Could you please spare me a moment? The Two-Stage Survey Game.

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Abstract

We study how survey respondents and public statistics institutes interact in a primary information market through a game-theoretic approach. Our model is a two-stage game in which a respondent has to provide data, accurately or not, to an statistics institute. The institute, in turn, must choose between accepting the initial obtained information or engaging in an auditing process. We find that provision of accurate information by survey respondents is dependent on their cost to acquire information, the auditing strategy employed by the institutes and fine sizes. We comment on how credibility regarding the threat of fines can impact the respondent's behavior. Finally, we explore how the existence of a public policy which uses the respondent's information as input can act as an indirect incentive towards accurate data provision.

Keywords: primary data; official surveys; auditing.

JEL classification: D01, D83, H49.

Resumo

Nesse trabalho nós avaliamos como institutos de estatística públicos e participantes de pesquisa interagem em um mercado primário de informações, a partir de uma abordagem de Teoria dos Jogos. O nosso modelo é constituído de um jogo em dois estágios, no qual um participante de pesquisa reporta uma informação, de forma precisa ou não, para um instituto de estatística. Por sua vez, o instituto deve escolher entre auditar ou não a informação recebida. Observamos que a provisão de informações precisas pelo participante depende de seu custo em obtê-las, da estratégia de auditoria do instituto e do valor das multas em caso de auditoria. A credibilidade quanto à ocorrência das multas também pode afetar o comportamento do participante. Por fim, exploramos como uma política pública, que utiliza as informações da pesquisa como insumo, pode servir como um incentivo indireto para o provimento de informações precisas.

Palavras-chave: dados primários, pesquisas oficiais, auditoria.

Área 8: Microeconomia, Métodos Quantitativos e Finanças.

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1 Introduction

Efficiently planning and executing both public and private policies is conditioned on having well-informed decision makers. It is essential that high quality information is produced and made promptly available for these decision makers (United Nations Statistic Division, 2015).

A policy that aims to redesign of an educational system, for example, requires a snapshot of the system's current infrastructure, as well as a correct assessment on the performance of its students and teachers. As these policies turns towards more specific goals, there is increasing demand for higher-frequency, disaggregated and accurate data. For example, having a socially-inclusive policy regarding disabled people requires regional indicators on accessibility, labor market as well as the specific demands from the several kinds of disabled people. These, in turn, require an advanced data collection infrastructure.

This information is initially, and mainly, produced by statistics institutes, which are responsible for gathering and processing raw data. Most countries have their own governmental institutions to produce and disseminate primary data, such as: United States Census Bureau, Statistics Canada, Bureau of Statistics (Japan) and the Brazilian Institute of Geography and Statistics.

However, recent studies have shown that these statistics institutions are facing rising difficulties in these data collecting processes. According to MEYER et al. (2015), there are mainly two classes of problems, non-response and lack of robustness. There has been an increase in non-response rates from surveyed units. In the National Health Interview Survey[1], for example, non-response has increased from 8% to 24% between 1997 and 2013. The second problem is due to lack of consistency among data collected by different institutions, which signals that data being collected needs to be more accurate.

Although there is constant improvement in statistical methods that aim to reduce the perverse effects of non-response, the latter continuous growth could result in long-term lower quality data (SINGLETON and STRAITS, 2017). The efficient usage of resources should also be a topic of concern. CURTIN et al. (2005) acknowledges that the decline in response rates is also generating growing data collection costs.

In this paper, we aim to present aspects of the strategic interaction between participants of a primary data collection process. There are primarily two factors that support our research agenda. First, from an empirical perspective, statistical institutions face daily routine problems in gathering information. Although several respondents comply with the proposed surveys, some of these participants still end up providing inaccurate data, while others opt out of the surveys altogether. Our paper comments on the possibilities that could prompt these actions during the data gathering interaction.

The second factor is related to a research gap on this particular topic. There is ample literature that analyzes the relationship between experts and lesser informed parts, for example the branch of Communication Games, more specifically the Sender-Receiver models, such as

[1] A survey conducted by the CDC, Centers for Disease Control and Prevention.

Cheap-Talk. Also, the Inspection Games literature has made several advancements regarding the design of monitoring mechanisms. Despite these advances, however, there has been no theoretical economic model specifically design to comment on to the primary data collection scenario[2].

We setup a survey environment in which an institute seeks to collect private data from a respondent, using a Game-Theoretic approach. We comment on the economic analysis possibly performed by survey respondents, which involves a comparison of the costs of gathering and transmitting the requested data in contrast with the consequences of non-complying, as well as the behavior and auditing strategies of statistics institutes.

We analyze non-compliance using two different approaches. First, we observe the impacts of punishment by the institute through the application of fines. We also comment on how the existence of public policies that are based on the collected data could have an indirect impact on the collection process itself. Finally, we also provide a short extension regarding time dynamics by implementing a discount rate, as well as possible issues regarding credibility.

This paper is structured in five sections, including this introduction. The next section presents a brief literature review. In the third section, our baseline model is introduced, while some extensions are shown in the fourth section. Our concluding remarks are in the fifth section.

2 Literature review

The primary data collection market depends on the interaction between two parts: statistics institutes and respondent agents. The institute selects participants, intentionally or randomly, to report private, unobservable data. This data is collected through methods such as surveys, and the resulting product is a set of primary statistics. These are then made available to the society[3].

