

Robert Torrens, the Quantitative Method and the General Glut Controversy

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This paper analyzes Robert Torrens's statement of Say's Law and the possible causes of economic distress based on the concept of a proportionate demand. Initially, we examine Torrens's thoughts on the best way of doing economics, showing how he opted for a full-blown deductive method to conduct his inquiries into political economy, but always anchored on a quantitative approach. Following that, his basic aggregate model for equilibrium prices and the profit rate is presented. Next, his proportional theory of Say's Law is described and articulated with a numerical illustration to exemplify how the correct supply of capital ingredients is taken by him as a necessary condition for the equilibrium of markets and also as a counterargument to Thomas R. Malthus' case for the general glut. The subsequent section considers the possible causes of a shortage in effectual demand and how it can quickly engulf the whole economy. The quantitative illustration developed by Torrens is reviewed on its effects on the transactions of the different economic sectors and synthesized in a disequilibrium matrix of exchanges. The final comments stress the originality of Torrens's theoretical contribution to the general glut controversy, marked by a peculiar numerical approach to economic theory.

Key words: classical methodology, quantitative approach, competitive equilibrium, effectual demand, general glut

O artigo analisa a formulação proposta por Robert Torrens para a Lei de Say e as possíveis causas de saturação geral com base no conceito de demanda proporcional. Inicialmente, examinam-se as ideias de Torrens relativas à melhor maneira de pensar a economia, mostrando como ele optou por um método essencialmente dedutivo para conduzir suas investigações em economia política, mas sempre apoiado num método quantitativo. A seguir, o seu modelo básico de preços de equilíbrio e da taxa de lucro é apresentado. Ainda, a sua teoria da Lei de Say baseada no conceito de proporções é descrita e articulada com uma ilustração numérica a fim de ilustrar como o correto suprimento do capital é considerado por Torrens como condição necessária ao equilíbrio dos mercados e também como contra-argumento às teses de Thomas R. Malthus sobre as crises de saturação geral. A seção seguinte considera as causas de uma deficiência localizada na demanda efetiva e como tal situação pode englobar toda a economia. O exemplo numérico desenvolvido por Torrens é revisto em seus efeitos sobre as transações dos diferentes setores e sintetizado numa matriz de trocas em desequilíbrio. Os comentários finais realçam a originalidade da contribuição teórica de Torrens ao debate sobre os mercados, marcada por um tratamento numérico peculiar da teoria econômica.

Palavras-chave: metodologia clássica, abordagem quantitativa, equilíbrio competitivo, demanda efetiva, saturação geral

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1. Introduction

Reconstructions of pioneer inroads into mathematical economics during the early 19th century have found that those attempts consisted mainly in applying calculus, basic algebra or geometry to problems such as gambling, the optimal amount of taxation, or even the proper balance among productive sectors (Theoharis 1993: 3–18; Robertson 1949; Jevons 1879: x–1). The group of intellectuals organized around William Whewell (1794–1866) at Cambridge, for instance, sought to scrutinize many aspects of Ricardian economics via mathematical and inductive methods (Theoharis 1993: 104–47; Henderson 1985; Strong 1955). This specific way of doing science conformed better to the Newtonian tradition of empirical knowledge that had been nurturing many British classical economists, at least until David Ricardo entered the scene. After the Napoleonic Wars, however, the prospect of building a completely deductive political economy seemed more promising than Whewell's or Thomas Robert Malthus's contrary claims, given the scarcity of reliable economic data and adequate instruments to extract relevant information from them.¹

One significant event on this score is that Ricardo was not by himself in his endeavor to create a purely abstract political economy bent on numerical analysis.² At the same time, Colonel Robert Torrens was developing a similar procedure with even greater conviction in his writings on economic theory. In fact, Torrens managed to elaborate a process of numerical determination of competitive prices and the profit rate for various branches of economic activity that would be proven immensely valuable in sharpening his arguments on several theoretical issues. Furthermore, he tirelessly pursued the logical implications of his elucubrations to the point of challenging his peers with unparalleled confidence. The fortunate aspect of his combative personality lies in the circumstance that Torrens, amid many controversies, ended up originating fundamental propositions that marked his work as unique among the constellation of classical economists. One of the most notable results of his mental exertions was the idea of a proportionate growth of the elements of capital as a necessary condition for the prevalence of Say's Law.

Notwithstanding the able accounts of Torrens's wavering thoughts on the equilibrium of aggregate markets (Kates 1998: 36–40; Sowell 1972: 129–31; Corry 1962: 99–100), they neglect the specificity of his methodological stance, and even the direct connection of his strictures with his peculiar quantitative approach to economic theory. This article seeks to offer a more comprehensive treatment of these coexisting aspects of Torrens's analytical work. For such, the following section deals with Torrens's deductive methodological standpoint. Section three presents his quantitative approach to economic theory. Section four links the arguments set forth in the previous sections with Torrens's views on Say's Law and effectual demand. Lastly, section five goes through his diagnostic

¹ Malthus was not an outright critic of deductive methods, although he indicated the limitations of Ricardian economics due to the erratic behavior of human passions, the risk of oversimplification in the study of complex phenomena, and the existence of exceptions to general laws in social sciences. Malthus also disagreed with the inductivists due to their unwillingness to follow a middle-ground between the two extreme positions (Cremaschi and Dascal 1998; Redman 1993: 259–320; De Marchi and Sturges 1973).

² Ricardo's contribution has never been taken as a concerted effort at expressing economic truths in mathematical terms, despite modern translations of it into a system of simultaneous equations (Kurz 2015; Garegnani 1987; Pasinetti 1960; for a critical assessment of such approach, see Peach 2009: 277–86). But in the *Principles of Political Economy and Taxation*, most notably in its first and second editions, one finds several numerical cases conceived to offer the reader a quantitative path across a rather sinuous theory of exchangeable-values (Ricardo [1817–21] 1951: 56–63; Cochrane 1970; Wilson & Pate 1968; St. Clair 1965: 20–35).

of the general glut and the corresponding numerical illustration provided by him to buttress his case. The closing remarks stress the originality of Torrens's so-called illustrative approach to classical economics.

