Preference, Utility, and Koopmans's Contributions to Modern Macroeconomics

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Abstract

Centered around a *dynamic stochastic general equilibrium* (DSGE) framework, modern macroeconomics was built on (independent) contributions of Frank Ramsey, Tjalling Koopmans and David Cass over a time span that lasted from the end of the 1920s until mid-1960s. Known also as the *Ramsey-Cass-Koopmans model*, a baseline theoretical structure of DSGE Macroeconomics, its central methodological device is an infinitely lived individual called the representative agent (consumer). The article examines the rise of that device in the 1950s-1970s period in the contributions to intertemporal economics by Tjalling Koopmans. In the first decade, the 1950s, Koopmans was an important figure in the Cowles Commission's (Foundation's) incursion into decision theory and, as an econometrician, an active participant in the debate on aggregation problems in economics. In the 1960s, Koopmans wrote the bulk of his contributions in the field of infinite horizon economies (including his famous optimal growth model) and it is in this decade that he fully articulated his views on the representative consumer. Finally, in the 1970s, Koopmans changed his research aim and began to contribute to the preference-based approach to individual decision-making, a move that led to a certain specification of the additively separable utility functions. Over these three decades, Koopmans went from an ambiguous stance toward the representative consumer to a more supportive one. I argue that part of this change is due to the ever increasing use of that device in macroeconomics, a movement that Koopmans did not initiate but helped intensify.

Keywords: Representative Agent, Aggregation Problems, Tjalling Koopmans

Resumo

Centrado em um arcabouço dinâmico estocástico de equilíbrio geral (DSGE), a macroeconomia moderna foi construída com contribuições (independentes) de Frank Ramsey, Tjalling Koopmans e David Cass em um período que durou do fim dos anos 1920s a meados dos anos 1960s. Conhecido também como modelo *Ramsey-Cass-Koopmans*, uma estrutura teórica base da macroeconomia DSGE, seu instrumento metodológico central é o agente (consumidor) representativo. Esse artigo examina o surgimento do agente representativo nos trabalhos sobre economia intertemporal de Tjaling Koopmans. Na primeira década, de 1950, Koopmans foi um nome importante na incursão da Comissão Cowles na teoria da decisão e, como econometrista, também participou ativamente do debate sobre problemas de agregação na economia. Na década de 1960, Koopmans escreveu a maior parte de suas contribuições para o subcampo da

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economia do horizonte infinito (incluindo seu modelo de crescimento ótimo) e é nessa década que ele articulou de forma completa suas visões sobre o agente representativo. Finalmente, na década de 1970, Koopmans mudou o foco de sua pesquisa e começou a contribuir para a teoria das decisões individuais baseada nas preferências, levando-o à elaboração das funções de utilidade intertemporalmente separáveis. Ao longo dessas três décadas, Koopmans passou de um posicionamento ambíguo em relação ao consumidor representativo para outro mais inclusivo. Eu argumento que parte dessa mudança se deve ao uso cada vez maior do dispositivo na macroeconomia, um movimento que ele não iniciou mas ajudou a intensificar.
Palavras-chave: Agente Representativo, Problema de agregação, Tjalling Koopmans

JEL Codes: B2, B21, B23

Área 1 - História do Pensamento Econômico e Metodologia

1 Introduction

A pivotal step in the development of modern macroeconomics consisted in the axiomatization of time preference and the ensuing specification of a utility function in discounted form.[1] In a series of papers, notably Koopmans (1950, 1953, 1960, 1964, 1972a, 1972b) and Koopmans, Diamond, and Williamson (1964), Tjalling Charles Koopmans made foundational contributions to the subfield of intertemporal choice theory. With varying degrees of technical as well as historical importance, in common, those articles extended the ordinary formulation of preferences and the utility functions over time. The following passage, from one of the aforementioned articles, exemplifies an important advancement in this research effort:

> In a previous article one of the authors [8] studied some implications of a set of postulates concerning a preference ordering of consumption programs for an infinite future. The preference ordering was assumed to be representable by a numerical utility function defined on the *space* of consumption programs, and the postulates were formulated as properties of that function. While these postulates themselves appeared to be concerned only with properties more immediate and elementary than any questions of timing preference, it was found that the postulates implied, at least in certain parts of the program space, a preference for advancing the timing of future satisfaction. This conclusion was expressed by the concept of *impatience* (Koopmans et al. 1964, p. 82, italics added).[2]

The article "[8]" mentioned above referred to Koopmans's 1960 "Stationary Ordinal Utility and Impatience." Notwithstanding impatience used to be regarded as an irrational behaviour, an economic disharmony according to Arthur Cecil Pigou, from that article onward it became a logical outcome out of a set of rational postulates underlying a utility function.[3]

As claimed by Christ and Hurwicz (2008), however, an important facet in Koopmans's contributions to intertemporal economics was the seemingly trivial change in his research agenda in the 1970s: from the postulates underlying utility functions, Koopmans (1972a, b) turned the focus to the postulates underlying preference relations instead. Such a re-orientation in his research affected the way the author approached the representative consumer, going from a less enthusiastic view to a more favourable one as his career progressed.

Founded on, but not limited to, the assumptions of continuity, stationarity, and total independence over time in preference relations, he then proved that a utility function over an *infinite program* had now a certain desirable *additive property*. In other words, not only did the existence problem of the continuous utility function appear to be solved, Koopmans now wished to expand its scope to focus on the intertemporal additive properties of preferences.[4]

As I will try to substantiate in this article, such an episode seems to have led to one important historical feature: the ever increasing explicitness of the representative agent in macroeconomics in the 1970s and 1980s.[5] As Kevin Hoover (2012) gauged using the JSTOR database and

[1] See Duarte (2016) on how mathematicians and economists strove to narrow their understanding of time discounting in the 1920s-1960s period, going from a nagging ethical subject to a pure technical requirement.

[2] The coinage of the term "impatience" owes first to the Austrian economist Eugen von Böhm-Bawerk, who introduced it into the theory of rate of interest (Christ and Hurwicz 2008) and, later, to Fisher (1930), for whom "(...) individuals facing the alternatives of having a given reward today or in the future opt for current reward" (Duarte 2016, p. 292).

[3] For an examination of Arthur Pigou's and other Cambridge economists' thoughts on welfare analysis, particularly on their stances against discounting future utilities, see Collard (1996).

[4] Debreu (1959), section 4.6, was among the first to prove the existence of a continuous utility function.

[5] It might not be a coincidence that the Sonneschein-Mantel-Debreu theorem, an important result contradicting the possibility of consumers aggregation, was also established in the 1970s. See, e.g., Rizvi (2006), Hands (2012), and their suggested references for a historical overview of that theorem.

reproduced in table 1, the number of mentions of the terms "representative agent," "representative consumer," and "representative household," for the periods "through 1969" and "1970 and after," went from 0, 1, and 0 to 157, 107, and 69, respectively.

