Investment Decision Under Inflation Targeting in Emerging Market Economies

Ricardo B. L. M. Oscar[a], Douglas Silveira[b], Silvinha Vasconcelos[b], Claudio R. F. Vasconcelos[b]

[a]*Department of Economics, Federal University of Rio Grande do Sul, Brazil.*
[b]*Department of Economics, Federal University of Juiz de Fora, Brazil.*

Abstract

This article is aimed at understanding in which conditions emerging market economies (EMEs) can find themselves with a high level of investment, given the announced inflation target rate. Thus, we extend the game proposed by Asako et al. (2017) and introduce a stochastic learning rule through an Agent-based Computational Economics (ACE) model. Entrepreneurs and workers iteratively play a stage game to make investment decisions. Investments are assumed to be complementary and more than one evolutionary equilibria can emerge as a solution. Thereby, the conditions for successfully guiding the EMEs toward the long-run equilibrium in which all players invest at the target inflation rate are: (i) investment must be demand-creating innovation; (ii) Central Bank must have credibility on the announced target inflation rate to increase players' expected profits. The main contributions of our EGT and ACE learning models are twofold: (a) a refinement of dynamic equilibrium to determine the level of investment in the economy for a given inflation targeting rate; (b) greater accuracy on the proportion of agents willing to invest in both physical and human capital, optimizing the implementation of the economic policy.

Keywords: Target inflation rate, Perfect-foresight and best-response dynamics, Demand-creating innovation
JEL classification: E52 · E58 · E22 · C73· C62

Resumo
Este artigo tem como objetivo compreender em que condições as economias de mercado emergentes (EMEs) podem se encontrar com um alto nível de investimento em regimes de metas de inflação. Estendemos o jogo proposto por Asako et al. (2017) e introduzimos uma regra de aprendizagem estocástica por meio de um modelo de Economia Computacional (ACE) baseado em Agente. Empresários e trabalhadores participam iterativamente de um jogo evolucionário para tomar decisões de investimento. Os investimentos são considerados complementares. Desse modo, as condições para que as EMEs convirjam para o equilíbrio de longo prazo no qual todos os participantes investem, dada a meta de inflação anunciada: (i) o investimento deve ser em inovação e expansão de demanda; (ii) O Banco Central deve ter credibilidade na meta de inflação anunciada. Nossas contribuições são duplas: (a) um refinamento do equilíbrio dinâmico para determinar o nível investimento na economia para uma dada taxa de inflação; (b) maior precisão na proporção de agentes dispostos a investir em capital físico e humano, otimizando a implementação das políticas econômicas.

Palavras-chave: Metas de Inflação, dinâmica de melhor resposta, expansão de demanda
Área 4 - Macroeconomia, Economia Monetária e Finanças

1. Introduction

The inflation-targeting (IT) mechanism has played a key role in macroeconomic stabilization in many emerging market economies (EMEs) and, in spite of large inflationary shocks, the inflation rate has been kept at a low level

Email addresses: ricardo.blm.oscar@gmail.com (Ricardo B. L. M. Oscar), douglas.sadsilveira@gmail.com (Douglas Silveira), silvinha.vasconcelos@ufjf.edu.br (Silvinha Vasconcelos), claudio.foffano@ufjf.edu.br (Claudio R. F. Vasconcelos)
URL: http://www.ufjf.br/poseconomia/ (Douglas Silveira)

(Gonçalves & Salles, 2008; Lin & Ye, 2009; de Mendonça & Souza, 2012; Ayres et al., 2014). Aguir et al. (2015) concluded that IT in EMEs has been a more challenging task than in developed economies. The conduct of monetary policy has to build credibility and reduce inflation rate levels, and deal simultaneously with a greater vulnerability to shocks . Besides that, de Mendonça (2018) stated that, to improvement monetary policy transmission channels, signaling and transparency is a relevant issue for the Central Bank in order to anchor inflationary expectations.

In fact, one task of Central Banks of EMEs has been to build credibility as a monetary authority committed to price stability in the context of large inflationary volatility. Mukherjee & Bhattacharya (2011) showed that changes in the short-term interest rate are related to the credibility of the monetary authority. There is a considerable set of variables capable of influencing investment decisions and the level of employment: real costs of capital, credit, exchange rate, wealth and expectations of entrepreneurs. As the fluctuations in real interest rates lead to significant changes in the costs of capital, it might reduce investments.

In other words, faced with an increase in the interest rate and higher prices volatility, there is a deterioration in the entrepreneurs' expectations; therefore, they set back the investment decision. Another concern of some central banks of EMEs derives from the low rate of interest practiced by the so-called developed economies. This may lead to an increase in the flow of risky capital to emerging economies, as shown by Bruno & Shin (2015). The authors stressed that the most effective way for the central bank to face this issue is to ensure a monetary policy transmission mechanism capable of signaling the commitment to a low interest rate. Once this path is clear and well delimited, the monetary authority can influence long term rates such as mortgage rates, corporate lending rates, as well as other prices that affect consumption and investment decisions.

According to Fraga et al. (2003), this requires actions consistent with the IT framework combined with high levels of transparency and communication with the public. An important element for the slowdown in the rate of inflation in Brazil[1], which is still building its credibility and is one of the most important EMEs countries that have adopted inflation targeting (since June 1999), is the feasibility and credibility of the inflation targeting rate (de Mendonça & Souza, 2009). The outcomes suggest that, despite small interest rate changes, when agents face a more credible environment, there is greater reliance on investing. Another interesting findings is showed in Barajas et al. (2014), which proposed a Markov-Switching methodology in order to evaluate the implementation of IT in Latin America[2]. Central banks pursued an inflation objective using a standard Taylor rule and by doing interventions in the foreign exchange market. Based on quarterly data on 16 EMEs under IT regime, Kempf (2018) adopted a panel data approach to show that IT helps creating a low and stable inflation environment, that favors a higher level of investment in the economy. The results were robust for Latin, Central European and some African countries. Ito & Hayashi (2004) details the IT frameworks adopted by Asian[3] countries and compares them to the established frameworks in the United Kingdom and New Zealand, concluding that IT has contributed to economic recovery by anchoring expectations.

In the face of what has been discussed so far, this paper is aimed at examining the necessary conditions for the EMEs to maintain inflation within its limits, and discussing the transmission mechanisms able to generate incentives in order to achieve a growth path with a high level of investment in the economy. More precisely, we intend to investigate the necessary conditions for the EMEs central banks to establish credibility on the inflation target rate.

In order to reach our results, we are aimed at developing a dynamic and stochastic learning rule based on Evolutionary Game Theory (EGT)[4], in accordance with Smith & Price (1973). Moreover, the selection from multiple

[1] On this matter, there is an extensive literature that evaluates the effects of the inflation targeting system in Brazil. Arestis et al. (2011) examine some features of the Brazilian experience with IT regime by asking whether it makes a difference in the fight against inflation if a country has adopted IT or not. de Mendonca (2007) analyses the use of the basic interest rate after the adoption of inflation targeting in Brazil and the credibility of this monetary regime through two indexes that consider the Cukierman & Meltzer (1986) definition for credibility. de Mendonça & Lima (2011) showed that the credibility of IT reduced the uncertainty in the economic environment and has led to a positive and statistically significant effect on private investment.

[2] Céspedes et al. (2014) review the recent experience of a group of Latin American (IT) nations, and have found that the restrictions on international capital movements and other non-conventional policy tools, especially changes in reserve requirements, have come into common use.

[3] Filardo et al. (2010) examined monetary policy institutional changes in Asia toward greater central bank focus on inflation control, institutional independence and transparency and concludes that IT is an important mechanism for delivering price stability. Prasertnukul et al. (2010) examines how IT influenced exchange rate pass-through and volatility in many Asian countries and shows that it caused a decline in exchange rate volatility.

[4] It is important to inform the reader that game theory models have been largely used in macroeconomics since the 1980's. For example, we can cite Vickers (1986), that proposes a game in order to capture the effects of signaling in a model of monetary policy with incomplete information. Morris & Shin (1998) demonstrate the uniqueness of equilibrium when speculators face a small amount of noise in their signals about the fundamentals and how they deal with the quantity of hot money in circulation and the costs of speculative trading. A global game modeling

equilibrium is made without considering the transition dynamics of the economy. Thus, we propose an Agent-based Computational Economics (ACE) approach to complement the EGT framework, as it allows an inference about the current state and the transition dynamics of the EMEs until the long-run equilibrium is reached. The transition mechanism of the EMEs will be analyzed based on both the best-response dynamics (Gilboa & Matsui, 1991), and the perfect-foresight dynamics (Matsui & Matsuyama, 1995). Thus, we can understand the adaptive expectations as a dynamic best-response and the equilibrium under rational expectations as a perfect-foresight dynamics.

As far as we know, this paper is pioneer in combining EGT and ACE models to evaluate the EMEs monetary policy issue. Strictly speaking, this is an approach that allows us to circumvent the problem of multiple long-run equilibria. Thus, it may provide a better understanding of the formation of expectations of the economic agents, since the consumption pattern and investment decisions in emerging economies have been changed. In this way, the Central Bank can measure how the inflation targeting can influence the investment made by entrepreneurs and workers.

