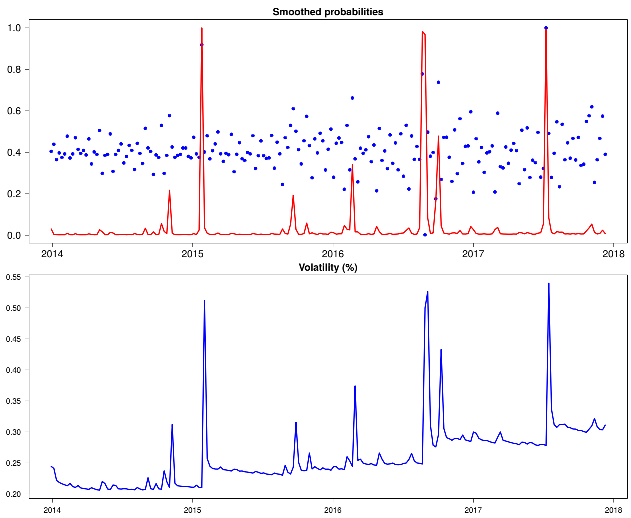
being in the two distinct market environments are approximately 52% and 48% in Brasília. For the gasoline market in Goiânia, the probability of being in the collusive or cartel market behaviour (regime 1) is about 86% and about 14% in the second one (competitive market behaviour). The probability of observing a cartel behaviour at time t and of this behaviour remaining at $t+1$ is 96% and 99% for Rio de Janeiro and São Paulo, respectively.



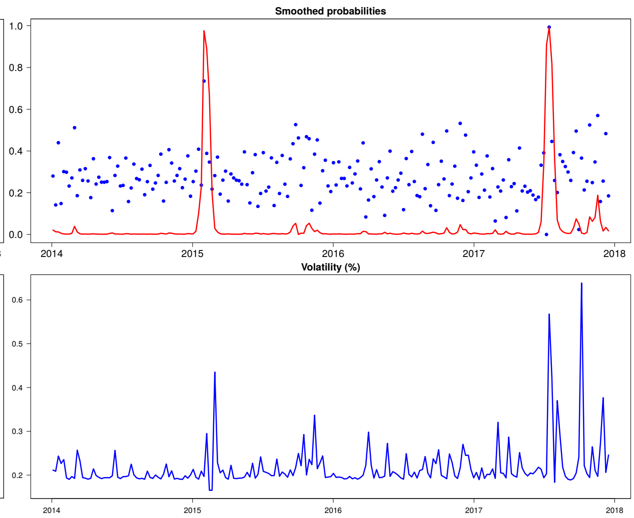
(a) (DF)



(b) (GO)



(c) (RJ)



(d) (SP)

Figure 2: Top: Estimated smoothed probabilities of the second regime, $\mathbf{P}[m_t = 2|\hat{\psi}, \Im_T]$, for $t = 1, ..., T$. The thin blue line depicts the log deviation of the gasoline sales price. Bottom: Filtered conditional volatilities.

It is important to report that the database used to generate the results shown in Figure (2a) was extracted from a case already analyzed and judged by CADE. In this sense, we can note that, since the first quarter of 2016, when the Brazilian antitrust authority condemned the practice of cartel in the gasoline retail market in Brasília, there has been an evident regime change which was duly captured by the Bayesian MCMC estimate.

In addition, when we combine the information in Table (1) with the descriptive analysis of Figure (2), we can understand that the Transition Matrix informs us that, if the gasoline market in Brasília is characterized as an anti-competitive regime at a given time instant t, the probability that it continues in that environment in $t+1$ is 90.85%. In Goiânia, this probability is equal to 86.47%. This value, shown in the 1st row and 1st column of Transition Matrix, can be used as a filter for signs of cartel formation.

Moreover, the results of the regime transition in Goiânia, Rio de Janeiro and São Paulo can be seen in Figures (2b), (2c) and (2d), respectively. We observe that the model was effective in differentiating the market behaviours according to the volatility. Although these markets have not been evaluated and judged by CADE, our outcomes suggest strong evidences that it has a cartel behaviour, once the smoothed probability of being in the second regime - that configures a competitive market - is equal (or closer) to zero in most of the time period analyzed.

We observed that during the period that characterizes collusive market behaviour in Brasília, i.e., from early 2014 to early 2016, the MSGARCH model seems to overestimate the probability of regime reversal in some brief time intervals, which suggests the need of a specific treatment in order to address these possible outliers and correct the bias of the estimators. This same procedure was applied to the other cities analyzed in this article. It is important to note that, as far as we are concerned, this literature is still in development and there are no studies applied to the Brazilian gasoline retail market debating cartel formation as shown in this article.

