Volatility Estimation for Bitcoin on Brazilian Market:
Speculative Trading Approach and GARCH Models Comparison

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Abstract
This study revisits the volatility around speculative trading and GARCH models of the Bitcoin Brazilian market. Besides providing an exploratory glace at the value and volatility about Bitcoin across time, we also test the ratio between volatility versus speculative trading and explore theoptimal heteroskedasticity model with regards to goodness-of-fit to Bitcoin price data. Results evidence that on the average day, trading activity in Bitcoin is speculative; and more, the analysis between volatility and return or speculative trading does not respond with a direct relation, indicating a high degree of asymmetric information in Brazilian market. Finally, it is found that the best conditional heteroskedasticity model is the AR-APARCH.
Keywords: Bitcoin, Currency Markets, Speculation, GARCH, Volatility

Resumo
Este estudo revisa a volatilidade em torno de negociação especulativa e modelos GARCH do mercado brasileiro de Bitcoin. Além de fornecer uma visão exploratória sobre o valor e a volatilidade do Bitcoin ao longo do tempo, também testamos a relação entre volatilidade versus negociação especulativa e exploramos o modelo de heterocedasticidade ideal em relação aos dados de preço do Bitcoin. Os resultados evidenciam que, na média diária, a atividade de negociação no Bitcoin é especulativa; e mais, a análise entre volatilidade e retorno ou negociação especulativa não responde com uma relação direta, indicando um alto grau de informação assimétrica no mercado brasileiro. Finalmente, verifica-se que o melhor modelo de heterocedasticidade condicional é o AR-APARCH.
Palavras-chave: Bitcoin, Mercados de Moeda, Especulação, GARCH, Volatilidade

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1. Introduction

Introduced in 2008 by a group of programmers, Bitcoin is a cryptocurrency or virtual money derived from mathematical cryptography. Compared to other traditional financial assets, Bitcoin provides investors a new instrument in portfolio management. Over the last few years, there have been many studies about Bitcoin market. This popularity has attracted the interest of researchers and practitioners alike, especially looking for a better understanding of the various characteristics of Bitcoin such as price volatility (Baek and Elbeck, 2015), speculative bubbles (Cheah and Fry, 2015), inneficiency (Nadarajah and Chu, 2017; Bariviera, 2017; Tiwari et al., 2018), price dinamics (Blau, 2018) and informed trading (Feng et al., 2018).

Feng et al. (2018) studied informed trade ahead of cryptocurrency-related events, through a novel indicator. Using a trade-level data of USD/BTC exchange rates, the authors found evidence of informed trading in Bitcoin market prior to large events: quantiles of buyer-initiated orders are abnormally high before large positive events, compared to quantiles of seller-initiated. The profits of informed trading in Bitcoin, could be considerede large. Thus, Blau (2018), kipping the signaling line, provides an exploratory research of Bitcoin's volatility across time, and also test the relationship between volatility and speculative trading. The author concludes a directly association with speculative trading and Bitcoin's unusual level of volatility.

The volatility of digital currency must be questioned: Bitcoin functions are currency? Bitcoin has certainly been used as a medium of exchange for many consumers, on the other hand we have the concern that Bitcoin is less of a currency and more a speculative investment. Extending Bitcoin as a investment, which suffer speculation impact, the virtual money reduces its viability as viable currency. As Blau (2018) affirms, approximating speculative trading is a difficult task given that the motives to trade are not observed.

Bitcoin has therefore a place in the financial markets and portfolio management (Dyhrberg, 2016; Kasiampa, 2017), being certain that examining its volatility is crucial. Moreover, the presence of long memory and persistent volatility (Bariviera, 2017) justifies the application of GARCH-type models.

Another discution point that volatility provides is related with the Efficient Market Hypothesis (henceforth, EMH). Being a market where exists a free negotiation and normal levels of volatility, we can consider that the EMH is weak, since the information is available to all participants and it has no impact on negotiation and price oscillation. The opposite can happen once there are large price oscillations, which may not be explained by news available in the market; and would eventually characterize Bitcoin as an asset of speculation, and not so much as currency.

The objective of this study is threefold. First, we look for provide some stylized facts about price dynamics of Bitcoin on Brazilian market. Second, we test the fact that speculative trading in Bitcoin is responsible for its unusual level of volatility. Third, we investigate which conditional heteroskedasticity model can describe the Bitcoin price volatility better.