Therefore, the main contributions of our EGT and ACE learning models are twofold: (a) a refinement of dynamic equilibrium analysis through both analytical derivation and numerical simulation to determine the trajectory of the level of investment in the economy for a given inflation targeting rate; (b) a more precise dimensioning of the proportion of agents willing to invest in demand expansion rather than cost reduction, optimizing the implementation of the economic policy by the central planner. In this sense, we extend the analysis provided by Asako et al. (2017) to evaluate the EMEs monetary policy. Thus, the game model is composed of two players[5] that are facing a market under monopolistic competition: (1) entrepreneurs and (2) workers. Let each entrepreneur be the owner of a firm and suppose that they hires a worker to produce a good. In sequence, the stage game is dynamically played by entrepreneurs and workers.

The available strategies are to (i) invest[6] and (ii) not to invest. These investments combine demand-creating and cost-reducing innovations (Gilbert, 2006). As a further matter, we consider that there is complementarity between entrepreneurs' and workers' investments and is in line with the complementarity between physical and human capital investments discussed in Blundell et al. (1999) and with the capital-skill complementarity analyzed by Krusell et al. (2000). For simplicity, our study does not take into account the accumulation of physical and human capitals. In accordance with Acemoglu (1997), due to the presence of complementarity, the following outcomes are expected: all players invest or all players do not invest. In these equilibria, it is not guaranteed that the inflation rate converges into its target. Nevertheless, when all players are investing, inflation is expected to converge into its target.

To achieve a long-run equilibrium at the target inflation rate, the following conditions are necessary: (i) entrepreneurs are willing to invest in demand-creating innovation rather than cost-reducing innovation. When entrepreneurs invest in cost-reducing innovation, the economy might observe a decline in inflation, once cost-reducing innovation leads to fierce competition among firms by decreasing prices and increasing their market shares. Therefore, when the investment is in cost-reducing innovation, in the long term, no player views the inflation target as credible; (ii) according to the analysis with the best-response dynamics, the share of entrepreneurs and workers investing is large enough. If only a few players are willing to invest in the current state of the economy, then the investment counterpart will decrease. As a consequence, the profits from their investment are expected to be lower.

In addition, the economy continues in its long-term equilibrium path, given by a scenario that combines lack of investment and absence of inflation. Under these circumstances, the inflation target is not credible and the target inflation rate is not achievable. These two conditions are needed. In this sense, policies to stimulate investments in demand-creating innovation are required to guide the economy successfully towards its long run equilibrium in which all players invest, and the inflation rate is at its target. The remainder of this paper is organized as follows. Section 2 shows the game model baseline. The Evolutionary Game and Agent-based Computational Economics algorithm is in Section 3. Section 4 discusses the outcomes of the stage game and evaluates the dynamics of the strategies using the EGT and ACE approaches. Section 5 shows the final remarks.

approach was proposed by Carlsson & Van Damme (1993), Morris & Shin (1998), and Katagiri et al. (2016). In this type of game, an equilibrium is determined solely on the basis of signals about future fundamentals regardless of the current state of the economy.

[5]In the model, only players have bounded rationality about the decision of the level of investment to be made in the economy. The Central Bank, in turn, always makes its decision in an optimal way.

[6]As workers incur costs to acquire general or firm-specific skills or make more effort to increase demand for goods or reduce production costs, it is reasonable to interpret these situations as workers' investments.

2. The Model

In order to better prompt the discussion in the previous section, we will show the bases of the strategic decision on the investment to be made by entrepreneurs and workers, in view of the inflation targeting rate announced by the Central Bank. Thus, we extend the model proposed by Asako et al. (2017) for EMEs and complement our analysis with the application of an ACE algorithm. In this sense, there is a population of workers interacting strategically with a population of entrepreneurs. It can be represented in a continuum space of mass equal to one. One player from each population is randomly selected to play the stage game. In this sense, the two-period stage game is played iteratively for an infinite number of rounds denoted by $\tau \in [0, \infty)$.

Stage Game	Decision on investment (t_0)	Profits (t_1)
Entrepreneur	physical capital k	$(1-\beta)\pi$
Worker	human capital h	$\beta\pi$
Firm	production and sales x_0	production and sales x_1

Table 1: The Stage Game. At time t_0, Entrepreneur and Worker invest in innovation activities. At time t_1, firms total profit, π, is paired between worker, who receive the payment ($\beta\pi$) and entrepreneur, who earns the portion $(1-\beta)\pi$ of the Firm's total profit π.

Table 1 shows the amount of investment chosen by the worker (entrepreneur) in human (physical) capital at time t_0, denoted by h $(k) \in \mathbb{R}_+$. The firm chooses the level of production in t_0 and t_1, which is given by the pair $(x_0, x_1) \in \mathbb{R}^2_+$. Both entrepreneur and worker make their decisions on the investment at the same time (t_0). Taking into account the current information available, the amount produced by the firm, x_1, in period t_1 may vary. At the instant of time denoted by t_1, the entrepreneur rewards the worker a fraction $\beta \in (0, 1)$ of the total profit earned by the firm after the production of the final good. The total real profit earned by the firm is represented by:

$$\pi = \frac{p_0 - \upsilon_0}{\gamma} x_0 + \frac{1}{1+r} \frac{p_1 - \upsilon_1}{\gamma_1} x_1,$$

where the pair $(p_0, p_1) \in \mathbb{R}^2_{++}$ corresponds to the prices of the goods produced by the firm in $t = 0, 1$. The variable cost of production[7] is given by $(\upsilon_0, \upsilon_1) \in \mathbb{R}^2_+$. The price levels observed in the economy are $(\gamma_0, \gamma_1) \in \mathbb{R}^2_{++}$ and the real interest rate is represented by $r \in \mathbb{R}$. The term γ_1 remains unobservable until the end of t_1. Each player sets $\gamma_1 = \tilde{\gamma}_1$. To calculate the value of $\tilde{\gamma}_1$, players use the price level observed at $t = 0$, γ_0 and the inflation target rate announced by the central bank. This assumption is in line with Sims (2003), being related to the implications of rational inattention issue. The agents are risk neutral. Thus, the payoffs earned by a representative entrepreneur and by a representative worker are given by $(1-\beta)\pi - k$ and $\beta\pi - h$, respectively. The demand functions for the good in $t = 0, 1$ are:

$$x_t = d_t - \epsilon(p_t - \gamma_t).$$

Note that if $p_t = \gamma_t$, then $d_t \in \mathbb{R}_{++}$ corresponds to the demand for the good, and $\epsilon \in \mathbb{R}_{++}$ is the elasticity of demand with respect to the relative price $(p_t - \gamma_t)$. In accordance with what has been exposed so far, equations (1) and (2) represent the inverse demand functions for the pair (p_0, p_1), respectively:

$$p_0 = \gamma_0 + \frac{1}{\epsilon}(d_0 - x_0), \quad (1)$$

$$p_1 = \gamma_1 + \frac{1}{\epsilon}(d_1 - x_1). \quad (2)$$

[7]Other costs than the payments to the workers.

We can linearly decompose these inverse demand functions into the *price-competition effect* (γ_0, γ_1) and the *goods-property effect* $(d_0/\epsilon, d_1/\epsilon)$. By (2), the expected price in $t = 1$, when the stage game is in $t = 0$, is given as follows:

$$E[p_1] = \tilde{\gamma}_1 + \frac{1}{\epsilon}(E[d_1] - x_1). \tag{3}$$

The worker's and entrepreneur's investments are assumed to be both comprised of demand-creating innovation and cost-reducing innovation. By doing so, the entrepreneurs face a continuous interval of innovation types $\omega \in [0, 1]$ to choose from. At the time $t = 0$, the pair (d_0, υ_0), which represents the fundamental demand and the variable cost, respectively, are constants. On the other hand, when $t = 1$, (d_1, υ_1) are random variables whose distributions are conditional to the amount of investments k and h. In addition, by assuming that when the amount of each of the possible forms of investment is greater than the respective threshold value $\underline{k} \in \mathbb{R}_+$ or $\underline{h} \in \mathbb{R}_+$, the investment stochastically leads to a decrease in the variable cost and to an increase in the fundamental demand. In other words, the pair (d_I, υ_I) corresponds to the expected fundamental demand and the variable cost when both players decide to invest, respectively. Otherwise, when both players decide not to invest, we have the pair (d_N, υ_N).

2.1. The Equilibrium Conditions of the Game

In order to find the equilibrium conditions, we must evaluate players' best response. By doing so, we will analyze separately the optimization problem of the entrepreneurs and the workers. The problems of each entrepreneur and each worker are consistent with the maximization problem of indirect utility functions derived from the log utility[8].

