Área 6 – Crescimento, Desenvolvimento Econômico e Instituições.

Generalizing Kaldor neo-Pasinetti model with Political Orientation and Open Economy.

João Gabriel de Araujo Oliveira[1]

Renato Nozaki Sugahara[2]

Joanilio Rodolpho Teixeira[3]

Abstract

We reconsider Kaldor (1966) model of growth and income distribution where he introduces some ideas of financial market in the “Cambridge Equation”, extending it to a general model with government activities and an Open Economy. We show how the implications of Political Orientation affect the income distribution. This paper corrects Charles (2007) about the negatively effects the income distribution when the government expands the consumption in favour to households. We also prove that the political choice, both in the case with and without an Open Economy, does not influence the essential nature of the Kaldor neo-Pasinetti dynamic equilibrium and the “Cambridge Equation”. Connections with the neo-Kaleckians or post-Kaleckians theories (wage-led and export-led growth) are considered. Next, applying the Olech’s Theorem we guarantee that the global equilibrium is stable in both cases.

Keywords: Distribution, Growth, Government, Open Economy, Stability.

JEL: D30, O40, P16

Resumo

Reconsiderando o modelo de crescimento e distribuição de renda de Kaldor (1966), onde ele introduz algumas ideias de mercado financeiro na “Equação de Cambridge”, nós estendemos aqui para um modelo geral com atividades governamentais e Economia Aberta. Mostramos quais os efeitos a distribuição de renda quando consideradas as Políticas Orientadas. Este artigo corrige Charles (2007), sobre os efeitos negativos encontrados na distribuição de renda quando o governo expande o consumo em favor das famílias. Nós também provamos que as escolhas políticas, para ambos os casos com e sem Economia Aberta, não afetam a natureza da dinâmica de equilíbrio Kaldor neo-Pasinetti e da “Equação de Cambridge”. São consideradas conexões com as teorias neo-Kaleckianas e pós-Kaleckianas (*wage-led* e *export-led*). Por fim, aplicamos o Teorema de Olech, garantindo que ambos os casos são globalmente estáveis no equilíbrio.

Palavras-chaves: Distribuição, Crescimento, Governo, Economia Aberta, Estabilidade.

[1] Department of Economics at State University of Londrina. Contact: +55 43 996260249 or joaogabrielaraujooliveira@gmail.com , to which all comments must be addressed.

[2] Department of Economics at State University of Londrina.

[3] Department of Economics at University of Brasilia. A research grant from the Brazilian Research Council (CNPq) is acknowledged. The authors thank to Jorge Thompson Araújo, Ricardo Araujo and Angelo Rondina for helpful comments.

1. Introduction

The theory of long-run growth macroeconomic analyses starts when Harrod (1939) and Domar (1947) present the "razor wire" problem, which implicates that the growth of the economy could be sustainable if the natural growth ratio is equal to the effective growth ratio. By analysing this problem, Solow (1956) and Kaldor (1956) search for an alternative solution. The latter creates a theory of growth based on the side of income distribution. This effort shows us that the growth rate will be given by the multiplication between the propensity to save and the profit rate of the economy. Such result was named as "Cambridge Equation" and all extensions from this theorem have to return to the original result if the modifications are not considered.

Studying the kaldorian approach, Pasinetti (1962) divide the economy in two classes, workers and capitalists, saying that Kaldor (1956) committed a "logical sleep" when he did not consider the division of classes. Pasinetti proves that the "Cambridge Equation" is not given by de propensity to save of the economy, but only from the propensity to save of the capitalists. This indicates how much the economy will grow.

Using this classes division, but named the capitalists as firms, Kaldor (1966) create the "Kaldor neo-Pasinetti Theorem". His model present two sides. It shows how the existence of the financial system implicates on the income distribution. The first side indicates the traditional profit rate and the second the valuation ratio of the firms in the financial market. The significant point of this theorem is that the profit rate affects negatively the valuation ratio, and the existence of financial assets leads to a reduction of the profit rate.

Concerned with to how the valuation ratio and the profit rate behave in Kaldor neo-Pasinetti model, Araújo (1995) reproduce graphically the theory and design the model as an IS-LM alternative. Another extension is presented by Panico (1997), which introduce the concept of capital gains to analyse how the existence of firms can impact the level of income and the implication on the income distribution with government activities. It is interesting to note that all the Political Orientations of the above extensions were assumed to be exogenous. Trying to endogenize these assumptions, Charles (2007) introduces the concept of investment increased by government expenditures involving two favouring choices. The first is to increase the consumption (households) and the second to expand the profit of the economy (firms).

The relevance of the Kaldor-Pasinetti theory persists in our days, Romero (2019) combine the kaldorian perspective with Schumpeter to present a new cumulative growth model. Another example is the empirical analyses to the case of the "Cambridge Equation". Pacheco-Lópes and Thirlwall (2014) verify the association between manufacturing output growth, export growth as well as between export growth and GDP growth in 89 developing countries. Bernardo, Stockhammer and Martínez (2016) shows the reinterpretation of the Tobin's q from the Kaldor neo-Pasinetti perspective. George (2018) made a positive analysis of the modern capitalism from the Pasinetti perspective to consider the implication of investors decisions in a long-run perspective.

The present article, focussing the kaldorian perspective, analyses the interaction between government, financial market and an Open Economy (especially introducing the Political Orientation to the model). In this case, we are dealing with a more realistic context in comparison with the most of the literature. This work is divided in five sections; the first one synthesizes the history and contextualizes the literature. The second presents the evolution of the "Kaldor neo-Pasinetti model" and their extensions, showing the development of the theory. Also corrects the formal version of the model published by Charles (2007), since it contained a number of mistakes.

In the third section we extend the "Kaldor neo-Pasinetti model" showing the effective expansion of the consumption by the government and the stability condition of this extension. This improved alternative has as consequence interesting results about the proper incentives (increasing consumption) of the Political Orientations to households. This leads to a positive relation with income distribution. In the fourth section we extend our model to the case of an Open Economy showing the impact of this new configuration, using the same approach as in the third section when we use the Olech's Theorem for the stability condition, as was presented by Garcia (1972). Section 5 contains the conclude remarks.