The main goal of statistics institutes is to be a source of unique information regarding different characteristics of a given society (United Nations Statistic Division, 2015). This encompasses social, economics and environmental features, for example. In order to met this goal, these institutes have to continuously adapt to changes in data needs, as well as ensure that the produced data is of high quality.

According to the United Nations Statistic Division (2012), quality is defined as a multidimensional attribute. It is composed of aspects such as: (A) relevance, the correct matching of the institutes supply and societies data demands, (B) accuracy, the statistics precision of data, (C) accessibility, regarding different methods of accessing data as well as the availability of microdata , (D) clarity, ease of reading and interpretation and (E) coherence, which encompasses the possibility of combining different types of statistics[4].

We restrict the multidimensionality of primary data quality, in our paper, by performing an economic analysis of market and agent behavior features that could have an effect on the

[2]To the best of our knowledge.
[3]Usually free of charge, in the case of public institutes.
[4]For a complete list of attributes, see http://unstats.un.org/unsd/dnss/gp/FP-New-E.pdf.

accuracy measurement. This is a different approach from the usual statistical method analysis, which is focused on index construction techniques, sample size and distribution among other aspects.

Our paper presents an applied Game Theory model consisting of two types of players, a survey respondent and a statistics institute. The respondent is selected by the institute to take part in a survey, and he has to decide whether to provide a true or an inaccurate information. After this information is transmitted, the institute chooses whether or not to audit this data. Although this specific analysis has not yet been performed, we utilized characteristics from different branches of the Game Theory literature, which we briefly comment below, as to provide further insights into our own modeling strategies.

A similar problem is presented by ALLINGHAM and SANDMO (1972), in which an agent has to report his yearly income to the Internal Revenue Service (IRS). If he chooses to omit part of his income on the report, less taxes will be due, however he will also be exposed to the possibility of an auditing process and subsequent fines.

In this model, the agent's actual income, W, is known to him, but is not observable by the IRS. The latter only has access to the reported income X, and has to act based on this information. Tax rates are defined by θ, such that θX is the amount paid to the IRS. The possibility of auditing is measured by p, and if the agent is caught in this process, he has to pay a fine that is based on the difference between actual and declared income, $\pi(W - X)$. The agent's expected utility function is then as follows, which is solved by standard static optimization methods.

$$E[U] = pU\left(W - \theta X - \pi(W - X)\right) + (1 - p)U(W - \theta X)$$

There are two main results in ALLINGHAM and SANDMO (1972). The first is that the agent's decision on omitting income is dependent on his degree of absolute and relative risk aversion, which the IRS has no control of. However, the increase of both the probability of detection, p, and penalty rate π have direct impacts on reducing income omission. Being the case that the IRS is able to affect those two variables makes this model particularly interesting to policy-makers.

In a survey environment, however, it is difficult to perceive, at least initially, any direct gains from intentionally providing inaccurate data. Also, we can not establish empirical evidence regarding the existence of penalties that are directly related to the distance between actual and reported information regarding primary data collection. This is significantly different to the direct relationship that exists in omitting a larger income parcel to the IRS and the perpetrator's gains.

Provision of inaccurate data is commonly explored in Communication Games, more specially the Cheap-Talk literature. Cheap-Talk focus is on models of information transmission between two parts, with costless signaling. These two parts are the sender (S), an expert which has access to private information, and the receiver (R), the uninformed part who does not observe the private information and depends on signaling provided by the sender in order to implement a policy.

In the seminal paper by CRAWFORD and SOBEL (1982), the sender costlessly observe a private information, m, with $f(m)$ and $F(m)$ being its PDF and CDF, respectively. After observing this information, he generates and transmits a signal (n) to the receiver, who interprets this signal and implements a policy, y. The participants utility functions are given by $U^R(y,m)$ and $U^S(y,m,b)$, which indicates that both participants depend on the information and the policy function. As the choice of n affects the policy function y, note that U^S also depends indirectly on n. Compatibility of interests between the two parts are measured by b .

The sender's problem is to choose a feasible signal n that maximizes his utility, taking both m and b as given. He uses a signaling rule function, $q(n|m)$, where the signal n is conditioned on m. The receiver problem is to maximizes his expected utility. Note that as the receiver observes the signal and not the actual information m, he utilizes a conditional probability of m given n, that is, $p(m|n)$, to weight his expected utility.

The main finding in CRAWFORD and SOBEL (1982) is that although signaling is costless, perfect communication does not happen unless the sender and receiver interests coincide. As bias regarding interests grows, less information is actually provided in the signal. One limiting case is that of a babbling equilibria, where the signal is entirely composed by noise.

Since then, the basic Crawford-Sobel model has received a number of extensions, of which we deem the results in ISHIDA and SHIMIZU (2016) particularly interesting to our own model. The paper analyzes the effects of a partially informed receiver, which is modeled as both the receiver and the sender initially receiving a costless signal regarding the true nature of the information. The main implication of this assumption is that as the receiver's information set gets more informative, there are less incentives for informative signals by the sender[5].

Finally, our modeling approach also utilizes insights from the *Inspection Games* framework. The setup is of an inspector who has to define an optimal auditing strategy over an inspectee. This branch of the literature started with the paper by DRESHER (1962), regarding arms control auditing processes. A stylized version is as follows[6].

