Human capital accumulation, income distribution and economic growth: A demand-led analytical framework

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Abstract: This paper incorporates human capital accumulation through provision of universal public education by a balanced-budget government to a demand-led analytical framework of distribution and output growth. Human capital accumulation positively impacts on workers' productivity in output production and their bargaining power in wage negotiations. In the long-run equilibrium, a rise in workers' (capitalists') bargaining power raises (lowers) the pre- and after-tax wage share, which raises (lowers) the rates of physical capital utilization, employment (which also measures the rate of human capital utilization) and output growth. Yet a rise in the tax rate (which also denotes the share of tax spending in public education in output) lowers the long-run equilibrium values of the pre- and after-tax wage share and rates of physical capital utilization, employment and output growth. Counterintuitively, in the long-run equilibrium, a higher share of public investment in human capital in output lowers the rate of human capital accumulation and thereby the rate of output growth. It follows that a higher level of investment in human capital impacts positively (negatively) on long-run output growth when measured relatively to the current stock of human capital (flow of output). Therefore, a strengthening in the bargaining power of workers is output growth-enhancing in the long-run equilibrium, given that it raises the rates of accumulation of both physical and human capital.

Keywords: Human capital; income distribution; economic growth; employment.

Resumo: Este artigo incorpora acumulação de capital humano através da provisão de educação pública universal por um governo com orçamento equilibrado a um arcabouço analíticoo de distribuição e crescimento liderado pela demanda. A acumulação de capital humano impacta positivamente a produtividade dos trabalhadores e seu poder de barganha nas negociações salariais. No equilíbrio de longo prazo, um aumento no poder de barganha dos trabalhadores (capitalistas) aumenta (diminui) a participação salarial pré e pós impostos, o que eleva (reduz) as taxas de utilização de capital físico, emprego (que também representa a taxa de utilização do capital humano) e crescimento do produto. Entretanto, um aumento de uma alíquota de imposto uniforme (que também denota a participação dos gastos com impostos na educação pública sobre o produto dessa economia) reduz os valores de equilíbrio de longo prazo da parcela dos salários na renda pré e pós impostos e as taxas de utilização de capital físico, emprego e crescimento. No equilíbrio de longo prazo, contraintuitivamente, uma maior proporção do investimento em capital humano no produto reduz a taxa de acumulação de capital humano e, portanto, a taxa de crescimento do produto. De fato, um nível mais alto de investimento em capital humano impacta positivamente (negativamente) o crescimento do produto no longo prazo quando medido relativamente ao estoque de capital humano (fluxo de produto). Assim, um fortalecimento do poder de barganha dos trabalhadores eleva o crescimento do produto no equilíbrio de longo prazo, ao aumentar as taxas de acumulação de capital físico e capital humano.

Palavras-chave: Capital humano; distribuição de renda; crescimento econômico; emprego.

J.E.L. Classification Codes: E12, E24, E25, O41.

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1. Introduction

The contribution of human capital accumulation to economic growth in the long-run has been extensively investigated both theoretically and empirically in neoclassical growth theory. In fact, the incorporation of human capital accumulation was seen as an early possible solution to the failure of the Solow model to predict the observed persistence of large differences in income per capita among countries. In one standard approach, developed by Mankiw, Romer and Weil (1992), human capital is included, together with physical capital and labor, as an additional factor in a production function exhibiting constant returns to scale. It turns out that the level of output per worker depends positively on the levels of physical and human capital per worker. Analogously to the accumulation of physical capital, the accumulation of human capital is fully and automatically governed by the availability of savings. However, since the accumulation of both physical and human capital is subject to diminishing returns, a permanent rise in the rate of investment in either of (or even both) these types of capital generates an increase in the level (but not the growth rate) of the output per worker in the long-run equilibrium. In another standard approach, Lucas (1988) assumes that individuals choose periodically how to allocate their non-leisure time between current production and schooling, with the latter raising labor productivity in future periods. As human capital accumulation is assumed to exhibit constant returns, it arises as a source of sustained long-run growth in output per worker.

Yet these mainstream approaches to long-run capital accumulation and economic growth, by invariably assuming that the economy operates at full capacity utilization (at least) in the long run, mistakenly ignore both the positive role of aggregate demand in growth dynamics and the positive impact of investment in human capital formation on aggregate supply *and* demand. Meanwhile, demand-led approaches to capital accumulation and economic growth, including developments in the Neo-Kaleckian literature, have typically relegated any closer attention to human capital formation through education or schooling as narrowly supply-sided. However, this relegation implies ignoring the potential impacts of human capital accumulation on labor productivity, the bargaining power of workers, and ultimately the functional distribution of income and components of aggregate demand. One notable exception in this regard is Dutt (2010), who explicitly formalizes the process of skill acquisition in a Neo-Kaleckian model in a way that both the number of high-skilled and low-skilled workers and their wages vary over time and affect the interaction between income distribution and economic growth. Relatedly, Carvalho, Lima and Serra (2017), motivated to some extent by the empirical significance of student loans to human capital accumulation in the U.S., incorporate debt-financed knowledge capital formation to a demand-led, Neo-Kaleckian model of capacity utilization and economic growth. Any increase rise in labor productivity ensuing from knowledge capital accumulation is fully passed on to the real wage, but insufficient aggregate effective demand, by producing unemployment, results in underutilization of the knowledge capital capacity. As it turns out, the stability properties and extent of financial fragility (in the Minskyan sense) of the long-run equilibrium depend on how the debt servicing of working households is specified.

In fact, the very scant attention that has been paid to human capital accumulation in the Neo-Kaleckian growth literature contrasts with the importance attributed to it by most probably the earliest follower and developer of the Neo-Kaleckian approach. Indeed, as mentioned in Guger and Walterskirchen (2012), in the late 1960s and early 1970s Josef Steindl wrote a book (in German) and several papers (in English) on education, as he argued that, ultimately, growth is limited only by the ability of a society to learn. While conceding that the learning process can not exceed a certain maximum pace, Steindl greatly emphasized education as a major engine of long-run growth. As recalled in Guger et al. (2006), Steindl once stated at a conference that the neglect of human capital is rather grave, as the bottlenecks for higher growth should be seen in qualified manpower, not in capital equipment. In his view, scarcity of a qualified labor force makes it impossible for investment to increase beyond a certain point. Consonant with this view, in the 1960s Steindl pushed for an educational policy in Austria by elaborating an educational planning study underlining the importance of skilled workers for long-run growth (Steindl, 1968).[1] Interestingly, a similar view was expressed about the same time by another follower (as Steindl himself) of Keynesian ideas, namely, Roy Harrod. As aptly described in Boianovsky (2017), in the 1960s and 1970s Harrod shifted the emphasis of his research in economic dynamics from the study of business cycles to that of economic growth. And one of the main results coming out of Harrod's shifted emphasis is the proposition that the maximum rate of growth of qualified workers represents a more significant limitation than the supply of saving in setting the maximum rate of economic growth in developing countries. In Harrod's view, the training and qualification of workers were related to education, the "most important problem in the whole range of development economics" (1962, p.10).