2. Review of the "Kaldor neo-Pasinetti model" and extensions

Kaldor (1966) starts with the assumptions that in this model we have two classes, firms and workers. The first agent is owned by capitalists and workers (households) and the profit is divided between then. [4] The last agent will also be remunerated with wages coming from his\her jobs. The sum of these two remunerations is equal to the national income. The savings functions are similar to the one used by Pasinetti (1962) when he made the distinction between workers and of capitalists. Following Kaldor (1966) and Charles (2007) assumptions:[5]

$$Y = W + P \tag{i}$$
$$S = S_w + S_f \tag{ii}$$
$$S_w = s_w W \tag{iii}$$
$$S_f = s_f P \tag{iv}$$
$$xI = s_w W - cG \tag{v}$$

The difference from the equations above and Kaldor (1956) is that he and Charles (2007) consider that the total saving is composed by the one of workers and that of firms. The new part of the investment function derived from the financial market is given by (v).An interesting property of this equation is to show that the existence of financial market will be direct affected from workers savings.
Following the assumptions by Kaldor (1966), Araújo (1992), Charles (2007) and Lavoie (2014), the equilibrium between investments and saving to maintain the full employment has to be written as in (vi) and (vii):[6]

$$I = s_f P + xI \tag{vi}$$
$$I = s_f P + s_w W - cG, \text{ been } 0 \leq c \leq 1 \tag{vii}$$

The capital gains in (v) and (vii) is provided from financial market equilibrium where the valuation ratio (v_r) is equal to the amount value of equities (pN) divided by the total capital stock (K). That is:

$$v_r = \frac{pN}{K} \tag{viii}$$

Deriving (viii) with respect to time, and applying some algebraic manipulations, we have the capital gains function. This results can be either positive (capital gain), zero or negative (capital loses). Thus:

$$G = (v_r - x)I \tag{ix}$$

Substituting (ix) in (vii), Kaldor (1966), Araujo (1995), Charles (2007) obtained the main results like as presented in Lavoie (2014). First of all, they find the profit rate "Cambridge Equation" (1), showing that the existence of financial market will make a decreasing impact on the profit rate. The other result presents the valuation ratio (2), which indicates the signal of capital gains. The rate of investment by financial market is impacted positive in the valuation ratio:

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[4] It is possible to consider that not all the workers own part of the firms, but those who have earn profits. This concept is maintained in the rest of this paper.
[5]The notations is standard (see Appendix 1).
[6] All the investments functions in this article are based on the Keynes perspective, where this variable is exogenous and the distribution is given by the saving function, as we can see in Bertola (2000).

$$\mathrm{r} = \frac{(1-\mathrm{x})\mathrm{g}_n}{\mathrm{s_f}} \tag{1}$$

$$\mathrm{v_r} = \frac{1}{\mathrm{c}}\left[\frac{\mathrm{s_w}}{\mathrm{g}_n\mathrm{v}} - \frac{\mathrm{s_w}}{\mathrm{s_f}}(1-\mathrm{x}) - \mathrm{x}(1-\mathrm{c})\right] \tag{2}$$

Following assumptions (i) to (ix), Panico (1997) introduced the government expenditures (x) which increase the investment function (vii). He assumed that the government budget is balanced as expressed by (x) and (xi):

$$\mathrm{G_e} = \mathrm{T} \tag{x}$$

$$\mathrm{T} = \mathrm{t_w W} + \mathrm{t_p P}, \text{ been } 0 \leq \mathrm{t_w} \leq 1 \text{ and } 0 \leq \mathrm{t_p} \leq 1 \tag{xi}$$

Assuming the existence of government and the respective direct taxation, we obtain the right landside of the equations (xii) and (xiii) which, in equilibrium, show that investment is equal to total amount of saving, as expressed by (xii):

$$\mathrm{I} + \mathrm{G_e} = \mathrm{s_f}(1-\mathrm{t_p})\mathrm{P} + \mathrm{s_w}(1-\mathrm{t_w})\mathrm{W} - \mathrm{cG} \tag{xii}$$

$$\mathrm{xI} = \mathrm{s_w}(1-\mathrm{t_w})\mathrm{W} - \mathrm{cG} \tag{xiii}$$

$$\mathrm{I} + \mathrm{G_e} = \mathrm{s_f}(1-\mathrm{t_p})\mathrm{P} + \mathrm{xI} \tag{xiv}$$

After some mathematical manipulations, Panico (1997) shows new extensions of Kaldor (1966) approach now with government activities, as presented in (3) and (4). The interesting part of these results is concerned with the existence of the government expenditures affecting positively the profit rate and negatively the valuation ratio as we can see in the equations below (we just have to make the partial derivate of r and $\mathrm{v_r}$ with the respect to $\mathrm{g_e}$):

$$\mathrm{r} = \frac{(1-\mathrm{x})\mathrm{g_n} + \mathrm{g_e}}{\mathrm{s_f}(1-\mathrm{t_p})} \tag{3}$$

$$\mathrm{v_r} = \frac{1}{\mathrm{c}}\left\{\frac{\mathrm{s_w}}{\mathrm{vg_n}}(1-\mathrm{t_w}) - \frac{\mathrm{s_w}}{\mathrm{g_n}}(1-\mathrm{t_w})\left[\frac{(1-\mathrm{x})\mathrm{g_n}+\mathrm{g_e}}{\mathrm{s_f}(1-\mathrm{t_p})}\right] - \mathrm{x}(1-\mathrm{c})\right\} \tag{4}$$

Note that $\mathrm{g_e} = \frac{\mathrm{G_e}}{\mathrm{K}}$. However, the government expenditures decisions are exogenous. Wondering about this, Charles (2007) elaborated a kaldorian extension model with Political Orientation which deserver especial attention. The section V of his article starts by differencing the government orientation in two ways. The first was to increase the consumption in favour to households and the second to increase profit favouring to firms. Being $0 \leq \alpha \leq 1$, we have:

$$\mathrm{G_e} = \alpha(\overline{\mathrm{C}} - \mathrm{C}), \overline{\mathrm{C}} > \mathrm{C}, \text{ been } 0 \leq \alpha \leq 1 \tag{xv}$$

$$\mathrm{G_e} = \alpha(\overline{\mathrm{P}} - \mathrm{P}), \overline{\mathrm{P}} > P \tag{xvi}$$