There exists an international arms agreement, in which its participants are hence forbidden to acquire certain weapons, for example, nuclear ones. The participant can follow to the agreement or he can stock illegal weapons. A third party is then responsible for monitoring the participants for possible breach of rules. It is assumed that the action is verifiable, that is, the third party can correctly identify existing weapons on the participants' military sites.

When the inspectee complies with the agreement, both players payoffs are null, whether there is an inspection or not. If illegal behavior goes undetected, the payoffs are $(-c,d)$, and if this behavior is detected, the payoffs are $(-a,-b)$. It is assumed that $0 < a < c$ and $b,d > 0$, so that the inspector prefers identifying illegal behavior. The verification process used is imperfect, where $1-\beta$ is the probability of illegal behavior detection.

A stable solution where legal behavior is attained is conditional on the non-detection prob-

[5] A literature review on Cheap-Talk is available in FARREL and RABIN (1996).

[6] As presented in AVENHAUS (2004). For an in-depth survey on Inspection Games, see AVENHAUS et al. (2002).

ability being sufficiently small, $\beta < \frac{1}{1+\frac{d}{b}}$. This value is dependent on the ratio of unverified illegal behavior (d), and punishment b. An increase in expected illegal behavior gains, which is represented by a larger $\frac{d}{b}$, for example, requires an even smaller β.

A game model in which the inspector faces a time-dependent loss function is presented in DIAMOND (1982). The loss function is given by $L = f(t_d - s)$, where $t_d - s$ is the number of periods the illegal behavior remained undetected. Adopting a monotonic function, the higher $t_d - s$ is, the costlier it gets to the inspector. The author was able to derive a explicit solution to the case of a linear loss function, as well as propose a solution method for the non-linear case.

This proposition is further developed in ROTHENSTEIN and ZAMIR (2002), which also uses a time-dependent loss function but also incorporates an imperfect monitoring technology. This allows for the existence of false alarms, in which the inspector may declare the inspectee guilty when he actually is not. The opposite may also happen, as the inspector consider the inspectee actions were legal when they were not. This approach is akin to incorporating type one and two errors in a classical statistics hypothesis testing approach, with these errors being dependent on agent behavior. A unique solution to this class of problems is also obtained.

Given this short introduction to the related literature, in the next section we present our baseline model.

3 Baseline model: A Two-Stage Survey Game

We will establish a simple communication game between two economic agents, a statistics institute and a respondent, in which the institute applies a survey to the respondent in order to obtain a certain private data.

We deemed appropriate to restrict the nature of statistic institute to a public or governmental firm. Private research institutes have a wide set of actions, and can, for instance, employ incentive mechanisms such as direct monetary payments to elicit the desired behavior of survey participants. If we had included these types of institutions, we would end up just mimicking results already explored in microeconomic theory.

A second assumption we make is on the nature of the private information. We shall limit it to knowledge that is not promptly available to the respondent. That is, we are dealing with an information good that has a level of finesse, for which an acquisition cost exists. Our assumption includes, for example, the case of firms reporting monthly produced units of a certain subgroup of products or producer prices, as well as individuals who have to fully disclose expenditure habits. Thus, our assumption excludes data that is already available to the respondent, such as her os his marital status, educational degree or the number of washrooms available at home.

Although these two assumptions may seem strict at first, they actually represent a major proportion of surveys being performed by official statistics institutes such as consumer expenditure surveys and industrial Production indexes.

The game's timeline is as follows: (1) an agent is chosen to take part in a survey by the statistics institute, (2) the respondent agent decide whether or not to incur in information acquisition costs and transmits the data, which may or not be accurate information, (3) the institute decides whether to audit this data, and (4) the payoffs are given to the participants. We can now fully define the structure and variables of our model:

- There is only one institute and one respondent. Their utility functions are U^I e U^R, respectively.
- The requested information is a random variable, θ, and its distribution, with PDF $f(\theta)$ and CDF $F(\theta)$, is common knowledge. This means that both the institute and the respondent have information regarding the expected interval of the random variable, but they do not know its actual true value, which is also defined as θ.
- In order to obtain the true value θ, the respondent has to pay a fixed cost K, where $K > 0$;
- The information sent by the respondent is $\hat{\theta}$, which may or may not coincide with the true value θ. In order to report the true value, that is, $\hat{\theta} = \theta$, he will need to incur in the data acquisition cost;
- The institute may audit the colected data. This procedure is able to correctly identify false data. Auditing has a fixed cost to the institute, which is A, where $A > 0$;
- If the respondent sent false data and is audited by the institute, a fine F is applied, with $F > 0$.

An extensive form representation of this game is despited in figure 1. An important aspect of this game is the information set of the institute: it is not certain of the previous move by the respondent, that is, if the respondent sent accurate or inaccurate information.