As in Dutt (2010), the model set forth herein also formally explores theoretical underpinnings and implications of human capital accumulation within a demand-led, Neo-Kaleckian dynamic model of growth and distribution, but it focuses on a rather different (and unexplored) set of transmission channels and mechanisms. The model features human capital accumulation as a source of aggregate effective demand alongside with consumption and investment in physical capital. The average human capital (or productive skills) of the available labor force, which in turn impacts on the average labor productivity of (employed and unemployed) workers, varies positively with the spending on universal public education by a balanced-budget government. As that the aggregate human capital stock is uniformly distributed in the labor force, which is always in excess supply, unemployed labor also means unutilized human capital. As it turns out, the economy operates with excess productive capacity not only in physical capital and labor quantities, as typically assumed in Neo-Kaleckian models, but also in human capital and hence labor skills.

Moreover, we consider a conflicting claims framework for the determination of the real wage, with workers' bargaining power depending positively and separately on the employment rate and the rate of human capital formation. The latter effect can be due, *inter alia,* to the growing of self-assurance and class-consciousness on the part of workers. In fact, the ambiguous result of an increase in government spending on universal public education on income distribution and economic growth in the long run generated by the model is (partially) a reflection of its specification of a more inclusive and complex relationship between demand-led growth and the supply side of the economy. Clearly, the resolving of such a theoretical ambiguity carries relevant empirical and policy implications as well.

As the model economy operates with excess capacity in labor quantity and skills, with the real wage being determined by means of a conflicting-claims mechanism, this paper is likewise somehow related to an expanding literature on overeducation (see, e.g., Borghans and Grip, 2000, and Skott, 2006). While overeducation is typically described in this literature as the extent to which an individual worker possesses a level of education in excess of that which is required for her particular job (i.e., occupational mismatch), in this paper one sense in which the labor force is overeducated is that not all of the aggregate human capital uniformly embodied in the labor force is fully utilized due to unemployment. Thus, as in the literature on overeducation as occupational mismatch, in the model herein macroeconomic performance is worse than would be the case if the skills of the educated workers were fully utilized in output production. Given that the literature on overeducation and occupational mismatches has implications for the wage distribution, our specification of a conflicting claims contest for the determination of the average real wage, with workers' bargaining power varying positively with the rates

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[1] According to Guger and Walterskirchen (2012), the introduction of vocational secondary schools in Austria was mainly a consequence of Steindl's 1968 book on educational planning. Steindl's ideas on growth-promoting policies over the 1960s to the 1980s are nicely and more extensively detailed in Guger et al. (2006). In addition to calling for technological innovation and education policies, Steindl stressed the positive demand-side effects of the public sector and the contribution of lower household savings and anticyclical policies.

of employment and human capital accumulation, can be seen as suggesting a possible contributing factor for the dynamics of the labor compensation for human capital accumulation. In fact, since the wage share is given by the ratio of the real wage to labor productivity, in our model such a share can be seen as a measure of the wage compensation received by the human capital. Thus, our conflicting-claims mechanism for the determination of the average real wage implies that a weakening of workers' bargaining power due to a fall in either the rate of employment or the rate of human capital accumulation (or both) may have a negative impact on the wage compensation that workers receive for their human capital. Recall, however, that the rates of capital capacity utilization, employment and growth are all determined by aggregate demand, and changes in aggregate demand will result from changes, *inter alia*, in the wage share and in the investment in human capital through expenditures on universal public education by a balanced-budget government. Meanwhile, given that there is taxation on wages and profits, the after-tax wage share (and consequently the after-tax wage compensation that workers receive for their human capital) is lower than the pre-tax wage share.

Therefore, as the accumulation of human capital is carried out through provision of universal public education by a government running a balanced budget, this paper is also related to (and lightly draws on) the literature that incorporates taxation and public expenditures in a Neo-Kaleckian framework broadly defined, such as Laramie and Mair (1996, 2000, 2003), Mair and Laramie (1997), Dutt (2010, 2013), Commendatore and Pinto (2011), Commendatore, Panico and Pinto (2011), and Tavani and Zamparelli (2016).

The remainder of this paper is structured as follows. Section 2 introduces the model structure. Section 3 solves for the short-run equilibrium configuration, assuming that the productivity of labor, the nominal wage, the price level and the stocks of physical and human capital are all given. Section 4 focuses on long-run dynamics by investigating the impact of a change in the bargaining power of capitalists and workers and in an uniform tax rate (and hence in the share of output dedicated to investment in human capital) on the long-run equilibrium configuration with the wage share and the physical-to-human capital ratio as stationary variables. Section 5 concludes the paper.

2. The structure of the model

The model deals with a closed economy that produces a single good/service for consumption and investment. We assume that the government holds a balanced budget and spends all tax revenues (out of wage and profit income) on the provision of universal public education, which raises the average human capital across the labor force. Two homogeneous factors of production are used in the production of the single good/service, physical capital and labor, and the aggregate stock of human capital is assumed to remain uniformly distributed in the labor force. These production inputs are combined through a fixed-coefficient technology:

$$X = min\left[K\nu, La(h)\right], \tag{1}$$

where X is the output level, K is the stock of physical capital, L is the employment level, $h \equiv H / N$ is the human capital stock to labor force ratio (or average human capital) and $a(h)$ is the output to labor ratio (or labor productivity), which varies endogenously with the average human capital. For simplicity and specificity of focus, the technical coefficient ν is normalized to one, and we measure the rate of physical capital capacity utilization, u, by the output to capital ratio, X / K. In the production function in (1), we also assume that $a(0) = 0$, $a'(h) > 0$ and $a''(h) \le 0$. Note that unemployed workers are as skilled (or human capital endowed) as employed ones, so that the rate of labor employment, which is determined by aggregate effective demand, also measures the degree of human capital utilization. Though we consider only the situation in which aggregate effective demand is insufficient to yield full utilization of the existing human capital capacity at the ongoing real wage rate, we abstract from human capital depreciation and labor deskilling.

The economy is composed of two social classes, firm-owner capitalists and workers, who earn profits and wages, respectively. The functional division of aggregate *pre-tax* income is then given by:

$$X = \frac{W}{P} L + R, \tag{2}$$

where W is the pre-tax money wage, P is the price level and R is the volume of pre-tax aggregate profits. From (1) and (2), the share of labor in pre-tax income, σ, is given by:

$$\sigma = \frac{V}{a(h)}, \tag{3}$$

where $V = W / P$ stands for the pre-tax real wage.

Firms produce (and hire labor) according to aggregate effective demand. As we model only the situation in which excess productive capacity (in labor and overall capital) prevails, labor employment is determined by production:

$$L = \frac{X}{a(h)}. \tag{4}$$

Firms operate in oligopolistic markets and set the price level as a markup factor over unit labor costs, as in Kalecki (1971):

$$P = z \frac{W}{a(h)}, \tag{5}$$

where $z = 1/\sigma(h) > 1$ is the markup factor (one plus the markup), which is inversely related to the wage share. The price level is given at a point in time. However, it varies over time at a (proportionate) rate which is equal to firms' desired (proportionate) rate of change in the price level, $\hat{P}_f = (dP_f / dt)(1/P_f)$, within an alternative (accelerationist) framework of conflicting claims on income. More precisely, we postulate an instantaneous adjustment of the rate of growth of the price level, $\hat{P} = (dP / dt)(1/P)$, to the firms' desired rate of growth of the price level given by:

$$\hat{P}_f = \theta u, \tag{6}$$

where $\theta > 0$ is a parameter measuring firm-owner capitalists' bargaining power in the distributive conflict. Therefore, the price inflation rate desired by firms is reasonably assumed to depend positively on the state of the product market. A higher rate of physical capital utilization, by reflecting more buoyant effective demand conditions, allows firms to desire a higher price inflation rate. Although there is non-evadable taxation on wages and profits, as described shortly, which implies that taxation has a prior claim on income, firm-owner capitalists' desired inflation rate in (6) does not feature the tax rate on profits as a separate, directly causal factor. As will be seen later, this specification will allow us to draw clear-cut implications regarding the impact of changes in the bargaining power of either workers or capitalists on the pre- and after-tax wage share, and thereby on the rates of physical capital utilization, employment and economic growth in the long-run equilibrium.