2. Torrens's methodology of economics

Biographical sketches of Torrens's career at the Royal Marines, the infantry branch of the British Royal Navy, are few and revolve around either his brave repelling of a Danish invasion of the Anholt island in 1811 or his longwinded quarrel with superiors over his rank and pay (Moore 2004; Fetter 1990, 1962). Torrens entered service as an ensign of the Royal Navy in February 1796, at 16 years of age. He was promoted to first lieutenant by the end of the following year, to Captain in 1806, and finally deployed to Anholt in 1810. He would fight other great battles during the Napoleonic Wars, retiring from the Royal Marines in 1834 as a major. Frank W. Fetter (1962) has argued that Torrens's military mind strongly influenced his economic policy propositions since he was always ready to prioritize Britain's interests on issues involving other nations, contrary to his contemporary free traders. However, there is another side of Torrens's military life worth of attention and related to the development of his intellectual skills during his initial years in active duty.

In 1677, a Royal Proclamation established a mandatory examination for promotion to lieutenantcy within the Royal Navy, when the young candidates should confirm their proficiency in seamanship and navigation. The usual manner of obtaining adequate learning to the exam during the 18th century was through teachers in small private navigation schools or sea-going instructors. An Admiralty Order of Council from 1702 set a modest payment to anyone proven able and willing to be a schoolmaster at sea in charge of educating young gentlemen in the theoretical and practical aspects of navigation. The scheme, however, seems to have been somehow deficient since the schoolmasters had to continuously scramble for teaching time amid the sailors' busy daily routine. In 1733 the Admiralty inaugurated the Portsmouth Naval Academy, an institution composed of highly qualified and well-paid headmasters and staff. Despite the substantial investment, the establishment never reached a satisfactory number of students, probably due to the long-held belief among experienced officers that the right way of learning about navigation was at sea (Scott 2011: 54–72). This understanding finally prevailed, and the Academy was shut down in 1837. Of interest here is the circumstance that, together with military and navigation skills, the program of studies at the institution displayed a robust mathematical content. As observed by naval historian Harry Dickinson:

The student's progress was recorded in an individual Plan of Learning, a sizeable and heavy notebook, extensively illustrated and sub divided into the sections of the syllabus. Fortunately a number of these have survived and provide a detailed record of the subjects followed, namely: arithmetic, geometry, spherical geometry, trigonometry, geography, astronomy, navigation, fortification and gunnery. In a typical Plan of Learning each subject merited a separate chapter and each of these followed a similar pattern starting with a statement of relative definitions and then proceeding rather laboriously through a list of worked examples (Dickinson 2007: 35).

Most likely, Torrens did not attend the Academy, for there are indications he was often boarded during his first years at the Navy (Moore 2004). But schoolmasters' manuals from the 18th century covered the same ground as the Academy. Among the most famous, Mango Murray's *The Rudiments of Navigation*, published in 1760, was made up of six chapters, comprising, respectively: proportions and logarithms; geometry and trigonometry;

trigonometrical and logarithmical tables; spherical geometry; Mercator's chart and the measurement of distances; and lastly, astronomy and the use of David's quadrant. The book is compact but filled with numerical illustrations and many rather intricate problems. Perhaps the most serious task for a young officer was the so-called "working the day's work", that is, calculating the course and distance sailed each day by the ship (Murray 1760: 103–11).³ One way or another, via the Academy or sea-going masters, we can safely assume that, in his early years in the Navy, Torrens got very much acquainted with arithmetic, proportions, trigonometry, logarithms and geometry, as well with the resolution of generic problems on these fields related to nautical abilities.⁴

There are not many writings by Torrens on methodological issues. In the short preface to his 1821 *Essay on the Production of Wealth*, he advocates the adoption of a mixed methodology for economics, something in between the Ricardian and Malthusian methods, both of which he judged as incomplete. Torrens felt that Ricardo was always prone to generalize without empirical support, whereas Malthus pushed his reflections all the way to the opposite end, conferring excessive weight to particulars without putting forth any trace of an abstract theory (Torrens 1821: iii–iv). What Torrens understood as the essential empirical touch to economic analysis at this stage of his career was, however, far from satisfying more sophisticated philosophers in search of a real-world basis for science. For Torrens, empirical support meant exactly numerical illustrations of abstract thoughts, and he did not spare the reader from going through a plethora of numbers over his 1821 book. When addressing complaints about his characteristic excessive appeal to quantitative arguments, which he called "demonstrative", the answer came as follows:

The science of Political Economy is analogous to the mixed mathematics. The data upon which it proceeds are furnished by observation and experience; while the conclusions to which it leads, are attained by a process of ratiocination self-evident in all its steps. To give this science, therefore, the exactness and certainty of which it is susceptible, it must be presented under the analytical and demonstrative form (Torrens 1821: x).

Over time, Torrens's thoughts about economic methodology definitely leaned to the abstract side after 1821. In an extensive introduction to his voluminous *The Budget*, published in 1844, one finds an interesting digression on what should be the most appropriate nature of economic reasoning. The book consists of several letters to politicians composed in previous years, and its chief goal is to prove, via a highly hypothetical trade pattern between England and Cuba, that reciprocity in tariffs, instead of unilateral free trade, would be the optimal commercial policy for the British Empire. According to Torrens, Ricardo's theory of exchangeable values was conceived to explain domestic trade, where capital and labor were fully transferable among economic sectors to equalize the profit rates when changes in supply and demand, production costs, and taxes are observed. That, however, is hardly the case when the analysis moves to an international dimension and there is little mobility of people and capital across national boundaries (Torrens 1844: v–viii).

³ Other 18th century manuals from well-known schoolmasters such as William Jones's 1702 *A New Compendium on the Whole Art of Practical Navigation* and John Collier's 1729 *Compendium Artis Nauticae* have contents equivalent to Murray's.

⁴ How much importance Torrens attached to military education can be inferred by a letter from James Mill to Ricardo, dated October 26, 1818 (Ricardo [1816–18] 1952: 313–18), where it is reported that Torrens was deeply worried about one of his sons being "too idle to get knowledge enough to pass" the exam at the Royal Military College, Bagshot. James Mill had enrolled the young John Mill there to attend a two-week course on Chemistry.