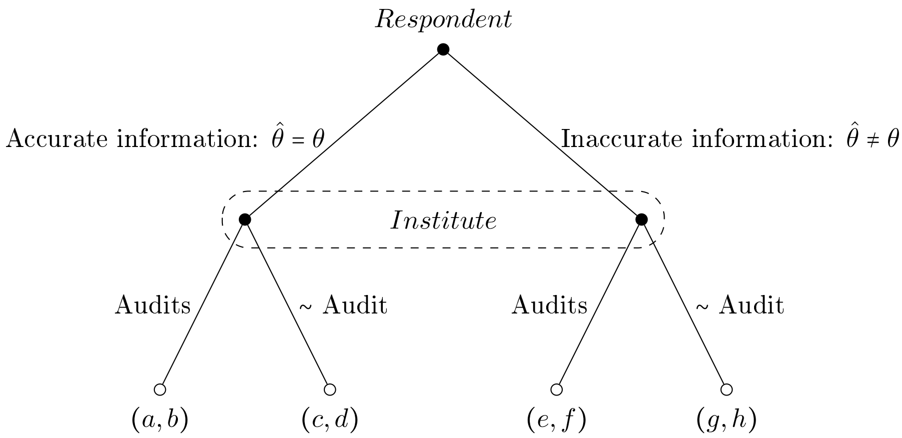

Fig. 1: A Two-Stage Survey Game

Let the utility functions of the institute and respondent be continuous maps, respectively $U^I : \mathbb{R} \to \mathbb{R}$ e $U^R : \mathbb{R} \to \mathbb{R}$. The institute's utility, equation (1), has a similar functional form as used in the Cheap-Talk literature. It is compromised of two cases, depending on whether he audits or not.

$$U^I = \begin{cases} -A, & \text{If it audits,} \\ -\left(\hat{\theta} - \theta\right)^2, & \text{if it does not audit.} \end{cases} \tag{1}$$

The utility of not auditing is measured by the difference between the reported and true value of the information. The underlying intuition is that when these values coincide, the term is nullified, while an inaccurate value generates disutility. The quadratic term assigns more weight to larger differences between these values.

However, this description is not entirely correct. Although $f(\theta)$ is common knowledge, at the second stage the institute only observes $\hat{\theta}$ and is not able to infer what the respondent's action in the earlier stage was. Thus, even when the respondent complies with the acquisition cost and reports an accurate value, $\hat{\theta} = \theta$, the institute has no means of acknowledging this situation unless an audit is performed. We then interpret this quadratic term as a measure of expected loss when not auditing, which depends both on $f(\theta)$ and the reported value.

If an auditing process is performed, false data is correctly identified, and the institute forcefully obtain the accurate information. In this case, $\hat{\theta} = \theta$, while at a cost A to the institute. As such, the respondent is obliged to now incur in the acquisition cost, if he had not done so, as well as pay a fine which amounts to F. One could argue that the respondent could, again, report a different value. We assume that the auditing process requires further reports, at a negligible cost to someone who has already incurred in the acquisition cost [7].

The respondent's utility, equation (2), is also composed of these two parts. If he opts to report the accurate value in the first place, he must incur in the acquisition cost, K. If he provides an inaccurate value, he is exposed to the possibility of a fine F and a forced acquisition cost. This is weighted by the probability of an audit, which is explained further along.

$$U^R = \begin{cases} -K, & \text{If } \hat{\theta} = \theta \\ -Pr(Audit)(F + K), & \text{If } \hat{\theta} \neq \theta. \end{cases} \tag{2}$$

Note that the fine amount F is exogenously to our model. We assume that the institute has no means of directly controlling this size, which is consistent with the actual process of legally defining fine amounts. Also, the institute does not receive monetary feedback from applying fines. If this were the case, it could elicit an undesired behavior profile from the institute[8]. An empirically consistent interpretation to this assumption is that collected fines are transfered to a central government branch.

This class of game is solved using Backward Induction (OSBORNE, 2003). The first step is to analyze the institute's decision on whether or not to audit. Equation (3) represents

[7]This additional cost would not change our main findings, so that we opted not to include it.

[8]A particularly interesting situation arises in ANDREOZZI (2004), which employs an applied Inspection Game model to crime deterrence. The theoretical results point that larger incentives for the inspector's behaviors eventually reduce their inspection frequency,

its expected utility when auditing. This process allows the institute to forcefully obtain (or confirm) the accurate information, which means that the reported information now equals the true information, $\hat{\theta} \to \theta$.

$$\begin{aligned} E[U^I] &= -E\left(\hat{\theta} - \theta\right)^2 - A \\ E[U^I | Audit] &= -E\left(\theta - \theta\right)^2 - A \\ &= -A \end{aligned} \tag{3}$$

As the distribution of the required information is common knowledge, we can also obtain the institute's expected utility of not auditing, equation (4)[9]. Non-auditing utility is dependent on the reported value $\hat{\theta}$ as well as on the the variables first and second moments.

$$\begin{aligned} E[U^I | \sim Audit] &= -E[\left(\hat{\theta} - \theta\right)^2] \\ &= E[2\hat{\theta}\theta - \hat{\theta}^2 - \theta^2] \\ &= 2\hat{\theta} \int_{-\infty}^{\infty} \theta f(\theta) d\theta - \int_{-\infty}^{\infty} \theta^2 f(\theta) d\theta \\ &= 2\hat{\theta} E[\theta] - Var(\theta) - E[\theta]^2 - \hat{\theta}^2 \end{aligned} \tag{4}$$

It is decreasing on the variance of θ. A higher variance means that there is a larger interval of possible values for the true information, which in turn also means that there is a larger possible gap between the reported and true variable. The effects of both the mean and reported value effects are ambiguous[10].