3.2. Statistical Treatment for the Markovian-Switching Regimes

The purpose of the statistical treatment performed in this section is to identify precisely at what point in time there was a statistically significant regime change. This is therefore justified, as can be seen by the smoothed probability of the city of Brasilia, available in Figure (2a). Due to the presence of some possible outliers, the procedure implemented to ensure the robustness of the results is described as follows.

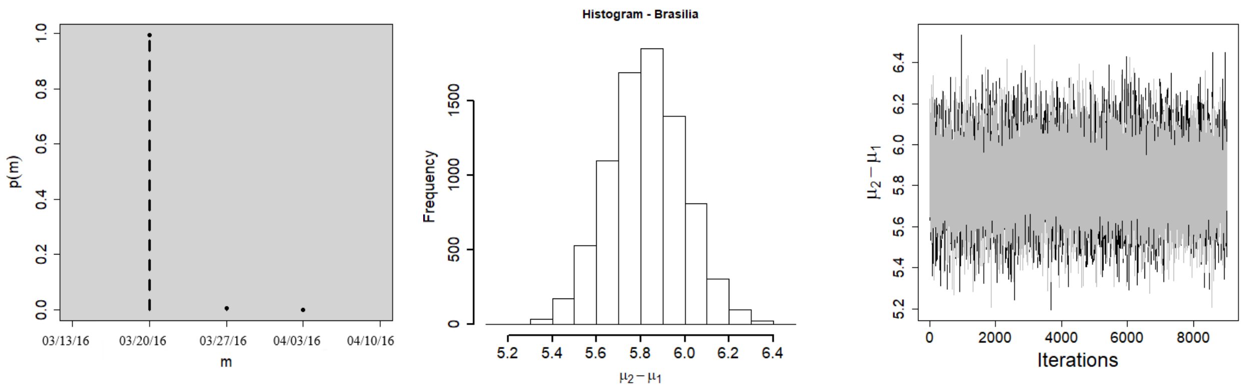
We apply a scale transformation to the smoothed probabilities so that the market be in a competitive environment (Regime 2) in each city evaluated. In this sense, a new variable, $\mathcal{P}_{k=2,t}$, is created and defined with the purposes of changing its scale of values, restricted to the interval $(0, 1)$, for a set of values in the real line, $\mathbb{R}$.

In this way, we can assume a Normal distribution for this new variable. More precisely, we use the transformation $w_t = \Phi^{-1}(\mathcal{P}_{k=2,t})$, in which Φ^{-1} is the inverse of the cumulative of the standard Normal distribution. In sequence, a model that establishes two regimes is formulated: an initial regime that goes to time instant represented by m, for which w_t follows a distribution $N(\mu_1, \sigma_1^2)$ and, after the change point, m, (to be estimated by the model), w_t follows a distribution $N(\mu_2, \sigma_2^2)$, characterizing the second regime.

In order to capture the moment of change in m, a Bayesian approach is used. By doing so, we formulate a priori uniform discrete distribution. Thus, all the instants of time have the same a priori probability. In summary, the model considered establishes that:

$$w_t \sim \begin{cases} N(\mu_1, \sigma_1^2) & \text{for} \quad i \leq \mathsf{m} \\ N(\mu_2, \sigma_2^2) & \text{for} \quad i > \mathsf{m} \end{cases}$$

To estimate the parameters of this model, we used Markov Chain Monte Carlo (MCMC). More specifically, we use the Gibbs sampler with steps from Metropolis Hastings to simulate the posterior distribution. In addition to the model parameters, we also simulated a posteriori samples of $\Delta = \mu_2 - \mu_1$ to identify how distinct (heterogeneous) the two regimes are and to assess whether there is actually a significant regime change.



(a) Changing point m (DF)

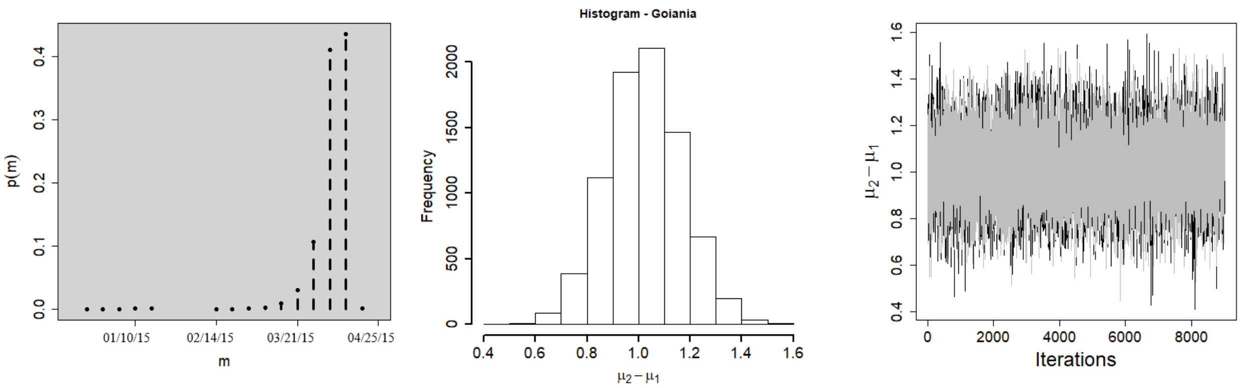
(b) Histogram of $\mu_2 - \mu_1$ (DF)

(c) Posterior distribution (DF)

Figure 3: Statistical Treatment for the Markovian-Switching Regimes in DF.