2.1.1. The Entrepreneur's Best Response

A representative entrepreneur faces the following maximization problem:

$$\max_{x_0, x_1, k} (1-\beta)\left(\frac{p_0 - \upsilon_0}{\gamma_0} x_0 + \frac{1}{1+r} E\Big[\frac{p_1 - \upsilon_1}{\tilde{\gamma}_1}\Big] x_1\right) - k.$$

As the optimal values attributed to x_0, x_1, and k can be defined independently, it is possible to separate this problem into three parts. First, the value of x_0^* (optimal) is determined so that $x_0^* \equiv \arg\max_{x_0} \frac{p_0 - \upsilon_0}{\gamma_0} x_0$, which gives equation (4):

$$\arg\max_{x_0} \left(1 + \frac{1}{\epsilon\gamma_0}(d_0 - x_0) - \frac{\upsilon_0}{\gamma_0}\right) x_0 = \frac{1}{2}[d_0 + \epsilon(\gamma_0 - \upsilon_0)]. \tag{4}$$

By replacing (1) with (4) we have:

$$p_0^* = \frac{1}{2}\left(\frac{d_0}{\epsilon} + \gamma_0 + \upsilon_0\right).$$

Assuming an equilibrium in which all players from each population adopt the same strategy, $p_0^* = \gamma_0$ must hold. Therefore, $p_0^* = \gamma_0 = (d_0 + \epsilon\upsilon_0)/\epsilon$. Knowing that $\tilde{\gamma}_1$ is exogenously determined and $(d_1, \tilde{\gamma}_1); (\upsilon_1, \tilde{\gamma}_1)$ are independent, the optimal amount (x_1^*) produced in t_1 is given by:

$$\arg\max_{x_1} E\left[\frac{p_1 - \upsilon_1}{\tilde{\gamma}_1}\right] x_1.$$

Which yields equation (5)

$$\arg\max_{x_1} \left(1 + \frac{1}{\epsilon\tilde{\gamma}_1}(E[d_1] - x_1) - \frac{E[\upsilon_1]}{\tilde{\gamma}_1}\right) x_1 = \frac{1}{2}[E[d_1] + \epsilon(\tilde{\gamma}_1 - E[\upsilon_1])]. \tag{5}$$

[8] A detailed discussion is shown in Asako et al. (2017).

By inserting the result of (5) into (2), we have:

$$E[p_1^*] = \frac{1}{2}\left(\frac{E[d_1]}{\epsilon} + \tilde{\gamma}_1 + E[\upsilon_1]\right).$$

Notice that, although k assumes continuous values, its optimal choice is binary, i.e., $k \in (0, \underline{k})$. When $k = 0$, it strictly dominates any given $k \in (0, \underline{k})$, as the level of investment does not affect d_1, and the entrepreneur has to pay k. Likewise, $k \in (\underline{k}, \infty)$ is strictly dominated by $k = \underline{k}$. Thereby, considering the binary choice $k \in (0, \underline{k})$ is sufficient. Analogously, each worker faces the choice $h \in (0, \underline{h})$. Let π_1 be the real profit of the firm at time $t = 1$, and we have:

$$E[\pi_1|k,h] \equiv \frac{1}{1+r}E\left[\frac{p_1 - \upsilon_1}{\tilde{\gamma}_1}\right]x_1 = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}(\epsilon\tilde{\gamma}_1 + E[d_1|k,h] - \epsilon E[\upsilon_1|k,h])^2$$

To facilitate the economic understanding, we will define A and B as below:

$$\begin{aligned} A &\equiv E[\pi_1|k = \underline{k}, h \in [0, \underline{h})] - E[\pi_1|k = 0, h \in [0, \underline{h})] \\ &= \frac{\alpha[(d_I - \epsilon\upsilon_I) - (d_N - \epsilon\upsilon_N)]}{4\tilde{\gamma}_1(1+r)\epsilon}[2\epsilon\tilde{\gamma}_1 + \alpha(d_I - \epsilon\upsilon_I) + (2-\alpha)(d_N - \epsilon\upsilon_N)], \end{aligned}$$

$$\begin{aligned} B &\equiv E[\pi_1|k = \underline{k}, h \in [\underline{h}, \infty)] - E[\pi_1|k = 0, h \in [\underline{h}, \infty)] \\ &= \frac{(1-\alpha)[(d_I - \epsilon\upsilon_I) - (d_N - \epsilon\upsilon_N)]}{4\tilde{\gamma}_1(1+r)\epsilon}[2\epsilon\tilde{\gamma}_1 + (1+\alpha)(d_I - \epsilon\upsilon_I) + (1-\alpha)(d_N - \epsilon\upsilon_N)]. \end{aligned}$$

The expression A provides the increase of the firm's expected profit as an effect of the entrepreneur's investment when $h \in [0, \underline{h})$, which is positive if $(1-\beta)A > \underline{k}$ and negative if $(1-\beta)A < \underline{k}$. The value of B represents that increase when $h \in [\underline{h}, \infty)$, which is positive if$(1-\beta)B > \underline{k}$ and negative if $(1-\beta)B < \underline{k}$. Assuming α[9] $\in (0, 1/2)$, we have $B > A$ by the following expression:

$$\begin{aligned} &B > A \\ \Leftrightarrow\ &\frac{1-\alpha}{\alpha} > \frac{2\epsilon\tilde{\gamma}_1 + \alpha[(d_I - \epsilon\upsilon_I) - (d_N - \epsilon\upsilon_N)] + 2(d_N - \epsilon\upsilon_N)}{2\epsilon\tilde{\gamma}_1 + \alpha[(d_I - \epsilon\upsilon_I) - (d_N - \epsilon\upsilon_N)] + [(d_I - \epsilon\upsilon_I) + (d_N - \epsilon\upsilon_N)]} \end{aligned}$$

as the left side of the inequality is greater than one and the right side is less than one.

For $(1-\beta)A < \underline{k} < (1-\beta)B$, the entrepreneur will choose $k = \underline{k}$ if the worker chooses $h \in [\underline{h}, \infty)$. Otherwise, the entrepreneur sets $k = 0$. There is still the possibility for entrepreneurs to adopt a mixed strategy in which $k = \underline{k}$ with the probability $\sigma_e \in [0, 1]$ and $k = 0$ otherwise. Similarly, the worker may choose a mixed strategy in which $h = \underline{h}$ with the probability $\sigma_w \in [0, 1]$ and $h = 0$ otherwise.

2.1.2. The Worker's Best Response

A representative worker faces a problem given by

$$\max_h \beta\left(\frac{p_0 - \upsilon_0}{\gamma_0}x_0 + \frac{1}{1+r}E\Big[\frac{p_1 - \upsilon_1}{\tilde{\gamma}_1}\Big]x_1\right) - h.$$

When $t = 0$, the term $\beta\left(\frac{p_0 - \upsilon_0}{\gamma_0}\right)x_0$ could be ignored since it is not affected by h. For a given amount of investment made by the workers, if $k \in [0, \underline{k})$, then the expected increase in firms' profit is denoted by:

$$E[\pi_1|k \in [0, \underline{k}), h = \underline{h}] - E[\pi_1|k \in [0, \underline{k}), h = 0] = A;$$

[9]The term α measure the degree of complementarity between players' investment. Thus, by assigning a low value to α, a greater degree of complementarity between the investment made by each of the players is obtained. In this sense, we consider that $\alpha \in (0, 1/2)$. In the situation in which only one of the players invests, the fundamental demand reacts with a rather timid growth, while the variable cost decreases marginally. It is only possible to observe substantial changes in these variables when both players invest.

when $k \in [\underline{k}, \infty)$, we have

$$E[\pi_1 | k \in [\underline{k}, \infty), h = \underline{h}] - E[\pi_1 | k \in [\underline{k}, \infty), h = 0] = B.$$

When $k \in [0, \underline{k})$ and if $\beta A > \underline{h}$, the increase in the worker's expected payoff is positive. On the other hand, if $\beta A < \underline{h}$, it is negative. Following this intuition, if $k \in [\underline{k}, \infty)$, the increase is positive if $\beta B > \underline{h}$ and is negative if $\beta B < \underline{h}$. If we consider that $\beta A < \underline{h} < \beta B$, the worker's best response is conditional to the entrepreneur's mixed strategy.

2.2. The Nash Equilibria

Making the conjecture on the best response of each of the players, we summarize in table 2 the Nash equilibria of the game[10]. When workers and entrepreneurs decide to invest (A and B) and the marginal increases in the expected payoffs are sufficiently high, the best response for both players is to invest. On the other hand, if the value of both investments is low, players do not invest. If $A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$, then a multiple equilibria solution arises and the complementarity between investments of both players is critical to determine the outcome of the game.

	Nash Equilibria		
	$\underline{h} < \beta A$	$A < \underline{h}/\beta < B$	$\underline{h} > \beta B$
$\underline{k} < (1-\beta)A$	$(\underline{k}, \underline{h})$	$(\underline{k}, \underline{h})$	$(\underline{k}, 0)$
$A < \underline{k}/(1-\beta) < B$	$(\underline{k}, \underline{h})$	$(\underline{k}, \underline{h}), (0,0), (\sigma_e^*, \sigma_w^*)$	$(0,0)$
$\underline{k} > (1-\beta)B$	$(0, \underline{h})$	$(0,0)$	$(0,0)$

Table 2: The Nash Equilibria outcomes.