Table 1: Examples of consumers aggregation

	Through 1969	1970 and after
Representative Agent	0	157
Representative Consumer	1	107
Representative Household	0	69

Note: Although "representative agent" differs slightly in meaning compared to "representative consumer" and "representative household," they are commonly used to mean aggregate demand of the economy.

A possible reason is that the additive properties of the utility function that Koopmans helped build might have now become an important complement to the first established—but sometimes neglected—proof of aggregation across individuals provided in Gorman (1953) and Samuelson (1956) and, as such, strengthened the overall technical justification of that methodological device.[6] Coupled with Samuelson's contributions regarding homothetic preferences (Hands 2016), it could have freed the representative agent to be used in many other subfields of macroeconomics as well, such as the stochastic optimal growth literature found in Brock and Mirman (1972).

While broad and epistemically distinct, since I consider relatively a long period and contemplate both micro and macroeconomics, this analysis is intended to contribute to the understanding of the ever higher explicitness of the representative consumer in the writings of Tjalling Koopmans in the postwar period, aided, also, by the parallel contributions to the problem of aggregation led by Paul Samuelson during the same period. Furthermore, I also attempt to show the widespread use of separable utility functions in macroeconomics to be a possible corollary of such a theoretical development.

This diversity of theoretical interpretation of preferences and utility functions across time and people showcases the challenge involved in tracing certain concepts in the history of ideas, particularly when they straddle different fields. In the present case, they began as a new approach to tackle decisions sciences in microeconomics, the offshoot of a specific Normative Turn at the Cowles Foundation for Research in Economics (Herfeld 2018), and turned out to make a splash in the subfield of growth theories in macroeconomics. To write this article I peruse (many of) the main articles published about the subject by Tjalling Koopmans, Paul Samuelson and other authors and also profit from interpretations made by important historians of thought. Finally, to supply additional evidence to my story I use archival material.

The article is organized as follows. In the next section, I present Koopmans's initial works in the subfield of intertemporal economics in the 1950s, a contribution marked by aggregation concerns and paralleled by his incursions into activity analysis. Section 1.3, focusing on Koopmans's writings in the 1960s, discusses consumer choices with an infinite horizon in a context of optimal growth problems. Section 1.4 describes Koopmans's research reorientation in the 1970s, when he shifted his attention to investigate postulates of preference relations. The possible outcome of this new research to modern macroeconomics is also delineated. Section 2.5 offers some concluding remarks.[7]

[6]Of the authors who consider Gorman (1953) and Samuelson (1956) to be the first proofs of the representative consumer are Muellbauer (1976), Deaton and Muellbauer (1980), Lewbel (1994), Blackborby et al. (2008), and Acemoglu (2009).

[7]With a view to giving greater clarity to Koopmans's writings, I chose to divide his articles into those published in the 1950s, 1960s and 1970s. A different way to discern a segmentation in his writings is through his

2 Koopmans's intertemporal economics in the 1950s

As explained by a long-time colleague, Koopmans's incursion into the field of infinite horizon economies took place as early as the 1950s with two important short publications, "Utility Analysis of Decisions Affecting Future Well Being" and "La Notion d'Utilité dans Le Cas de Décisions Concernant le Bien-Être Futur" (Chipman 2006, p. 531).[8] In Christ and Hurwicz's (2008) view, this research is probably Koopmans's most important contribution in theoretical economics, a subject intimately linked to his understanding of the "optimal allocation of resource" (observed, e.g., in his 1965 growth model) and for which he would be awarded the 1975 Nobel Memorial Prize in Economic Sciences.[9]

Initially presented at the 1949 Boulder (Colorado) meeting of the Econometric Society, and published in 1950 as a report of the organization's journal *Econometrica*, "Utility Analysis of Decisions Affecting Future Well Being" laid down the initial reasoning that would later serve the author's long-term engagement with utility analysis. With the utility function based originally upon a complete and static ordering of objects, Koopmans contended that its sought-after dynamic generalization oftentimes ignored the desire for postponement of decisions in cases of unpredictable events. To deal with it, he understood that:

> Assets can then be entered in the utility function as representative of the *sets of consumption sequences* they give access to, through direct enjoyment, resale and purchase of other assets or consumption flows, alternatively or in succession, subject to later decision (Koopmans 1950, p. 175, italics added).

Koopmans continued by drawing special attention to the role played by borrowing, or consuming financing, to the well-being of consumers, since "(...) it gives access to the flow of services associated with the possession of durable consumer's goods at an earlier time than would otherwise be possible at the same rate of saving" (ibid.).

In the 1953 edition of *Cahiers du Séminaire d'Économétrie*, a special volume dedicated to choice theory, Koopmans contributed a small article entitled "La Notion d'Utilité dans Le Cas de Décisions Concernant le Bien-Être Futur," in which he extended the central idea pursued in his previous article.[10] By contrasting the ordinary concept of "preferences" to that of "possibilities" he enabled the introduction of a set Q, or ordered sequences, into the utility function; and via the replacement of points by sets he then permitted uncertainty to play an explicit role in choice theory.[11] In the discrete case it is possible to represent the sets in *tree form* whose

optimal growth paper (Koopmans 1963). Even though the final versions of the articles published in the 1970s came out as early as the mid-1960s they were, nonetheless, written after his growth article was finalized. In other words, the aforesaid *change* in the emphasis of his papers took shape only after his foray into the realm of normative growth theory (Christ and Hurwicz 2008). I expect to demonstrate why in the subsequent sections.

[8] John Chipman completed his PhD at Johns Hopkins University in 1951 and during the 1950-51 academic year was a post-doctoral fellow at the University of Chicago's economics department and a guest fellow at the Cowles Commission; his close contact with Koopmans took place while the latter served as a professor at the University of Chicago and had an appointment as research director at Cowles around the same period. Chipman has important publications in international trade theory, econometric theory and microeconomics, especially in demand theory. He has also written history of economics papers. Chipman taught at the University of Minnesota (Economics) from 1955 until his retirement in 2007.

[9] In addition to Tjalling Koopmans, the *Royal Swedish Academy of Science* also awarded the Russian mathematician Leonid V. Kantorovich (1912-1986) for their (independent) contributions to the theory of optimum allocation of resources.

[10] Although the 1950 and 1953 articles had very similar titles, they focused on different aspects of choice theory. I thank Pedro Duarte for pointing this out. The French journal *Cahiers du Séminaire d'Économétrie* is published nowadays under the title *Annals of Economics and Statistics*.

[11] *Decision-making under uncertainty* became part of a broader research "package" in *decision theory* after Koopmans joined the Cowles Commission in 1943 and "reengineered" it under the influence of von Neumman. According to Mirowski (2002, chapter five), other packages included in this new wave were *linear programming*, *activity analysis*, and *Arrow-Debreu general equilibrium*.

ramifications portray every possible choice which varies according to different contingencies.[12]

In light of Koopmans's early treatment of preferences over time in the 1950s, it is possible to suggest that those articles led him to rewrite the utility function as a *time aggregator* whose parts consisted of an *one-period utility* and a *prospective utility*, as seen in the articles "Stationary Ordinal Utility and Impatience" and "Stationary Utility and Time Perspective" published in the 1960s. In the next section, I will discuss some of the technical aspects that enabled Koopmans to attain this outcome.