By Figure (3), we can have a better dimension of how the market behavior of the gasoline retail price in Brasilia changes over time. In Figure (3a), we might infer that, from the week of 03/20/2016 onwards, with a high probability ($\mathsf{p}(\mathsf{m}) \approxeq 1$), there was a regime change captured by m in the market under analysis. Thus, when comparing the difference between the means, $\mu_2 - \mu_1$, in Figure (3b), we can notice that these are quite different.

This suggests, in fact, that there are two distinct market behaviors for the retail gasoline price in Brasilia. Figure (3c) shows the number of interactions used for the simulation of the posterior distribution. In this way, we have a more robust evidence that the oscillations observed before 2016 for the city of Brasilia, shown in Figure (2a), does not characterize a change of market behavior for Brasília.



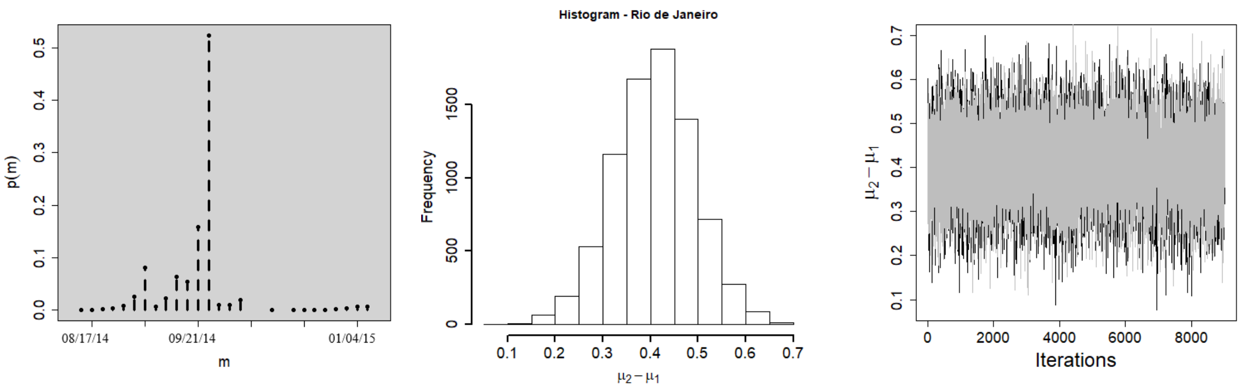
(a) Changing point m (GO)

(b) Histogram of $\mu_2 - \mu_1$ (GO)

(c) Posterior distribution (GO)

Figure 4: Statistical Treatment for the Markovian-Switching Regimes in GO.

By submitting the same evaluation to the city of Goiania, we see from Figure (4a) that there is a probability $\mathsf{p}(\mathsf{m}) \approxeq 0.45$ associated with a change of regime in the period that corresponds to the instant of time between the weeks of 03/21/2015 and 04/25/2015. On the other hand, on Figure (4b), we see that the difference between $\mu_2 - \mu_1$ is not as expressive as that observed in Brasilia. Figure (4b), in turn, shows the interactions of the simulation process used to obtain the posterior distribution.



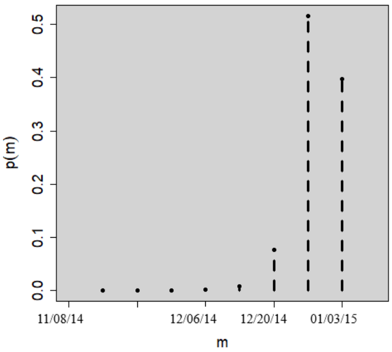
(a) Changing point m (RJ)

(b) Histogram of $\mu_2 - \mu_1$ (RJ)

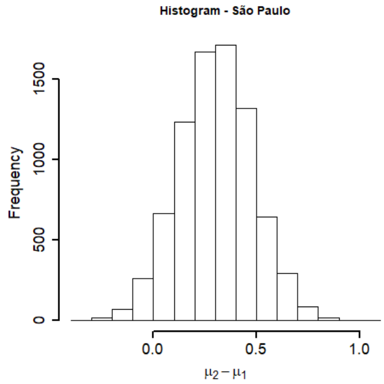
(c) Posterior distribution (RJ)

Figure 5: Statistical Treatment for the Markovian-Switching Regimes in RJ.