The specification in (6) can be interpreted as assuming that the desired growth rate of the markup factor is procyclical. Even though the literature on markup determination typically features the level of the markup as depending on the level of economic activity as measured by different indicators, some of the provided rationales may plausibly apply to the growth rate specification assumed here. For instance, Eichner (1976) argues that expansions are times in which firms may want to invest more by generating higher internal savings and consequently desire a higher markup. Rowthorn (1977) claims that higher capacity utilization allows firms to raise prices with less fear of being undercut by (either existing or potential) competitors, who would gain little by undercutting due to higher capacity constraints. Meanwhile, Gordon, Weisskopf and Bowles (1984) suggestively argue that marked-up prices are inversely related to the perceived price elasticity of demand, which in turn is negatively related to the industry concentration and the fraction of potential competitors that are perceived to be quantity-constrained and hence not engaged in or responsive to price competition. In the downturn, as it turns out, the markup will fall because the attendant decline in capacity utilization results in a smaller share of potential competitors being perceived to be under capacity constraints, and therefore in an increase in the perceived elasticity of demand facing the firm.

At a point in time the pre- and after-tax money wage are both given, and with labor being always in excess supply, employment is determined by labor demand and therefore ultimately by aggregate effective demand. Over time, however, the pre-tax money wage varies at a rate which is equal to workers' desired (proportionate) rate of change in the pre-tax money wage, $\hat{W}_w = (dW_w / dt)(1/W_w)$, within such an alternative framework of conflicting income claims. More precisely, we assume an instantaneous adjustment of the rate of growth of the pre-tax nominal wage, $\hat{W} = (dW / dt)(1/W)$, to the workers' desired rate of growth of the nominal wage given by:

$$\hat{W}_w = \lambda(e + \hat{h}), \tag{7}$$

where $\lambda > 0$ is a parameter measuring workers' bargaining power in the distributive conflict, $e = L / N$ is the rate of employment and $\hat{h} = (dh / dt)(1/h)$ is the rate of growth of the average human capital. Therefore, both capitalists and workers are always able to have their desired inflation rate fully and instantaneously translated into the respective actual inflation rate (price inflation rate in the case of capitalists, wage inflation rate in the case of workers) within such an accelerationist conflicting-claims inflation dynamics. The wage inflation rate desired – and successfully bargained for – by workers is postulated to vary positively with their bargaining power in the wage negotiations as represented by the parameter λ in (7). A higher rate of either human capital accumulation or employment, given workers' bargaining power, allows workers to desire and obtain a higher wage inflation rate. Meanwhile, the employment rate is linked to the state of the product market in the following way:

$$e = uk, \tag{8}$$

where k stands for the ratio of physical capital stock to labor force in productivity units, that is, $k = K / (Na(h))$. This formal link between u and e is necessary since the fixed-coefficient nature of the technology implies that an increase in output in the short run will necessarily be accompanied by an increase in employment. Moreover, as the aggregate human capital stock is uniformly distributed in the labor force, the employment rate as well measures the degree of utilization of the aggregate human capital. Note in addition that with the rates of employment and physical capital utilization so moving together in the short run, the positive effect of a rise in the former on wage inflation in (7) can also be seen as reflecting workers' realization that a rise in price inflation is under way.

Firms make decisions to accumulate physical capital independently from available savings as described by a standard Neo-Kaleckian-Steindlian desired investment function, so that firms' desired growth rate of the stock of physical capital, assuming no depreciation, is given by:

$$g^i = \alpha + \beta u + \gamma r, \tag{9}$$

where α, β and γ are all positive parameters, $u = X / K$ is the rate of (physical capital) capacity utilization and r is the after-tax rate of profit on physical capital, which is the after-tax flow of money profits divided by the value of the physical capital stock at output price. The after-tax profit rate r is then given by:

$$r = (1-\tau_p)(1-\sigma(h))u \,, \tag{10}$$

where $0 < \tau_P < 1$ is the tax rate on profit income. Substituting expression (10) for r in (9) yields:

$$g^i = \alpha + [\beta + \gamma(1-\tau_p)(1-\sigma(h))]u \,. \tag{9'}$$

Given that we are dealing with a single good/service economy, the 'production' of human capital (or labor skills) does not constitute another production process or productive sector. Indeed, we assume here that the single good/service that can be used for both physical capital accumulation and consumption can also be used for human capital accumulation. In the long-run equilibrium, therefore, the growth rate of output can be measured by the growth rate of either kind of capital, given that both physical and human capital grow at the same rate in the long-run equilibrium.[2]

At a point in time, the technological parameters are given, having resulted from previous human and physical capital accumulation. Over time, however, human capital accumulation takes place, which results in labor productivity growing at a proportionate rate $\hat{a}$. Formally:

$$\hat{a} = \phi(\hat{h}) \,, \tag{11}$$

where $\hat{h}$ is the growth rate of the human capital to labor force ratio. For simplicity, we assume that the level of labor productivity has a one-to-one correspondence with the average human capital, so that $a(h) = h$ and hence $\hat{a} = \hat{h}$. Besides, given (3), the wage share becomes a measure of the wage compensation received by workers for their human capital: a constant wage share means that the result of the distributive conflict between workers and capitalists is such that any increase in labor productivity arising from human capital accumulation ends up being fully passed on to the real wage.

Following Kalecki (1971), Kaldor (1956), Robinson (1956, 1962) and Pasinetti (1962), we assume that workers and capitalists have different consumption behavior. Moreover, the costs of human capital accumulation are entirely funded by the government through the collection of taxes levied on wage and profit income, with the government's propensity to save being equal to zero. Workers provide labor and earn wage income, which is taxed at an exogenous rate $0 < \tau_w < 1$, and consume a constant fraction of their disposable income. The propensity to save out of after-tax wages is given by $0 \leq s_w < 1$. Although we assume that the available labor force, N, is a constant normalized to one, workers are always in excess supply. Firm-owner capitalists receive profit income, pay an exogenous fraction $0 < \tau_p < 1$ of it as

[2] A more inclusive Neo-Kaleckian model of distribution and growth with human capital accumulation could consider the presence of heterogeneous labor – e.g. by incorporating the distinction between low-skilled and high-skilled workers, as in Dutt (2010). In fact, these two groups of workers could be taxed at different rates. In light of the connection between workers' human capital and skill levels and their wage bargaining power established in this paper, heterogeneity in skill levels could be a source of heterogeneity in workers' wage bargaining power and even intra-working-class conflict over wage income distribution. In this regard, we are grateful to Fábio Freitas of the Federal University of Rio de Janeiro, Brazil, for drawing to our attention that in the classical (political economy) approach to occupational wage rates developed in Gleicher and Stevans (1991) more skilled workers have stronger wage bargaining power than less skilled workers. However interesting a more inclusive specification along these lines, we leave it for future research.

taxes, and have a higher saving propensity out of disposable income than workers. The propensity to save out of disposable profit income is represented by $0 \le s_w < s_p \le 1$, which means that we further assume that the sum of consumption expenditures and tax payments does not exceed income for both classes, as there is no borrowing.