As the conditions of supply and demand in international trade are constantly changing, says Torrens, conclusions about the most suitable commercial policy can be found either by inductive or deductive methods. When following the former one, the economist will look over the statistics of trade, for instance, between French wine and British yarns, and find that the imposition or remission of duties affects the quantities and the exchangeable values of both commodities. On the one hand, such conclusion does not specify the several channels whereby these effects will manifest themselves; on the other, no information is gained about the intensity that the new tariff will have over trade and values. These limitations, though, do not apply to the deductive method. Its key advantage, argues Torrens, comes from the possibility of estimating the exact consequences personal interest will have upon producers and costumers after, for instance, the imposition of a duty. Furthermore, the relationship thus obtained would be valid not only in abstract, but also in the factual world if the economist were prudent enough to introduce modified assumptions to conform one's theory to alternative scenarios. "Hypothesis is to economical science what experiment is to physical science" (Torrens 1844: 1).

In other words, Torrens is arguing that by purely mental operations – by changing the premises –, one can build abstract experiments at will in political economy. From this perspective, the outcome of every cause acting upon an economic phenomenon could be isolated much more precisely than by the induction method, since multiple forces are always concurrently exercising their influence over social events. Historical and empirical critics of Ricardian economics were all in the wrong, claims Torrens. They had mistakenly expected from Ricardo's theory of profits a conformity with an environment distinct from the one he based his predictions on, but without ever conceding the necessity of adjusting their own assumptions to fit the changed surroundings they were working with. Once this provision is accepted, observes Torrens (1844: xxxviii) with a touch of irony, all economists could be considered Ricardian. Thus, the only way through the maze of ever-changing conditions in the social world, e.g., when dealing with the determinants of supply and demand in political economy, was to boldly ratiocinate in terms of fixed quantities as the best approximation to a real situation.

The mathematician, finding no perfect lines, squares, and circles in nature, is obliged to assume their existence as the basis of his reasonings; and, in like manner, the Political Economist, finding that in the actual circumstances of society the quantities of commodities offered in exchange for each other are in a constant state of fluctuation, is obliged to assume the existence of definite quantities, in order to arrive at definite conclusions (Torrens 1844: xlvii).

But Torrens does not intend to find general propositions on political economy and, consequently, on economic policy, either by the development of pure syllogisms or by means of functional relations or algebraic equations. Instead, he chooses the application of what he calls "illustrative cases", framed in terms of numerical examples, to achieve progress in economic knowledge. How to leave from one specific point and get to another, that is, a procedure akin to working the day's work is how Torrens handles in a single process theoretical and, therefore, practical problems of economic nature. Thus, having seen how he conceived the best route to make progress in political economy, we may now proceed with the exam of his quantitative framework to deal with issues related to prices, profits and capital accumulation.

3. A basic model of general equilibrium

The main weapon employed by Torrens in his disquisitions on value and capital accumulation consisted in his circular model of the economy, where both prices and the profit rate are simultaneously determined for a certain productive structure. The foundation for this approach lies in Adam Smith's analysis of how the division of labor works within a market-oriented society, articulating several interdependent productive branches in a single whole (Torrens 1821: iii, 20, 38). It did take Torrens some time and effort to master the technique, but once he got there, he did not spare ammunition. The first attempt already appears in his anonymously published article on "Mr. Owen's Plan for Relieving the National Distress", which despite having its authorship questioned by some scholars, can be safely assigned to Torrens.⁵ The reason lies on the numerical model presented in the article, still in very crude form, wherewith the profit rate is directly calculated as a ratio between the material expenditures on production as advances to workers and the final output in terms of corn and clothing. As the surplus of both commodities bear the same proportion to their total respective input in both sectors, no prices are needed to ascertain the common profit rate as a physical quotient. Although still crude, this approach immediately allows Torrens to specify the determinants of the overall profit rate as, first, the quality of the soil under cultivation; second, the efficiency of labor in both agriculture and the manufactures, and, lastly, the real wage paid as advances to laborers (Torrens 1819: 454–57).

The following year, in the second and very much enlarged edition of his *Essay on the External Corn Trade*, firstly published in 1815, Torrens admits the existence of an opposition between wages and profits, which he had not thoroughly considered previously, alerting the reader in the preface that Ricardo's theory now pervades the whole book (Torrens 1820: xx). What the added pages show is that Torrens had now successfully managed to calculate competitive prices for his productive structure even under the more complicated case when the physical surplus is different among the productive sectors. He clearly feels very much enthusiastic about this breakthrough and reels off 24 illustrations of his model (Torrens 1820: 350–412). Although quite novel and thought-provoking from a purely analytical standpoint, the whole argument is in no small measure hard to follow because of the unfortunate descriptive and tiresome form of presentation adopted by Torrens, which quickly bores even the most sympathetic reader. Regardless, a single example will be enough to provide the drift of his quantitative views on value and capital accumulation.

Consider x_{ij} as the quantity of commodity i used as an input, in the form of advances to labor, and j the respective sector of production (denoted by S_j); y_{ij} is also given and represents the output of commodity i in sector j ; p_i is unknown and denotes the price of commodity i , whereas r is also unknown and represents the common profit rate, with $i, j = 1, 2, \dots, n$ during an ordinary production cycle.⁶ Competition prevails all around and each

⁵ The authorship of the *Edinburgh Review* article is a matter of contention. Dennis O'Brien & Adrian Darnell (1980), Peter Gronewegen (1993: vi, xvii–xix) and others attributed the work to Torrens. Jacob Viner (1937: 194) thought it was written by John Ramsey McCulloch. William Thweatt (1980) proposed joint authorship. The controversy reached a resolution of sorts with evidence offered by Giancarlo de Vivo in a passage written by McCulloch in *The Scotsman*: "The greater part of a speech made by Colonel Torrens at a public meeting in London is incorporated in the excellent article on Mr. Owen's plans, furnished by that gentleman for the 64th number of the Edinburgh Review" (quoted from De Vivo 2000: vi).