While Koopmans's writings on the topic of preferences over time in the 1960s and later featured more abstract mathematical models, which he would often use to analyze concerns related to the representative consumer, his 1950s endeavors in the field had just recently been marked by what Catherine Herfeld (2018) identified as a Normative Turn, i.e., the transition from an empirical to an axiom-based approach to human behaviour. It does not follow from this that the problem of aggregation was no longer for Koopmans a nagging concern. In a series of letters exchanged between him and Marc Nerlove in January of 1957, the issue of aggregation emerged not only as a difficulty limited to a field, but a more general concern within economics.[13]

On January 10, 1957, Koopmans asked Nerlove about his recent research activities at the Agricultural Marketing Service of the United States Department of Agriculture (USDA).[14] Nerlove replied with a rather detailed description of his inquiries into, *inter alia*, the statistical properties of distributed lags models, their applications to the estimation of supply functions for agricultural commodities, their intertwined relationship with demand functions (in which Milton Friedman's permanent income hypothesis drew most of his interest) and, essentially, some problematic instances in the case such as the inapplicability of Friedman's consumption function "to cover individual items of consumption."[15,16]

Then, referring specifically to those as well as other concerns raised by Nerlove within econometrics, Koopmans responded with the following overall intuition:[17]

> I have not given much thought to the problem of distributed lags in the last few years. However, I have developed a general feeling that the clue to a number of different problems in econometrics lies in further exploration of aggregation problems,

[12] As I will show in section 1.4, the notion of choices representable in tree forms, also called *separable utility functions*, would be formalized (axiomatized) by Koopmans only in the articles published in the 1970s. Interestingly, even though it took Koopmans almost twenty years to re-engage with the formal properties of such utility functions, they continued to be studied by Robert Strotz (1957, 1959) and William Gorman (1959a, b). How Koopmans's contributions can be understood considering Strotz's and Gorman's will then be discussed in the same section.

[13] In the summer of 1953, as a graduate student, Marc Nerlove worked as a research assistant for Tjalling Koopmans and Jacob Marschak at the Cowles Commission. In 1956, under the supervision of Carl Christ, he earned a PhD in Economics from Johns Hopkins University with a dissertation measuring the dynamics and elasticity of the U.S. agricultural supply. In 1956-1957, the period of the correspondence, Nerlove worked as an analytical statistician at the United States Department of Agriculture (USDA).

[14] Letter from Koopmans to Nerlove, January 10, 1957, TKP, box 16, file folder 304.

[15] Letter from Nerlove to Koopmans, January 27, 1957, TKP, box 16, file folder 304.

[16] Milton Friedman's intuitive notion of the *permanent income hypothesis* is derived from chapter three of his famous book *A Theory of the Consumption Function,* published in 1957. The gist of his argument is that agents' consumption behaviours are better predicted with a non-directly observable "mean of the expected level of income in the very near-term" (Carroll 2001, p. 24), labeled "permanent income," instead of the usual "recorded income" as postulated by Keynes (De Vroey 2016). What should be clear to better comprehend Nerlove's interpretation of the Friedman's consumption function and, by extension, the story presented in this article, is that both Friedman's model and the neoclassical theory of consumer' choice were initially theorized at the individual levels. For an empirical economist like Nerlove, whose works came down to estimating aggregate relationships, the incongruence between both approaches overshadowed his analyses. It is precisely this frustration that prompted Henri Theil (see footnote 18 for a brief sketch of his biography) to write the opening paragraph of his 1957 book, which I chose to quote at full length below.

[17] Letter from Koopmans to Nerlove, January 31, 1957, TKP, box 16, file folder 304.

> that ties our notions of individual decision making with somewhat more aggregated relationships. I doubt that this reflection is of much value to you in regard to distributed lags, but it does seem to me that this is one of the problems that may stand to gain if progress should be made on aggregation. Theil's book is excellent as far as it goes. My main reservation about it is that it studies primarily the implications of current econometric procedures, rather than trying to answer the deeper question what procedures we should be using in view of aggregation problems.[18]

Even though the exchange of views on the aggregation problems between the authors arose in the field of applied econometrics, there was also a clear comprehension of its theoretical nature as captured in Koopmans's indication of the need to handle such a more difficult "deeper question" and in Theil's (1954, p. 1) own explanation of the problem:

> A serious gap exists between the greater part of rigorous economic theory and the pragmatic way in which economic systems are empirically analysed. Axiomatically founded theories refer mostly to individuals, for instance the consumer or the entrepreneur. Empirical descriptions of economic actions in large communities, on the other hand, are nearly always extremely global: they are confined to the behaviour of groups of individuals. The necessity of such a procedure can scarcely be questioned: How could one handle the tremendous mass of relations governing the actions of millions of individuals? But the introduction of relations pretending to describe the reactions of groups of individuals instead of single individuals raises questions of fundamental importance, which are not very well understood: What are the connections between the functional relationships postulated by the economic theories of individual households and the relationship for groups of individuals postulated by the empirical research worker? If the analysis shows that these relations are not of a "desirable" type, is it possible to indicate rules for the construction of group relationships that must be obeyed in order that certain optimum criteria are fulfilled? And how must these criteria been chosen?
>
> These are questions that must be answered by the theory of aggregation.[19]

In what follows, I shall make a few remarks concerning Koopmans's publications in the 1950s reviewed in this article vis-à-vis his overall contributions to the subfield. As far as I am aware, among all appreciations written so far about Koopmans's contributions to economics or about his professional life, such as Koopmans's own Nobel Memorial Lecture written in 1975, Werin and Jungenfelt (1976), Niehans (1990), Scarf (1995), Chipman (2006), and Christ and Hurwicz (2008), only Chipman (2006) cited the two articles I mentioned above in the body of his text

[18]The book mentioned by Koopmans in the correspondence was Theil's *Linear Aggregation of Economic Relations*, published in 1954. Henri Theil was born in 1924 in Amsterdam and passed away in the United States in 2000. After earning his doctorate in Economics (with distinction) from the University of Amsterdam in 1951, Theil began his career as a macroeconomic analyst and forecaster for the Central Planning Bureau (CPB) of the government of the Netherlands under the directorship of Jan Tibergen. Between 1953 and 1966 he taught econometrics at Erasmus University Rotterdam (formerly the Netherlands School of Economics in Rotterdam), where he also founded and directed The Econometric Institute. From 1966 to 1981 he was a member of the University of Chicago's economics department. From 1981 until his passing, Theil remained affiliated with the University of Florida. Over a period of five decades he published more than 250 articles and 17 books, of which the one mentioned by Koopmans was his very first (Bewley 2000).