Aggregate investment in human capital accumulation by the government running a balanced budget (normalized by the stock of physical capital) is consequently fully induced by wage and profit income given the differential tax rates:

$$\frac{I_H}{K} = \left[\tau_w \sigma + \tau_p (1-\sigma)\right] u \,. \qquad (12)$$

It is important to underline that any spending in public education provision by the government will increase the human capital endowed by the entire available labor force, and not only that of employed workers.[3]

3. Short-run equilibrium

The short-run is defined as the time period in which the stock of physical capital, K, the stock of human capital, H, the output-labor ratio, a, the price level, P, and the pre-tax money wage, W, can all be taken as given. The supply-demand equilibrium in the product market can be expressed by:

$$X = C + I_H + I_K \,, \qquad (13)$$

where C stands for aggregate consumption, and I_H and I_K for investment in human and physical capital, respectively. Therefore, human capital accumulation is a source of aggregate demand alongside with investment on physical capital and consumption. The rationale is that the provision of universal public education requires government expenditures on the single good/service which can be used for consumption and investment purposes.

Or, since aggregate taxes $T = I_H$ for a balanced budget of the government, we have:

$$S = I_K \,, \qquad (13')$$

where $S = X - T - C$ stands for aggregate savings.

Normalizing (13') by the physical capital stock yields:

$$g^s = g^i \,, \qquad (13'')$$

where aggregate savings as a proportion of the physical capital stock is given by:

$$g^s = \left[s_p (1-\tau_p)(1-\sigma) + s_w (1-\tau_w)\sigma\right] u \,. \qquad (14)$$

Substituting g^i from (9') and g^s from (14) into (13'') yields:

$$\left[s_p (1-\tau_p)(1-\sigma) + s_w (1-\tau_w)\sigma\right] u = \alpha + \left[\beta + \gamma (1-\tau_p)(1-\sigma)\right] u \,. \qquad (15)$$

[3] A more inclusive Neo-Kaleckian model of distribution and growth with human capital accumulation could also incorporate investment in entrepreneurial human capital. For instance, as in Ehrlich, Li and Liu (2017), such an investment could result in improvements in the capitalist-entrepreneurs' industrial and commercial knowledge which, in this paper, could conceivably positively affect physical capital accumulation. In fact, these authors find empirical evidence that investment in entrepreneurial human capital may contribute positively to long-run economic growth.

Since aggregate output is given by aggregate effective demand, and labor is always in excess supply, the rate of (physical capital) capacity utilization fully adjusts for the product market short-run equilibrium in (15) to obtain. The short-run equilibrium value of capacity utilization is thus given by:

$$u^* = \frac{\alpha}{(s_p - \gamma)(1-\tau_p)(1-\sigma) + s_w(1-\tau_w)\sigma - \beta}, \tag{16}$$

where $\partial u^* / \partial a = u^* / \alpha$ is the multiplier of autonomous investment demand. For stability of the short-run equilibrium value of capacity utilization, we assume that $(\partial g^s / \partial u) > (\partial g^i / \partial u)$, which in turn is equivalent to a positive denominator in (16):

$$(s_p - \gamma)(1-\tau_p)(1-\sigma) + s_w(1-\tau_w)\sigma - \beta > 0 .$$

The substance of this Keynesian stability condition is that after-tax saving (out of wages and profits combined) must react more than after-tax investment in physical capital to changes in capacity utilization, so that excess demand or supply is eliminated rather than exacerbated by changes in capacity utilization. As in the standard Neo-Kaleckian model, the paradox of thrift holds: a rise in the propensity to save of workers or capitalists reduces the level of economic activity as measured by the rate of capacity utilization in (16). As this is a one-good/service economy and the government spends all its tax revenues on the provision of universal public education, which in turn requires government expenditures on the single good/service, a rise in any tax rate, by raising the multiplier of autonomous demand, increases the rate of capacity utilization in the short run. In fact, as the running of a balanced budget by the government is equivalent to the latter's propensity to spend out of tax revenues being equal to one, a rise in any tax rate lowers effective demand leakages. In this balanced-budget context, then, taxation can be seen as a mechanism of what we would dub "forced dissaving" or "forced spending".

The impact of an increase in the wage share on aggregate effective demand (and hence on the rate of capital capacity utilization) can be examined through the following partial derivative:

$$\frac{\partial u^*}{\partial \sigma} = \frac{\alpha\left[(s_p - \gamma)(1-\tau_p) - s_w(1-\tau_w)\right]}{\left[(s_p - \gamma)(1-\tau_p)(1-\sigma) + s_w(1-\tau_w)\sigma - \beta\right]^2} . \tag{17}$$

Given that $\alpha > 0$, the parametric condition for the model economy to operate in a (pre-tax) wage-led effective demand (and hence capacity utilization) regime, $\partial u^* / \partial \sigma > 0$, is then given by:

$$s_p - s_w\left[\frac{1-\tau_w}{1-\tau_p}\right] > \gamma .$$

Therefore, everything else constant, we need a relatively large difference between marginal propensities to save between capitalists and workers and/or a relatively small sensitivity of investment in physical capital to the after-tax profit rate for the economy to be in a (pre-tax) wage-led effective demand regime. Moreover, everything else constant, as the government spends all its tax revenues in the provision of universal public education, if the tax rate is higher (lower) on profits than on wages, the term in brackets will be greater (lower) than one, which will raise the likelihood of a pre-tax profit-led (wage-led) effective demand regime.

Consequently, from (8), since the ratio of physical capital stock to labor force in productivity units, k, is given in the short run, a pre-tax wage-led (profit-led) effective demand regime implies that the rate of employment (and hence the rate of human capital utilization) is (pre-tax) wage- (profit-)led as well.

From (10) and (16), the short-run equilibrium value of the after-tax rate of profit on physical capital is then given by:

$$r^* = \frac{\alpha(1-\tau_p)(1-\sigma)}{(s_p-\gamma)(1-\tau_p)(1-\sigma)+s_w(1-\tau_w)\sigma-\beta}. \tag{18}$$

The effect of increases in the wage share on the short-run equilibrium value of the after-tax rate of profit on physical capital is given by:

$$\frac{\partial r^*}{\partial \sigma} = \frac{\alpha(1-\tau_p)\left[\beta - s_w(1-\tau_w)\right]}{\left[(s_p-\gamma)(1-\tau_p)(1-\sigma)+s_w(1-\tau_w)\sigma-\beta\right]^2}. \tag{19}$$

It can be checked that a paradox of costs, namely a positive effect of increases in the wage share on the rate of profit on physical capital, may or may not hold, depending on the sign of the numerator in (19). A lower propensity to save by workers and a higher tax rate on wages, with the latter increasing spending in education, increases the likelihood of occurrence of the paradox of costs.