The equation of the consumption (xvii) incorporates the taxation:

$$\mathrm{C} = \mathrm{c_w}(1-\mathrm{t_w})\mathrm{W} + (1-\mathrm{s_f})(1-\mathrm{t_p})\mathrm{P} + \mathrm{cG} \tag{xvii}$$

Mathematical manipulations in the Appendix 2 indicate that Charles (2007) committed some mathematical mistakes. Our first contribution corrects his results in relation to the consumption incentives. His model disappeared with the difference between the valuation ratio and the share of investments financed by the financial market $(v_r - x)$. The actual results are given by (3) and (4):

$$r = \frac{g_n[1-\alpha c(v_r - x)] + \alpha\bar{c} - \frac{\propto}{v}(1-t_w)}{(1-t_p)[\alpha(1-s_c)-s_c] - \propto(1-t_w)} \tag{5}$$

$$v_r = \frac{x g_n \theta(1-c) + s_w(1-t_w)\left\{\frac{\theta}{v} g_n + \alpha\left[\frac{(1-t_w)}{v} - \bar{c} - g_n c x\right]\right\}}{c g \theta - s_w(1-t_w) g \alpha c} \tag{6}$$

where $\bar{c} = \frac{\bar{C}}{K}$ and $\theta = (1-t_p)[\propto(1-s_c) - s_c] - \alpha(1-t_w)$.
With the same manipulations, but considering (xvi) we have the equations favouring profits (firms), as expressed by (7) and (8). These two results are expressed correctly by Charles (2007):

$$r = \frac{\alpha\bar{r} + g_n(1-x)}{s_c(1-t_p) + \alpha} \tag{7}$$

$$v_r = \frac{1}{c g_n}\left[\frac{s_w}{v}(1-t_w) - g_n(1-c)x - s_w\frac{\alpha\bar{r} + g_n(1-x)}{s_c(1-t_p)+\alpha}\right] \tag{8}$$

Equations (5) and (6) show us that the government expenditure to incentive the households will affect the income distribution negatively. In addition, the equation (7) and (8) indicate that government expenditures tend to increase profit and the profit ratio will be bigger, favouring the firms.

A number of extensions considers some cases and empirical applications for the Kaldor neo-Pasinetti approach, published by Park (2002), Ryoo (2016) and Ryoo (2018). Thinking about the incentives and fiscal policy we realized that some issues were not considered in the extensions above. One of our main contribution is to present how much bigger it will be $\bar{C} > C$. From this assumption we develop a new extension of the "Kaldor neo-Pasinetti Theorem" with proper Political Orientation. We formulate this new approach in section 3 of this paper.

3. Extension of the Kaldor neo-Pasinetti Model With Proper Political Orientations

This part of our article focus on a generalization for the "Kaldor neo-Pasinetti Theorem", concerning governmental economic Political Orientation favouring consumption (benefit to households) and/or profit (benefit to firms), from government preferences determinate by the political power. Our contribution is composed in two characteristics: the first is that we are considering that the government can incentive simultaneously both classes in the model and what happens in our new extension when the policymakers decide to increase consumption. This is defined here as the sum of the consumption of the economy and the incentive derived from the government expending (income transfer).

Assume the assumptions below:

$$G_e = \beta\alpha(\bar{C} - C) + (1-\beta)\alpha(\bar{P} - P) \tag{xviii}$$

$$0 \le \beta \le 1 \tag{xix}$$

$$\bar{P} > P \tag{xx}$$

$$\bar{C} > C \tag{xxi}$$

$$\bar{C} = C + G_i \tag{xxii}$$

For each economy, we have the coefficient β determined, and the decision of how much it will be spend as incentive to consumption and/or profit, defined by the maximization function $G_e =$

$\max\{\alpha G_i, \alpha(\overline{P} - P)\}$. From these values the government will distribute the incentives using (xviii)[7]. Expression (xxii) introduce the increased amount of consumption by the current policy favouring households (G_i). Following Kaldor (1966), Panico (1997) and Charles (2007) we have:

$$S = S_w + S_f = s_f(1 - t_p)P + s_w(1 - t_w)W - cG \quad \text{(xxiii)}$$
$$T = t_w W + t_p P \quad \text{(xxiv)}$$

Substituting (xxii) in (xviii):

$$G_e = \beta\alpha(G_i) + (1 - \beta)\alpha(\overline{P} - P) \quad (9)$$

Which represents the government expenditures considering Political Orientations to both consumption and profit, where the first is designated to households and the second to firms. From some mathematical manipulations, as we can see in the Appendix 3 (a), we develop our extension of the "Cambridge Equation" to the "Kaldor neo-Pasinetti Theorem".

$$r = \frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i}{\left[(1-\beta)\alpha + s_f(1-t_p)\right]} \quad (10)$$

It is easy to see that the "Kaldor neo-Pasinetti Theorem" can be obtained by assuming that $\alpha = 0$. Furthermore, the extension of the "Cambridge Equation", by Steedman (1972), is obtained if $\alpha = x = 0$. We get Pasinetti (1962) if $\alpha = x = t_p = 0$.

Relaxing the assumption of full capacity utilization, we take into account the implications of non-competitive markets and their imperfections[8]. To determine an investment function, as in models *à la* Kalecki, it is possible to construct a new version of our extension with wage-led and profit led growth view. Another accomplishment to consider, instead of to propose the incentives to consume, we determinate income transfers to workers and the analysis of how will behave the fluctuation of the income in short, medium[9] as well as to determinate the equilibrium in long-run term from Kalecki's and Kaldor's visions.

A desirable extension, linking the idea of Political Orientation and Kaleckians views, could be supported by the following sentence in Lavoie and Stockhammer (2012, p. 1): "Income distribution can be modified or influenced by appropriate government policies that act both on primary income distribution, for instance by reinforcing the bargaining power of labour unions or securing low real interest rates and on secondary income distribution, by modifying the tax code". This is something to think in another work.

To deal with the interrelation between neo-kaleckian (post-Kalekian[10]) and kaldorian models, in the case of small closed economy for both theories, Araujo and Teixeira (2015) as well as Araujo and Teixeira (2016) reinterpreted the "Cambridge Equation", which can be viewed as particular case of the Kalecki

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[7] For an example, consider the following parameters: $\beta = \frac{1}{2}$, $\alpha = 1$, $G_i = 4$ and $\overline{P} - P = 2$. From these values we have $G_e = \max\{4, 2\}$ and using (xviii), $G_e = 0{,}5 * 4 + 0{,}5 * 2 = 2 + 1 = 3$, which determinate the amount of 2 for incentives to consumption and 1 for incentives to profit.