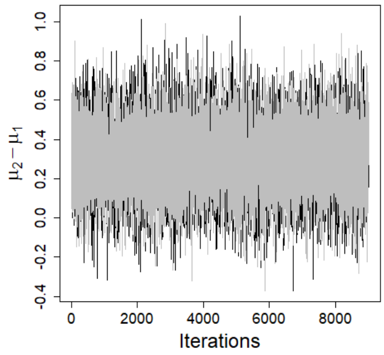
For Rio de Janeiro, from Figure (5a), we see a possible regime change with probability $\mathsf{p}(\mathsf{m}) \approxeq 0.5$ around week 09/21/2014. On the other hand, when we observe the difference between the averages, $\mu_2 - \mu_1$, through Figure (5b), we infer that, given its small size, there may not be a significant change in the gasoline retail price behavior in RJ.



(a) Changing point m (SP)



(b) Histogram of $\mu_2 - \mu_1$ (SP)

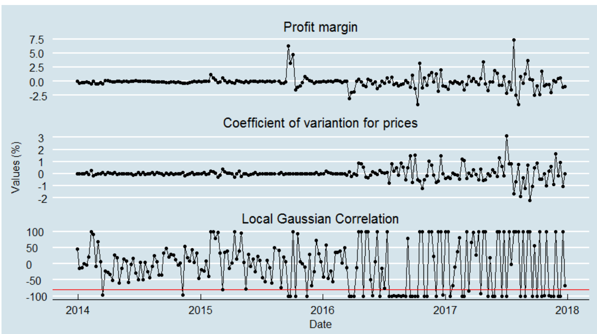


(c) Posterior distribution (SP)

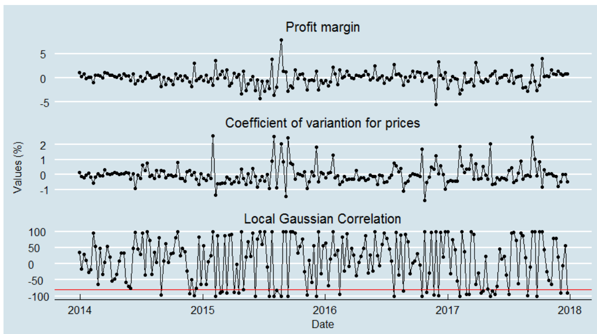
Figure 6: Statistical Treatment for the Markovian-Switching Regimes in SP.

Finally, when we give the same statistical treatment to the city of São Paulo, from Figure (6a), we see that the probability of a change in price behavior between weeks 12/20/2014 and 01/07/2015 is given by $\mathsf{p(m)} \approxeq 0.5$. However, with the result shown in Figure (6b), we see that $\mu_2 - \mu_1$ is practically unperceivable. Thus, it may not be possible to state that the São Paulo market showed competitive market characteristics for the period under analysis.

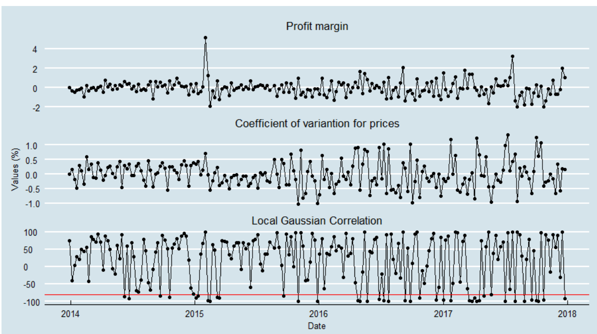
3.3. The Local Gaussian Correlation (LGC) Outcomes



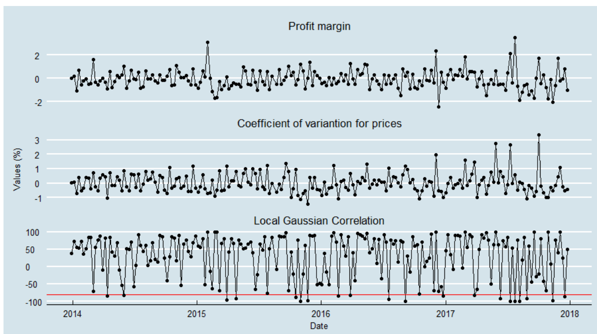
(a) (DF)



(b) (GO)



(c) (RJ)



(d) (SP)

Figure 7: The Local Gaussian Correlation outcomes for the weekly gasoline sales price between January 2014 and December 2017. The profit margin is equivalent to the resale price margin.