2.3. The Effects of Innovation on the Inflation Rate at the Equilibrium

To evaluate the observed effect of each type of innovation on the inflation rate we must recall that $(d_1, \upsilon_1) = (d_I, \upsilon_I)$ is the equilibrium outcome for the pair $(k^*, h^*) = (\underline{k}, \underline{h})$ and that $(d_1, \upsilon_1) = (d_N, \upsilon_N)$ is the equilibrium outcome for the pair $(k^*, h^*) = (0, 0)$. So, $\frac{(\tilde{\gamma}_1 + d_I/\epsilon + \upsilon_I)}{2}$ is the expected price level in the first equilibrium. In the second equilibrium, we have that $\frac{(\tilde{\gamma}_1 + d_N/\epsilon + \upsilon_N)}{2}$. For the case in which workers and entrepreneurs invest in the economy, the expected price level should be equivalent to that in the situation where the expectations are rational:

$$\tilde{\gamma}_1 = \frac{d_I + \epsilon \upsilon_I}{\epsilon}.$$

The expected price level in the economy for the case in which players do not invest is:

$$\tilde{\gamma}_1 = \frac{d_N + \epsilon \upsilon_N}{\epsilon}.$$

Taking into account the possibility of the players to choose different strategies, $\tilde{\gamma}_1$ should be defined in the range

$$\tilde{\gamma}_1 \in \left[\min\left\{\frac{d_N + \epsilon \upsilon_N}{\epsilon}, \frac{d_I + \epsilon \upsilon_I}{\epsilon}\right\}, \max\left\{\frac{d_N + \epsilon \upsilon_N}{\epsilon}, \frac{d_I + \epsilon \upsilon_I}{\epsilon}\right\}\right].$$

In this sense, the expected inflation rate between $t = 0$ and $t = 1$, $(\tilde{\gamma}_1/\gamma_0) - 1$, corresponds to

$$\frac{\tilde{\gamma}_1}{\gamma_0} - 1 \in \left[\min\left\{\frac{d_N + \epsilon \upsilon_N}{d_0 + \epsilon \upsilon_0}, \frac{d_I + \epsilon \upsilon_I}{d_0 + \epsilon \upsilon_0}\right\} - 1, \max\left\{\frac{d_N + \epsilon \upsilon_N}{d_0 + \epsilon \upsilon_0}, \frac{d_I + \epsilon \upsilon_I}{d_0 + \epsilon \upsilon_0}\right\} - 1\right].$$

[10] See Asako et al. (2017) for the proof of each equilibrium. The case where $A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$ will be discussed in Section 3.

In order to better understand how the types of innovation and the target inflation rate determine equilibrium conditions, let us suppose the following cases:
Case I: If firms choose demand-creating innovation, then $\frac{d_1-d_0}{\upsilon_0-\upsilon_1} > \epsilon$ holds. At the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$ a positive inflation rate is observed. Thus, the target inflation rate is assumed to be credible;
Case II: If firms choose cost-reducing innovation, then $\frac{d_1-d_0}{\upsilon_0-\upsilon_1} < \epsilon$ holds. Therefore, we do not observe inflation in both equilibria. Thus, no player views the target inflation rate as credible.

In other words, when entrepreneurs invest in demand-creating innovation in place of cost-reducing innovation, the high value of d_I guarantees the inequality $d_I - d_0 > \epsilon(\upsilon_0 - \upsilon_I)$ and the announced target inflation rate is said to be credible. Otherwise, when entrepreneurs choose the cost-reduction innovation, the low value of υ_I guarantees that $d_I - d_0 < \epsilon(\upsilon_0 - \upsilon_I)$ and no player views the target inflation rate as credible. This is because in this case the upper bound of $\tilde{\gamma}_1$ is equal to zero. The condition $d_N + \epsilon\upsilon_N \leq d_0 + \epsilon\upsilon_0$ always holds based on the assumptions $d_N \leq d_0$ and $\upsilon_N \leq \upsilon_0$. In such case, the policymaker must encourage investments in demand-creating innovation, such as structural reforms, tax incentives and subsidies.

When the elasticity of demand (ϵ) increases, firms are more prone to cost-reducing innovations. This happens because if the demand for a good is elastic, a reduction in price causes revenue to increase, which provides an incentive for the entrepreneurs to makes cost reduction. When demand is inelastic, entrepreneurs may increase prices and observe an increase in demand for their products through demand-creating innovation (Kamien & Schwartz, 1970; Spence, 1975). When ϵ is low[11], entrepreneurs have an incentive to choose demand-creating innovation. Otherwise, policies to raise the value of d^* could hold the condition given by $d^* - d_N > \epsilon(\upsilon_N - \upsilon^*)$.

3. Evolutionary Game and Agent-based Computational Economics (ACE)

According to Smith & Price (1973), evolutionary games imply a convergence to the dominant long-run equilibrium. In this equilibrium, achieved after a period of dynamic interaction, players must have adopted an evolutionary stable strategy (ESS) that is a strategy where players have no incentive to abandon, unless some external force disturbs the underlying conditions of the game. Then, if classical game theory can be defined as the science that studies strategic behavior, with the theory of evolutionary games it takes a step forward, since we now have the science that studies the robustness of strategic behavior. In order to achieve this robustness, in subsection 3.1, we present the Replicator Dynamics (RD) analysis, obtained from a non-linear differential equation system, and complement the study with an Agent-based Computational Economics (ACE) model, presented in subsection 3.2, which implements a stochastic component in the evolutionary dynamics of the game.

3.1. The Replicator Dynamics

In an evolutionary game, a bounded rationality, a large population, n, of players ($n \to \infty$) and an implicit recognition that agents learn are assumed. Every period, a player is randomly matched with another player and they play a two-player game. Each agent is assigned a strategy and they cannot choose it. In other words, they are "programmed" to play a strategy in the initial period ($t = 0$) and it may not maximize their utility function. However, the systematic interaction with other agents will lead them to modify or update their behavior over time by choosing a given strategy. Thus, one player can imitate other players' strategies.

So, in this section, the evolutionary dynamics of the players' strategies between investment and non-investment is evaluated by using the perfect-foresight dynamics according to the same approach used by Gilboa & Matsui (1991) and Matsui & Matsuyama (1995). By doing so, players are assumed to be forward-looking. The best-response dynamics approach is also applied. Here, we focus on the best-response dynamics because, according to Asako et al. (2017) there is a lack of understanding on how an economy, given its initial condition[12], will evolve to the long-term equilibrium. Thence, our ACE model remedies some of these drawbacks.

Our approach follows the literature that compares results under rational and adaptive expectations. The first one has been largely used, although it is sustained by the strong assumption that economic agents use all the set of information available in order to build their expectations. We highlight and follow the intuition of Woodford (2002),

[11] A sufficiently low ϵ is a necessary condition for positive inflation in an equilibrium.
[12] The initial state corresponds to the period of the stage game right after the implementation of a policy.

Mankiw & Reis (2002) and Sims (2003), who evaluate the macroeconomic environment by considering the fact that agents may deviate from rational expectations. On this matter, knowing that the target inflation rate could not be achievable in the long term, there is a clear difference between the perfect-foresight and the best-response dynamics behavior for guiding the economic agents into the equilibrium. When considering the perfect-foresight approach, the long-term equilibrium is never affected by the current state of the economy. On the other hand, the current state of the economy, as well as the equilibrium conditions are affected when considering the best-response dynamics.

In order to derive the ESS, we first determine π_H, π_M and π_L as follows:

$$\pi_H = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}(\epsilon\tilde{\gamma}_1 + d_I - \epsilon\upsilon_I)^2;$$
$$\pi_M = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}[\epsilon\tilde{\gamma}_1 + \alpha(d_I - \epsilon\upsilon_I) + (1-\alpha)(d_N - \epsilon\upsilon_N)]^2;$$
$$\pi_L = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}(\epsilon\tilde{\gamma}_1 + d_N - \epsilon\upsilon_N)^2.$$

π_H corresponds to the expected total profit when workers and entrepreneurs decide to invest. If only one player decides to invest, the expected total profit is given by π_M, and if no player invests they receive π_L. The payoff matrix of the stage game[13] is presented in (6), and we introduce the following parameters $\phi \equiv (\pi_H - \pi_M)$ and $\theta \equiv (\pi_L - \pi_M)$. This procedure will not affect the best response structure of the game and will simplify the analysis. Following the evolutionary algorithm, players are randomly matched and compete against each other and a one-shot game is played. The level of aggregate strategies of the populations does not change all at once. In fact, they continuously update their strategic behavior over time.

$$\begin{array}{cc} & \begin{array}{cc} I(h=\underline{h}) & NI(h=0) \end{array} \\ \begin{array}{c} I(k=\underline{k}) \\ NI(k=0) \end{array} & \begin{pmatrix} (1-\beta)\phi - \underline{k}; \beta\phi - \underline{h} & 0;0 \\ 0;0 & (1-\beta)\theta + \underline{k}; \beta\theta + \underline{h} \end{pmatrix} \end{array} \tag{6}$$

Let $s_w \in [0,1]$ be the share of workers using strategy I, so the share of workers adopting NI is $1-s_w$. Analogously, for the population of entrepreneurs, the share of players using strategies I and NI are $s_e \in [0,1]$ and $1-s_e$, respectively. The observed variations in the proportion of players adopting each one of the strategies reflects their evolutionary process within each population. Note that this relative frequency can be understood as the probability that a player will play a given strategy. Both the evolution of the game and the strategic behavior of the firms is conditioned to the fitness of their strategies. The fitness, according to Binmore & Samuelson (1992) and Samuelson (2002), depends on the player's payoff of a given strategy and on the relative frequency of the strategies observed in both populations. That is, players make decisions based on the expected utility of their payoffs.