⁶ When fixed capital is introduced as a component x_{ij} , its residual value at the end of the productive cycle may be added to its respective output $p_i y_{ij}$: "[W]hen capitals equal in amount, but of different degrees of durability, are employed, the articles produced, together with the residue of capital, in one occupation, will be equal in

sector produces a single y_{jj} good. Then, Torrens's system may be represented by the following array of equations:

$$\left. \begin{aligned} (p_1x_{11} + p_2x_{21} + \dots + p_nx_{n1})(1 + r) &= p_1y_{11} \\ (p_1x_{12} + p_2x_{22} + \dots + p_nx_{n2})(1 + r) &= p_2y_{22} \\ &\vdots \\ (p_1x_{1n} + p_2x_{2n} + \dots + p_nx_{nn})(1 + r) &= p_ny_{nn} \end{aligned} \right\} \quad (1)$$

Torrens never laid out his examples in a mathematical setting like above, but it will be shown that system (1) is what he really had in mind when talking about prices and Say's Law.⁷ At this point we should stress that this intellectual construction must be considered a real landmark in British classical economics, even in view of the scanty attention it received at the time. Such structure is indeed a nonlinear system with n equations, one for each of the i commodities produced in sector j , and $n + 1$ unknowns, namely, the n prices p_i and the profit rate r . However, once that has been recognized, a huge obstacle surfaces immediately since there is no established method for solving this kind of system. How did Torrens get around this difficulty? Before delving into that, it is instructive to go through a quantitative example included in the second edition of his *External Corn Trade* to have a sample of how the system worked in its creator's hands.

With Torrens, we shall consider a two-sector model in which agriculture, sector S_1 , uses 50 suits and 100 quarters of corn as advances to wages, obtaining a harvest of 250 quarters of corn from that expenditure. In turn, manufactures – sector S_2 – advance 50 suits and 100 quarters of corn as wages to its laborers to produce 150 suits. A tabular representation of the situation can be registered as follows.

Table 1. Torrens's equilibrium example

Sectors	S_j	Inputs x_{ij}		Output y_{ij}		Sectoral surplus $\Sigma_j y_{ij} - \Sigma_j x_{ij}$	Prices p_j	Profit rate $r \times 100$
		Corn $i = 1$	Clothing $i = 2$	Corn $i = 1$	Clothing $i = 2$			
Corn	S_1	100	50	250	0	50	1.00	38%
Clothing	S_2	100	50	0	150	50	1.66	38%
Total		200	100	250	150			

Source: Torrens (1820: 399–401). The exact profit rate is 36.4%, which Torrens calculated as 38% due to his rounding the price of suits to 1.6.

exchangeable value to the things produced, and the residue of capital, in another occupation" (Torrens 1821: 29).

⁷ In modern matrix notation, system (1) may be rewritten by $(1 + r)\mathbf{px} = \mathbf{py}$, where $\mathbf{p} = (p_1 \dots p_n)$, $\mathbf{x} = \begin{pmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \dots & x_{nn} \end{pmatrix}$, and $\mathbf{y} = \begin{pmatrix} y_{11} & & 0 \\ & \ddots & \\ 0 & & y_{nn} \end{pmatrix}$. Torrens's system is formally equivalent to the one developed by Piero Sraffa (1960: 5–6).

The composition of the final output is enough to ensure that the economy can reproduce itself in an enlarged scale. A purely physical calculation of the profit rate, however, is impossible, since the surplus of corn over its total consumption is $(50 \div 200) \times 100 = 25\%$, whereas the same ratio for suits equals $(50 \div 100) \times 100 = 50\%$. Therefore, a different procedure must be devised. What is needed at this point is a pair of prices that permit the cost of inputs to be set against the total revenue for each kind of output in such a way that the profit rate results are identical for both sectors by the utter pressure of competition. The years Torrens spent solving generic mathematical problems in the Navy are now quite useful since he ingeniously defines identical physical costs for both corn raising and clothing manufacturing, so that their total capital expenditure is necessarily the same for any possible combination of prices. This fundamental problem-solving artifice also elicits an economic principle that Torrens rapidly introduces against Ricardo's labor theory of value, stating that capitals of equal amount invested during the same time, under a regime of overall competition, must yield the same profit rate.⁸ As Torrens remarks about the workings of capitalistic markets in the third edition of the *External Corn Trade*:

In this state of society the competition of capitalists will have a constant tendency to equalise the rate of profit; and it is a necessary consequence of the equality of profits, that the products of equal capitals, employed for equal times, should be of equal value. But the products of equal capitals are scarcely ever the products of equal quantities of labour; and consequently, as the quantity of producing labour is no longer the circumstance on which competition turns, it ceases to be a regulating cause of value (Torrens 1826: 58).

Torrens thus understands competition as the full mobility of capital and labor over the economy to explore eventual differences in profit rates. Returning to our example and following the principle of a common profit rate for capitals of equal magnitude, all economic sectors must have identical sales revenues, so an additional equation for system (1) comes into play in the form $p_1y_{11} = p_2y_{22} = \dots = p_ny_{nn}$ whenever the technical combination among the different ingredients of capital is alike for the existing branches of production. By assuming any price as numéraire, one can easily calculate the remaining ones and, by extension, the profit rate. This is precisely what Torrens did to solve the above illustration. Then, if the price of corn is taken as $p_1 = 1.00$, the price of each suit will be equivalent, in terms of quarters of corn, to $p_2 = 1.66$, providing a common profit rate of $r \times 100 = 36.4\%$ when both costs and total revenues are measured in quarters of corn (Torrens 1820: 399–401; refer to the observation in Table 1).

In addition to his tenacity to parade one numerical illustration after another in the second edition of his *External Corn Trade*, Torrens applies with advantage his system to several theoretical questions such as the invariable measure of value, the effect of the productivity of agriculture on the profit rate, and the convenience of free trade. As these issues have already been dealt with in previous occasions (Arthmar 2014; Hisamatsu 2009; see also Noguchi 1986), our intent in the remaining sections is to analyze how Torrens applied his

⁸ De Vivo (1985, 2001) and Samuel Hollander (1998) have engaged in a debate over the true nature of Torrens's theory of profits, the former maintaining that it consisted in a pure corn-ratio model, and the latter defending that it assumed the heterogeneity of capital in agriculture. Although Hollander's view seems more in line with Torrens's conception, none of them acknowledged that a system of equations underpinned the multiple numerical examples formulated in the second edition of the *External Corn Trade*. This limitation renders their arguments about Torrens's theory of profits somewhat flimsy. De Vivo fails to perceive that Torrens uses physical illustrations of the profit rate just for simplicity of exposition, keeping the proportion between corn and manufactured goods constant to ease up calculations in most cases, although that is not always true. Regarding Hollander's observation, the mixed composition of capital goods is an assumption by Torrens not only for agriculture, but for all other economic sectors as well.

system of aggregate production to explain not only the prevalence of Say's Law but also the events that could push the economy towards a general glut.