The research contributes to literature in important ways. First, we provide some initial findings about exchange rate dynamics of Bitcoin on Brazilian market. Second, we demonstrate that the level of speculative trading, considering the procedure adopted by Llorente et al. (2002), occurs frequently on Bitcoin Brazilian market into the current period; going against the EMH with strong evidence - once that volatility presents different behavior if compared to returns, which can be affected by this event. Third, we found evidence that the optimal model in terms of goodness-of-fit to the data is the AR-APARCH, which suggests the importance of having both component of conditional variance.

2. Background on Bitcoin and Speculative Trading

As detailed previously, around the objetives on this paper, is necessary to present some stylized facts about the historic price dynamics of Bitcoin and analyze causuality between Bitcoin's volatility and speculative trading. In order to further motivate our research, we discuss the background of Bitcoin and speculative trading aligned to Efficient Market Hypothesis.

2.1. ***Bitcoin***

Academic interests in anonymous comunications researches date back to early 1980s, and first digital currency, DigitalCash, was launched in 1990 which offered anonymity through cryptographic protocols. The peer-to-peer electronic moneraty system was initially described by Nakamoto (2008) with the objective to explain how the digital currency could be created and implated. In a short paper, Nakamoto (2008) dicusses the weaknesses of the existing electronic payment system and identifies the high costs of mediating disputes around the system. To overcome the trust issues regarding the electronic payment system, Nakamoto (2008) argues that is needed an electronic payment system based on cryptographic proof instead of trust, allowing any two willing parties to transact directly with each other without the need for a trusted third party.

The first Bitcoin transactions occurred in January 2009. More than two years later, various reports estimated the circulation of Bitcoin to be more than 6.5 million with about 10000 users (Blau, 2018). While the firts transactions in Bitcoin appeared to function according to the initial intentions, soon reports began to appear that Bitcoin was being used to purchase illegal drugs. Policy makers around the world became concerned with the anonymity afforded by Bitcoin. Beyond the potential to fund criminal activity, some researchers have voiced concerns that, because of the price dynamics, Bitcoin functions more as a speculative asset than as a medium of exchange. Considering its anonymity, Bitcoin may be a target by speculators. Reports have attempted to link the rise and subsequent collapse in the value of Bitcoin through speculative trading.

Today, this has manifested itself into a growing Cryptocurrency community which now include banks, hedgefunds and even Government. The most popular Cryptocurrency, which has the biggest mark capitalization, is Bitcoin. A $1000 USD investment in Bitcoin in July of 2010 would have returned $81000000 just seven years later (Phillip et al., 2018). Bitcoin is genereal treated as speculative (Cheah and Fry, 2015; Baek and Elbeck, 2015). Thus, some evidences suggest the Cryptocurrency market is still inefficient (Nadarajah and Chu, 2017; Bariviera, 2017; Tiwari et al., 2018), with properties such as volatility (Urquhart, 2017), infomed trading (Feng et al., 2018) and price dinamics (Blau, 2018).

Reports have foccused to link the meteoric rise and subsequent collapse in Bitcoin's value to speculative trading. These, seem to have maerit as the theoretical literature nicely describes the link between speculation and bublles in different asset markets (the most popular, stock market). For example, Stein (1987) proves that the presence of speculation can inhibit arbitrage and lead destabilized asset prices. Thus, Shiller (1981) gives some additional insight, showing the link between speculation and destabilization of prices in equity markets; the author suggests that the observed excess volatility in speculative prices contradicts the EMH. The idea motivating this paper is that as a network the flow of contracts and negotiations (speculative trading) is key determinant for Bitcoin's price behavior.

Cheah and Fry (2015) test for evidence of speculative bubbles in Bitcoin returns. The authors find that as with other asset classes, Bitcoin prices are prone to speculative bubbles, and the bubble component contained within Bitcoin prices is substantial. In a more recent study, Cobert et al. (2018) analyse through time and frequency domain the relationships between three popular cryptocurrencies and a variety of other financial assets, with concluding results that support the view that cryptocurrencies may offer diversification benefits for investors with short investment horizons. Blau (2018) test the volatility of Bitcoin and speculative activity; showing the debate behind cryptocurencies be a speculative investment asset or currency.