The Brazilian antitrust authority uses the correlation between the resale price margin and the variability of prices for the fuel market as a signalling of collusive behaviour. In other words, the LGC approach suggests that local negative correlations that are persistent for three consecutive weeks and are below -0.80 might indicate a cartel agreement in the gasoline sales market. Thus, for Brasília (DF), we observe some points where the local correlation is negatively and persistently stronger. It is situated below the red line in Figure (7a), which indicates that the local correlation is lower than the value of the pocket rule (-0.8). The persistence of this value corresponds to the period between July and November of 2016, and between June and November of 2017. Figure (7b) presents the LGC results for Goiânia (GO). We note some points below the red line in May of 2017.

Considering the results of the LGC approach for Rio de Janeiro, it is possible to note in Figure (7c) that in April 2017 the local correlation is negative and less than -0.8, suggesting a cartel behaviour. For São Paulo, although some time interval in which the local correlation is lower than -0.8 has been observed, it does not show persistence during the evaluated period. Therefore, for the city of São Paulo, as shown in Figure (7d) the result of the LGC approach does not reject the condition that the gasoline retail market has competitive characteristics.

4. Conclusion

This paper analyzed the behaviour of the gasoline sales market in the cities of Brasília, Goiânia, Rio de Janeiro and São Paulo. The first approach was based on the Markov-Switching GARCH model. The results suggested that, in each market, there are evidences of cartel behaviour. The accuracy of the model is partially validated by the data analysis of Brasília, a city that has already been evaluated and judged by CADE due to the formation of Cartel in the gasoline market. In addition, the MSGARCH model indicates that all the other markets evaluated should be better investigated, since there is a substantial probability that it would have practiced cartel between 2014 and 2017.

The second approach was the Local Gaussian Correlation. The idea of adopting two models was to compare their fit quality to the data and to evaluate if they could be used in a complementary way. In this sense, even though the Local Gaussian Correlation model indicated that the gasoline retail markets in Brasilia, Goiânia and Rio de Janeiro are considerably more likely to have practiced cartels in the observed period, the dates in which this collusive behaviour would have occurred did not match with the MS GARCH analysis.

In other words, for the Brasília gasoline retail market, the LGC model indicates cartel formation from July to November 2016 and from June to November 2017. However, during this time, CADE had already judged and condemned the collusive practice in the gasoline market. Another result that affects the accuracy of this model is the fact that it did not identify the cartel period along the years 2014 and 2015, where the weekly price volatility was quite low. Through the MS-GARCH models, it was possible to infer that the gasoline retail market in Goiânia, Rio de Janeiro and São Paulo showed a behaviour that was persistently cartelized. On the other hand, by the LGC approach, the same collusive behaviour was identified sporadically in Goiânia and Rio de Janeiro. The most striking result - and the one that most prevents the dialogue between the applied methodologies - was the one observed for São Paulo, given that, according to the methodology used by CADE, the gasoline retail market fits the characteristics of a competitive economic environment.

Finally, through this article, we hope to have emphasized the relevance of the analysis on price variation, which is a strategic variable of the firms with regard to the decisions on whether to adhere to a collusive agreement or not. In addition, it is important to recognize the limitation of the MSGARCH model with regard to the treatment of possible outliers. Another issue that may limit the validity of our results is the quality of the sample available. Thus, for future research, it may be useful to design a sample plan that guarantees the consistency of the estimators and also a way to treat the outliers departing from a MCMC approach.