4. Effectual demand and Say's Law

In 1821 Torrens published his *Production of Wealth*, in which he methodically employs his analytical system of the aggregate economy, advising the reader right away about the originality of his theoretical reflections on Say's Law, which would not be found in either Jean-Baptiste Say's or James Mill's writings (Torrens 1821: vi–ix). Torrens's first step is to define capital essentially in a physical way, or as that collection of things which, under a system of general division of labor and reciprocal exchanges, are used as inputs in the production of articles of utility. But in a self-reproducing system, he explains, the sustainability of the whole economy depends on the critical condition that each ingredient of capital x_{ij} is produced in such quantity y_{ij} capable to replace with a surplus the whole amount $\sum_{j=1}^n x_{ij}$ absorbed by all economic sectors during the ongoing productive cycle. And such theoretical condition, albeit derived in abstract and independently of actual prices, has notwithstanding an immediate concrete appeal, as Torrens is always ready to cross the narrow bridge between mental abstractions and reality:

The great practical problem in economical science is, so to proportion production that supply and demand shall be in the relation of equality; or, to express the same thing in particular rather than in general terms, that the quantity of the ingredients of capital brought to market to exchange against other commodities, shall be equal at the least to the quantity of these ingredients expended on the other commodities. So long as this proportion is preserved, every article which the industrious classes have the will and power to produce, will find a ready and a profitable vend (Torrens 1821: 370).

Torrens thus defines supply and demand in a very particular way, as the adequate provision of the ingredients of capital necessary to keep the whole economy operating without disturbances and in a harmonic state of self-reproduction, or even at an amplified scale at each new productive cycle. As long as corn, clothing, and all other items consumed by the laborers, as well as the raw materials and elements of fixed capital, are all supplied in adequate amount and proportionate to each other – as required by the prevailing division of labor, the efficiency of production and the level of real wages –, there would be no objective reason why a wide-reaching shortage of demand might ever occur. If the self-sufficiency and proportionality conditions are achieved, the whole economy can thus expand itself continuously and every commodity would automatically open a market for another one, as predicted by Say's Law.⁹ Were it not for the limited availability of fertile lands and population, observes Torrens, that tend to raise wages and squeeze the profit rate until it falls below that barely minimum required by capitalists for the continuance of capital accumulation, the process could go on unimpeded while effectual demand would never be missing.

For every article brought to market there would be a profitable vend. Each class would find that a part of the things it produced would replace the whole of the things it expended in

⁹ In his *Commerce Defended*, James Mill (1808: 82) had contended that since a market economy is ultimately a system of barter, once the overall assortment of commodities has been brought to the market, one half is always exchanged for the other, so that no excess supply would be ever observed. The statement, however, is framed in very general terms and presupposes an automatic conformity among the various types of commodities, something completely absent in Torrens's theory, as shown in section 5 below.

production. But this is exactly what is meant by effectual demand; and the more accurately we analyse the operations of industry and the transactions of the market, the more clearly we shall perceive, that while the due proportions are preserved between the quantity of the ingredients of capital and of other things, increased production is the one and only cause of extended demand (Torrens 1821: 377).

By appealing to his system of aggregate production, Torrens is able to quickly dismiss Malthus’s perpetual concern with the prospect of too much love of parsimony generating an overaccumulation of capital that would bring about a general glut of commodities (Malthus 1820: Chapter VII; Winch 2013: 84–95; Paglin 1961: 115–55). At this stage, introducing the illustrative case conceived by Torrens is expedient to show how even within an economy in a state of simple reproduction – i.e., without any net investment, and where all profits are expended in luxury goods – there could be a situation of mutual equilibrium among the many productive sectors when the aggregate production of capital is proportionate to the technical and social conditions of the economy. But, before that, since luxury goods for capitalists do not enter the productive process as elements of capital, Torrens correctly advises the reader that their supply does not play any role in the determination of effectual demand.¹⁰

No increased production of silks, for example, can give rise to an increased effectual demand either for muslins or for corn. The reason is obvious. In fabricating muslin or in raising corn, a great variety of articles of capital, such as food and clothing, material and implements, must be expended, and these ingredients of capital no supply of silks or of other superfluities can replace. Whatever may be the quantities of the ingredients of capital expended in production, they can be replaced only by the same quantities of themselves (Torrens 1821: 350).

Consider a hundred cultivators of corn in sector S_1 , a hundred manufacturers of clothing in S_2 , another hundred cultivators of sugar and tobacco in S_3 , and another hundred manufacturers of ribbons and lace in S_4 . Each group of producers S_j uses as ingredients of capital 100 quarters of corn and 100 suits as advances to their laborers. The final output is, respectively: $y_{11} = 400$ quarters of corn; $y_{22} = 400$ suits, $y_{33} =$ an indefinite amount of sugar and tobacco (thenceforward sugar only); and lastly, $y_{44} =$ another indefinite amount of ribbons and lace (thenceforward lace only). With such numerical configuration, Torrens can develop his reasoning without having to find out equilibrium prices, and such quantitative system can be presented as follows:

Table 2. Torrens’ equilibrium example: Say’s Law

Sectors	S_j	Inputs x_{ij}				Output y_{ij}			
		Corn $i = 1$	Clothing $i = 2$	Sugar $i = 3$	Lace $i = 4$	Corn $i = 1$	Clothing $i = 2$	Sugar $i = 3$	Lace $i = 4$
Corn	S_1	100	100	0	0	400	0	0	0
Clothing	S_2	100	100	0	0	0	400	0	0
Sugar	S_3	100	100	0	0	0	0	y_{33}	0
Lace	S_4	100	100	0	0	0	0	0	y_{44}

¹⁰ The necessities, corn and clothing, are Sraffian basic commodities, while the luxuries, sugar and lace, are non-basic commodities.