4. Long-run equilibrium

In the long run we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stock of physical capital, K, the stock of human capital, H, the output to labor ratio, or labor productivity, a, the price level, P, and the pre-tax money wage rate, W. Therefore, one way of following the behavior of the system over time is by examining the dynamics of the short-run state variables σ, the wage share (and consequently its after-tax counterpart, which is given by $(1-\tau)\sigma$), and k, the ratio of physical capital to labor supply in productivity units. From the definition of these variables, and recalling that the labor force is constant, we have the following state transition functions:

$$\hat{\sigma} = \hat{W} - \hat{P} - \hat{a} = \hat{V} - \hat{h}, \tag{20}$$

and

$$\hat{k} = \hat{K} - \hat{a} = \hat{K} - \hat{h}. \tag{21}$$

In the long-run equilibrium characterized by $\hat{\sigma} = \hat{k} = 0$, therefore, the pre- and after-tax real wage will grow at a constant rate which is equal to the common growth rate of the stocks of physical and human capital, which in turn is equal to the rate of growth of this one-good economy, g^*.

4.1 Long-run dynamics with analytical solutions

In order to facilitate our analytical study of the model dynamics and steady-state properties, in this sub-section we will make two further simplifying assumptions. First, we will assume that the exogenous tax rate on wages and profits is exactly the same, so that $\tau_w = \tau_p = \tau$, thus avoiding direct effects of changes in income distribution on human capital formation. Second, we will assume that workers consume all their after-tax wage income, so that $s_w = 0$, while profit earners save all of their after-tax income, so that $s_p = 1$.

Adopting these simplifying assumptions yields the following new expressions for the short-run equilibrium level of capacity utilization in physical capital and its response to changes in the wage share, from (16) and (17):

$$u^* = \frac{\alpha}{(1-\gamma)(1-\tau)(1-\sigma)-\beta}, \qquad (16')$$

and

$$\frac{\partial u^*}{\partial \sigma} = \frac{\alpha(1-\gamma)(1-\tau)}{\left[(1-\gamma)(1-\tau)(1-\sigma)-\beta\right]^2} > 0 . \qquad (17')$$

Since $0<\tau<1$, and a necessary condition for a positive and stable equilibrium value for u^* is $\gamma<1$, it follows that the numerator in (17') is positive and physical capital utilization varies positively with the pre-tax wage share. Note, however, that an increase in the tax rate raises physical capital utilization in the short-run equilibrium, as the corresponding tax collection is entirely spent on financing human capital accumulation. As the pre-tax wage share is given in the short run, an increase in the tax rate, which then implies a fall in the after-tax wage share, nonetheless brings about a rise in capital capacity utilization. Intuitively, since investment in human capital adds to aggregate effective demand and the tax collection which is fully spent on such an investment demand (public saving is zero) has a prior claim on both profit and wage income, an increase in the tax rate has a positive net effect on effective demand. Thus, it follows from (8) that, since k is given in the short run, the employment rate (which is also the rate of human capital utilization) varies positively with the wage share, the tax rate and the parameters of the physical capital accumulation function in (9). It turns out, then, that the rates of utilization and growth of the aggregate human capital stock both vary positively with the wage share and the tax rate in the short-run equilibrium. Per (10), meanwhile, the pre-tax rate of profit on physical capital also varies positively with the tax rate, and the accompanying impact on the after-tax value of such a rate is subject to two opposite forces in operation: a positive one coming through physical capital utilization and a negative one operating through the direct impact of the tax rate on the after-tax profit rate on physical capital. However, we have:

$$\frac{\partial r^*}{\partial \tau} = \frac{\alpha\left(1-\sigma\right)\beta}{\left[\left(1-\tau\right)\left(1-\gamma\right)\left(1-\sigma\right)-\beta\right]^2} > 0 .$$

Interestingly, therefore, the model features the occurrence of another kind of 'paradox of costs', now applying to the higher costs (to firm-owners capitalists) associated with a higher tax rate. The reason is that the after-tax rate of profit on physical capital varies positively with the tax rate in the short-run when the wage share and the physical-to-human capital are given. Later we will check whether the same positive relationship holds in the long run as well.

Meanwhile, given (8) and (11) and the assumption that all the investment in human capital accumulation, I_H, is carried out and funded by a government running a balanced budget, so that $I_H = T = \tau X$, it follows that the aggregate (and average) stock of human capital grows over time according to:

$$\hat{h} = \tau e = \tau u k, \qquad (22)$$

where u is given by (16'). Recall that the human capital stock is uniformly distributed in the labor force, so that the employment rate also measures the rate of utilization of such a stock. As a result, the specification in (22) can be interpreted as incorporating an accelerator effect, but in this case applied to the investment in human capital instead of physical capital. Recall also that we are assuming that the stock of human capital does not depreciate due to either the passage of time or new human capital turning existing one obsolete. However, the expression in (22) could be interpreted as referring to the growth rate of the *net* stock of human capital in the presence of de-skilling due to unemployment. In this alternative interpretation, a higher rate of employment generates a higher rate of growth of the *net* stock of human capital by causing less de-skilling (or, 'unlearning by not doing') of the existing labor force. Yet another interpretation is that human capital accumulation involves on-the-job learning externalities.

Extending the simplifying assumptions above to (14), aggregate saving as a proportion of the capital stock is given by:

$$g^{s} = (1-\tau)(1-\sigma)u. \tag{14'}$$

Substitution of (14') and (22) into (21) yields:

$$\hat{k} = \left[(1-\tau)(1-\sigma) - \tau k\right]u, \tag{23}$$

where u is given by (16'), so that (14') also represents the nonetheless demand-led growth rate of the physical capital stock.

Meanwhile, substituting from (8) and (22) into (7), and from the ensuing expression, (6) and (22) into (20), we obtain:

$$\hat{\sigma} = \left[\lambda(1+\tau)k - \theta - \tau k\right]u, \tag{24}$$

where u is again given by (16').

Equations (23) and (24), after using (16'), constitute a planar autonomous two-dimensional system of differential equations in which the rates of change of σ and k depend on the levels of σ and k and on the parameters of the system.

Solving (23) for the long-run equilibrium with $\hat{k} = 0$ gives a locus of points relating the wage share and the ratio of physical capital to human capital:

$$k = \frac{(1-\tau)(1-\sigma)}{\tau}. \tag{25}$$

The slope of this isocline can thus be computed as:

$$\frac{dk}{d\sigma} = -\frac{(1-\tau)}{\tau} < 0,$$

so that along the $\hat{k} = 0$ locus higher ratios of physical-to-human capital are associated with lower levels of wage share. Moreover, the higher the tax rate, the more (less) a higher wage share (physical-to-human capital ratio) is associated with a lower physical-to-human capital ratio (wage share). Meanwhile, solving for $\hat{\sigma} = 0$ using (24) yields the following locus of points also relating the wage share and the ratio of physical capital to human capital:

$$k = \frac{\theta}{\lambda(1+\tau) - \tau}. \tag{26}$$

Hence, an economically meaningful positive sign for the physical-to-human capital ratio in (26) requires that its denominator is positive, which in turn requires that $\lambda > \tau/(1+\tau) \equiv \bar{\lambda}$. Given that $\bar{\lambda} \equiv \tau/(1+\tau) < 1$ and λ is a parameter measuring workers' bargaining power in the distributive conflict, a positive sign for the physical-to-human capital ratio in the long-run equilibrium requires that workers' bargaining power is not too low or is sufficiently high (and the higher, the higher is the tax rate, given that $(d\bar{\lambda}/d\tau) > 0$). Note that it follows from (26) that $(dk/d\sigma) = 0$. Therefore, the $\hat{\sigma} = 0$ isocline is vertical in the (k, σ)-space and the value of the physical-to-human capital ratio in the long-run equilibrium, k^{*}, is represented by the expression in (26). We can use (25) and (26) to solve for the corresponding unique value of the pre-tax wage share in the long-run equilibrium:

$$\sigma^{*} = 1 - \Omega , \tag{27}$$

where $\Omega \equiv \theta\tau / (1-\tau)[\lambda(1+\tau)-\tau]$. Therefore, an economically meaningful value for the pre-tax wage share represented by $0 < \sigma^{*} < 1$ in (27) (and for its after-tax counterpart, $(1-\tau)\sigma^{*}$) requires that $0 < \Omega < 1$. Note that $\Omega > 0$ is automatically ensured by our assumption made above that $\lambda > \bar{\lambda}$, which is required for a positive sign for the physical-to-human capital ratio in (26). Meanwhile, the condition that $\Omega < 1$ is equivalent to:

$$\lambda > \bar{\bar{\lambda}} \equiv \bar{\lambda}\left[1 + \frac{\theta}{1-\tau}\right]. \tag{28}$$

Therefore, the minimum magnitude of the parameter measuring workers' bargaining power required to ensure a positive pre-tax wage share in the long-run equilibrium is higher than the minimum magnitude of the same parameter required to ensure a positive ratio of physical to human capital ratio in the long-run equilibrium (and such a minimum magnitude is the higher, the higher is either the parameter denoting capitalists' bargaining power, as $(\partial\tilde{\lambda} / \partial\theta) > 0$, or the tax rate, as $(\partial\bar{\bar{\lambda}} / \partial\tau) > 0$). As a result, an economically meaningful value for the unique long-run represented by (k^{*}, σ^{*}) requires that $\lambda > \bar{\bar{\lambda}} > \bar{\lambda}$, whose substance is that workers' bargaining power should not be too low or should be sufficiently high. Recall that a positive sign for the physical-to-human capital ratio in (26) requires that $\lambda > \tau / (1+\tau) \equiv \bar{\lambda}$, where $\bar{\lambda} < 1$. In principle, therefore, it seems possible that the condition above for an economically meaningful value for the unique long-run equilibrium is satisfied with $1 > \lambda > \bar{\bar{\lambda}} > \bar{\lambda}$. Yet it seems also possible that such a condition is only satisfied with $\lambda > \bar{\bar{\lambda}} \geq 1 > \bar{\lambda}$, which is clearly a more stringent condition with respect to the required extent of workers' bargaining power. In fact, it follows from (28) that $\bar{\bar{\lambda}} \geq 1$ ($\bar{\bar{\lambda}} < 1$) if $\theta \geq \bar{\theta}$ ($\theta < \bar{\theta}$), where $\bar{\theta} \equiv (1-\tau)/\tau$. Hence, the higher is capitalists' bargaining power, the higher workers' bargaining power has to be to ensure an economically meaningful value for the unique long-run equilibrium. Note, however, that $\bar{\theta}$ varies negatively with the tax rate.

As shown in the Appendix, the unique long-run equilibrium represented by (k^{*}, σ^{*}) is locally stable. Let us then explore the impact on such a long-run equilibrium of a change in each one of the parameters on which it depends. This will allow us to investigate how these parametric changes ultimately affect the after-tax wage share (which also measures the after-tax wage compensation that workers receive for their human capital) and the rates of physical capital utilization, employment (which also measures human capital utilization) and output growth in the long-run equilibrium. First, note that a higher bargaining power of capitalists as measured by θ raises k^{*} (per (26)) and lowers σ^{*} (and $(1-\tau)\sigma^{*}$) (per (27)), so that physical capital utilization and output growth fall. The impact of such a rise in capitalists' bargaining power on the rate of employment in the long-run equilibrium solution, which is therefore given by $\partial e^{*} / \partial\theta = (\partial u^{*} / \partial\sigma^{*})(\partial\sigma^{*} / \partial\theta)k^{*} + (\partial k^{*} / \partial\theta)u^{*}$, is the result of two effects acting in opposite directions. A rise in capitalists' bargaining power exerts an upward (a downward) pressure on the long-run equilibrium employment rate by raising (lowering) the physical-to-human capital ratio (pre- and after-tax wage share). In fact, it follows from (27) that:

$$\frac{\partial\sigma^{*}}{\partial\theta} = -\frac{\partial\Omega}{\partial\theta} = -\frac{\tau}{(1-\tau)\left[\lambda(1+\tau)-\tau\right]} < 0 , \tag{29}$$

so that, using (17’), we obtain that:

$$\frac{\partial u^*}{\partial\theta}=-\frac{\alpha(1-\gamma)(1-\tau)}{\left[(1-\gamma)(1-\tau)(1-\sigma^*)-\beta\right]^2}\frac{\tau}{(1-\tau)\left[\lambda(1+\tau)-\tau\right]}<0. \tag{30}$$

Recall that in the long-run equilibrium characterized by $\hat{\sigma}=\hat{k}=0$, the pre- and after-tax real wage grows at a constant rate which is equal to the common growth rate of the stocks of physical and human capital, which is equal to the growth rate of the economy, g^*. But given that the growth rate of the economy varies positively with the pre-tax wage share in the long-run equilibrium, a fall in the pre-tax wage share lowers the common growth rates of the stocks of physical and human capital in the long-run equilibrium. Meanwhile, although it follows from (25) that $(\partial k^*/\partial\sigma)=-(1-\tau)/\tau<0$, the employment rate also varies positively with the pre-tax wage share in the long-run equilibrium:

$$\frac{\partial e^*}{\partial\sigma}=\frac{\partial u^*}{\partial\sigma}k^*+\frac{\partial k^*}{\partial\sigma}u^*=\frac{\alpha(1-\tau)\beta}{\tau\left[(1-\gamma)(1-\tau)(1-\sigma)-\beta\right]^2}>0, \tag{31}$$

which confirms, upon substitution of (16’) and (25) into (22), that the growth rate of the stock of human capital, and hence the growth rate of output, varies positively with the pre-tax wage share in the long-run equilibrium:

$$\frac{\partial g^*}{\partial\sigma}=\frac{\partial(\hat{h})^*}{\partial\sigma}=\tau\frac{\partial e^*}{\partial\sigma}>0. \tag{32}$$

Therefore, given that the growth rate of the stock of human capital in the long-run equilibrium is given by $\hat{h}^*=te^*=\tau u^*k^*$, in the long-run equilibrium the employment rate varies positively with the pre-tax wage share and therefore negatively with the bargaining power of capitalists as measured by θ in (6):

$$\frac{\partial e^*}{\partial\theta}=\frac{\partial u^*}{\partial\theta}k^*+\frac{\partial k^*}{\partial\theta}u^*=-\frac{\alpha\beta}{\left[(1-\gamma)(1-\tau)(1-\sigma)-\beta\right]^2\left[\lambda(1+\tau)-\tau\right]}<0. \tag{33}$$