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5. Appendix

```
        FITTED PARAMETERS (ML) DF
----------------------------------------
Specification type: Markov-switching
Specification name: sGARCH_sstd sARCH_sstd
Number of parameters in each variance model: 3 2
Number of parameters in each distribution: 2 2
----------------------------------------
Fixed parameters:
None
----------------------------------------
Across regime constrained parameters:
nu xi
----------------------------------------
Fitted parameters:
         Estimate Std. Error  t value  Pr(>|t|)
alpha0_1   0.0000     0.0000   7.1205 5.376e-13
alpha1_1   0.0012     0.0002   5.4853 2.064e-08
beta_1     0.0010     0.0090   0.1164 4.537e-01
nu_1       2.1181     0.0176 120.3435    <1e-16
xi_1       1.0540     0.0040 265.6320    <1e-16
alpha0_2   0.0017     0.0002   6.7795 6.030e-12
alpha1_2   0.9918     0.0130  76.2233    <1e-16
P_1_1      0.9181     0.0024 377.3940    <1e-16
P_2_1      0.0729     0.0021  34.6878    <1e-16


----------------------------------------
Transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9181  0.0819
t|k=2  0.0729  0.9271
----------------------------------------
Stable probabilities:
State 1 State 2
  0.471   0.529
----------------------------------------
LL: 834.9691
AIC: -1651.9382
BIC: -1621.9873
----------------------------------------
```

```
        FITTED PARAMETERS (ML) GO
----------------------------------------
Specification type: Markov-switching
Specification name: eGARCH_sged sARCH_snorm
Number of parameters in each variance model: 4 2
Number of parameters in each distribution: 2 1
----------------------------------------
Fixed parameters:
None
----------------------------------------
Across regime constrained parameters:
None
----------------------------------------
Fitted parameters:
         Estimate Std. Error  t value Pr(>|t|)
alpha0_1  -3.4757     0.0006  -5573.6   <1e-16
alpha1_1   0.2121     0.0006    331.4   <1e-16
alpha2_1  -0.4536     0.0007   -682.2   <1e-16
beta_1     0.6013     0.0002   3297.1   <1e-16
nu_1       0.7011     0.0000 929277.0   <1e-16
xi_1       1.0461     0.0003   3456.6   <1e-16
alpha0_2   0.0054     0.0000   2713.8   <1e-16
alpha1_2   0.0719     0.0001    495.3   <1e-16
xi_2      61.4125     0.0157   3900.4   <1e-16
P_1_1      0.9805     0.0001   9241.1   <1e-16
P_2_1      0.1995     0.0000  15709.1   <1e-16
----------------------------------------
Transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9805  0.0195
t|k=2  0.1995  0.8005
----------------------------------------
Stable probabilities:
State 1 State 2
  0.911   0.089
----------------------------------------
LL: 595.6091
AIC: -1169.2183
BIC: -1132.5584
----------------------------------------
```

```
        FITTED PARAMETERS (ML) RJ
----------------------------------------
Specification type: Markov-switching
Specification name: gjrGARCH_sstd gjrGARCH_sstd
Number of parameters in each variance model: 4 4
Number of parameters in each distribution: 2 2
----------------------------------------
Fixed parameters:
None
----------------------------------------
Across regime constrained parameters:
nu
----------------------------------------
Fitted parameters:
         Estimate Std. Error   t value  Pr(>|t|)
alpha0_1   0.0000     0.0000   63.6199    <1e-16
alpha1_1   0.0123     0.0003   39.6330    <1e-16
alpha2_1   0.0002     0.0001    1.6756 4.691e-02
beta_1     0.9857     0.0003 2993.8053    <1e-16
nu_1      15.4872     0.8777   17.6457    <1e-16
xi_1       1.0554     0.0067  157.4801    <1e-16
alpha0_2   0.0007     0.0000   98.9147    <1e-16
alpha1_2   0.0064     0.0074    0.8727 1.914e-01
alpha2_2   0.0373     0.0368    1.0142 1.553e-01
beta_2     0.8398     0.0016  530.7282    <1e-16
xi_2       0.0625     0.0133    4.6881 1.378e-06
P_1_1      0.9705     0.0012  827.7495    <1e-16
P_2_1      0.7658     0.0142   54.0448    <1e-16
----------------------------------------
Transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9705  0.0295
t|k=2  0.7658  0.2342
----------------------------------------
Stable probabilities:
State 1 State 2
 0.9629  0.0371
----------------------------------------
LL: 613.7238
AIC: -1201.4475
BIC: -1158.1222
----------------------------------------
```

```
        FITTED PARAMETERS (ML) SP
----------------------------------------
Specification type: Markov-switching
Specification name: sARCH_sged eGARCH_sged
Number of parameters in each variance model: 2 4
Number of parameters in each distribution: 2 2
----------------------------------------
Fixed parameters:
None
----------------------------------------
Across regime constrained parameters:
nu
----------------------------------------
Fitted parameters:
         Estimate Std. Error  t value Pr(>|t|)
alpha0_1   0.0001     0.0000   2589.7   <1e-16
alpha1_1   0.1620     0.0001   3060.5   <1e-16
nu_1       1.5515     0.0003   4836.9   <1e-16
xi_1       1.3051     0.0005   2623.1   <1e-16
alpha0_2  -5.1649     0.0067   -767.5   <1e-16
alpha1_2  -0.4807     0.0012   -414.8   <1e-16
alpha2_2  -1.9143     0.0010  -1898.1   <1e-16
beta_2     0.2141     0.0006    371.0   <1e-16
xi_2       0.1193     0.0000   2802.3   <1e-16
P_1_1      0.9866     0.0000 191476.6   <1e-16
P_2_1      0.3915     0.0001   4214.8   <1e-16
----------------------------------------
Transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9866  0.0134
t|k=2  0.3915  0.6085
----------------------------------------
Stable probabilities:
State 1 State 2
 0.9669  0.0331
----------------------------------------
LL: 613.3483
AIC: -1204.6966
BIC: -1168.0367
----------------------------------------
```