Total	400	400	0	0	400	400	y_{33}	y_{44}
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Source: Torrens (1820: 375–78).

The pattern of exchanges or the operation of effectual demand, in Torrens’s description, would play out, firstly, by capitalists in S_1 giving up 100 quarters of corn for 100 suits from S_2 ; another 100 quarters of corn for a fourth part of sugar, i.e., $y_{22}/4$ units from S_3 ; and 100 quarters of corn for a fourth part of lace, or $y_{44}/4$ units from S_4 . The final collection of goods accruing to the capitalists in S_1 will be: 100 quarters of corn; 100 suits; $y_{22}/4$ units of sugar, and $y_{44}/4$ units of lace. It is easily verifiable that this composition will be reproduced for the capitalists of the remaining sectors. The upshot is that all profits, whatever their actual rate to capital disbursements and without even having to calculate their respective prices, will be fully spent in y_{22} and y_{44} , while each group of capitalists will have back in their hands the required amount of corn and suits to start the productive cycle all over again at the previous scale. For Torrens, if the profit rate stays above the bare minimum required to keep the productive process going on, and the supply of material capital is duly proportionate among its several items, as depicted in the illustration above, no problem of effectual demand would ever occur. Such observation is precisely what Torrens considered as his personal contribution to enrich the meaning of Say’s Law in classical economics (Torrens 1821: 341, 349, 352, 362, 371).¹¹

Having expressed the necessary condition for aggregate equilibrium, Torrens then begins to perform the mental experiments mentioned in section two by contemplating alternative scenarios for the economy. Firstly, he does admit of the possibility of a quick process of capital accumulation creating an oversupply of commodities and a deficiency in effectual demand – as insisted upon by Malthus –, but only if the growing need of labor occasions an increase in wages that overtakes the supply of ingredients of capital available to continue the productive process at an enlarged scale. Nevertheless, the mismatch between the demand and supply of labor would be self-correcting, since the consequent lowering of the profit rate would ease the passion for accumulating capital, and a bigger fraction of profits would then be spent back in those luxuries capitalists used to indulge themselves on before the desire for parsimony got stronger (Torrens 1821: 389–93).

In addition, if the love of ease prevails instead of accumulating capital, which was another possible cause for a lack of demand raised by Malthus (1820: 358–59), then manufacturing and cultivation would contract to a minimum, just enough to allow capitalists, landlords and workers to survive. A situation this extreme, however, could very well be attained with a due proportion being observed among the elements of capital so that no engorgement or glut would necessarily come about. Or, in Torrens’s (1821: 397) own words: “Vary our suppositions as we will, increased production, provided it be duly proportioned, is the one and only cause of extended demand, and diminished production the one and only cause of contracted demand”. More revealing, though, is his belief that when

¹¹ For the sake of historical accuracy, the first instance on which Torrens adds this proviso occurs in the mentioned 1819 *Edinburgh Review* article, when he talks about the effects of the introduction of machinery in production which, according to him, had been wrongly criticized by Robert Owen: “The alterations which occasionally take place in the distribution of industry, may lead to temporary embarrassment; but *after the readjustment has been effected, and commodities are brought to market in quantities duly proportioned to each other*, the increased supply will be accompanied by increased demand” (Torrens 1819: 472; italics added).

the markets are left alone to adjust by themselves, they would reach the best possible distribution of capital and labor by the pressure of competition on people and capital, thus, restoring the equilibrium of the whole economy.

Thus, in every conceivable case, a redundant supply of a commodity occasioned by an improvement in the productive powers of industry, after occasioning temporary embarrassment and distress, would be succeeded by a rectifying process, and would terminate in conferring solid advantages upon the society at large (Torrens 1821: 417).

We shall now inspect how Torrens articulated his explanation for the general glut, which many voices at the time proclaimed to be the effective state of the British economy during the years following Waterloo.

5. The general glut and its progression

In the *Production of Wealth*, Torrens does not challenge the contention that economic crises do exist. In fact, he remarks that even the most complex theory, if not in conformity with the facts of social life, should be discarded without further ado. And his own disquisitions on the working of the markets, he announces with his usual self-assurance, should be no exception to such methodological rule.

If the theory of effectual demand, which I have ventured to unfold, does not explain in a satisfactory manner that overstocking of the market, and want of profitable vend for commodities, the existence of which is matter of general experience, I am ready to admit that such theory must be essentially defective and incorrect (Torrens 1821: 399).

Being true to his previous exposition, Torrens (1821: 400) then introduces two reasons for the occurrence of a general glut within a competitive economy: first, the miscalculations of some producers, who bring to the market an amount of certain ingredients of capital different from that one the consumers and remaining producers are willing to trade for their own commodities; and second, the fluctuation in harvests, which time and again throw upon the market a quantity of corn and other agricultural commodities either above or below its adequate proportion with the existing supply of the other elements of capital. But the task of explaining how a general glut or engorgement develops is now somewhat more difficult, since Torrens is forced to tread into the treacherous terrain of economic disequilibrium, through which effectual demand, prices, quantities and the profit rate are in disarray and undergoing drastic changes over time. But his characteristic tenacity does not fail the former Royal Navy Colonel, and Torrens begins a long digression on how a bountiful harvest could bring down the whole economy in its path.

Since for Torrens, as seen, an explanation would only be complete when anchored on a numerical illustration, an accompanying and sophisticated example is elaborated, in a situation which unfolds itself over successive production periods: t , $t + 1$ and $t + 2$. And to make matters even more involved, he introduces money as the mean of exchange into a hypothetical economy like the one presented in the previous section. But along with the conditions already specified, each sector is supposed to own now a sum of £100 at the beginning of the productive cycle and, under equilibrium, end the cycle with the same amount in its account after all exchanges have been processed. Since the case under consideration is one of simple reproduction, corn and clothing have their unitary price at the initial period t set as $p_{1,t} = p_{3,t} = 1.0$, while each sector starts off with a capital of £300 (corn, clothing and cash) and closes its operations with the same £300 plus £200 in luxury

goods. The common profit rate, hence, is $r_t \times 100 = \left(\frac{500}{300} - 1\right) \times 100 = 66.6\%$ for all capitals at the end of the productive cycle, as indicated in Table 3.1.