[19]Along the line put forward by Tjalling Koopmans and Henri Theil, it is interesting to observe that many theoretical problems seem to surface more commonly in applied works than in theoretical ones proper. For example, aggregation issues were more of a problem in the "Modeling Resource Group" (MRG) than in any other academic communities Koopmans belonged to. The MRG was an association of economists and other experts created to provide the Committee on Nuclear and Alternative Energy System (CONAES) with economic information obtained through mathematical models. In addition to Tjalling Koopmans, William Nordhaus, Hendrik Houthakker and Kenneth Arrow took part in the discussions of the group.

as indication of their importance to economics (although Werin and Jungenfelt (1976) did at least list the 1953 piece in their references). In the two-volume *Scientific Papers of Tjalling C. Koopmans*, published in 1970 and 1985, none of those 1950s papers were included even though in the preface to the first volume, the editors, Martin Beckmann, Carl Christ, and Marc Nerlove (1970, p. v) wrote: "In making our selection of his papers to be included in this volume, we have attempted to include significant contributions in each of the major areas of Koopmans' work (...)," intertemporal economics being one of them.[20]

As to why the profession overlooked Koopmans's contributions in intertemporal economics at the time, a possible answer is that the subject could have been upstaged by the parallel development of activity analysis whose 1949 conference, "Activity Analysis of Production and Allocation," set a new era in economics research and, hence, drawing most of the author's attention for the remaining years of the 1950s.[21] This can be observed, e.g., in the ensuing publications of the conference volume (Koopmans 1951) and, especially, of his celebrated *Three Essays on the State of Economic Science* (1957), both of which featured novel developments in the subfield. To complement and, thus, get a more reliable answer to the question though, bibliometric research would be worth conducting, helping us to shed light on such related questions as how have the citations of Koopmans's writings evolved since he published those papers in the 1950s.

By bringing further details into the works of the Dutch economist in the 1950s and, more important, by later tightening them up with his writings in subsequent decades, I intend to contribute with a narrative of Koopmans's works in the subfield of intertemporal economics. Such a narrative shall therefore aid in the understanding of his thoughts regarding the representative consumer.

3 Koopmans's Intertemporal Economics in the 1960s: articulation of the representative agent

The bulk of Koopmans's contributions to infinite horizon economies appeared in a series of papers concentrated in the first half of the 1960s. The intertemporal utility function with time discounting that appeared in Koopmans's 1960 "Stationary Ordinal Utility and Impatience" had the form[22]

$$U = \sum_{t=1}^{\infty} \alpha^{t-1} u(x_t), \tag{1}$$

where $\alpha \in (0, 1)$ denoted a (constant) exponential discount factor, x_t a vector of goods, and $u(\cdot)$ an instantaneous utility function.[23] Interpreted as a preference ordering of infinite program, equation (1) had the following comment by the author (Koopmans 1960, p. 288):

> Flexibility of interpretation remains as to whether this ordering may serve as a first approximation to the preferences of an individual consumer, or may perhaps be an "impersonal" result of the aggregation of somewhat similar individual preferences

[20] The other two areas considered by the editors were econometrics and activity analysis.

[21] The importance of the 1949 conference to the development of economic science in the second half of the twentieth century was only recently made more clear by Düppe and Weintraub (2014).

[22] The first version of this work was the Cowles Foundation Discussion Paper No. 81, dated November, 1959.

[23] The instantaneous utility function is an essential concept in both macroeconomics and growth theory, and a description of the function is thus worth quoting at length from a major textbook on growth theory: "The instantaneous utility function captures the utility that an individual derives from consumption at time t. It is therefore *not* the same as a utility function specifying a complete preference ordering over all commodities - here consumption levels in all dates. For this reason, the instantaneous utility function is sometimes referred to as the 'felicity function'" (Acemoglu 2009, p. 180, italics added).

(interpreting "consumption" as "consumption per head" in the case of a growing population), or finally may guide choices in a centrally planned economy. In each of these interpretations further modifications and refinements may be called for.

Jointly written with Peter Diamond and Richard Williamson, and published in *Econometrica* in 1964, "Stationary Utility and Time Perspective" consisted primarily of direct generalizations of the notion of *impatience* found in Koopmans (1960) by way of a discovery of a deeper property of preference ordering, called *time perspective* (hence the titles of both articles), applicable to a larger part of the commodity space.[24,25] Among other goals, the article demonstrated that in comparing two consumption programs of the same period, for example, programs A and B, where the first is strictly preferred to the second, the more the consumer postponed his or her consumption, the smaller would be the difference assigned by the utility function; put another way, time mattered in the authors' account of preference and utility.

An example shall illustrate this point. Suppose an individual prefers apples to oranges today. A utility function then would assign a higher value for the first fruit. In Koopmans, Diamond, and Williamson's interpretation, since consumers are impatient, the preference for apples over oranges would be diluted when he/she tried to compare both fruits in, say, twenty months from today, leading to an ever-lower difference assigned by the utility function as time passes. Yet, to achieve this outcome, a real-valued utility function had to be capable of representing an ordering of sequences over time of commodity bundles in the first place.

If ${}_1x = (x_1, x_2, ..., x_t)$ represents an infinite-dimensional sequence (over time) of a particular commodity bundle, it can alternatively be written ${}_1x = (x_1, {}_2x)$, where ${}_2x = (x_2, x_3, ..., x_t)$ is a sequence of commodity bundles starting at $t = 2$. Thus, the initial notation can be rewritten as ${}_1x \equiv (x_1, {}_2x)$ or, considering only two periods, ${}_1x \equiv (x_1, x_2)$. In this definition x_1 and x_2 constitute the same commodity vector (consumption bundle) in the consumption set X but for periods 1 and 2, respectively.[26] The authors' intention was to justify the formulation of $U({}_1x)$ as a function of the instantaneous utility function $u_n(x_n)$ and, therefore, attain an intertemporal utility function such as $U({}_1x) = (u_1(x_1), u_2(x_2), ..., u_n(x_n))$. By accepting the postulates P1 (existence and continuity), P2 (sensitivity), P3 (limited complementarity), P4 (stationarity), and P5 (extreme programs), and undertaking various proofs, $U({}_1x)$ could then be written in the form

$$U({}_1x) = V(u(x_1), U({}_2x)), \tag{2}$$

where $V(u, U)$ is continuous and increasing in its two arguments u and U. As $u(x_1)$ and $U({}_2x)$ can be defined, respectively, as an *one-period* (or an *immediate*) *utility* and a *prospective utility*, $U({}_1x)$ can be redefined as an *aggregate utility*, where the function $V(\cdot)$ is the *time aggregator* (over all future time periods). Related to this function, Koopmans (1960, p. 292) stated that it "(...) indicates how any given pair of utility levels, immediate (u_1) and prospective (U_2) stacks up against any other pair in making choices for the entire future."[27] By iteration, (1.2) can be

[24] Following Mas-Colell et al. (1995, p. 19), the consumption set is the set of all nonnegative bundles of commodities $X = \mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_l \geq 0 \text{ for } l = 1, ..., L\}$, where $\mathbb{R}^L$ is the commodity space.