[8] The study more precisely the relation between capacity utilization and non-competitive markets, imperfections in the labour market (unemployment), we indicate Hein (2014, p. 241-271), which treat about the Kaleckians basic models.

[9] As was treated by Ros (2016), when he distinguish both short and medium term to determinate the impact of a wage-led growth theory in an small developing economy with two sectors.

[10] The difference between neo and post-Kaleckian are that the first one has as a precursors Dutt (1984) and Rowthorn (1981) works, which is known as wage-led theories and the second one start from Bhaduri and Marglin (1990) and Kurz (1990) works, as we can see in Hein (2017).

School, if we consider that the sensibility of the growth rate of investment been zero. It is necessary determinate the valuation ratio of the model, which is expressed by (9):

$$v_r = \frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{[(1-\beta)\alpha+s_f(1-t_p)]}\right] - x(1-c)g_n\right\} \quad (11)$$

In the same way that r was analysed from (10), we can obtain from (11) the original model if $\alpha = t_p = t_w = 0$. Our expressions (10) and (11) has as a especial case, considering $\beta = 0$, the Charles extension with Political Orientation to firms[11]. These equations show a new general extension of the "Kaldor neo-Pasinetti Model". To see how the r and v_r behave we can apply the partial derivations: $\frac{\partial r}{\partial g_i} > 0$; $\frac{\partial r}{\partial \bar{r}} > 0$ and $\frac{\partial v_r}{\partial g_i} < 0$; $\frac{\partial v_r}{\partial \bar{r}} < 0$. These results show that if the government activities favour the consumption (households) and/or profit (firms), it follows that the income distribution will be affected positively. Consequently the profit ratio will increase. Note that we have been dealing with a model for small closed economy. In order to generalize our approach, we extended it to the case of an Open Economy in section 4.

3.1. Stability Analyse of the Model Without an Open Economy.

In this subsection we apply the Olech's Theorem to analyse the stability of our extension. From the equation (2A)[12] we obtain:

$$\frac{dr}{dt} = E(r, v_r) = \delta\left[g_n - \frac{s}{v}\right] = \delta\Big\{g_n - \frac{s_w}{v}(1-t_w) - [(1-\beta)\alpha + s_f(1-t_p) - s_w(1-t_w)]r + cg_n v_r - cxg_n + (1-\beta)\alpha\bar{r} + \beta\alpha g_i\Big\}, \delta > 0 \quad (12)$$

Considering that the net demand for placements (xxv) is equal to the workers savings less the consumption from capital gains and the supply of new securities issues by the corporation (xxvi), as in Davidson (1968) and Araújo (1995), we have:

$$D_p = s_w(1-t_w)W - cG \quad (xxv)$$

$$S_p = xI\left(\frac{K}{K}\right) = xg_n K \quad (xxvi)$$

Following the conventional IS-LM stability analyses, we postulate the equilibrium adjustment between r and v_r, represented by the excess demand function below:

$$\frac{dv_r}{dt} = E(r, v_r) = \varphi\left[\frac{D_p}{K} - \frac{S_p}{K}\right] = \varphi\left[\frac{s_w}{v}(1-t_w) - s_w(1-t_w)r - cg_n v_r + cxg_n - xg_n\right] \quad (13)$$

being $\varphi > 0$

[11] Based on an erroneous assumption to increase consumption, determinate by Charles it is not possible to obtain from our equations (10) and (11) his results.
[12] "A" refer to an equation in the Appendix 3 (a).

From (12) and (13) we can analyse the stability condition, considering the first term of the Taylor expansion. From this, we are allowed to determinate the matrix system: and in the middle we have the Jacobian Matrix:

$$J[E(r, v_r)] = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right) - s_w(1-t_w)\right] & \delta c g_n \\ -\varphi s_w(1-t_w) & -\varphi c g_n \end{bmatrix} \tag{14}$$

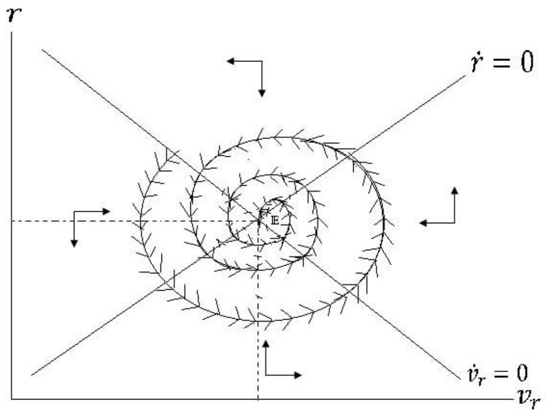
Applying the Olech's Theorem in (14), we have all the tools to analyse the stability in a Matrix 2x2. This is a necessary and sufficient condition, if the trace is negative and the determinant is positive, thus:

$$Tr(J) = -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right) - s_w(1-t_w)\right] - \varphi c g_n < 0 \tag{15}$$

$$|J| = \delta\varphi c g_n\left[(1-\beta)\alpha + s_f\left(1-t_p\right)\right] > 0 \tag{16}$$

These results show us that with all the assumptions assumed above, the model is stable as required. Furthermore, we conclude that our extension satisfied the stability conditions, as indicated in Figure1.

Figure 1: Dynamic Equilibrium without an Open Economy.



Source: Elaborated by the authors.

Equalizing the equation (10) and (11), we can determinate the value of the equilibrium β^* to our extension:

$$\beta^* = \frac{\frac{s_w}{v}(1-t_w)\left[\alpha + s_f\left(1-t_p\right)\right] - x(1-c)g_n\left[\alpha + s_f\left(1-t_p\right)\right] - [g_n(1-x) + \alpha\bar{r}][cg_n + s_w(1-t_w)]}{(\alpha g_i - \alpha\bar{r})[cg_n + s_w(1-t_w)] + \frac{s_w}{v}(1-t_w)\alpha - x(1-c)g_n\alpha} \tag{17}$$

With some numerical exercise, it is possible to conclude that a changes in the parameters α , s_f or t_p , in the equilibrium, only affects the value of β^*. However, if we modify the value of s_w , v , t_w , g , x or c , both values of β^* and the equilibrium between r and v_r will be altered.