From the normalized payoff matrix (6) and considering $(1-\beta)\phi > \underline{k}$ and $\beta\theta > \underline{h}$, there are simultaneously three Nash equilibria for the game: $\Theta^{NE} = \{(\underline{k}, \underline{h}); (0,0); (\sigma_e^*, \sigma_w^*)\}$. Using the values of π_H, π_M and π_L, the mixed-strategy equilibrium is given by $(\sigma_e^*, \sigma_w^*) = \left(\frac{\underline{h}/\beta+\theta}{\phi+\theta}, \frac{\underline{k}/(1-\beta)+\theta}{\phi+\theta}\right)$. Θ^{NE} represents the set of Nash equilibria of the game. The following questions arise: departing from an initial game condition, which equilibrium will be reached? In fact, which is the game played? Without loss of generality, to answer these questions, we first analyze the evolution and the robustness of the players' strategic behavior with the analytic solution of the RD system proposed by Taylor & Jonker (1978), which is a very general ordinary differential equation (ODE) system in the evolutionary game theory. As shown in Hirth (2014), in a dynamic system, the growth rate $\dot{s_e}/s_e$ equals the strategy I's fitness $e^1 A(s_w, 1-s_w)^T$ less the average fitness $(s_e, 1-s_e)A(s_w, 1-s_w)^T$ of population of entrepreneurs, where $\dot{s_e} = ds_e/dt$ and $e^1 = (1,0)$ represents that all entrepreneurs from the population choose the pure strategy $I(k = \underline{k})$.

Let $A = \begin{bmatrix} (1-\beta)\phi - \underline{k} & 0 \\ 0 & (1-\beta)\theta + \underline{k} \end{bmatrix}$ be the payoff matrix of a representative entrepreneur. After some trivial matrix algebra, the replicator dynamic equation for the population of entrepreneurs is $\dot{s_e} = s_e((e^1 - (s_e, 1-s_e))A(s_w, 1-$

[13] For further details, see Asako et al. (2017).

$s_w)^T$. Substituting the values of A (payoffs earned by the representative entrepreneur), in order to derive the dynamic replicator system for the population of entrepreneurs e and workers w:

$$\dot{s_e} = s_e(1-s_e)\Big\{s_w\Big[(1-\beta)\phi - \underline{k}\Big] - (1-s_w)\Big[(1-\beta)\theta + \underline{k}\Big]\Big\} \tag{7}$$

$$\dot{s_w} = s_w(1-s_w)\Big\{s_e\Big[\beta\phi - \underline{h}\Big] - (1-s_e)\Big[\beta\theta + \underline{h}\Big]\Big\} \tag{8}$$

$\dot{s_e}$ and $\dot{s_w}$ represent the growth rate of the proportion of entrepreneurs and workers that adopt the first pure strategy I within each population. The stability of the system is a coordinate $(s_e, s_w) \in [0,1] \times [0,1]$, in which $\dot{s_e} = \dot{s_w} = 0$ is a necessary condition for the stationarity of (7) and (8). To check the stability of the points candidates for an ESS, i.e., an asymptotically stable steady state for the two-population game, we must use the Jacobian matrix (Ω). Thus, for the system (7) and (8):

$$\Omega(s_e, s_w) = \begin{bmatrix} (1-2s_e)\{s_w[(1-\beta)\phi - \underline{k}] - (1-s_w)[(1-\beta)\theta + \underline{k}]\} & s_e(1-s_e)[(1-\beta)(\theta+\phi)] \\ s_w(1-s_w)[\beta(\theta+\phi)] & (1-2s_w)\{s_e[\beta\phi - \underline{h}] - (1-s_e)[\beta\theta + \underline{h}]\} \end{bmatrix}.$$

For the stationary point to be asymptotically stable, the eigenvalues $\lambda_{1,2}$ of the matrix (Ω) evaluated at points that hold the condition $\dot{s_e} = 0$ and $\dot{s_w} = 0$ must have negative real parts. Thus, the phase space becomes the unit square $[0,1] \times [0,1] \in \Re^2$ and the stationary points are the corners of the phase space $(s_e, s_w) = \{(0,0), (0,1), (1,0), (1,1)\}$ as well as the interior stationary point $(s_e^*, s_w^*) = \left\{\frac{\beta\theta + \underline{h}}{\beta(\theta+\phi)}, \frac{(1-\beta)\theta + \underline{k}}{(1-\beta)(\theta+\phi)}\right\}$. Analyzing the linearized system in the neighborhood of the stationary points, the eigenvalues of $\Omega(s_e, s_w)$ evaluated at the corners of the phase space are $\lambda_1 = \frac{\partial \dot{s}_e}{\partial s_e}$ and $\lambda_2 = \frac{\partial \dot{s}_w}{\partial s_w}$. For the coordinates (0,0);(0,1);(1,0) and (1,1) the partial derivatives are, respectively:

$$\frac{\partial \dot{s}_e}{\partial s_e}\Big|_{(0,0)} = -[(1-\beta)\theta + \underline{k}]; \quad \frac{\partial \dot{s}_w}{\partial s_w}\Big|_{(0,0)} = -(\beta\theta + \underline{h}); \quad \frac{\partial \dot{s}_e}{\partial s_e}\Big|_{(0,1)} = [(1-\beta)\phi - \underline{k}]; \quad \frac{\partial \dot{s}_w}{\partial s_w}\Big|_{(0,1)} = -[-1(\beta\theta + \underline{h})];$$

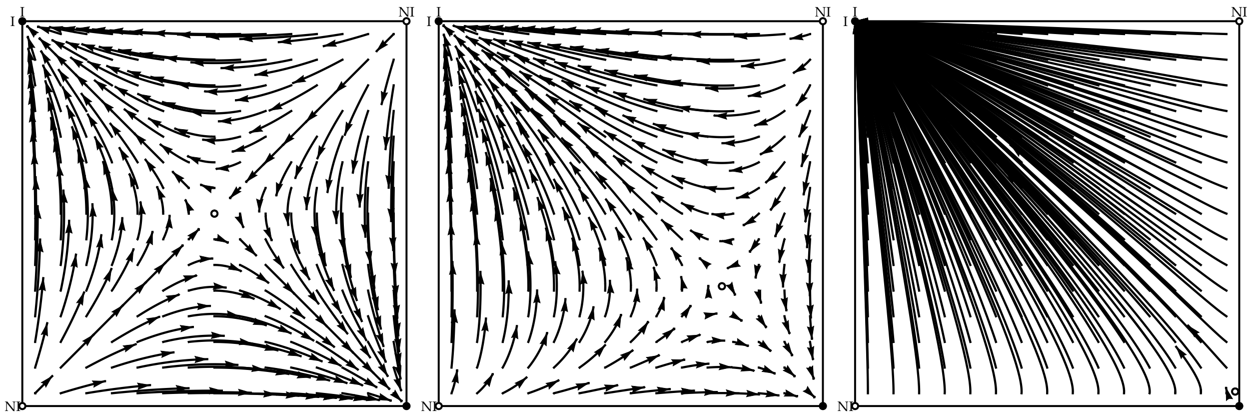
$$\frac{\partial \dot{s}_e}{\partial s_e}\Big|_{(1,0)} = -\{-1[(1-\beta)\theta + \underline{k}]\}; \quad \frac{\partial \dot{s}_w}{\partial s_w}\Big|_{(1,0)} = (\beta\phi - \underline{h}); \quad \frac{\partial \dot{s}_e}{\partial s_e}\Big|_{(1,1)} = -[(1-\beta)\phi - \underline{k}]; \quad \frac{\partial \dot{s}_w}{\partial s_w}\Big|_{(1,1)} = -(\beta\phi - \underline{h}).$$

Observe that the eigenvalues at the corners of the phase space are both negative at (0,0) and (1,1) and both positive at (0,1) and (1,0). On the other hand, the eigenvalues of the Jacobian matrix evaluated at $\left\{\frac{\beta\theta + \underline{h}}{\beta(\theta+\phi)}, \frac{(1-\beta)\theta + \underline{k}}{(1-\beta)(\theta+\phi)}\right\}$ are $\lambda_{1,2} = \pm\sqrt{\frac{[(1-\beta)\theta + \underline{k}][(1-\beta)\phi - \underline{k}](\beta\theta + \underline{h})(\beta\phi - \underline{h})}{\beta(1-\beta)(\theta+\phi)^2}}$. Thus, the later is a saddle point, (1,0) and (0,1) are unstable points and (0,0) and (1,1) are attractor points. On this matter, it is possible to infer that a coordination game is formed.

The analytical solution presented here suggests that in the long run a rise in the expected inflation may lead to an increase in the nominal interest rate. However, these results remain unchanged as long as it is assumed that the nominal interest rate is fixed for a while. Once the economy embarks on the path toward the equilibrium with an inflation rate closer to the target, they still moves closer to the equilibrium over time.

Therefore, even if the nominal interest rate rises and entrepreneurs and workers expect payoffs to decline in the long run, the economy should still evolve to the equilibrium with a higher inflation rate, because players expect the probability of successful coordination between entrepreneurs and workers to be high.