[25] The first version of this paper was a Cowles Foundation Discussion Paper No. 142, dated August, 1962. In his 2010 Nobel memoir Peter Diamond recounted, with inescapable gratitude, how his mathematical ability helped reorient Koopmans's research while working as his research assistant in the summer of 1960 (when Diamond had not even started graduate studies at MIT), a fact that stirred Koopmans to promote him as a co-author of the 1964 paper.

[26] Throughout this article, I follow the notations adopted by Koopmans in his examination of the subject. To clarify, a consumption vector x_t can also be understood as $x_t = (x_{t1}, x_{t2}, ..., x_{tl})$, that is, assuming a $l-$dimensional commodity space.

[27] The passage refers indeed to an expression obtained prior to equation (1.2), namely, $U({}_1x) = V(u_1(x_1), U_2({}_2x))$, where the difference consists in the appearance of time subscripts in both arguments of the function V. Nevertheless, since my concern is to focus on the aggregative feature of $U({}_1x)$, both expressions play the same role. See Phlips (1983, chapter XI) for further clarifications on this point.

generalized to

$$U({}_1x) = V(u(x_1), u(x_2), u(x_3), ..., u(x_\tau), U({}_{\tau+1}x)) \tag{3}$$

for all τ. Bearing on a result attributed to Debreu ([1959] 1960), in which postulate 3′ (independence) is added to the set of postulates, a monotonic transformation can be performed on (1.3) such that (using $t = 3$)

$$U({}_1x) = u_1(x_1) + u_2(x_2) + U_3({}_3x). \tag{4}$$

Finally, as Koopmans (1960) and Phlips (1983) argued, equation (1.4) is usually written in the literature as

$$U({}_1x) = \sum_{t=1}^{\infty} \kappa^{t-1} u(x_t), \quad \kappa = \frac{1}{1+\gamma}, \quad 0 < \kappa < 1 \tag{5}$$

or

$$U({}_1x) = \int_0^{\infty} e^{-\rho t} u(x_t) dt \tag{6}$$

for discrete and continuous time, respectively.[28] The significance of the additively separable preference assumption (postulates 3 and 3′) cannot be underestimated. It is indispensable in an intertemporal problem solved by either a household or a central planner: in a workhorse Ramsey-Cass-Koopmans model, each reaches an Euler equation where the marginal rate of substitution in consumption between periods t and $t + 1$ is independent of future periods.[29]

Koopmans's ingenious way of working out an intertemporal utility function depended unavoidably on how he aggregated the time-varying instantaneous utility function. Although he never explicitly identified such a function with the (aggregate) expression "representative consumer," one could argue that the author always had the latter construct in mind: as is the case with intertemporal preference and intertemporal utility, the representative agent also enabled a correspondence between consumer and central planner.

In a 1964 sole-authored paper entitled "On Flexibility of Future Preferences," an examination of the effects of uncertainties on agents' preferences in coming, yet distant periods, Koopmans adopted an open stance regarding whom should such a (future) preference belong to by stating from the outset: "The economist's traditional model of choice - *whether consumer's choice or planner's choice* - is based on an analytical separation of preferences and opportunity" (Koopmans 1964, p. 469, italics added).[30] In another passage, Koopmans alluded to the usefulness (and therefore ability) of a central planner acting as some sort of aggregator of preferences. According to him:

> (...) taste evolves with experience. A model that freezes preferences by the adoption at an initial point in time of an ordering of programs for a future period of indefinite duration is likely to become an unacceptable straight jacket as time proceeds. At what age would the individual consumer be supposed to embrace the ordering that is to guide all his consumption choices for the remainder of his life? *And the economic planner, who presumably attempts to aggregate the preferences of the population - perhaps with some admixture of his own values - wouldn't he wish to retain flexibility so that he could respond in the future to newly perceived currents of taste and desire?* In fact, would it be morally defensible for one generation to pre-

[28] Koopmans stated that equation (1.5), where "aggregate utility is a discounted sum of all future one-period utilities, with a constant discount factor κ" (1960, p. 308), had been used "extensively" before, such as in Ramsey (1928), Samuelson and Solow (1956), and Strotz (1957), but with $\kappa = 1$ imposed in the first two cases.

[29] In such problems, an intertemporal utility function of the form (1.5) or (1.6) is optimized subject to a feasibility or technology constraint (in the centralized case) or subject to a wealth constraint (in the decentralized one).

[30] This short paper appeared first as a Cowles Foundation Discussion Paper dated No. 150, December, 1962.

sume, and act on, a very definite specification of the next generation's preferences? (ibid., p. 472, italics added).

As pointed out by Duarte (2016), some controversies involved in the analysis of an individual's intertemporal allocation problem compared to the planner's were already raised by Amartya Sen and Stephen Marglin in the 1950s and 1960s; Koopmans's words quoted above could be a reaction to this discussion and, in this sense, his treatment of individual vis-à-vis centralizer figures, such as "the subsistence farmer" or the "the economic planner" (Koopmans 1964, p. 470) might be taken as an example of the care Koopmans granted to the issue as well.

In a particular study centered on the axiomatization of time preference and the subsequent incorporation of time discounting in optimal growth models in the 1960s, Duarte (2016, p. 294) claimed: "While Cass advocates an infinite horizon, discounted optimization problem, he does not make an argument for basing social welfare on the representative agent's utility function." In contrast, "(...) Koopmans blurred the lines between individual and social intertemporal choices" and, as such, made "(...) more room for discounting to become a rather technical requirement" (ibid.).

In his famous optimal growth article written in 1963, "On the Concept of Optimal Economic Growth," Koopmans introduced a slightly modified version of (1.1), but now applied to examine normative features of economic growth:[31]

$$U = \sum_{t=1}^{\infty} \kappa^{t-1} u(x_t), \tag{7}$$

where $\kappa \in (0,1)$ is the constant discount factor. Looking for mathematical simplicity, his analysis into the existence and characteristics of optimal growth paths resorted to the finite-horizon utility function $U = \int_0^T u(x_t)dt$, where the simple integration of the instantaneous utility flow $u(x_t)$ "(...) implies *noncomplementarity* between consumption in any two or more parts of the future" (Koopmans 1963, p. 20). Applying the discount factor $e^{\rho t}$ and pushing time horizon to infinity, the latter expression could then be rewritten as

$$V(\rho) = \int_0^{\infty} e^{-\rho t} u(x_t)dt, \tag{8}$$

where ρ is a positive instantaneous discount rate strictly greater than 0, whilst other variables remain with the same definitions as in (1.1). Equation (1.8) is therefore almost identical to (1.6). Initially presented with a finite horizon, where "(...) the choice of the terminal capital stock is as much a part of the problem to be solved as the choice of the path" (ibid., p. 5), the optimal growth problem was re-framed with an infinite horizon perspective since in Koopmans's view it was "(...) perhaps a more natural specification in many formulations of the growth problem" (ibid., p. 6).