4. An extension of the "Kaldor neo-Pasinetti model" with Political Orientation and an Open Economy

This section will concentrate on the implications of an Open Economy in a "Kaldor neo-Pasinetti Model" with economic policy. We consider what happens if our extension takes into account the financial international market and how this can improve the income distribution. Such formalized preoccupation came from Kaldor (1966). We are considering an income function presented by Metcalfe and Steedman (1979) and also used by Teixeira & Araújo (1997) when the latter introduced the idea of foreign bounds. Thus:

$$\mathrm{Y} = \mathrm{W} + \mathrm{P} + \mathrm{F} \qquad \text{(xxvii)}$$

Being, $\mathrm{F} = \mathrm{iZ}$, $\mathrm{i} = \mathrm{r}$, the new assets are holding by firms and will be introduced in the general saving of an Open Economy impacting the aggregate investments. Let (xxviii) be the saving function and (xxix) the aggregate investment. Thus:

$$\mathrm{S} = \mathrm{S_w} + \mathrm{S_f} = \mathrm{s_w}(1 - \mathrm{t_w})\mathrm{W} + \mathrm{s_f}\left(1 - \mathrm{t_p}\right)(\mathrm{P} + \mathrm{F}) - \mathrm{cG} \qquad \text{(xxviii)}$$

$$\mathrm{AI} = \mathrm{I} + \mathrm{G_e} + \mathrm{M} - \mathrm{X} + \mathrm{rZ} \qquad \text{(xxix)}$$

From these new conditions we develop our extension considering an Open Economy, as we can see below:

$$\therefore\ \mathrm{r} = \frac{\mathrm{g_n}(1-\mathrm{x}) + (1-\beta)\alpha\bar{\mathrm{r}} + \beta\alpha\mathrm{g_i} + \mathrm{g_n}\mathrm{z}}{(1-\beta)\alpha + \mathrm{s_f}\left(1-\mathrm{t_p}\right)(1+\mathrm{z}) - \mathrm{s_w}(1-\mathrm{t_w})\mathrm{z}} \qquad (18)$$

The crucial point to notice is that (18) is an extension of the "Cambridge Equation" with Political Orientation and an Open Economy. This extension can be linked to the neo-Kaleckian theory, when the export-led growth model is considered. Such literature was started by Blecker (1989) and Bhaduri and Marglin (1990). The former, was concerned with the relationship between income distribution and international competitiveness, being extended by the later, which introduced the real exchange rate to the model.

Hein (2014, Chapter 7) shows that the exchange rate is the cause of redistribution in this kind of models and he also presents the positive relationship between profit share and international competitiveness. This assumption can be relevant if linked to our model and considers the effects[13]. In order to introduce the financial sector, it is possible to follow the investment function presented by Arestis, González-Martinez and Dejuán (2016), when they consider the relation between capital accumulation and financial market.

Therefore, as was done by Araujo and Lima (2007), it is possible to analyse the implications of the balanced-payments-constrained growth in different economic structures and/or structural changes. Following their proposal one may consider to construct a new Kaleckian extension to verify some

[13] Bhaduri and Marglin (1990) and Hein (2014, p. 290) show that we have to consider a domestic and foreign capacity utilization to deal with the demand of import and export in the export-led growth models.

implications of our assumptions on Political Orientation and Open Economy with Financial Globalization[14]. We leave this to another opportunity.[15]

The Kaldor neo-Pasinetti model also presents the valuation ratio.

$$v_r = \frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i+g_n z}{(1-\beta)\alpha+s_f(1-t_p)(1+z)-s_w(1-t_w)z}\right] - x(1-c)g_n\right\} \quad (19)$$

The equations (18) and (19) show that in the case of an Open Economy and Political Orientation we will have a positive result for the profit ration. On the other hand, like in Kaldor (1966), the implication of the profit ratio will be negative to the valuation ratio. The partial derivative of profit ratio in relation to the foreign equities is negative $\left(\frac{\partial r}{\partial z} < 0\right)$. However to the valuation ratio it will be positive $\left(\frac{\partial v_r}{\partial z} > 0\right)$. One important issue here is that both total income to capitalists and workers are direct influence from the Current Balance of Payments (difference between exports and imports) and that is a sensitive issue, because in the case of a sustainable deficit in the long-run perspective, both classes will be damaged as we can interpreted in Teixeira and Araújo (1997).

One important issue here is that both total income to capitalists and workers are direct influence from the Current Balance of Payments (difference between exports and imports) and that is a sensitive issue, because in the case of a sustainable deficit in the long-run perspective, both classes will be harmed as we can see in Teixeira and Araújo (1997).

4.1. Stability analyse of the model with an Open Economy.

Using the procedure to the stability from extension without Open Economy, we now have to apply it to this case. From (1B)[16] we have:

$$\frac{dr}{dt} = E(r, v_r) = \delta\left[g_n - \frac{s_w}{v}\right] = \delta\left\{g_n + g_n z - \frac{s_w}{v}(1-t_w) - \left[(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)(1+z)\right]r + \beta\alpha g_i + (1-\beta)\bar{r} + cgv_r - cxg_n\right\}, \delta > 0 \quad (20)$$

Take into account the assumptions (xxv) and (xxvi), but considering the national income from (xxvii), we have:

$$\frac{dv_r}{dt} = E(r, v_r) = \varphi\left[\frac{D_p}{K} - \frac{S_p}{K}\right] = \varphi\left\{\frac{s_w}{v}(1-t_w) - r\left[s_w(1-t_w) - s_w(1-t_w)z\right] - cg_n v_r + cxg_n - xg_n\right\}, \varphi > 0 \quad (21)$$

To analyse the stability condition, we have to consider the first term of the Taylor expansion and from this we obtain the matrix system and the matrix in the middle is Jacobian Matrix (J). Thus:

$$J[E(r, v_r)] = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)(1+z)\right] & \delta cg_n \\ -\varphi\left[s_w(1-t_w) - s_w(1-t_w)z\right] & -\varphi cg_n \end{bmatrix} \quad (22)$$

[14] It is essential to know that, one important difference between the Kaldorians perspectives and the neo-Kaleckians approaches on the capacity utilization is that in the first case is assumed full employment and in the second one is not. These imply in a great difference in studies about structural change and economic structures. We recommend to read Palley (2013).