Second, a higher bargaining power of workers as measured by λ lowers k^* (per (26)) and raises σ^* (and $(1-\tau)\sigma^*$) (per (27)), so that both physical capital utilization and output growth increase in the long-run equilibrium. Thus, the effect on the long-run equilibrium employment rate, given by $\partial e^*/\partial\lambda=(\partial u^*/\partial\sigma^*)(\partial\sigma^*/\partial\lambda)k^*+(\partial k^*/\partial\lambda)u^*$, is the result of two effects that operate in opposite directions. An increase in workers’ bargaining power exerts an upward (a downward) pressure on the long-run equilibrium employment rate by lifting (lowering) the pre- and after-tax wage share (physical-to-human capital ratio). In fact, (27) implies that:

$$\frac{\partial\sigma^*}{\partial\lambda}=-\frac{\partial\Omega}{\partial\lambda}=\frac{\tau\theta(1+\tau)}{(1-\tau)\left[\lambda(1+\tau)-\tau\right]^2}>0, \tag{34}$$

so that, using (17’), we obtain that:

$$\frac{\partial u^*}{\partial\lambda}=\frac{\alpha(1-\gamma)(1-\tau)}{\left[(1-\gamma)(1-\tau)(1-\sigma^*)-\beta\right]^2}\frac{\tau\theta(1+\tau)}{(1-\tau)\left[\lambda(1+\tau)-\tau\right]^2}>0. \tag{35}$$

Given that the growth rate of output similarly varies positively with the pre-tax wage share in the long-run equilibrium, it follows that a higher bargaining power of workers will also result in a higher output growth in the long-run equilibrium. Meanwhile, as shown above, although it follows from (25) that the physical-to-human capital ratio varies negatively with the pre-tax wage share in the long-run equilibrium, the rate of employment nonetheless varies positively with the pre-tax wage share in the long-run equilibrium. Hence, in the long-run equilibrium the employment rate (which also measures the human capital utilization rate) varies positively with workers' bargaining power as measured by λ in (7):

$$\frac{\partial e^*}{\partial \lambda} = \frac{\partial u^*}{\partial \lambda} k^* + \frac{\partial k^*}{\partial \lambda} u^* = \frac{\alpha\theta(1+\tau)\beta}{\left[(1-\gamma)(1-\tau)(1-\sigma)-\beta\right]^2 \left[\lambda(1+\tau)-\tau\right]^2} > 0. \tag{36}$$

Finally, let us explore the impact of an increase in the uniform tax rate on physical capital utilization, human capital utilization (or employment rate) and output growth. In the balanced-budget context of this paper, note that the tax rate can be alternatively interpreted as the share of tax spending in public education in output or the share of investment in human capital in output. Using (26), the impact of a higher tax rate on the horizontal intercept of the vertical $\hat{\sigma}=0$ isocline, and therefore on the long-run equilibrium physical-to-human capital ratio, is given by:

$$\frac{\partial k^*}{\partial \tau} = \frac{\theta(1-\lambda)}{[\lambda(1+\tau)-\tau]^2}. \tag{37}$$

Thus, the $\hat{\sigma}=0$ isocline will shift to the right (left) if $\lambda<1$ ($\lambda>1$), and will not be displaced in any direction in the case of $\lambda=1$. Meanwhile, we can make use of (25) to evaluate the impact of a higher tax rate on the negatively sloped $\hat{k}=0$ isocline, which is given by:

$$\frac{\partial k}{\partial \tau} = -\frac{1-\sigma}{\tau^2}. \tag{38}$$

Thus, the negatively sloped $\hat{k}=0$ isocline shifts to the left and becomes steeper in response to an increase in the tax rate. It follows that if the ability of workers to translate a rise in either the employment rate or the growth rate of the average human capital stock into a higher rate of growth of the nominal wage is not sufficiently high, so that $\lambda \le 1$, both the pre- and after-tax wage share unambiguously fall as a result of an increase in the tax rate in the long-run equilibrium. However, even if the $\hat{\sigma}=0$ isocline shifts to the left in response to an increase in the tax rate because workers' bargaining power is relatively higher ($\lambda>1$), it is still the case that both the pre- and after-tax wage share fall as a result of such an increase in the tax rate in the long-run equilibrium. In fact, we can use (27) to obtain that:

$$\frac{\partial \sigma^*}{\partial \tau} = -\frac{\partial \Omega}{\partial \tau} = -\frac{\theta\left\{(1-\tau)\lambda + \tau\left[\lambda(1+\tau)-\tau\right]\right\}}{(1-\tau)^2\left[\lambda(1+\tau)-\tau\right]^2} < 0. \tag{39}$$

Moreover, the absolute value of the negative response of the pre-tax wage share to an increase in the tax rate in the long-run equilibrium varies positively with workers' bargaining power:

$$\frac{\partial\left|(\partial\sigma^*/\partial\tau)\right|}{\partial\lambda} = \frac{\theta\left\{(1-\tau)\left[\tau+(1+\tau)\lambda\right]+(1+\tau)\tau\left[\lambda(1+\tau)-\tau\right]\right\}}{(1-\tau)^2\left[\lambda(1+\tau)-\tau\right]^3} > 0. \tag{40}$$

Meanwhile, given that it follows from (27) that $(1-\tau)\sigma^* = (1-\tau)(1-\Omega)$, the after-tax wage share in the long-run equilibrium also varies negatively with the tax rate:

$$\frac{\partial\left[\left(1-\tau\right)\sigma^*\right]}{\partial\tau}=-\sigma^*-\left\{\frac{\theta\left(1-\tau\right)\lambda+\theta\tau\left[\lambda\left(1+\tau\right)-\tau\right]}{\left(1-\tau\right)\left[\lambda\left(1+\tau\right)-\tau\right]^2}\right\}<0\,. \tag{41}$$

In the long-run equilibrium, therefore, unlike the result in the short-run equilibrium when the pre-tax wage share is given, physical capital utilization varies negatively with the tax rate:

$$\frac{\partial u^*}{\partial\tau}=-\frac{\alpha\theta\lambda\left(1-\gamma\right)}{\left[\left(1-\gamma\right)\left(1-\tau\right)\left(1-\sigma^*\right)-\beta\right]^2\left[\lambda\left(1+\tau\right)-\tau\right]^2}<0\,. \tag{42}$$

Thus, the employment rate also varies negatively with the tax rate in the long-run equilibrium:

$$\frac{\partial e^*}{\partial\tau}=\frac{\partial u^*}{\partial\tau}k^*-\frac{\left(1-\sigma\right)}{\tau^2}u^*<0\,, \tag{43}$$

so that it follows from (14’), after using (16’) and (27), or alternatively from (22), after using (16’), (25) and (27), that the output growth rate also varies negatively with the tax rate in the long-run equilibrium:

$$\frac{\partial g^*}{\partial\tau}=-\frac{\alpha\theta\lambda\beta}{\left\{\left(1-\gamma\right)\theta\tau-\beta[\lambda(1+\tau)-\tau]\right\}^2}<0\,. \tag{44}$$

Meanwhile, the absolute value of the negative response of the output growth rate to a rise in the tax rate in the long-run equilibrium varies negatively with workers’ bargaining power:

$$\frac{\partial\left|(\partial g^*/\partial\tau)\right|}{\partial\lambda}=-\frac{\alpha\theta\beta\left\{\left(1-\gamma\right)\theta\tau+\beta\left[\lambda(1+\tau)+\tau\right]\right\}}{\left\{\left(1-\gamma\right)\theta\tau-\beta[\lambda(1+\tau)-\tau]\right\}^3}<0\,. \tag{45}$$

In the long-run equilibrium, therefore, a rise in the uniform tax rate generates a decrease in the rates of physical capital utilization, employment and output growth regardless of the strength of workers’ bargaining power to translate a rise in either the rate of employment or the rate of human capital accumulation into a higher rate of growth of the nominal wage. However, per (45), the absolute value of the negative response of the output growth rate to a rise in the tax rate in the long-run equilibrium varies negatively with the bargaining power of workers.