Baek and Elbeck (2015) find evidence to suggest that Bitcoin returns are driven by buyers and sellers internally, and not by fundamental economic factors. Using de-trended ratios, the authors determine Bitcoin returns to be 26 times more volatile than those of the SP 500 index, suggesting that Bitcoin is a speculative investment vehicle. The authors however, determine that this classification may change as usage grows, volatility decreases and Bitcoin attracts market and economic influence. In doing so, Bitcoin may become a more balanced investment vehicle, driven both internally and externally. Finally, Kasiampa (2017) explores the ability of several competing GARCH-type models to explain the Bitcoin price volatility, and find that Bitcoin is different from any other asset on the financial market and thereby creates new possibilities for stakeholders with regards to risk management, portfolio analysis and consumer sentment analysis.

2.2. Speculative Trading and Efficient Market Hypotehsis

The EMH is a fundamental concept of financial economics. Over a century ago, Bachelier (1900) developed the first mathematical model of security prices, applying the arithmetic Brownian motion model to French bonds. The systematic study of informational efficiency begun on 60s, when financial economics was borns as a new area within economics. The classical definition due to Fama (1970) which afirms that a market is informationally efficient if it can reflect all avaible information on circulation. Briefly, the EMH requires that returns of financial assets follow a memoryless stochastic process with respect to the underlying information set.

Therefore, the key element in assessing efficiency is to determine the appropriate set of information that impels prices. According to the author, informational efficiency can be considered into three categories: (i) weak efficiency, if prices reflect the information cointed in the past series of prices, excluding the possibility of finding, systematically, profitable trading strategies; (ii) semi-strong efficiency, if prices reflect all public avaiable information; and (iii) strong efficiency, when prices reflect all public and private information. A part of literature focused on lon-range dependence study. Considering a financial market as a dynamical structure, short term memory can exist without contradicting the EMH. In fact, the presence of some mispriced assets is the necessary stimulus for individuals trade and reached an arbitrage free situation. On the other hand, the presence of long range memory is at odds with the EMH, because it would allow an stable trading rule to beat the market.

There are several studies that find long memory on financial assets time series, using different methods. For example, McCarthy et al. (2009) find long memory in yields of corporate bonds and spread of returns of corporate bonds and treasury bonds. Another issue in the literature is the time varying beahvior of the market efficiency. In this knowledge area, Ito and Sugiyama (2009) find that inefficiency varies through time in the US stock market. Kim et al. (2011) find that returns predictability is altered by political and economic crises but not during market crashes. Bariviera (2017), using the Hurst exponent, find that the long memory content of daily volatility is stronger than in daily returns; in particular, volatility clustering is a key features of the Bitcoin market.

3. Data and Methodology

We collect data from www.bitcoincharts.com which provides complete history of various Bitcoin exchanges denoted in various exchanges. The data consists of daily closing prices of Brazilian operators (FoxBit, LocalBitcoin, Mercado Bitcoin, Bitcoin to You, Brasil Bitcoin Market) from 1st August 2011 to 28th February 2018 therefore capturing almost seven 7 years of Bitcoins prices.

We compute the results for the usual daily logharithmic return:

$$r_t = (lnP_t - lnP_{t-1}) * 100 \quad (1)$$

and the daily price volatility, definined as the logharithmic difference between intraday highest and lowest price:

$$ReturnVolatility = lnP_t^{high} - lnP_t^{low} \tag{2}$$

Objectifying to measuring speculative trading, we follow Blau (2018) and Llorente et al. (2002) and use a time series model that identifies the dynamic relation between volume and prices. We estimate daily turnover on day t in the following equations:[1]

$$logturn_t = log(turn_t + 0.000025) \tag{3}$$

$$vt = logturn_t - \frac{1}{50}\sum_{s=50}^{-1} logturn_t \tag{4}$$

v_t is the (50-day) de-trended measure of trading acitivity. Llorente et al. (2002) then estimates the following time-series equation.

$$R_{t+1} = \beta_0 + \beta_1 R_t + \beta_2 R_t x v_t + \epsilon_{t+1} \tag{5}$$

The dependent variable R_{t+1} is the daily returns for Bitcoin on day $t + 1$. Llorente et al. (2002) argue that when β_2 is positive, volume is likely to represent speculative trading. Under this circumstance, trading volume directly affects the serial correlation in asset returns. When β_2 is negative, trading volume inversely affects return autocorrelation and can be thought of as hedging activity.