[15] If the reader has the intention to deepen in others kinds of methodologies issues on an Open Economy, we advise to read Romero and McCombie (2017), especially for the case considering Thirlwall's Law.

[16] "B" refers to equation (2B) in the Appendix 3 (b).

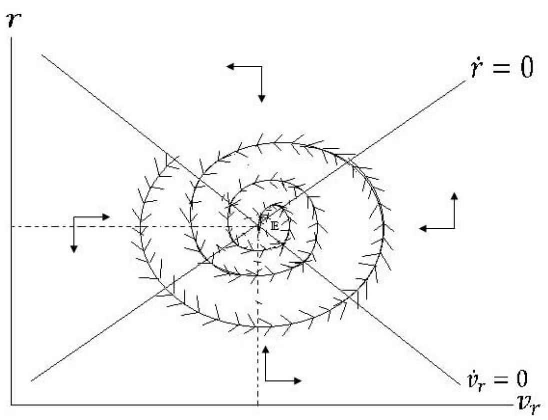
Applying the Olech's Theorem in (22), we have all the tools to analyse the stability in a Matrix 2x2. As already stated above, this is a necessary and sufficient condition if it trace is negative and the determinant is positive, thus:

$$\mathrm{Tr}(J) = -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z) - s_w(1-t_w)(1+z)\right] - \varphi c g_n < 0 \quad (23)$$

$$|J| = \delta\varphi c g_n\left[(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z)\right] > 0 \quad (24)$$

These results show us that, with all the assumptions assumed, the model continue stable as required. From this we conclude that our extension satisfied the stability conditions, as we present in Figure 2.

Figure 2: Dynamic Equilibrium with an Open Economy.



Source: Elaborated by the author.

Equalizing the equation (16) and (17), we determinate the value of the equilibrium β to our extension. Thus:

$$\beta^* = \frac{\left[\frac{s_w}{v}(1-t_w) - x(1-c)g_n\right]\left[\alpha + s_f\left(1-t_p\right)(1+z) - s_w(1-t_w)z\right] - \left[g_n(1-x) + \alpha\bar{r} + g_n z\right]\left[c g_n + s_w(1-t_w)\right]}{(\alpha g_i - \alpha\bar{r})\left[c g_n + s_w(1-t_w)\right] + \frac{s_w}{v}(1-t_w)\alpha - x(1-c)g_n\alpha} \quad (25)$$

This expression determinates β when $r = v_r$, as is presented in the Figure 2. An interesting issue about this stability analysis is that, if we consider a small increase in z, the equilibrium between r and v_r does not modified, but the value of β^* grows. Which means that the government will be forced to increase consumption (incentive to households) more than the profit to maintain the equilibrium condition in long-run.[17]

Note that some care is essential concerning the use of a Political Orientation based on the strategy of export-led growth. It is not difficult to raise the point that such approach necessarily suffers a fallacy of composition. Of course, not all countries can pursue export-led growth simultaneously, unless the domestic economy can expand as a result of a general international expansion of trade. We leave this issue to another opportunity in which we intend of deal with special requirements involving the relevant equations in the

[17] It is possible to prove this affirmation, making a numerical exercise considering in the second extension $z = 0$ and increasing the value of z.

present work, income distribution, credit, savings and investments, somewhat take, for convenience, simplified formal specifications from a behavioural perspective.

5. Concluding remarks

The present paper contains three important issues. The first, presented in the second section is that kipping Charles (2007) hypotheses and correcting his mathematical mistakes, we have the same conclusion that favouring households, the impact continues to be negative in relation to the income distribution.

As a second issue, we presented in the section three a new version of the "Kaldor neo-Pasinetti Theorem" with Political Orientation. One relevant contribution of this part of our article is that the government can decide how much will be designated as incentives to profit (firms) and/or to consumption (households). As $0 \leq \beta \leq 1$, our approach is relevant to the post-Keynesian theory since we consider a new way to deal with government decision: the interval of political choice given by β. Another accomplishment is that we established the difference between increased consumption by government expenditures and the natural consumption of the economy (G_i). Therefore, the impact of profit ratio is positive independently of the governmental choice.

At the subsection 3.1 we present the stability of the model. From this we can conclude that in the long-run the extension will always converge to equilibrium in steady-state. This new extension can be linked to the neo-Kaleckian theory with respect to the wage-led growth model.

As was showed in the section 3 and 4, it is important to think, with the intend to develop some interesting ideas, show how is it possible to link the Kaldor-Passinetti and neo-Kaleckian views, as we can see in Fonseca and Araujo (2018), these models have lots of common principles.

The third issue, in section four, is an extension of the "Kaldor neo-Pasinetti Model" with Open Economy, government activities and Political Orientation. It shows that the globalization affect the income distribution. The existence of international financial system increase the valuation ratio, which is positive to the firms growing the value of their bonds by the introduction of foreign equities, one the other hand, decrease the profit rate, however, increase the value of β. Implying that, in this condition, it is necessary that the government grow the incentive to households to maintain the equilibrium. Especially the latter, where we concluded that the profit ratio (with respect to foreign equities) has positive results. Subsection 4.1 show us that our model with an Open Economy, as in the subsection 3.1 is also stable. We showed that our model can be connected with the export-led growth theory in the post-Kaleckian framework.