Similar to all papers that dealt with the topic of infinite horizon economies seen above, the 1963 growth article also left open the interpretation by which equations (1.5) and (1.6) could be the utility function of some generic preference, that is, "(...) no discussion of whether these preferences were of individuals, a representative agent, or a social planner" (Duarte 2016, p. 292-3).

Koopmans's famous optimal growth paper was then published in definitive form in a 1965 volume that wrapped up the proceedings of a conference held in Vatican City.[32] But, as can be implied from the 1965 article, Koopmans's concern regarding the technical appropriateness

[31] Cowles Foundation Discussion Paper No. 163, dated December, 1963.

[32] Under the theme "Study Week on the Econometric Approach to Development Planning," the conference was organized by the Pontifical Academy of Sciences in 1963. In addition to Koopmans, who discussed his 1963 version of the paper, other important economists at the time also presented articles and participated in the

of (1.7) as an optimality criterion did not encompass an explanation of whether it purported the preferences of an individual, a social planner, or a representative agent either; he carried on his investigation solely on the grounds that the most basic (and suitable) mathematical formulation of an optimality criterion is simply "(...) that of a preference ordering of growth paths" (Koopmans 1963, p. 2).

Not specifying whose preference is (1.7) could be a practice inherited from his previous work "Stationary Ordinal Utility and Impatience" (Koopmans 1960). In this article—following the derivation of equation (1.1), an expression almost identical to equation (1.7)—although Koopmans asserted that such an equation had already been used extensively in the literature, a perusal of the papers mentioned therein suggests that they did not include time discounting.[33]

We can derive two implications from this fact. First, as examined in Duarte (2016), it substantiates the claim by which Koopmans pioneered the axiomatization of time preference, for a clear difference can be observed between his 1960 work and other contemporaneous works analyzed in Duarte's article. Second, due to Koopmans's early treatment of the discounted intertemporal utility function, it would be reasonable to expect a timid stance toward the wording of the term "representative consumer" as well. This posture, nevertheless, did not go unscathed as seen in the quotation below:

> The effort economists such as Koopmans made to construct a behavioral basis of an ordinal intertemporal discounted utility function that characterized either the behavior of an individual or of a social planner did not prevent serious criticisms either to employing a social discount factor or to treating individual and social decisions in the same way (Duarte 2016, p. 296).

Irrespective of the technical reason behind Koopmans's treatment of individual and social decisions, there was still a noticeable vagueness in the way he viewed the aggregation of preferences and of utility functions in the 1950s and 1960s. Although I have tried to show such ambiguity through a close reading of Koopmans's writings and the perusal of the letters he exchanged with Marc Nerlove, a further investigation may answer *why* that was the case.

4 Koopmans's intertemporal economics in the 1970s: expansion of the representative agent appeal in macroeconomics

Initially part of a single 1966 Cowles Foundation discussion paper entitled "Structure of Preference Over Time," the articles "Representation of Preference Orderings with Independent Components of Consumption" (1972a) and "Representation of Preference Orderings Over Time" (1972b) appeared as separate pieces in a single volume in honor of Koopmans's long-time friend and mentor Jacob Marschak.[34] In common, and in opposition to his writings from the 1950s and 1960s, the 1970s articles signified a (re)orientation of his research interest toward the postulates of preference relations (which he called *ordering)* under which he sought the

overall discussion during the occasion such as Richard Stone, Walter Isard, Ragnar Frisch, Wassily Leontief, Luigi Pasinetti, and Michio Morishima, among others. The changes that occurred between the 1963 and 1965 versions and the role played by the French economist Edmond Malinvaud are discussed in detail in Spear and Young (2014).

[33]The articles mentioned were Ramsey (1928), Samuelson and Solow (1956), and Strotz (1956).

[34]On Koopmans's meandering career before joining the Cowles Commission in July 1944, Mirowski (2012, p. 152) said: "Were it not for Marschak, one can easily imagine that the dissatisfied chap bouncing from one brief job to another, never really fitting in anywhere, might have left no mark whatsoever on the history of economics."

possibility of a more complete representation in terms of utility functions.[35]

If in the 1960 article Koopmans proved the existence of a (continuous) utility function at the very outset, with the reformulated postulates set out in the 1970s, the author endeavored "(...) to construct a representation of $\succsim$ on the entire program space ${}_1\mathcal{X}$, or on as large a subspace of it as we can" (Koopmans 1972b, p. 108). Stated even more clearly, and in connection to the point I want to emphasize, the author made this complementary point in another passage:

> The postulates are modeled after those used in two earlier studies by KOOPMANS (1960) and by KOOPMANS, DIAMOND AND WILLIAMSON (1964). The main difference is that the former studies presupposed the existence of a continuous representation. In the present study, the postulates refer to a continuous ordering, and the proximate aim of the study is to derive the existence of a continuous representation. Further differences will be noted in connection with the *third* and fifth *postulates* (Koopmans 1972b, p. 105, italics added).

Aligned with the main results predicated on the assumption of *separable preferences* obtained in the 1972a article, the re-examination of the independence (third), monotonicity (fifth) and, especially, stationarity (fourth) postulates in the 1972b paper led to the continuous representation of the utility function with certain desirable additive properties.[36] The ultimate goal of such re-appraisals was therefore to find a theoretical device that would enable a better comparison of growth paths for an indefinite future or, in other words, a technical device that once applied to growth theories could broaden the understanding of infinite-time growth processes. Such was the direct importance of such an elaboration that it can be found, for example, in a prominent recent textbook *Introduction to Modern Economic Growth*, where the author states:

> The Solow growth model is predicated on a constant savings rate. Instead, it would be much more satisfactory to specify the *preference orderings* of individuals, as in standard general equilibrium theory, and derive their decisions from these preferences. This will enable us both to have a better understanding of the factors that affect savings decisions and also to discuss the "optimality of equilibria" - in other words, to pose and answer questions related to whether the (competitive) equilibria of growth models can be "improved upon." The notion of improvement here will be based on the standard concept of Pareto optimality, which asks whether some household can be made better off without others being worse off. Naturally, we can only talk of individuals or households being "better off" if we have some information about well-defined preference orderings (Acemoglu 2009, p. 215, italics added).

The conceptual and mathematical refinements Koopmans sought within intertemporal choice theory, especially with regard to its implications for the additivity properties of utility functions, turned out to have important sway in the profession's effort to integrate optimal growth theory into general equilibrium theory. In the next two subsections, in addition to providing a relatively thick outline of Koopmans's writings, I show how he dealt with aggregation problems and, by extension, with the representative agent.

[35] A subtle, yet difference worth noting between the two articles was that rather than studying the postulates underlying preference orderings *on a prospect space*, in the second paper he focused on the postulates underlying preference orderings *over time*, that is, a space ${}_1\mathcal{X}$ of programs, of infinite-time sequences.