The institute will only audit if its expected utility when doing so, given by (3), is larger than (4). This is expressed in condition (5), which requires that its auditing cost, A, is sufficiently low. A larger variance implies a higher auditing probability. Although this condition also depends on the reported value and the distribution's mean, their effects remain ambiguous.

$$\begin{aligned} \text{Institute audits} \iff E[U^I | Audit] &\geq E[U^I | \sim Audit] \\ -A &\geq 2\hat{\theta} E[\theta] - Var(\theta) - E[\theta]^2 - \hat{\theta}^2 \\ A &\leq Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta} E[\theta] \end{aligned} \tag{5}$$

Let G be the CDF of the auditing costs, A. We can rewrite condition (5) as follows, which represents the institute's auditing probability.

$$\begin{aligned} A &\leq Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta} E[\theta] \\ Pr(Audit | \hat{\theta}) &\equiv G\left(Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta} E[\theta]\right) \end{aligned} \tag{6}$$

We now solve the respondent's problem, in which he decides whether to incur in the acqui-

[9]In the forth step we used the variance in terms of moments expression: $Var(X) = E[X^2] - E[X]^2$.

[10] $\frac{\partial [U^I | \sim Audit]}{\partial \hat{\theta}} = 2E[\theta] - 2\hat{\theta}$, which is positive when $E[\theta] > \hat{\theta}$ and negative otherwise. The inverse case happens on the partial derivative with respect to $E[\theta]$.

sition cost and report accurate information, equation (7), or to transmit false data, equation (8).

$$U^R(\hat{\theta} = \theta) = -K \tag{7}$$

$$\begin{aligned} E[U^R|\hat{\theta} \neq \theta] &= -Pr(Audit|\hat{\theta})(F + K) \\ &= -Pr\left(A \leq Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta}E[\theta]\right)(F + K) \\ &= -G\left(Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta}E[\theta]\right)(F + K) \end{aligned} \tag{8}$$

If the respondent is audited but had transmitted accurate data, he incurs in no additional costs and no fine is applied. If false data is reported, he is exposed to the possibility of an auditing process. If this is the case, the respondent must incur in both the acquisition cost and a subsequent fine. Through equation (8) we can derive the optimal inaccurate data to be reported by the respondent, which is given by the variable's distribution mean, equation (9). This reporting strategy is such that it minimizes his exposure to audits.

$$\frac{d[U^R|\hat{\theta} \neq \theta]}{d\theta} = 0 \rightarrow \tilde{\theta} = E[\theta] \tag{9}$$

Substituting (9) into (6), we obtain the probability of being audited when the mean of the random variable is reported. As the variance of the information grows, the least likely the actual value will be its mean.

$$Pr(Audit|\hat{\theta} = \theta) = G(Var(\theta)) \tag{10}$$

Finally we can now define a decision rule for the respondent agent, which reports accurately when his acquisition cost, K, is sufficiently small. This is expressed in (11), as follows. Note that we used the previously derived result that when the respondent opts to provide inaccurate information, his optimal choice is to use the mean of the random variable.

$$\begin{aligned} \text{Respondent reports accurate information } \iff& \\ U^R(\hat{\theta} = \theta) &\geq E[U^R|\hat{\theta} \neq \theta] \\ U^R(\hat{\theta} = \theta) &\geq E[U^R|\hat{\theta} = E[\theta]] \\ -K &\geq -P(Audit|\hat{\theta} \neq \theta)(F + K) \\ K &\leq G\left(Var(\theta)\right)(F + K) \end{aligned} \tag{11}$$

When the terms in equation (11) are equal, the respondent is indifferent to reporting true or false data. An interesting result is derived from this condition, which is the minimum fine amount that drives accurate information provision, F^{MIN}:

$$K = G\left(Var(\theta)\right)\left(F^{MIN} + K\right) \rightarrow F^{MIN} = \frac{K[1 - G\left(Var(\theta)\right]}{G(Var(\theta)} \tag{12}$$

As information acquisition costs grow, there are fewer incentives for the respondent to incur in them, such that the minimum fine must be larger. Auditing costs appear indirectly in the cumulative probability function G: higher auditing costs imply a smaller probability of its occurrence, which in turn must be compensated by an increase in fine size. In similar fashion, an increase in the random variable's variance also prompts a higher auditing probability, resulting in a smaller fine. These results are expressed in equation (13). The partial derivatives are presented in Appendix A.

$$F^{MIN} = f(\underset{+}{K}, \underset{-}{Var(\theta)}, \underset{+}{A}) \tag{13}$$

Although we do not explicitly define the parameters of the acquisition costs, they could be related to two main aspects. The first is the complexity of the requested data, which could range from a simple to an intricate report, meaning a lower or a higher K, respectively. A promptly available information, for example, would translate to a $K = 0$, a case which we purposely left out of our model. Secondly, the respondent's size could also affect acquisition costs. A small start-up company could present a higher acquisition cost than its already established counterpart. The auditing costs could also depend on similar characteristics, such as complex information being harder to audit, the institute's size and its available auditing technology.

Finally, note that there is a trade-off between auditing probabilities and fine sizes. This result is empirically consistent, as institutes hardly have the necessary budget to audit every single survey participant. So, in designing the optimal fine size, this could be taken into consideration.

In the next section, we explore a few extensions to our baseline model, such as: (a) analyze the effects of indirect incentives, such as the implementation of a public policy that use the gathered information and (b) the significance of discount rates and credibility.

4 Public policy, credibility and dynamics in a single respondent model

4.1 Public policy function

Suppose that an exogenous third party uses the institutes's collected information to design and implement a public policy, which affects our respondent's payoff. This could be the case where there is provision of an in-demand public good, such as further infrastructure made available in a certain region.