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APPENDIX 1: NOTATIONS

α speed adjustment of government policy
β percentage destination of the Political Orientation to increase consumption
AI aggregate investment
c marginal propensity to consume of the capital gain/loses
c_w workers propensity to consume
$\bar{c}$ consumption increased by the Political Orientation in capital terms
C general consumption
$\bar{C}$ general consumption increased by the Political Orientation
E Excess Demand
F financial international market
g_n natural growth rate
G capital gains/loses
G_e government expending with Political Orientation
g_i real value from consumption increased by Political Orientation in capital terms
G_i real value from consumption increase b Political Orientation
i nominal interest rate
I domestic investment
J Jacobian Matrix
$|J|$ Determinant of the Jacobian Matrix
K capital stock
M import
N share of the firm in the financial market
p price level
P profit
$\bar{P}$ profit increased by the Political Orientation
r profit rate
$\bar{r}$ profit increased by the Political Orientation in capital terms
S saving
s_f marginal propensity to save of the firms
S_f firms savings
s_g marginal propensity to saving of the government
S_g government saving
S_p supply of new securities issued by the corporations
s_w marginal propensity to save of the workers
S_w workers saving
t time
T amount tax
$T_r(J)$ Trace of the Jacobian Matrix
t_p marginal tribute to the profit
t_w marginal tribute to the wages
v technology
v_r valuation ratio of the share in financial markets
X export
x share of the investment financiered by the existence of the financial market
W wages amount
Y income
z international amount of the international share in the economy in capital terms
Z international amount of the international share in the economy

APPENDIX 2: THE CORRECT EQUATION OF INCENTIVE TO CONSUMPTION FOR CHARLES (2007)

Considering the following assumptions:

a) $G_e = \alpha(\bar{C} - C)$;

b) $C = c_w(1 - t_w)W + (1 - s_c)\big(1 - t_p\big)P + cG$

Substituting (a) in (b) we have:

c) $C = c_w(1 - t_w)Y - c_w(1 - t_w)P + \alpha(1 - s_c)\big(1 - t_p\big)P + \alpha c(v_r - x)I$

Considering the (xiv) and substituting (c) in (a):

$$I + \alpha\bar{C} - \alpha\big[c_w(1 - t_w)Y - c_w(1 - t_w)P + \alpha(1 - s_c)\big(1 - t_p\big)P + \alpha c(v_{rw} - x)I\big] = s_c\big(1 - t_p\big)P + s_w(1 - t_w)Y - s_w(1 - t_w)P - c(v_{rw} - x)I$$

Dividing the equation per K:

$$g_n + \alpha\bar{c} - \frac{\alpha}{v}(1 - t_w) - \alpha(1 - t_w)r_w + \alpha(1 - s_c)\big(1 - t_p\big)P + \alpha c(v_{rw} - x)g_n = s_c\big(1 - t_p\big)r_w + \frac{s_w}{v}(1 - t_w) - s_w(1 - t_w)r_w - c(v_{rw} - x)g_n$$

Isolating $\frac{s_w}{v}$:

d) $\frac{s_w}{v} = \frac{1}{(1 - t_w)}\big[g_n + \alpha\bar{c} - \frac{\alpha}{v}(1 - t_w) - \propto (1 - t_w)r_w + \alpha(1 - s_c)\big(1 - t_p\big)r + \alpha c(v_{rw} - x)g_n - s_c\big(1 - t_p\big)r_w + s_w(1 - t_w)r + c(v_{rw} - x)g_n\big]$

Now considering (9) equal to (d):

$$\frac{1}{(1 - t_w)}[xg + c(v_{rw} - x)g_n + s_w(1 - t_w)r_w] = \frac{1}{(1 - t_w)}\big[g_n + \alpha\bar{c} - \frac{\alpha}{v}(1 - t_w) - \alpha(1 - t_w)r_w + \alpha(1 - s_c)\big(1 - t_p\big)r_w + \alpha c(v_{rw} - x)g_n - s_c\big(1 - t_p\big)r_w + s_w(1 - t_w)r_w + c(v_{rw} - x)g_n\big]$$

Isolating r and we have the (3)

$$r = \frac{g_n[1 - \alpha c(v_r - x)] + \alpha\bar{c} - \frac{\alpha}{v}(1 - t_w)}{\big(1 - t_p\big)[\alpha(1 - s_c) - s_c] - \propto (1 - t_w)}$$

Substituting in (xiv) and manipulating algebraically we can obtain (4):

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left(xg_n + c(v_r - x)g_n + s_w(1-t_w)\left\{\frac{g_n[1-\alpha c(v_r-x)]+\alpha\bar{c}-\frac{\alpha}{v}(1-t_w)}{(1-t_p)[\alpha(1-s_c)-s_c]-\alpha(1-t_w)}\right\}\right)$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_rg_n + s_w(1-t_w)\left\{\frac{g_n[1-\alpha c(v_r-x)]+\alpha\bar{c}-\frac{\alpha}{v}(1-t_w)}{(1-t_p)[\alpha(1-s_c)-s_c]-\alpha(1-t_w)}\right\}$$

Assuming $\theta = (1-t_p)[\alpha(1-s_c)-s_c] - \alpha(1-t_w)$

$$\frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_rg_n + \frac{s_w(1-t_w)g_n[1-\alpha c(v_r-x)]}{\theta} + \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta}$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_rg_n + \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cv_r}{\theta} + \frac{s_w(1-t_w)g_n\alpha cx}{\theta} + \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta}$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = cv_rg_n - \frac{s_w(1-t_w)g_n\alpha cv_r}{\theta}$$

From now, we have to isolate the v_r :

$$\frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = v_r\left[cg_n - \frac{s_w(1-t_w)g_n\alpha c}{\theta}\right]$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = v_r\left[\frac{cg_n\theta - s_w(1-t_w)g_n\alpha c}{\theta}\right]$$

$$\therefore \frac{\frac{s_w(1-t_w)\theta}{v} - xg_n\theta + cxg_n\theta - s_w(1-t_w)g_n - s_w(1-t_w)g_n\alpha cx - \alpha\bar{c} - \frac{\alpha}{v}(1-t_w)}{cg_n\theta - s_w(1-t_w)g_n\alpha c} = v_r$$

Rearranging the last equation we have the correct valuation ratio:

$$v_r = \frac{xg_n\theta(1-c) + s_w(1-t_w)\left\{\frac{\theta}{v} - g_n + \alpha\left[\frac{(1-t_w)}{v} - \bar{c} - g_ncx\right]\right\}}{cg_n\theta - s_w(1-t_w)g_n\alpha c}$$

APPENDIX 3: MATHEMATICAL MANIPULATIONS OF OUR NEW EXTENSIONS

a) Extension without an Open Economy.