[36] Koopmans used the term *independence of different components of consumption* instead of *separable preference.*

4.1 Representation of preference ordering with independent components of consumption

Koopmans strove to fulfill two aims set out for his 1972a paper: first, summarized under *proposition 1*, provide proofs of the existence of continuous utility functions and, second, under *proposition 2*, provide proofs of an (additively) separable utility function from the assumption of (additively) separable preferences. As a starting point, in section 2, Koopmans defined and described the mathematical properties of a complete preference ordering on a prospect space. In section 3, drawing on the results found in Debreu (1959), Koopmans announced proposition 1 for continuous as well as non-continuous preference relations.[37]

In section 4, building on the previous works by Leontief (1947a, b), Samuelson (1947, chapter seven) and, especially, Debreu ([1959] 1960), Koopmans proved preliminarily the existence of a separable utility function under the assumption of two independent components of consumption.[38,39] After the definitions of some particular mathematical properties related to preference relations, the author attained what he called *Result A*:[40]

$$U(x) = F(u(x_P), v(x_Q)). \tag{4.4}$$

According to Koopmans (1972a, p. 88), "A function of this form has been called a *utility tree* by STROTZ (1956, 1959), and a *separable utility function* by GORMAN (1959a, b)."[41] The idea of separability in the utility function, therefore, implies that "(...) instead of one function U of $n_P + n_Q$ variables (...)," there is "(...) a triple of functions, one (F) of two variables, one (u) of n_P, and one (v) of n_Q" (Koopmans 1972a, p. 88).

In section 5, mathematically the most challenging one, he began tackling the subject of an *additively* separable utility function under a Euclidean (n-dimensional) commodity space, partitioned into three independent subsets ($n = 3$). Unlike proposition 1, where Koopmans resorted to a definition found in Debreu (1959), the arguments used to attain proposition 2 were drawn from Debreu ([1959] 1960) instead, related now to the definition of a cardinal utility function applied to "independent factors of the action set."[42] Koopmans then reached the following additive separable utility function (proposition 2):

$$U^*(x) = u^*(x_P) + v^*(x_Q) + w^*(x_R), \tag{5.4}$$

which is said to be unique up to a linear transformation.

In section 6, Koopmans extended the existence proof of an additively separable utility functions for the case of additively separable preferences defined in a commodity space with more than three variables (that is, the case of more than three independent components of consumption, $n \geq 3$). His important conclusion, called *result C*, presents the following continuous utility function,

$$U(x) = u_1(x_1) + u_2(x_2) + ... + u_k(x_k), \tag{6.3}$$

unique up to an increasing linear transformation.

In section 7, the author basically discussed why proving additively separable preferences in the case with two commodities ($n = 2$) is harder than in the case where there exists three or

[37] The actual announcement of proposition 1 was based on the assumption of continuity of $\succsim$.

[38] It is interesting to observe that Debreu himself referred to Samuelson's (1947) *Foundations of Economic Analysis* on the subject.

[39] "Preliminarily" because Koopmans will return to this case in section 7 of his article.

[40] Henceforth, I keep Koopmans's (1972a) equation numbering.

[41] There is a typo in Koopmans's text: Robert Strotz's papers that dealt with the concept of *utility tree* were his 1957 and 1959 articles, not his 1956 paper.

[42] Besides this case, Debreu ([1959] 1960) also studied the concept of cardinal utility in two other situations: "stochastic objects of choice" and "stochastic acts of choice."

more commodities ($n \geq 3$). In Koopmans's (1972a, p. 101) words, the latter case "(...) leads to a more special class of representation than the case $k = 2$." The whole point is tightened up to the question of why not every function of separable form, such as that obtained in *result A*, $U(x) = F(u(x_P), v(x_Q))$, can be expressed as an *additively* function of separable form

$$U^*(x) = u^*(x_P) + v^*(x_Q), \tag{7.1}$$

This discussion matters because this development would turn out to be important in macroeconomics.

4.2 Representation of preference ordering over time

While the appraisals of preference orderings moved from an emphasis on prospect space (Koopmans 1972a) to an emphasis over time (Koopmans 1972b), the *propositions* obtained in the first article were used to attain the goal set out in the second one. I briefly discussed both propositions above, which concerned, first, the existence of a continuous utility function and, second, the existence of an additively separable utility function (first, for the $n = 3$ case, and later extended to $n \geq 3$).

If the first article fulfilled an essential role in introducing all necessary ingredients for a rigorous discussion of preferences on a prospect space, namely the propositions and their corresponding proofs, the greatest merit of the second article consisted in its *de facto* re-assessment of the *postulates* or axioms (of preference relations) in light of such novel developments (the propositions). It is worth noting that Koopmans had already discussed painstakingly the very same postulates in a series of papers in the 1960s, including, of course, his paper on optimal growth (Koopmans 1963). By rescuing such discussions in a time setting in the 1970s, he intended to provide proof of existence of an additively separable utility function. Furthermore, in the author's own words, "(...) both propositions are applied in discussing the choice of a criterion for the evaluation of growth paths, starting from postulates about a preference ordering of such paths" (Koopmans 1972a, p. 81-2).

In section 2, titled "Postulates Concerning a Preference Ordering over Time," Koopmans sought to re-analyze all five postulates previously studied in Koopmans (1960) and Koopmans et al. (1964), this time enumerating them in the following arrangement: continuity (P1), sensitivity (P2), complete independence (P3) (consists in a stronger case, in which both the "limited" independence, P3′, and the "extended" independence, P3″, postulates hold), stationarity (P4), and monotonicity (P5) (which includes a stronger case denominated "extreme programs"); Koopmans aimed at re-framing them as axioms of choices in a time setting and no longer taking their algebraic representations as given.[43] In the article an important emphasis is then placed on the postulate of independence by encompassing P3′ and P3″, for they "(...) facilitate explorations of the implications of the fourth postulate, the real objective of this study (...)" (Koopmans 1972b, p. 107).

Specifically, instead of focusing on P3′, as he did in the aforementioned articles, the focus now rested on the implications of P3. The modification in the weight given to a crucial axiom was meant to "(...) preclude all complementarity between the consumption of different periods" (ibid.), that is, to ensure a separable utility function. Put in another way, if stationarity disregards whatever past values a consumption vector ${}_1x_{t-1}$ assumed before a certain time (in any consumption program), and disregards the exact time those changes began, then it can be attained through the validity of postulate P3 (P3′ and P3″ hold). With all the postulates propounded, Koopmans sought to build a representation of preference relations on the entire program space ${}_1\mathcal{X}$ "(...) or on as large a subspace of it as we can" (ibid.).

[43] According to Koopmans, the postulate P3 is attributable to Gorman (1968).