Table 3.1. Matrix of equilibrium in t (money values)

Sectors	S_j	S_1	S_2	S_3	S_4	Revenue (Demand)	Yield (Supply)	Cash	Unsold goods	Profit rate
Corn	S_1	100	100	100	100	400	400	100	0	66.6%
Clothing	S_2	100	100	100	100	400	400	100	0	66.6%
Sugar	S_3	100	100	100	100	400	400	100	0	66.6%
Lace	S_4	100	100	100	100	400	400	100	0	66.6%
Expenditure		400	400	400	400	1,600	1,600	400	0	

Source: Torrens (1821: 403–4). For the sake of simplification, we assume $y_{33} = y_{44} = 400$ units.

The state of general equilibrium, however, undergoes a commotion once the usual corn crop, due to exceptional favorable circumstances, goes up to 500 quarters for the same advances in period $t + 1$, marking the first step in the march of the general glut. We must then inspect how the economic disequilibrium surfaces in Torrens's thought-experiment. The first consequence of such bountiful harvest would be, according to his supposition, a reduction in half of the money price of corn due to the one-fourth increase in supply, so that $p_{1,t+1} = 0.5$ -pound per quarter. But as the demand for corn is supposedly inelastic, Torrens continues, consumers will buy the same quantity as before, although at a very lowered expense.¹² From the clothiers, corn farmers purchase £100 in clothing, but receive back now just £50 for 100 quarters of corn. The reproduction of both sectors, despite the asymmetry in prices, is thus materially assured at this second productive stage. But from sugar growers, corn farmers receive only £50 from 100 quarters of corn and are thus constrained to reducing their consumption of these commodities to the same amount of money received. Identical situation takes place in the exchanges between corn farmers and lace manufacturers.

The market transactions under disequilibrium during period $t + 1$ by producers in S_1 , as conceived by Torrens, are registered in the matrix below, over the first row (revenue) and the first column (expenditure) of Table 3.2. As total expenditures of £250 surpasses the corn farmers' total revenue of £200, a £50 reduction in their cash holdings takes place in detriment of the clothiers, as already indicated. Having thus began the productive cycle with £300 capital in value, corn farmers finish their operations owning £50 in intra-sectoral trade in corn, £100 in clothing, £100 in luxury goods, and £50 in cash, for £300 in total, but still having £50 in unsold corn. Consequently, their profit rate for the second productive period is $r_{1,t+1} = 0$ (Torrens 1821: 403–6).

¹² It could be argued that corn growers lower their price in expectation of selling an additional quantity of their larger stocks, but that this does not materialize because the demand for food depends also on the supply of the other ingredients of capital which, by assumption, have not suffered any increase.

Table 3.2. Matrix of disequilibrium in $t + 1$ (money values)

Sectors	S_j	S_1	S_2	S_3	S_4	Revenue (Demand)	Yield (Supply)	Cash	Unsold goods	Profit rate
Corn	S_1	50	50	50	50	200	250	50	50	0.0%
Clothing	S_2	100	100	100	100	400	400	150	0	66.6%
Sugar	S_3	50	100	71	71	292	332	100	40	30.6%
Lace	S_4	50	100	71	71	292	332	100	40	30.6%
Expenditure		250	350	294	294	1,186	1,314	400	130	

Source: Torrens (1821: 403–8). Some rounding has been done to facilitate presentation. The transactions between S_3 and S_4 , as well as the profit rates, have been estimated from Torrens's own figures.

For the clothiers in S_2 , however, the situation is, at first, the most convenient, for they can now buy their 100 quarters of corn for only £50 and sell their clothing to corn farmers for £100. Thus, the clothiers' account accumulates £150 in cash-balance. Their transactions with producers in S_3 and S_4 are not affected in period $t + 1$, they keep selling £100 in clothing to each group and buying back the same amount of luxury goods, as depicted in the corresponding row and column of Table 3.2 for S_2 . The clothiers' initial capital amounted to £300, as indicated, but now they reach the end of the productive cycle with assets worth £50 in corn, £100 in clothing, £150 in cash and £200 in luxury goods. Their profit rate thus remains unchanged at $r_{3,t+1} = 66.6\%$ during the period, with no piling up of unsold suits (Torrens 1821: 406).

Finally, as the demand for luxury goods from corn farmers has contracted in half, the price of luxury goods, according to Torrens, experiences a one-sixth drop to $p_{3,t+1} = p_{4,t+1} = 0.83$ -pound, in addition to a given amount of unsold goods of the same proportion for both sectors. Thus, the whole supply of sugar, which formerly returned £400 in sales, will now fetch $£332 = \left(1 - \frac{1}{6}\right) \times £400$ minus £40 of unsold goods at the new price, for a total revenue of just £292. From this amount, £50 comes from corn farmers, £100 from clothing manufacturers, and the remaining £142 from the intra-sectoral trade of £71, accompanied by another £71 from the trade among sugar farmers and lace producers. Identical sums, says Torrens, apply to the commercial transactions by lace producers in S_4 . At the end of the productive cycle, sugar farmers will then possess £50 in corn, another £100 in clothing, £100 in cash, £71 in sugar and £71 from the intra-sectoral trade in lace, with £40 of unsold goods, as shown in Table 3.2 for farmers in S_2 . Their profit rate plummets to $r_{2,t+1} \times 100 = \left(\frac{392}{300} - 1\right) \times 100 = 30.6\%$ during the period, the same gain obtained by lace producers.