We assume that this public policy is solely dependent on the quality of the information used, not being affected by characteristics of the third party[11], and is given by $\gamma(\theta)$. An inaccurate information could elicit the provision of an ineffective service, which renders no actual returns to the respondent. Let $\gamma(\theta) \in \{\underline{\gamma}, \bar{\gamma}\}$, with $\bar{\gamma} > \underline{\gamma}$, with:

$$\gamma(\theta) = \begin{cases} \bar{\gamma}, & \text{if } \hat{\theta} = \theta \\ \underline{\gamma}, & \text{otherwise.} \end{cases} \tag{14}$$

The respondent's utility is now given by:

$$U^R = \begin{cases} \bar{\gamma} - K, & \text{If } \hat{\theta} = \theta \\ (1 - P_A)\underline{\gamma} + P_A(\bar{\gamma} - F - K), & \text{If } \hat{\theta} \neq \theta. \end{cases} \tag{15}$$

As the institute's problem is not changed, we can turn directly to the respondent's[12].

Respondent reports accurate information $\iff$

$$\begin{aligned} U^R(\hat{\theta} = \theta) &\geq E[U^R|\hat{\theta} \neq \theta] \\ \bar{\gamma} - K &\geq (1 - P_A)\underline{\gamma} + P_A(\bar{\gamma} - F - K) \\ \bar{\gamma} - K &\geq \underline{\gamma} + G\left(Var(\theta)\right)\left(\bar{\gamma} - \underline{\gamma} - F - K\right) \\ K &\leq \bar{\gamma} - \underline{\gamma} - G(Var(\theta))(F + K - (\bar{\gamma} - \underline{\gamma})) \end{aligned} \tag{16}$$

From which we can also recover the respective minimum fine required to drive the provision of accurate information, equation (17).

$$F_{pp}^{MIN} = [K - (\bar{\gamma} - \underline{\gamma})]\frac{1 - G\left(Var(\theta)\right)}{G\left(Var(\theta)\right)} \tag{17}$$

Comparing (17) with (13), we can see that $F_{pp}^{MIN} < F^{MIN}$. The public policy function acts as an additional incentive for survey respondents. The policy function also manages to mitigate the risk of non-accurate information if the fine amount is somehow limited by an exogenous upper bound, such as $F^{MIN} > \bar{F} > F_{pp}^{MIN}$.

Valuing how much a public policy is worth is not without its own problems, however, and is one of the objects for applications of the Contingent Valuation literature, where agents try to attribute value to goods whose price is not directly observed (Hausman, 2012). Hence, one could argue that the total effect of public policy is actually both itself and the public's perception of it. Bringing awareness to results of information usages to a society could be a possible way to drive this perception rating up, hence, improving its effect on survey compliance behavior.

Finally, note that with the addition of a public policy in our model, we could also generate a cyclical feature. As policies are implemented or are better perceived by the respondent, they act as an incentive for him to provide accurate data, which in turn, could further increase the

[11] By doing so, we simplify our analysis by excluding the possibility of a conflict of interests, as well as the efficiency of the third party.

[12] We shortened $Pr(Audit)$ to P_A to make the notation cleaner.

efficiency of a public policy. This endogenous cycle could be implemented in a dynamic setting and a continuous $\gamma(\theta)$, for instance.

4.2 Discount rate and credibility

An interesting addition to our baseline model is to consider a more complex approach to the auditing process. Suppose that, as it begins, the respondent has to incur in the acquisition cost, but the fine is only applied at a latter period. This dynamic captures the delay that exists between the penalty notices and actual payments.

If this were the case, the utility function in equation (2) would be adjusted by a discount factor β^t, with $\beta \in [0,1]$, where t represents the delay, in periods, between notice and payment.

$$U^R = \begin{cases} -K, & \text{If reporting an accurate value,} \\ -Pr(Audit)(\beta^t F + K) & \text{If reporting non-accurate value.} \end{cases} \tag{18}$$

Condition (11) has to be updated, which respectively impacts the minimum fine amount in equation (13). Now, the minimum fine amount also has to compensate for higher impatience, a lower β, as well as lags between notice and payment, that is, a higher t.

$$F_\beta^{MIN} = \frac{K}{\beta^t}\frac{[1 - G(Var(\theta)]}{G(Var(\theta)}$$

Note that if $\beta = 1$ we return to our baseline model. In the limiting case, if β is arbitrarily low, we end up with $\lim_{\beta^t \to 0} F^{MIN} = \infty$. In this latter case, there is not much that the institute can do, as there is no feasible fine size that is able to drive accurate data provision.

A different interpretation of this type of formulation is that it can capture the institute's enforcement capability, or credibility, as perceived by the respondent. Suppose that in an one-round game, that the respondent has lower expectations about the fine being actually applied if he is caught in an auditing process. In this scenario, which can be represented by an arbitrarily high t, the fine has little to no impact on his decision.

Thus, our model hints that credibility regarding punishment could play a role in understanding general survey behavior in different environments. A proxy for this variable, could be, for example the confidence level that the respective respondent has regarding Courts: while in Japan, the category "None at all" and "Not very much" were combined to a total of 15,9%, in Brazil these categories amounted to 49% (World Value Surveys, 2014).