In section 3, the two propositions discussed in Koopmans (1972a) will then finally aid in the construction of such representations. The first outcome, called *result D*, churned out the following utility function for all time T:

$$U_T({}_1x_T) = u(x_1) + \alpha u(x_2) + ... + \alpha^{T-1}u(x_T), \qquad 0 < \alpha < 1, \tag{3.2}$$

where $u(x)$ is a continuous utility function defined on $\mathcal{X}$ and independent of T, while α is also independent of T.

In section 5, Koopmans reached probably one of his most (and last) important results in the field of infinite horizon economies, the attainment of a representation of preference ordering (a utility function) on the space of programs bounded in utility.[44,45] Formally, any consumption program ${}_1x = (x_1, x_2, ..., x_t, ...)$ is bounded in utility if there exist vectors $\overline{x}$ and $\overline{\overline{x}}$ in $\mathcal{X}$ such that $\overline{\overline{x}} \succsim x_t \succsim \overline{x}$, for all t.

Hence, on the large subspace of the program space bounded in preference (made possible with the application of propositions 1 and 2), for example, ${}_1\mathcal{X}^*$, the ordering $\succsim$ can therefore be represented by the continuous utility function

$$U({}_1x) \equiv \sum_{t=1}^{\infty} \alpha^{t-1}u(x_t), \qquad 0 < \alpha < 1, \tag{5.1}$$

which is a result that consists in his *proposition 3*.

As acknowledged by Koopmans in section 6, the utility function (5.1) surprisingly exhibited strong implications of the axioms used to derive it. Moreover, he emphasized the roles played by two mathematical concepts in the studies of program changes in future periods and, therefore, of direct concern to the evaluation of growth paths: while the function $u(x)$ allowed the comparison of utility differences in the same period, the discount factor α enabled the comparison of utility differences in different periods. Koopmans then went on to discuss cases with and without the discount factor (see Duarte (2016) for an overall analysis).

In section 7, Koopmans (1972b, p. 117) synthesized the results he had found thus far: "If instead of complete independence (P3) we postulate only limited independence (P3′), Proposition 2 is not available, and we must fall back on Result A." That is, without the assumption of separable preferences, guaranteed by the axiom of complete independence (P3), an *additively* separable utility function (proposition 2) would have been impossible and, accordingly, the best offshoot would be a separable utility function as obtained under *Result A* (equation 4.4). He nevertheless pointed to an interesting result obtained by Diamond (1965) (Existence Theorem) in which an *aggregate utility function* identical to expression (1.2) above was obtained through a different set of postulates, all applied to a preference ordering as well.[46]

Finally, as I anticipated, in the concluding part of the article the author highlighted the special role played by the stationary assumption (P4) in producing:

> (...) interesting special forms for the utility function $U({}_1x)$ in terms of simpler functions $u(x)$ and possibly $V(u, U)$, that facilitate the use of $U({}_1x)$ in models of *optimal economic growth*, and may perhaps suggest further parametrization or other specialization for econometric studies of individual consumption plans over time (Koopmans 1972b, p. 121-2, italics added).

In other words, the passage referred to the equation obtained in proposition 3, that is, the

[44] I will skip directly to the principal arguments of section 5 since is more general and, furthermore, comprises the main result found in section 4.

[45] As pointed out by Christ and Hurwicz (2008), the term "bounded in preference" is preferable to the term "bounded in utility," used by Koopmans, because no numerical utility is involved in his definition.

[46] The postulates were P1′, P2, P3′, P4, and P5′.

expression (5.1), or the expression (1.2), which were previously obtained in Koopmans (1960) and Koopmans et al. (1964). Such expressions are also known to be in *recursive stationary* form, where the preference ordering would remain the same even if the timing of all periods were moved one period into the future.

I have discussed in this section two important changes in Koopmans's works in the 1970s. The first one was how he changed the emphasis from the axioms/postulates concerning utility functions to those concerning preference orderings/relations. The second change was the extension of such an analysis over time. An important similarity between the 1970s articles and his earlier writings consisted, again, in the author's preoccupation with aggregation issues and, as a result, the representative consumer.

In essence, by asking "(...) [if] one may wish to examine whether, or under what conditions, an aggregate preference ordering over time can be imputed, on an 'as if' basis, to a society of individual decision-makers each guided by his own preference ordering over time" (Koopmans 1972b, p. 104), and by considering it "(...) the ultimate goal of a theory of preference over time for an economy with private wealth (...)" (ibid.), Koopmans indeed tried to tackle the same Post-World War II theoretical challenges raised by Kenneth Arrow, James Buchanan and Gordon Tullock in such different fields as Social Choice Theory and Public Choice Theory.[47]

Interestingly, due to the key role of economic planning in the postwar context, and given the debates over whether the difference between social choice and public choice theories could be framed as positive or normative interpretations, Koopmans might have felt compelled to address the same issue.[48] In his words, "In regard to preference over time, the simplest interpretation of the orderings that have been studied most thus far is a normative one" (ibid.). In contrast, he also stated that: "Another possible interpretation is that one wishes to study descriptively the preference ordering of an individual with regard to his life-time consumption program, assuming that such an ordering is implicit in his decisions" (ibid.).

5 Concluding Remarks

The Dutch physicist and economist Tjalling Koopmans wrote seminal articles in three areas of research in economics: activity analysis, econometric theory and intertemporal economics (infinite horizon economies). For his several contributions to the theme of optimal allocation of resources, he was awarded the 1975 Nobel Prize in Economics.

This article investigated Koopmans's contributions to the intertemporal economics literature, also considered to be within the realm of optimal allocation of resources, in an attempt to uncover the factors that might have influenced his stance toward a methodological device used in optimal growth models: the representative consumer. To do so, I focused specifically on his contributions to the subfield in the 1950s, 1960s and 1970s.

By separating his contributions into those subperiods, I was able to observe in more detail the evolution of his thinking. From a greater concern with aggregation problems in the 1950s, he shifted to a more practical emphasis in the 1960s. In the 1970s, Koopmans's works became more interrelated with the instrumental needs of macroeconomists, understood as the use of additive separable utility functions. In the works of all those periods, a certain vagueness is noticeable in the use of the term "representative consumer."

The analysis in this article can be enhanced in at least one direction. Koopmans's con-

[47] During the Cold War new approaches to decision sciences emerged, based mostly on the role played by a rational economic agent. The literature on Social Choice Theory (Arrow ([1951] 1963) and Public Choice Theory (Buchanan and Tullock 1962) were parts of this movement (see Amadae 2003).

[48] As explained by Judy Klein (2007), both World War II and the Cold War forced the US government into planning (or "programming"), even though such conflicts represented, on the one side, capitalism and the free-market and, on the other side, state corporatism and central planning.

struction of additive separable utility functions in the 1970s seems to have followed the overall change in economics that took place after the adoption of homothetic preferences in the 1950s (in an effort that took Paul Samuelson almost thirty years). These joint facts seem to have favoured a higher use of the representative agent in macroeconomics in the 1980s with the advent of the Real Business Cycle School and, later, in the 1990s, with the emergence of the so-called New Neoclassical Synthesis.

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