It should be emphasized that the negative impact of a higher tax rate (which is equivalent to a higher share of output dedicated to investment in human capital formation by the balanced-budget government) expressed in (44) does not mean that human capital accumulation is not output growth-enhancing in the long-run equilibrium. In fact, in the long-run equilibrium with a constant physical-to-human capital ratio the stocks of physical and human capital grow at a common rate which is therefore the growth rate of output of this one-good/service economy. Yet, since investment in human capital exerts a positive impact on both aggregate supply and aggregate demand, it turns out that a higher share of output dedicated to investment spending in human capital formation reduces the rates of utilization and accumulation of human capital in the long-run equilibrium. Alternatively, a strengthening in the bargaining power of workers is output growth-enhancing in the long-run equilibrium by virtue of raising the pre- and after-tax wage share and thereby the rates of utilization and accumulation of physical and human capital.

5. Conclusion

This paper incorporates human capital accumulation through provision of universal public education by a balanced-budget government to a Neo-Kaleckian analytical framework of physical and human capital utilization, income distribution and output growth. The overall level of education, as represented by the stock of human capital, affects positively workers' productivity in output production and thereby arguably their bargaining power in the labor market. The wage share in income is therefore also a measure of the wage compensation that workers receive for their human capital accumulated through education.

In the short-run equilibrium, a time span in which the stocks of physical and human capital as well as pre- and after-tax income distribution are all given, standard Neo-Kaleckian results arise, and the difference in tax rates on wage and profit income has distributive implications for consumption and investment behavior and determine the demand regime of the economy. In particular, if the tax rate on profit income is higher than that on wage income, a (pre-tax) wage-led economy may become (pre-tax) profit-led. Assuming a uniform tax rate, the paper also investigates the dynamics of a two-dimensional system featuring the wage share and the physical-to-human capital ratio. In the long-run equilibrium, a rise in workers' (capitalists') bargaining power raises (lowers) the pre- and after-tax wage share, which raises (reduces) the rates of physical capital utilization, employment (which measures the rate of human capital utilization as well) and output growth. Meanwhile, an increase in the uniform tax rate (and hence in the share of output dedicated to tax spending in public education) reduces the long-run equilibrium values of the pre- and after-tax wage share, which in turn lowers the rates of physical capital utilization, employment and output growth. Yet the absolute magnitude of the negative response of the output growth rate to a rise in the tax rate in the long-run equilibrium varies negatively workers' bargaining power. Counterintuitively, in the long-run equilibrium, a higher share of public investment in human capital in output lowers the rate of human capital accumulation and thereby the rate of output growth. It follows that a higher level of investment in human capital impacts positively (negatively) on long-run output growth when measured relatively to the current stock of human capital (flow of output). Therefore, a strengthening in the bargaining power of workers is output growth-enhancing in the long-run equilibrium, given that it raises the rates of accumulation of both physical and human capital.

Given that the wage share also measures the wage compensation that workers receive for their human capital, a rise in such wage compensation is then conducive to higher rates of human capital utilization and accumulation as well as output growth in the long-run equilibrium. The wage compensation received by workers for their human capital, however, is established in a contentious context of accelerating-inflation conflicting claims on available income by workers and capitalists instead of being fully and automatically granted to workers in a friendly wage negotiation.

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APPENDIX

The Jacobian matrix of partial derivatives for the two-dimensional dynamic system composed by (23) and (24), when evaluated at the unique long-run equilibrium (k^*, σ^*) by using (16''), is given by:

$$J_{11} = \frac{\partial \hat{k}}{\partial k} = -\tau u^* < 0 \tag{A-1}$$

$$J_{12} = \frac{\partial \hat{k}}{\partial \sigma} = -(1-\tau)u^* + \left[(1-\tau)(1-\sigma^*) - \tau k^*\right]\frac{\partial u^*}{\partial \sigma^*} \tag{A-2}$$

$$J_{21} = \frac{\partial \hat{\sigma}}{\partial k} = \left[\lambda(1+\tau) - \tau\right]u^* \tag{A-3}$$

$$J_{22} = \frac{\partial \hat{\sigma}}{\partial \sigma} = \left[\lambda(1+\tau)k^* - \theta - \tau k^*\right]\frac{\partial u^*}{\partial \sigma^*} \tag{A-4}$$

All of these partial derivatives can be unambiguously signed. The sign of J_{11} is negative: the growth rate of the ratio of physical capital stock to labor supply in productivity units varies negatively with its level. The reason is that although a change in this level leaves the growth rate of physical capital unchanged, it impacts positively on the employment rate and thereby on the growth rate of the human capital stock. Meanwhile, a change in the wage share seems to have an ambiguous impact on the growth rate of the ratio of physical capital to labor supply in productivity units, as it varies the growth rates of the two stocks of capital in the same direction. However, given (25), the term in brackets in (A-2) is equal to zero, so that J_{12} is negative. In fact, it follows from (25) that the slope of the $\hat{k} = 0$ locus is negative, and given that such a slope can be expressed as $-J_{11}/J_{12}$, it turns out that J_{12} is negative. Recall that the term in brackets in (A-3) has to be positive to ensure an economically meaningful positive sign for the physical-to-human capital ratio in the long-run equilibrium, which implies that the sign of J_{21} is positive. Finally, given that the term in brackets in (A-4) is equal to zero, given (26), it follows that $J_{22} = 0$. Therefore, given that the trace of the Jacobian matrix in (A-1)-(A-4), which is represented by $Tr(J) = J_{11} + J_{22}$, is negative, whereas the respective determinant, which is given by $Det(J) = J_{11}J_{22} - J_{12}J_{21}$, is positive, it follows that the unique long-run equilibrium (k^*, σ^*) represented by (26)-(27) is locally stable.

One noteworthy feature of this result regarding the existence, uniqueness and stability of a long-run equilibrium is that it *does not* depend on how the rates of physical capital utilization, employment and output growth vary with the pre- or after-tax wage share. In fact, the long-run equilibrium values of the pre- and after-tax wage share and the physical-to-human capital ratio given by (27) and (26), respectively, depend uniquely on the parameters which measure workers' and capitalists' bargaining power and the tax rate. However, the ultimate impact of a change in any of these parameters on the rates of physical capital utilization, human capital utilization (or rate of employment) and output growth in the unique long-run equilibrium *does* depend on how these demand-led macroeconomic variables respond to a change in the pre- or after-tax distribution of income.