We can easily notice that the general glut, began in the corn-growing activity, quickly spread to the luxury goods sectors, followed by falling prices, the accumulation of unsold commodities and a drop and dispersion in profit rates. The clothing producing sector, as seen, has escaped the debacle unharmed, and despite its severity, the glut is not properly general at this point. Perhaps, most importantly, all sectors manage to replace their physical capital used up during the current production cycle. To this overall picture of the economy, Torrens then adds a second abundant corn harvest in $t + 2$, raising its total supply to 600

quarters, including the unsold amount stored from the previous period. There is no need to go through the whole sequence of transactions among the distinct sectors for this third stage of the crisis to reach what Torrens sought to show. Since corn prices drop once again, its producers have their receipts further squeezed. The luxury goods sectors also undergo a similar receding in prices and profits, forcing all these producers to restrict their scale of production for lack of profitable outlets. Sooner or later, the narrowed effectual demand will hit the clothing producing sector equally as hard.¹³ According to Torrens, this is when the glut grows deeper and becomes a general phenomenon.

But as soon as the farmers, with the growers of sugar and the manufacturers of lace, ceased to be able to sell their commodities at prices which would repurchase the ingredients of capital expended in their production, they would from interest no less than necessity, employ less capital than before, and consequently a less quantity of clothing, the second great ingredient of capital, would be demanded and consumed. The class of clothiers, after a temporary prosperity, would now participate in the general stagnation. For a considerable part of their products no profitable or desirable sale could be effected (Torrens 1821: 412).

While money is part of the scenario concocted by Torrens, the crisis is truly ignited by a real factor, i.e., the disproportion among the supply of the ingredients of capital or, in terms of system (1), among the various commodities y_{jj} composing the aggregate supply. Regardless of the specific values and proportions attributed by him to changes in prices and effectual demand, Torrens believed his numerical illustration provided a useful picture of what happens in the real world since price reductions during a glut would have a more damaging effect over the moderate profit rates prevailing in the actual economy. Even something resembling a kind of cobweb oscillation could be observed during the adjustment in agriculture. For after a succession of bountiful harvests, as capital there leaves toward more profitable activities, crops might even fall below the requirements of the whole economy, creating now a reverse disproportion in effectual demand (Torrens 1821: 417–18).

Although the foundation of the general glut is located in the real economy in Torrens's theory, it does not preclude the crisis being intensified by the presence of money. Indeed, as he argues, monied capitalists would benefit the most during a deflationary episode, first, by the increased purchasing power of their cash-holdings; second, by the higher real interest rate obtained from loans, and lastly, by the intrinsic quality of money as the universal equivalent in times of distress. This last factor makes having cash in hand the main goal of business when the derangement of markets is running loose. A kind of preference for liquidity emerges, and the exchanges grow biased towards holding money instead of commodities, a factor that greatly intensifies the original drop in prices:

Hence, on every occasion of glut or general stagnation, the desire of turning goods into money is rendered more intense than the desire of turning money into goods, and the proportion in which prices will fall, will be much greater than that in which the relation between the quantity of commodities and the amount of currency will be altered (Torrens 1821: 420).

¹³ Closer inspection of Table 3.2 reveals that the contraction in the production scale of sectors S_1 , S_3 and S_4 is not even needed to get the clothiers in S_2 engulfed by the crisis. Since corn farmers in $t + 2$ have now only £50 in cash to buy £100 in clothing, if credit is not introduced into the model, that would mean a severe retrenchment in the effectual demand for the supply from producers in S_2 . Torrens believed that interest rates would get higher than profit rates in times of deflation; thus, precluding the counterbalancing action of this possible channel of adjustment. "The multiplied failures in agriculture, manufactures, and trade, would strike a panic into the holders of floating capital, and they would refuse to grant accommodation upon securities, which in more prosperous times they would be disposed to consider unobjectionable" (Torrens 1821: 425).

As a matter of practical policy, which is the final step of reasoning on economic issues for Torrens, four main initiatives were suggested to keep the markets well-adjusted and prevent the recurrence of general gluts. First, he condemns the fluctuations in corn prices and its deleterious effect on wages. Being the main component of the laborers' food, corn availability deserved special attention for its role as the key ingredient of capital. Instability in its supply would upset the whole effectual demand; thus, impairing prosperity. The end of protection and the adoption of free trade would be the best way to overcome this hindrance and keep a stable price for corn. Second, a uniform value for money would go a long way towards preserving the stability of the markets, more so in the context of Britain at the time, when a deliberate policy of monetary contraction to reevaluate the pound was being pursued by the government with the purpose of resuming gold convertibility (see Cannan 1919: vii–xlvi). Torrens is fully against the deflationary policy produced by the tightening of the money supply on the grounds that all economic contracts have already adjusted to the devalued purchasing power of money brought about by the Napoleonic Wars. Third, to facilitate the unavoidable displacement of capital from one sector to another in times of gluts, the prevailing usury laws had to be revoked, since they have been retarding the necessary transferences of capital and labor within the economy. Lastly, and perhaps the most important principle of all for Torrens, is the prudential rule of always introducing legal changes gradually, instead of replacing one law after the other. The uncertainty coming from an ever-shifting course of action would have as its only effect the confusion of producers, inducing them to err in their forecasts of the future effectual demand (Torrens 1821: 427–30).

6. Concluding remarks

Despite some claims to the contrary (Fetter 1962; Robbins 1958: v), Torrens may be considered one of the best thinkers of the classical school (Gerkhe 2016: 150–56; O'Brien 2004: 5). His works in economic theory displayed a clear appeal to pure abstraction that, albeit not expressed in full mathematical notation, had constant resource to quantitative illustrations to support his arguments on economic policy. In so doing, he was able to connect pure theory with pressing economic issues of his time, indicating what he understood as the best measures to address contemporary problems of the British economy. That was certainly the case with his treatment of Say's Law and the possibility of a general glut, which he blamed on protection of the corn trade and the efforts to resume gold convertibility through deflation.

The foregoing considerations make it clear that Torrens's implicit system of equations was the base for his concept of proportional effectual demand. Both in his 1819 and 1821 works he praised Jean-B. Say and James Mill for their advancement of the idea that supply is the true source of demand but qualified his praise by noting that they did not specify the particular conditions under which that proposition would be true. More to the point, the overall equilibrium of the aggregate economy required not only the full mobility of labor and capital, but also the proportionate supply of the ingredients of capital. By assuming a preestablished structure of production, Torrens indicated that, whenever such arrangement was violated, either by miscalculations of producers or bumper crops, the economy could go through a cumulative process of engorgement of the markets, although for reasons quite apart from those devised by Malthus or Owen.

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