Figure 1 summarizes the standard strategic behavior. As mentioned, a coordination game is formed. Therefore, what varies between nations is the effort that each Central Bank will have to make in order to reach the equilibrium in which both workers and entrepreneurs invest in the economy. In Figure 1b, for instance, based on the information provided by Table 3 and assuming that the Central Bank of South Korea will be intolerant of any positive value for the inflation rate, the effort to achieve the steady state given by the strategies (I, I) is greater than that made by the Central Bank of Mexico to reach the inflation target announced. This is due to the basins of attraction and the intensity of the eigenvectors in the vicinity of the interior stationary (saddle) point in the phase diagrams. Thus, following the economic intuition of our model, to achieve the balance in which both players invest, the initial fraction of players investing should be greater in South Korea than in Mexico. Moreover, it is important to capture precisely this order of magnitude to assess and compare the effect of the inflation targeting regimes on the investment decision in EMEs.



(a) Pure coordination game (b) $fe_{zero}; fw_{zero}$ - South Korea (c) $fe_{IT}; fw_{IT}$ - Mexico

Figure 1: Evolutionary dynamics according to the announced inflation target. In Figure 1a we show the phase diagram with the typical dynamics of a pure coordination game. In Figure 1b the phase diagram shows the dynamics when the monetary authority of South Korea seeks zero inflation rate. The dynamics of the game when the Central Bank of Mexico pursue the inflation target rate is shown in Figure 1c. The white (black) dots represents the unstable (stable) stationary points. To generate the dynamics in Figure 1b we use the following values: $\epsilon = 1, d_0 = 100, d_I = 325, d_N = 100, \upsilon_0 = 9900, \upsilon_I = 9800, \upsilon_N = 9900, \alpha = 0.45, \beta = 0.49, \frac{1+i}{\gamma_1/\gamma_0} = 1.0125, \underline{k} = 2, \underline{h} = 2$. To generate the dynamics in Figure 1c, we use the following values: $\epsilon = 1, d_0 = 100, d_I = 900, d_N = 100, \upsilon_0 = 9900, \upsilon_I = 9800, \upsilon_N = 9900, \alpha = 0.45, \beta = 0.49, \frac{1+i}{\gamma_1/\gamma_0} = 1.04, \underline{k} = 10.5, \underline{h} = 10.5$.

Nevertheless, once the stability of the strategic coordination on the investment decision made by workers and entrepreneurs has been demonstrated analytically, it is important to move forward and evaluate how the credibility of signaling by the EMEs central bank can increase the level of investment in the economic system. For this purpose, the next subsection brings the numerical simulation model, which will allow us to infer about the fraction of firms and workers willing to invest in many EMEs, given the inflation target announced by the central bank. The countries selected to evaluate this issue are: Brazil, Chile, Colombia, Peru, Mexico, Russia, India, South Africa, South Korea, the Philippines and Thailand. Based on the results obtained for these eleven countries, we aim to find a pattern of investment behavior by the agents. This qualitative evaluation is important, since it allows the central planner of the EMEs to understand the mechanisms of monetary policy transmission that will guarantee a steady state in which there is a low and stable inflation rate and a high level of investments.

3.2. The ACE Algorithm: Modelling the Economic System

A stochastic component is implemented[14] in the analysis of evolutionary equilibrium using the ACE method. In general, Agent-based Modeling has been largely used in the understanding of the evolution of cooperative behavior in social dilemmas, as can be seen in Chan et al. (2013) and Da Silva Rocha (2017). In this section, an algorithm to complement the evolutionary game and to guide the dynamic interaction among players is presented. We consider the competition in which the two populations are distributed in a well-mixed network. A well-mixed arrangement may be consistent with our framework, once we assume that all players have access to the same level of information and that there is no local constraint capable of generating some exogenous noise in the communication between policymaker and entrepreneurs or workers. To implement the computational simulation, the intuition presented by Da Silva Rocha (2017) is followed. In this sense, at a time $t = 0$, we establish an initial proportion of entrepreneurs and workers, (s_e, s_w), that plays the (I) strategy.

Evolutionary dynamics are introduced in sequence and at each Monte Carlo time Step (MCS), a Focal Agent i, which can update[15] its strategy, is chosen randomly. This occurs simultaneously in both populations. Focal Agent i, in turn, plays against a random opponent from the rival population and starts the game presented in (6). Thus, Focal Agent i will obtain a payoff V_i. At the same time, an agent j is randomly chosen as a Reference Agent, which

[14] To implement the algorithm, we used the Java programming language.

[15] The update mechanisms used in ACE models are synchronous and asynchronous. Here, we use the second, since it allows the overlapping generations interactions. See Hauert (2002) and Chan et al. (2013).

randomly plays against an opponent from the rival population and obtains a payoff V_j. Notice that focal and reference players belong to the same population. The Focal Agent i compares V_i and V_j to analyze the possibility of updating his strategy in two stages as stated ahead: (i) If $V_i \geq V_j$, focal player keeps his strategy; (ii) If $V_i < V_j$, the focal player might update his strategy to the one adopted by the reference player with a probability given by the variable η:

$$\eta = \frac{V_j - V_i}{max.payoff - min.payoff} \tag{9}$$

The maximum and minimum payoffs are obtained from the game matrix in (6). By doing this procedure, we guarantee that $\eta \in (0, 1)$. To establish a decision criterion whether Focal Agent i updates his strategy or not, we use a random number generator named $rnd \in (0, 1)$. In this way, a stochastic component on the dynamics of the game is implemented. Focal Agent i compares rnd with the probability η, so that: (iii) If $\eta \geq rnd$, Focal Agent i updates his strategy and imitates the Reference Agent j; (iv) If $\eta < rnd$, Focal Agent i does not update his strategy.

At every MCS, randomly selected individuals from both populations have the opportunity to change strategy at least once, on average, comparing their payoffs with the Reference Agent j. We say that, on average, individuals can update strategies once, because within a MCS the same player may be invited to play many times, and other players may not, since the process of players' selection is random. Thus, when all players in both populations, have the opportunity to update their strategies, on average, a MCS is completed and a new MCS starts in order to repeat the dynamics of the game. This procedure characterizes the ACE model.

4. Results

In this section, we present the results[16] obtained by the ACE algorithm. For this purpose, we insert the inputs of each EMES shown in Table 3 into the payoff matrix given by (6). Thus, the evolutionary dynamics no longer rely on the analytical solution derived from replicator dynamics. Now, the strategy update criterion follows the rule presented in equation (9). To compare and evaluate the strategic behavior of players, we group the EMEs as follows: Brazil, Chile, Colombia, Mexico and Peru form the group of Latin American countries. India, Russia and South Africa make up the members of BRICS. Finally, the Philippines, South Korea and Thailand form the set of Asian countries. Subsections 4.1 and 4.2 discusses the efforts made by the Central Bank of EMEs to increase investors' expected profits.

[16]Since the ACE mechanism is based on a stochastic rule, we repeat the simulation for a hundred (100) times for each country under analysis. As South Africa does not have a specific value for the IT, the results were generated from the average considering 50 simulations with its lower bound plus 50 simulations considering its upper bound. In this way, Figures 2, 3 and 4 represent the average behavior pattern of entrepreneurs and workers in relation to investment decision.

Countries	IT (%)	LB (%)	UB (%)	i (%)	d_i	$\underline{h}$	$\underline{k}$	$s_{e_{IT}}; s_{w_{IT}}$	$s_{e_{zero}}; s_{w_{zero}}$
Brazil (BR)	4.25	2.75	5.75	6.50	850	9.75	9.75	0.09	0.13
Chile (CH)	3.00	2.00	4.00	2.50	450	3.25	3.25	0.13	0.16
Colombia (CO)	3.00	2.00	4.00	5.50	750	7.75	7.75	0.08	0.10
India (IND)	4.00	2.00	6.00	6.50	850	9.75	9.75	0.07	0.11
Mexico (ME)	3.00	2.00	4.00	7.00	900	10.5	10.5	0.05	0.09
Peru (PE)	2.00	1.00	3.00	3.75	575	4.75	4.75	0.08	0.09
Philippines (PHI)	3.00	2.00	4.00	3.00	500	3.75	3.75	0.08	0.11
Russia (RUS)	4.00	-	-	7.25	925	11.25	11.25	0.08	0.10
South Africa (SAF)	-	3.00	6.00	6.75	875	10.25	10.25	0.08	0.11
South Korea (SKO)	2.00	-	-	1.25	325	2.00	2.00	0.30	0.37
Thailand (THA)	2.50	1.00	4.00	1.50	350	2.00	2.00	0.13	0.19

Table 3: List of countries used in the ACE simulations. IT represents the announced Inflation Target. LB and UB corresponds to the lower and to the upper bound, respectively. The interest rate is given by i. The term d_i represents the expected fundamental demand and depends on the amount of investments in human ($\underline{h}$) and physical ($\underline{k}$) which depends on the expected inflation rate, $(\bar{\gamma}_1/\gamma_0) - 1$. The pair $(s_{e_{IT}}; s_{w_{IT}})$ represents the share of entrepreneurs and workers investing in physical and human capital, respectively, when the Central Bank is signaling that will pursue the announced inflation target. In contrast, the pair $(s_{e_{zero}}; s_{w_{zero}})$ represents the share of entrepreneurs and workers investing in physical and human capital, respectively, when the Central Bank is signalling that will pursue a zero inflation rate. Information available at http://www.centralbanknews.info/p/inflation-targets.html.