Figure 8: Fitted Parameters using ML estimation.

```
      FITTED PARAMETERS (MCMC) DF
------------------------------------------
Specification type: Markov-switching
Specification name: sGARCH_sstd sARCH_sstd
Number of parameters in each variance model: 3
Number of parameters in each distribution: 2 2
------------------------------------------
Fixed parameters:
None
------------------------------------------
Across regime constrained parameters:
nu xi
------------------------------------------
Posterior sample (size: 2500)
            Mean     SD     SE   TSSE    RNE
alpha0_1 0.0000 0.0000 0.0000 0.0000 0.1632
alpha1_1 0.0112 0.0272 0.0005 0.0010 0.3117
beta_1   0.0211 0.0780 0.0016 0.0049 0.1012
nu_1     2.1274 0.0287 0.0006 0.0018 0.1051
xi_1     1.0591 0.0606 0.0012 0.0027 0.1985
alpha0_2 0.0018 0.0005 0.0000 0.0000 0.1888
alpha1_2 0.9085 0.1586 0.0032 0.0066 0.2303
P_1_1    0.9085 0.0327 0.0007 0.0014 0.2311
P_2_1    0.0978 0.0313 0.0006 0.0014 0.1934
------------------------------------------
Posterior mean transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9085  0.0915
t|k=2  0.0978  0.9022
------------------------------------------
Posterior mean stable probabilities:
State 1 State 2
 0.5169  0.4831
------------------------------------------
Acceptance rate MCMC sampler: 27.4%
nmcmc: 12500
nburn: 5000
nthin: 5
------------------------------------------
DIC: -1655.997
------------------------------------------
```

```
      FITTED PARAMETERS (MCMC) GO
------------------------------------------
Specification type: Markov-switching
Specification name: eGARCH_sged sARCH_snorm
Number of parameters in each variance model: 4 2
Number of parameters in each distribution: 2 1
------------------------------------------
Fixed parameters:
None
------------------------------------------
Across regime constrained parameters:
None
------------------------------------------
Posterior sample (size: 2500)
             Mean     SD     SE   TSSE    RNE
alpha0_1 -2.6321 0.6349 0.0127 0.0461 0.0757
alpha1_1  0.2600 0.1117 0.0022 0.0069 0.1044
alpha2_1 -0.2768 0.0736 0.0015 0.0038 0.1523
beta_1    0.7225 0.0704 0.0014 0.0053 0.0711
nu_1      0.7023 0.0011 0.0000 0.0001 0.0759
xi_1      1.0395 0.0296 0.0006 0.0016 0.1348
alpha0_2  0.0035 0.0008 0.0000 0.0001 0.0608
alpha1_2  0.9916 0.0193 0.0004 0.0010 0.1405
xi_2     35.2050 7.3896 0.1478 0.6373 0.0538
P_1_1     0.8647 0.0382 0.0008 0.0027 0.0779
P_2_1     0.8620 0.0816 0.0016 0.0057 0.0810
------------------------------------------
Posterior mean transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.8647  0.1353
t|k=2  0.8620  0.1380
------------------------------------------
Posterior mean stable probabilities:
State 1 State 2
 0.8643  0.1357
------------------------------------------
Acceptance rate MCMC sampler: 28%
nmcmc: 12500
nburn: 5000
nthin: 5
------------------------------------------
DIC: -1184.7751
------------------------------------------
```

```
      FITTED PARAMETERS (MCMC) RJ
------------------------------------------
Specification type: Markov-switching
Specification name: gjrGARCH_sstd gjrGARCH_sstd
Number of parameters in each variance model: 4 4
Number of parameters in each distribution: 2 2
------------------------------------------
Fixed parameters:
None
------------------------------------------
Across regime constrained parameters:
nu
------------------------------------------
Posterior sample (size: 2500)
             Mean      SD     SE   TSSE    RNE
alpha0_1  0.0000  0.0000 0.0000 0.0000 0.0863
alpha1_1  0.0188  0.0082 0.0002 0.0005 0.1252
alpha2_1  0.0003  0.0008 0.0000 0.0001 0.0865
beta_1    0.9673  0.0141 0.0003 0.0009 0.0900
nu_1     41.5919 25.2145 0.5043 1.2117 0.1732
xi_1      1.1027  0.1057 0.0021 0.0051 0.1728
alpha0_2  0.0012  0.0014 0.0000 0.0001 0.1212
alpha1_2  0.1037  0.1072 0.0021 0.0051 0.1740
alpha2_2  0.0049  0.0077 0.0002 0.0005 0.1021
beta_2    0.7447  0.1361 0.0027 0.0073 0.1388
xi_2      5.6774  5.1156 0.1023 0.2853 0.1286
P_1_1     0.9617  0.0269 0.0005 0.0018 0.0934
P_2_1     0.6203  0.1624 0.0032 0.0094 0.1186
------------------------------------------
Posterior mean transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9617  0.0383
t|k=2  0.6203  0.3797
------------------------------------------
Posterior mean stable probabilities:
State 1 State 2
 0.9419  0.0581
------------------------------------------
Acceptance rate MCMC sampler: 28%
nmcmc: 12500
nburn: 5000
nthin: 5
------------------------------------------
DIC: -1202.0117
------------------------------------------
```

```
      FITTED PARAMETERS (MCMC) SP
------------------------------------------
Specification type: Markov-switching
Specification name: sARCH_sged eGARCH_sged
Number of parameters in each variance model: 2 4
Number of parameters in each distribution: 2 2
------------------------------------------
Fixed parameters:
None
------------------------------------------
Across regime constrained parameters:
nu
------------------------------------------
Posterior sample (size: 2500)
             Mean     SD     SE   TSSE    RNE
alpha0_1  0.0001 0.0000 0.0000 0.0000 0.0507
alpha1_1  0.1509 0.0644 0.0013 0.0042 0.0953
nu_1      1.5058 0.3040 0.0061 0.0277 0.0481
xi_1      1.3191 0.1228 0.0025 0.0089 0.0768
alpha0_2 -0.8058 9.1974 0.1839 2.1562 0.0073
alpha1_2  0.7438 2.3238 0.0465 0.1460 0.1013
alpha2_2 -2.0514 1.6139 0.0323 0.1645 0.0385
beta_2    0.0611 0.4817 0.0096 0.0451 0.0456
xi_2      0.2181 0.2209 0.0044 0.0171 0.0668
P_1_1     0.9881 0.0111 0.0002 0.0016 0.0193
P_2_1     0.5246 0.1902 0.0038 0.0132 0.0830
------------------------------------------
Posterior mean transition matrix:
      t+1|k=1 t+1|k=2
t|k=1  0.9881  0.0119
t|k=2  0.5246  0.4754
------------------------------------------
Posterior mean stable probabilities:
State 1 State 2
 0.9778  0.0222
------------------------------------------
Acceptance rate MCMC sampler: 27%
nmcmc: 12500
nburn: 5000
nthin: 5
------------------------------------------
DIC: -1204.8968
------------------------------------------
```