4.3 Public policy function in a multiple respondent environment

In this section we extend our base model with two distinct aspects. First, we increase the number of survey respondents to two: i and j, each with their respective access to private information, that is θ_i and θ_j, respectively. Then we build on a indirect incentive mechanism in the form of a public policy function, γ, that is implemented by an exogenous third party, but is affected by information gathered from survey respondents.

With two participants, the expected utility of the institute when not auditing either of them is given by[13]:

$$
\begin{aligned}
E[U^I| \sim A_{i,j}] &= E[-(\hat{\theta}_i - \theta_i)^2 - (\hat{\theta}_j - \theta_j)^2] \\
E[U| \sim A_{i,j}] &= 2\hat{\theta}_i E[\theta_i] - Var(\theta_i) - E[\theta_i]^2 - \hat{\theta}_i^{\,2} \\
&\quad + 2\hat{\theta}_j E[\theta_j] - Var(\theta_j) - E[\theta_j]^2 - \hat{\theta}_j^{\,2} \\
E[U| \sim A_{i,j}] &= W_i + W_j
\end{aligned}
\tag{19}
$$

Where W_i and W_j are measures of expected loss by the institute:

$$
\begin{aligned}
W_i &\equiv 2\hat{\theta}_i E[\theta_i] - Var(\theta_i) - E[\theta_i]^2 - \hat{\theta}_i^{\,2} \\
W_j &\equiv 2\hat{\theta}_j E[\theta_j] - Var(\theta_j) - E[\theta_j]^2 - \hat{\theta}_j^{\,2}
\end{aligned}
$$

The other cases of expected utilities are obtained by combining different auditing compositions, which are:

$$
\begin{aligned}
E[U|A_i, \sim A_j] &= W_j - A \\
E[U| \sim A_i, A_j] &= W_i - A \\
E[U|A_i, A_j] &= -2A
\end{aligned}
$$

In this case, we will use a particular case of the public function γ,, in which its effectiveness is proportional to the amount of accurate information collected by the institute. This is expressed in equation (20).

$$
\text{Effectiveness of additive public policy} = \frac{\sum_{i=1}^{n} T_i}{n}\gamma \tag{20}
$$

In this case, T_i is a counter variable which accounts for the total of accurate data collected. This occurs in two different scenarios: (1) the survey respondent initially provided the institute with an accurate value, that is, $\hat{\theta}_i = \tilde{\theta}_i$ and (2) the respondent provided inaccurate data but was audited (A_i or A_j)

$$
T_i = \begin{cases} 1, & \text{if } (T_i) \text{ or } (\sim T_i) \wedge (A_i) \\ 0, & \text{otherwise} \end{cases} \tag{21}
$$

The institute's decision on whether or not to audit a respondent is similar to previous model:

$$
\begin{aligned}
\text{Institute audits } i \text{ if:} E[U^I|A_i] &\geq E[U^I| \sim A_i] \\
-C &\geq W_i \\
C &\leq -W_i \\
P(A_i) &= G(-W_i)
\end{aligned}
$$

[13] *Audit* and $\sim$ *Audit* where shortened to A and $\sim A$ in order to provide a clearer notation.

From which we obtain the probability of auditing respondent i when he reports the expected value of θ, which is the same as in the previous model.

$$\begin{aligned}
P(A_i) &= G(-W_i) \\
P(A_i) &= G\left(\hat{\theta}_i E[\theta_i] - Var(\theta_i) - E[\theta_i]^2 - \hat{\theta_i}^2\right) \\
P(A_i|\hat{\theta}_i = E[\theta]) &= G\left(P(A_i|\hat{\theta}_i = E[\theta])\right) \\
&= G(Var(\theta_i))
\end{aligned}$$

The major change happens when obtaining the respondent's expected utility, now dependent on the public policy function:

$$\begin{aligned}
&\text{Transmits accurate information:} \\
E[U^{R_i}|T_i,\theta_j] &= -K + \frac{\gamma}{2} + \frac{E[T_j]\gamma}{2} \qquad (22) \\
&\text{Transmits inaccurate information:} \\
E[U^{R_j}|\sim T_i,\theta_j] &= -P(A_i)\left(F + K - \frac{\gamma}{2}\right) + \frac{E[T_j]\gamma}{2} \qquad (23)
\end{aligned}$$

The term $\frac{\gamma}{2}$ captures the public policy parcel which i will certainly receive when reporting accurate information. Similarly, $\frac{E[T_j]\gamma}{2}$ is the expected value received from i due to information provided by j. Also note that if i is audited when providing inaccurate information, he again has to incur in two costs: obtaining the true data and paying the fine, but now this cost is mitigated by public policy returns.