Assuming the assumptions from Kaldor (1966) and Panico (1997) and considering the following new proposes:

$$G_e = \beta\alpha(\bar{C} - C) + (1 - \beta)\alpha(\bar{P} - P) \quad \text{(iA)}$$
$$0 \leq \beta \leq 1 \quad \text{(iiA)}$$
$$\bar{P} > P \quad \text{(iiiA)}$$
$$\bar{C} > C \quad \text{(ivA)}$$
$$\bar{C} = C + G_i \quad \text{(vA)}$$

By saving and amount taxes function, we have:

$$S = S_w + S_f = s_f(1 - t_p)P + s_w(1 - t_w)W - cG \quad \text{(vA)}$$
$$T = t_wW + t_pP \quad \text{(viA)}$$

Substituting (vA) in (iA):

$$G_e = \beta\alpha(G_i) + (1 - \beta)\alpha(\bar{P} - P) \quad \text{(1A)}$$

Substituting (1A) and (ix) in (xii), then dividing such equation by K given that: $g_n = \frac{I}{K}$; $r = \frac{P}{K}$; $\bar{r} = \frac{\bar{P}}{K}$; $v = \frac{K}{Y}$ and $g_i = \frac{G_i}{K}$, it follows

$$g_n + (1 - \beta)\alpha\bar{r} - (1 - \beta)\alpha r + \beta\alpha g_i = s_f(1 - t_p)r + \frac{s_w}{v}(1 - t_w) - s_w(1 - t_w)r - c(v_r - x)g_n \quad \text{(2A)}$$

Isolating $\frac{s_w}{v}$ and rearranging the equation:

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left[g_n + (1 - \beta)\alpha\bar{r} - (1 - \beta)\alpha r + \beta\alpha g_i - s_f(1 - t_p)r + s_w(1 - t_w)r + c(v_r - x)g_n\right] \quad \text{(3A)}$$

Take into account (xiii), dividing the equation by K and isolating $\frac{s_w}{v}$, we obtain:

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}[xg_n + s_w(1 - t_w)r + c(v_r - x)g_n] \quad \text{(4A)}$$

Equating equations (3A) and (4A) and following steps (a) to (c), below, we obtain r expressed by equation (10):

$$\frac{1}{(1-t_w)}\left[g_n+(1-\beta)\alpha\bar{r}-(1-\beta)\alpha r+\beta\alpha g_i-s_f\left(1-t_p\right)r+s_w(1-t_w)r+c(v_r-x)g_n\right]=\frac{1}{(1-t_w)}\left[g_n+s_w(1-t_w)r+c(v_r-x)g_n\right] \quad \text{(a)}$$

$$g_n+(1-\beta)\alpha\bar{r}-(1-\beta)\alpha r+\beta\alpha g_i-s_f\left(1-t_p\right)r=xg_n \quad \text{(b)}$$

$$g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i=r\left[(1-\beta)\alpha+s_f\left(1-t_p\right)\right] \quad \text{(c)}$$

$$r=\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{\left[(1-\beta)\alpha+s_f\left(1-t_p\right)\right]} \quad (10)$$

Substituting (10) in (4A) we obtain (11):

$$\frac{s_w}{v}=\frac{1}{(1-t_w)}\left[xg_n+s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{\left[(1-\beta)\alpha+s_f\left(1-t_p\right)\right]}\right]+c(v_r-x)g_n\right]$$

Isolate the v_r :

$$v_r=\frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w)-s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{\left[(1-\beta)\alpha+s_f\left(1-t_p\right)\right]}\right]-x(1-c)g_n\right\} \quad (11)$$

b) Extension with an Open Economy.

Considering the assumption below:

$$Y=W+P+F \quad \text{(iB)}$$

Being, $F = iZ$, $i = r$, the new assets are holding by firms and will be introduced in the general saving of an Open Economy impacting the aggregate investments. Let (iiB) be the saving function and (iiiB) the aggregate investment. Thus:

$$S=S_w+S_f=s_w(1-t_w)W+s_f\left(1-t_p\right)(P+F)-cG \quad \text{(iiB)}$$
$$AI=I+G_e+M-X+rZ \quad \text{(iiiB)}$$

In equilibrium we have:

$$I+G_e+X-M+rZ=S_w+S_f=s_w(1-t_w)W+s_f\left(1-t_p\right)(P+rZ)-cG \quad \text{(ivB)}$$

After some mathematical manipulation, as Appendix 2 (a). By substituting (1A), (ix) and (iB) in (ivB), we obtain:

$$I + \beta\alpha(G_i) + (1-\beta)\alpha(\overline{P} - P) + X - M + rZ = S_w + S_f = s_w(1-t_w)W + s_f(1-t_p)(P + rZ) - c(v_r - x)I \quad (1B)$$

That is, how the government can be allowed to choose the percentage designated to the Political Orientation. Dividing the equation (1B) by K, considering: $\frac{Z}{K} = z$ and $\frac{X-M}{K} = (g_n - i)z$, isolating $\frac{s_w}{v}$ we have:

$$\frac{s_w}{v} = \frac{1}{1-t_w}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - s_f(1-t_p)r - s_f(1-t_p)rz + s_w(1-t_w)r + s_w(1-t_w)rz + c(v_r - x)g_n\right] \quad (2B)$$

Equalizing (2B) with (4A), after some algebraic manipulations we obtain the profit rate (18):

$$\frac{1}{1-t_w}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - s_f(1-t_p)r - s_f(1-t_p)rz + s_w(1-t_w)r + s_w(1-t_w)rz + c(v_r - x)g_n\right] = \frac{1}{(1-t_w)}\left[xg_n + s_w(1-t_w)r + c(v_r - x)g_n\right]$$

$$\therefore g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - r\left[s_f(1-t_p) + s_f(1-t_p)z - s_w(1-t_w)z\right] = xg_n$$

$$\therefore\ r = \frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i + g_n z}{(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)z} \quad (18)$$

But the Kaldor neo-Pasinetti model also presents the valuation ratio. Substituting (18) in (1A), after some mathematical procedure we obtain (19):

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left\{xg_n + s_w(1-t_w)\left[\frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i + g_n z}{(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)z}\right] + c(v_r - x)g_n\right\}$$

$$\therefore\ cv_r = \frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i + g_n z}{(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)z}\right] - x(1-c)g_n$$

$$\therefore v_r = \frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i + g_n z}{(1-\beta)\alpha + s_f(1-t_p)(1+z) - s_w(1-t_w)z}\right] - x(1-c)g_n\right\} \quad (19)$$