On the left side of Figure 2, we show the ACE results for the EMEs of Latin America, considering that the Central Bank pursues the target inflation rate. For Mexico, in order for the equilibrium to be reached with both players investing, it is necessary that at time $t = 0$, the share of entrepreneurs (ME_e) and workers (ME_w) willing to invest to be greater than or equal to 5%. Otherwise, the equilibrium obtained is with both players not investing. Although it requires the lowest initial share of investor among EMEs, it should be emphasized that it has the highest: (a) expected fundamental demand ($d_i = 900$); (b) expenditure with investments in both human and physical capital ($\underline{h}; \underline{k}$) = (10.5, 10.5); (c) interest rate ($i = 7.00\%$) of Latin American EMEs. Therefore, given the high costs associated with the investment decision, which could inhibit players' investment, together with an expected increase in the inflation rate, the great task of the Mexican Central Bank is to increase its credibility. The best way to achieve this goal is through signaling in monetary policy. In this sense, if the Central Bank is quite transparent, i.e., if the announced inflation target is actually reached, we might observe a coordination in players' expectations, leading workers and entrepreneurs to invest.

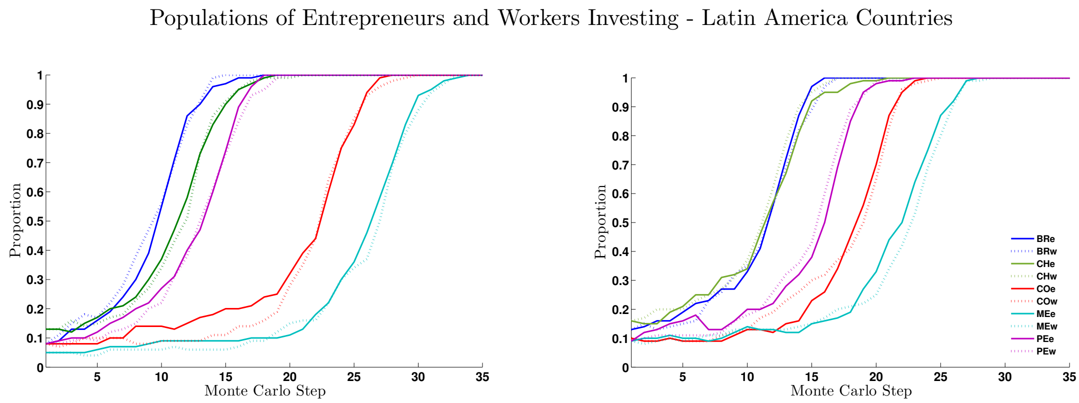


Figure 2: ACE. **Left**: The Central Bank pursues the target inflation rate. **Right**: The Central Bank pursues a zero inflation rate. The solid lines (dashed lines) correspond to the fraction of entrepreneurs (workers) playing strategy I in each of the Latin America countries.

Note that the ACE simulation for Mexico required the greatest amount of MCS to reach the steady state, i.e., MCS $\lesssim$ 35. The outcome showed that, once the Central Bank is committed to the inflation target, entrepreneurs make the first move towards investment decision and, in sequence, workers coordinate their strategy. Only in Brazil this pattern was not observed, since the population of workers (BR_w) reaches the steady state before the population of entrepreneurs (BR_e). Brazil's initial share (9%) is almost twice the share of Mexico and the convergence to the steady state occurs around MCS $\lesssim$ 15. This is the less time-consuming simulation. Thus, once confidence is established the coordination between investment decision quickly takes place.

The similarity lies in the amounts of d_i, $\underline{k}$, $\underline{h}$ and i. What determines the difference between the equilibrium paths is the level of effort each Central Bank must make to be credible. Note that Brazil's inflation target is significantly higher than that of Mexico and this may justify the difference between the initial shares ($s_{e_{IT}}$; $s_{w_{IT}}$) required for each country to reach the equilibrium. Chile requires the largest initial share of players willing to invest (13%). This difference in relation to the other Latin American EMEs might be justified by its (i) low real interest rate, (ii) low investment costs in physical and human capital and (iii) low expected fundamental demand. The combination of these three factors shortens the distance (in terms of the payoff) between the strategies *NI* and *I*. In this way, since the Central Bank of Chile seeks mechanisms to further solidify its institutional arrangement, the convergence to the steady state in which both players invest is rapidly attained, especially if compared to Colombia and Mexico. This intuition partially explains the equilibrium path of the Peruvian economy. Compared with Chile, the lower share of players willing to invest at $t = 0$ in Peru to achieve the steady state can be justified by their different expected fundamental demand.

On the right side of Figure 2, we show the results when the monetary authority seeks a zero inflation rate. We can observe that in Mexico and in Peru, if the share of entrepreneurs and workers willing to invest is less than 9%, the equilibrium reached is with both players not investing. For Colombia, this value is equal to 10%. Note again that the economies of Brazil and Chile require a greater effort from the monetary authority to gain investor confidence. For Brazil (Chile), if the proportion of workers and entrepreneurs willing to invest is less than 13% (16%), the ESS is reached with both players adopting the strategy (*NI, NI*). Despite of the particularities of each Latin American EMEs, the need for a higher initial share of players willing to invest when the Central Bank seeks a zero inflation rate is justified by a decreasing in the distance between the payoffs of the coordinated strategies. Hence, the monetary authority needs to intensify its effort to establish confidence in the economic environment. By doing so, agents are more likely to incur the cost of investing, on the belief that they will acquire the benefits of this strategic decision.

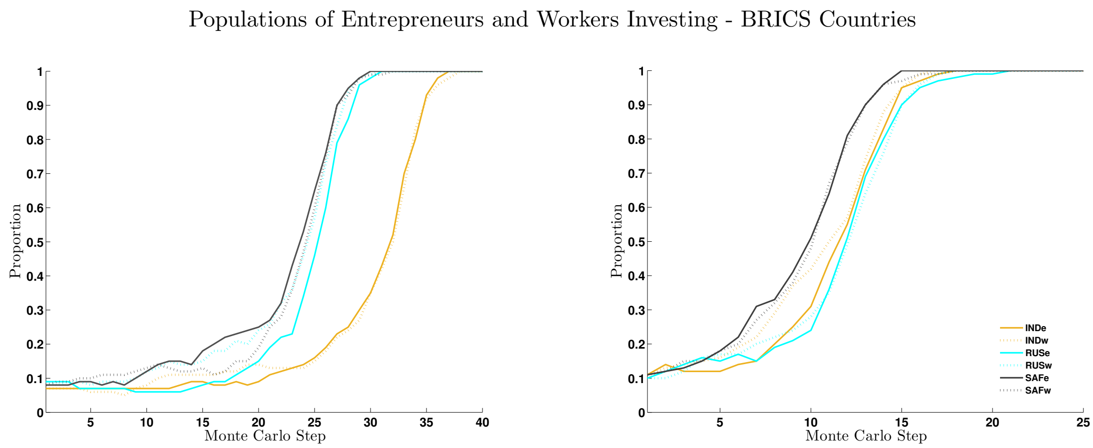


Figure 3: ACE. **Left**: The Central Bank pursues the target inflation rate. **Right**: The Central Bank pursues a zero inflation rate. Solid (dashed) lines correspond to the fraction of entrepreneurs (workers) playing strategy I in each of the BRICS countries.

Comparing the dynamics observed in Figure 3 with Figures 2 and 4, we see that BRICS countries form a homogeneous group in terms of the initial proportion of players to achieve the steady state in which both players invest. This behavioral pattern can be justified by the high (and similar) values for the variables that make up the payoff earned from each of the strategic interactions. On the left side of Figure 3, we see that South Africa and Russia reach the balance of the game around the MCS $\lesssim$ 30. In turn, India reaches equilibrium around the MCS $\lesssim$ 40. It can be explained by the smaller payoff difference when both players coordinate (NI, NI) and when they coordinate (I, I). Note that this difference in the speed of convergence is less evident on the right side of Figure 3. Finally, it is possible to observe that when the Central Bank seeks a zero inflation rate, the coordination between the investment decision occurs more quickly, reinforcing the importance of the credibility of monetary authority discussed above.

The EMEs of Asian countries have the lowest cost of investing in $\underline{h}$ and $\underline{k}$. Combined with an expectation of low inflation and, consequently, a low real interest rate, this makes the Central Bank performance even more challenging in order to avoid stagnation. This can be seen from Figure 4. Note that in South Korea the share of players willing to invest at $t = 0$ for the economy to reach its steady state is the highest among all countries surveyed (30%) - even more so when the Central Bank announces that its target is a zero inflation rate (37%). For both scenarios, the balance is reached around MCS $\lesssim 15$.