Figure 9: Fitted Parameters using MCMC estimation. SD:Standard Deviation; SE:Standard Error; $TSSE$:Time Series Standard Error; RNE:Relative Numerical Efficiency - $(SE/TSSE)^2$.

CONVERGENCE OF THE CHAIN (DF)

	Stationarity test	start iteration	p-value
alpha0_1	passed	1	0.6246
alpha1_1	passed	1	0.3402
beta_1	passed	751	0.0995
nu_1	passed	1	0.5198
xi_1	passed	1	0.0983
alpha0_2	passed	1	0.7784
alpha1_2	passed	1	0.0974
nu_2	passed	1	0.5198
xi_2	passed	1	0.0983
P_1_1	passed	1	0.6803
P_2_1	passed	1	0.2705

CONVERGENCE OF THE CHAIN (GO)

	Stationarity test	start iteration	p-value
alpha0_1	passed	1	0.6220
alpha1_1	passed	1	0.2773
alpha2_1	passed	1	0.0609
beta_1	passed	1	0.6596
nu_1	passed	1	0.6885
xi_1	passed	1	0.9428
alpha0_2	passed	1	0.9645
alpha1_2	passed	1	0.6182
xi_2	passed	1	0.3769
P_1_1	passed	1	0.5953
P_2_1	passed	1	0.3054

CONVERGENCE OF THE CHAIN (RJ)

	Stationarity test	start iteration	p-value
alpha0_1	passed	1	0.1273
alpha1_1	passed	1	0.7490
alpha2_1	passed	1	0.5252
beta_1	passed	1001	0.0951
nu_1	passed	1	0.9154
xi_1	passed	251	0.2268
alpha0_2	passed	1	0.7957
alpha1_2	passed	1	0.8165
alpha2_2	passed	1001	0.2748
beta_2	passed	1	0.7904
nu_2	passed	1	0.9154
xi_2	passed	1	0.3689
P_1_1	passed	1	0.0842
P_2_1	passed	1	0.0688

CONVERGENCE OF THE CHAIN (SP)

	Stationarity test	start iteration	p-value
alpha0_1	passed	1	0.351
alpha1_1	passed	1	0.225
nu_1	passed	1	0.653
xi_1	passed	1	0.800
alpha0_2	passed	1	0.245
alpha1_2	passed	1	0.327
alpha2_2	passed	1	0.339
beta_2	passed	1	0.176
nu_2	passed	1	0.653
xi_2	passed	1	0.681
P_1_1	passed	1	0.640
P_2_1	passed	1	0.866

Figure 10: Convergence of the Markov Chain.