In order for i to provide accurate data:

$$\begin{aligned}
E[U^{R_i}|T_i,\theta_j] &\geq E[U^{R_i}|\sim T_i,\theta_j] \\
-K + \frac{\gamma}{2} + \frac{E[T_j]\gamma}{2} &\geq -P(A_i)\left(m + K - \frac{\gamma}{2}\right) + \frac{E[T_j]\gamma}{2} \\
K &\leq G(Var(\theta_i)\left(F + K - \frac{\gamma}{2}\right) + \frac{\gamma}{2} \qquad (24)
\end{aligned}$$

As the public policy function is additive, note that the term $\frac{E[T_j]\gamma}{2}$ cancels out: what j is doing does not play a role in i's decision process. The minimum fine that drives i to provide accurate information follows from (24):

$$F_i^{MIN} = \frac{K[1 - G\left(Var(\theta_i)\right)]}{G(Var(\theta_i)} + \frac{\gamma}{2} \qquad (25)$$

Our model does not allow for the designation of different fines per respondent, such that the minimum size amount required to drive accurate data transmission by both respondents is given by (26). We could adapt this to control for different fines, but such a measure would only complicate the model.

$$F^{MIN} = \max\left(\frac{K[1 - G(Var(\theta_i))]}{G(Var(\theta_i)} + \frac{\gamma}{2}, \frac{K[1 - G(Var(\theta_j))]}{G(Var(\theta_j)} + \frac{\gamma}{2}\right) \tag{26}$$

There is, however, an important factor that we are not taking into consideration, which is the interaction between fines and firm sizes: even though a certain fine amount can act as huge driving force for small firms, for example, it can not be so for larger corporations. In order to incorporate this, we would need to remodel our respondents with a higher degree of heterogeneity.

5 Concluding remarks

The main contribution of this paper is to offer a novel game-theoretic approach to analyze the strategic interaction between survey applicants and respondents, more specifically the case of a public statistics institute. Public institutes, being limited to a stricter set of actions while compared to their private counterparts, provide a rich environment for agent behavior commentary and policy analysis.

In our baseline model, provision of accurate information is dependent on the true information's acquisition costs, and is largely affected by both the auditing policy by the institute, and fine sizes. A public policy that utilizes information as input can also serve as an indirectly reward for accurate survey compliance. The respondent's credibility regarding the applicability of fines also play a role in their decision process.

We were able to derive a mathematical condition that relates fine amounts and accurate responses on surveys. While we acknowledge our result is far from being directly applicable, it can represent a first step towards a more robust theoretical background when designing fines.

Our baseline model also provides a few possible extensions. The case of credibility can be further explored by setting a repeated game environment. There is also the possibility of implementing a strategic interaction between survey respondents, by further exploring the multi-participant game. Finally, although some of the main driving variables to our baseline model are currently exogenous, extensions could allow for the institute to affect both internal auditing costs and the respondent's transmission costs through investment in research and development.

Appendices

A Partial derivatives of F^{MIN}

From the minimum fine amount, equation (12), we have the following partial derivatives:

$$F^{MIN} = \frac{K[1 - G\,(Var(\theta)]}{G(Var(\theta))}$$

- Partial derivative w.r.t K:

$$\frac{\partial F^{MIN}}{\partial K} = \frac{1 - G\,(Var(\theta))}{G(Var(\theta))} > 0 \iff G(Var(\theta)) < 1 \tag{27}$$

- Partial derivative w.r.t $Var(\theta)$:

$$\begin{aligned} \frac{\partial F^{MIN}}{\partial Var(\theta)} &= \frac{-g(Var(\theta)G(Var(\theta) - g(var\theta)[1 - G(Var(\theta))}{G(Var(\theta))^2} \\ &= \frac{-g(Var(\theta))}{G(Var(\theta))^2} < 0 \end{aligned} \tag{28}$$

- Partial derivative w.r.t A:

  From condition (5), suppose we add an arbitrarily small positive number, ϵ, to A, we have:

$$\begin{aligned} A + \epsilon &\leq Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta}E[\theta] \\ A &\leq Var(\theta) + E[\theta]^2 + \hat{\theta}^2 - 2\hat{\theta}E[\theta] - \epsilon \end{aligned}$$

  Such that the respective minimum fine is:

$$F^{MIN}_{A+\epsilon} = \frac{K[1 - G\,(Var(\theta - \epsilon)]}{G(Var(\theta - \epsilon))}$$

  We want to show that $F^{MIN}_{A+\epsilon} > F^{MIN}$:

$$\begin{gathered} F^{MIN}_{A+\epsilon} > F^{MIN} \\ \frac{K[1 - G\,(Var(\theta) - \epsilon)]}{G(Var(\theta) - \epsilon)} > \frac{K[1 - G\,(Var(\theta)]}{G(Var(\theta))} \\ G(Var(\theta)) - G(Var(\theta))G(Var(\theta - \epsilon)) > G(Var(\theta - \epsilon)) - G(Var(\theta) - \epsilon)G(Var(\theta)) \\ G(Var(\theta)) > G(Var(\theta) - \epsilon) \end{gathered}$$

  As G is increasing, the above condition holds. Thus, $\frac{\partial F^{MIN}}{\partial A} > 0$.

B Independence of information provided under additive policy

From the derived expected utilities, the only term that could generate spillovers from j to i is $\frac{E[T_j]\gamma}{2}$.

Where

$$E[T_j] = \begin{cases} 1, & \text{if } (T_j \wedge A_j) \cup (T_j \wedge \sim A_j) \cup (\sim T_j \wedge A_j) \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

By symmetry, respondent j opts to transmits accurate data if:

$$K \leq \frac{\gamma}{2} + P(A_j)(F + K - \frac{\gamma}{2})$$
$$K \leq \frac{\gamma}{2} + G(-W_j)(F + K - \frac{\gamma}{2})$$

Which does not depend on respondent i's provided information, that is, θ_i. So that:

$$\frac{dE[T_j]}{d\hat{\theta}_i} = 0$$

The same argument could be made from i to j, so both i and j actions end up being independent from each other.

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