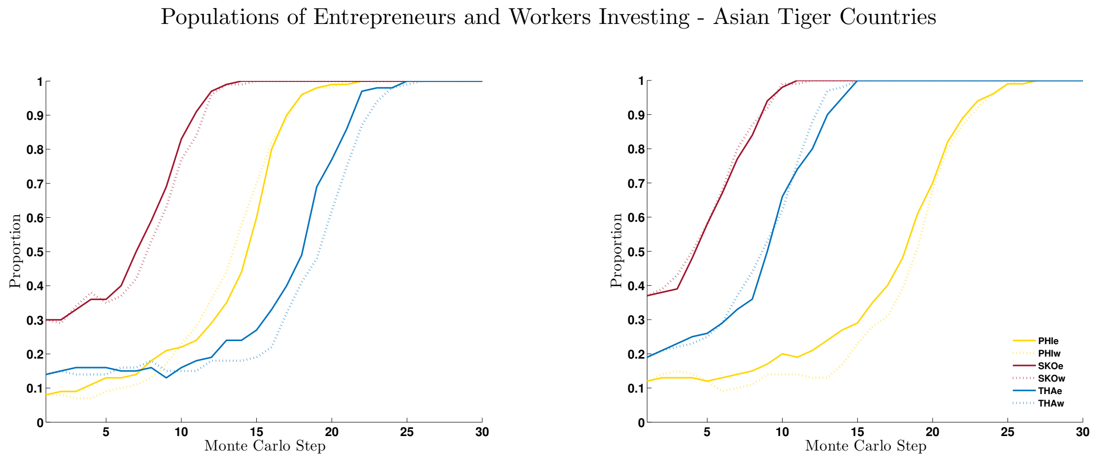


Figure 4: ACE. **Left**: The Central Bank pursues the target inflation rate. **Right**: The Central Bank pursues a zero inflation rate. Solid (dashed) lines correspond to the fraction of entrepreneurs (workers) playing strategy I in each of the Asian Tiger countries.

In Thailand, the costs associated with investment, the amount of the expected fundamental demand and the nominal interest rate resemble those observed in South Korea. However, when the Thai Central Bank announces that it will meet the IT, the expected inflation rate in the economy makes the share of workers and entrepreneurs willing to invest at $t = 0$ significantly lower than those in South Korea. This effect on the expected inflation rate is even more evident when we evaluate the equilibrium path for the Philippines, whose share of players willing to invest at $t = 0$ is lower than that observed in Thailand and in South Korea.

The results shown on the right side of Figure 4 suggests that, when the Central Bank is pursuing a zero inflation rate, there may exist a correlation between the costs associated with $\underline{h}$ and $\underline{k}$ and the time of convergence to the steady state. We emphasize that the Philippines have the highest investment costs among Asian countries and, perhaps not by chance, it is the last to reach the steady state (MCS $\lesssim 30$) in this particular case. This same pattern was observed for Mexico and Russia on the right side of Figures 2 and 3, respectively.

From what we presented here, it is possible to visualize that, for a level of inflation that corresponds to the inflation target, a smaller share of players that are willing to invest is necessary to guide the economy to the steady state. In addition, we can see that the results obtained by the ACE give more robustness to those obtained by the replicator dynamics. In the next subsection, we will present some theoretical derivations to support the results of the simulation.

4.1. The Policy Channels: The Efforts to Increase Players' Expected Profits

To examine the signaling and transmission mechanisms of monetary policy, we evaluate the effects of the target inflation rate. To explore the policy channels for reducing the nominal interest rate, i, and for announcing the target inflation rate, $\tilde{\gamma}_1/\gamma_0$, we decompose the real interest rate of the EMEs into its nominal interest[17] and the expected inflation rates:

$$1 + r = \frac{1 + i}{\tilde{\gamma}_1/\gamma_0}$$

In this way, by considering the medium term, i is the lowest rate possible to maintain inflation between the limits of the target announced by the Central Bank. The expectation of rising prices by the entrepreneurs leads to an increase in the level of equilibrium prices. This drives to an increase in expected profit. The comparative statics of $\tilde{\gamma}_1$ are presented in Corollary 1.

Corollary 1. *(a) A and B increase with $\tilde{\gamma}_1$. (b) The share of players investing in the economy, s_I, increases with $\tilde{\gamma}_1$.*

Proof. (a) can be deduced by the definition of A and B given in subsection 2.1. (b) $\pi_H > \pi_M > \pi_L$ and $\frac{\partial \pi_H}{\partial \tilde{\gamma}_1} > \frac{\partial \pi_M}{\partial \tilde{\gamma}_1} > \frac{\partial \pi_L}{\partial \tilde{\gamma}_1} > 0$ hold. In addition, $\alpha \in (0, 1/2)$ and π_H, π_M and π_L are convex. Thus, the following inequalities hold:

$$\frac{\partial \pi_H}{\partial \tilde{\gamma}_1} - \frac{\partial \pi_M}{\partial \tilde{\gamma}_1} > \frac{\partial \pi_M}{\partial \tilde{\gamma}_1} - \frac{\partial \pi_L}{\partial \tilde{\gamma}_1} > 0$$

Thus, $\frac{\partial \sigma_e^*}{\partial \tilde{\gamma}_1} < 0$ and $\frac{\partial \sigma_w^*}{\partial \tilde{\gamma}_1} < 0$ are obtained, and s_I increases with $\tilde{\gamma}_1$. ■

From this corollary, we observe that the increase in $\tilde{\gamma}_1$ makes the evolutionary dynamics of equilibrium more likely to be the one in which both entrepreneurs and workers invest. Specifically, through the increase in $(A + B)$ under the perfect-foresight dynamics and through the decrease in σ_e^* and σ_w^* under the best-response dynamics, the economy operates with an inflation rate closer to the upper bound of the target announced.

4.2. The Key Role of EMEs Central Banks

Here, we try to evaluate the possible incentives that can be given to the players, so that they invest more in the economy, even if the signaling of the Central Bank is to seek an inflation rate closer to zero. To achieve our purposes, we depart from the following question: do players always expect the Central Bank of EMEs to meet the inflation target? In our analysis, the agents are rational, i.e., for the sake of credibility, they expect that the price level to be reached in t + 1, $\tilde{\gamma}_1$, will be feasible if and only if its value belongs to the interval established as credible. On the other hand, if players perceive uncertainty about $\tilde{\gamma}_1$, the policymaker announcement may have a minor effect.

Through the presented results, the effect of an increase in $\tilde{\gamma}_1$ keeps the payoff structure of the game more favorable to those players who are willing to invest. However, there is a limit for $\tilde{\gamma}_1$ to have an effect on rational agents. Therefore, policy is not always efficient in stimulating investment in the economy. In addition, the current state of the economy is of utmost importance in analyzing policy effectiveness, especially when considering that players are backward-looking. As the game is dynamic, and there is a rule of learning and updating beliefs, the balance is said to be "path-dependent". In other words, the fraction of players who invested in the most recent rounds exert great influence on the equilibrium outcome. In contrast, the current state does not matter if players are forward-looking.

Consider that, for a given initial condition, the relative frequency of entrepreneurs and workers willing to invest (s_e, s_w) is already near the target inflation rate equilibrium. To increase the level of investment by agents, $\tilde{\gamma}_1$ should be raised to the upper bound inflation rate equilibrium. On this matter, the following policy measures could be implemented. First, the government may grant subsidies or tax incentives in order to expand s_I by reducing the cost of investment, $\underline{k}$. Second, it should undertake initiatives to synchronize interests between workers and entrepreneurs, with the objective of reducing $\underline{h}$. By doing so, if workers expect that they will be in the workforce for a sufficiently long period, the greater will be their incentives to invest in human capital. Another potential measure is the so-called networking innovation activities, i.e., the promotion of collaboration among industry, academia, and the government leads to a reduction in investment costs and an increase in expected returns (Nelson & Shaw, 2003).

[17] For the numerical simulations, we use the values provided by the Central Bank News: `http://www.centralbanknews.info/p/inflation-targets.html`.

5. Conclusion

In this paper we proposed an ACE model to evaluate in which conditions of equilibrium the EMEs economies could find themselves with a high level of investment, given the inflation target rate announced by the monetary authority. Our approach consists of an iteratively played stage game among entrepreneurs and workers, which assumes that the presence of complementarity between the investments decisions implies the existence of two equilibria.

Therefore, the main contributions of ACE learning models are twofold: (a) a refinement of dynamic equilibrium analysis through numerical simulation to determine the trajectory of the level of investment in the economy for a given inflation targeting rate; (b) a more precise measuring of the proportion of agents willing to invest in demand-creating rather than cost-reducing, optimizing the implementation of the economic policy by the central planner. In one of the balances, both entrepreneurs and workers invest. In this case, there are two conditions necessary to ensure that the economy converges to the inflation target: (i) entrepreneurs should signal their intention to invest in demand expansion; (ii) the proportion of workers and entrepreneurs investing at the current moment should be large enough. If these conditions are not observed, policies need to be developed to drive the economy into such favorable environment. On the other hand, according to the proposed model, there is a balance in which players coordinate their actions so that there is no investment in the economy.

Furthermore, our results do not differ from the mainstream economics, once we are stating that policymaker hardening on pursuing the inflation target results in a faster convergence to the equilibrium in which both players are investing and concurrently reduces investments costs in $\underline{h}$ and $\underline{k}$. In this sense, an important extension to the model for future researches is to consider the accumulation of physical and human capital in the economy. Settling these issues is not a trivial task from the evolutionary game theory perspective, but it expands the power of analysis and understanding of the ideal conditions to ensure that the inflation rate converges to the announced target inflation rate.

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