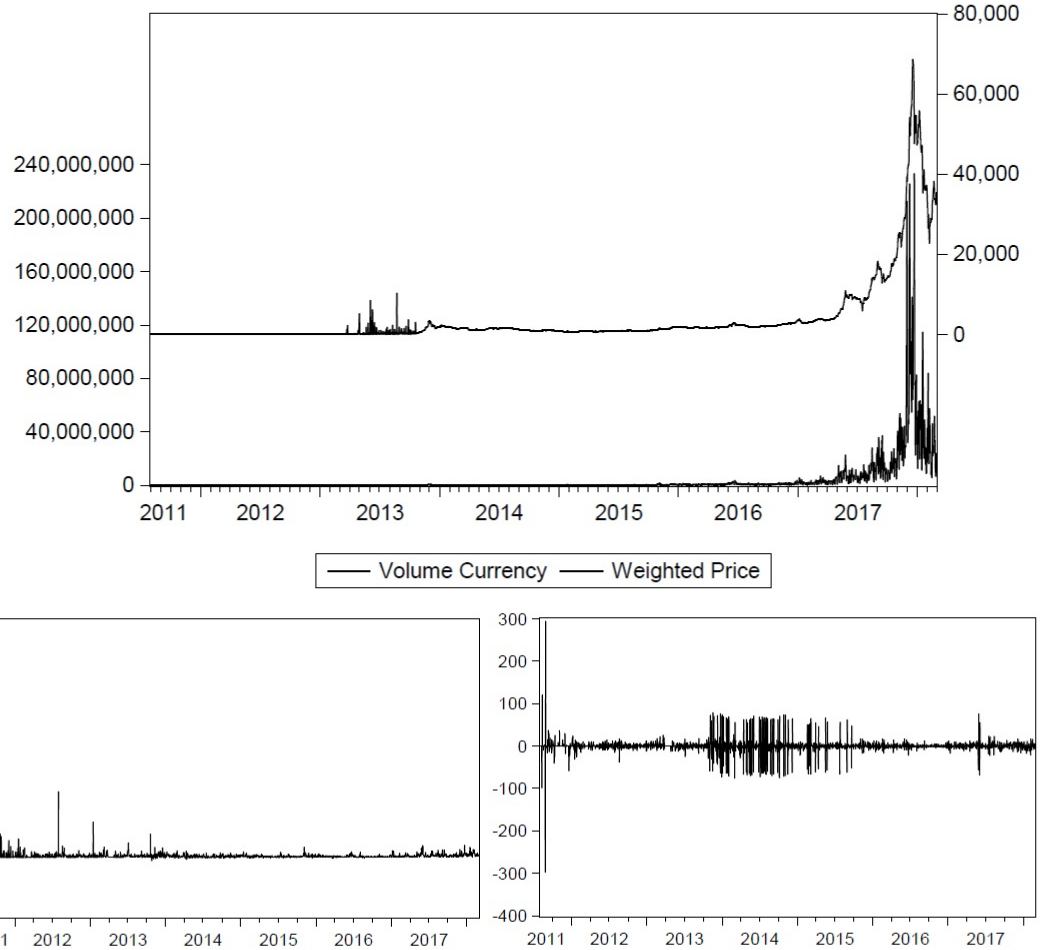
A plot of the data is shown in Figure (1), which detailed: BTC/BRL volume currency and weighted price, BTC/BRL daily returns and BTC/BRL daily returns volatility. We clearly notice that the long bull market lasted almost 1 year before it ended in July 2017 - July and August are the months during which we captured a major structural break in Bitcoin prices.

If compared to USA bull market (Balcilara et al., 2017), we notice that lenght Brazilian's bull is $\frac{1}{3}$ that USA's bull. This phenomenon may have several explanations. The high degree of speculative trading, guided in the context of information asymmetry in the Brazilian market, may lead to greater price sensitivity. In this way, we will have a lower peak of high prices, followed by sudden fluctuations, which will not be accompanied by asset price returns - since these are market failures and speculations, not real oscillations of the investment in question.

[1]Following prior research (Llorente et al., 2002) we use turnover as the ratio of daily Bitcoin volume and the number of outstanding Bitcoins. We note that we add a small constant (0.00000255) to volume to account for days without trading volume (fact that occurs consistently between the years 2011 and 2014). This constant is further shown to normalize the distribution of trading volume in Llorente et al. (2002) and Blau (2018).

Figure 1: **BTC/BRL: (i) volume currency and weighted price, (ii) daily returns, and (iii) daily returns volatility**

Note: Figure reports some important time series: volume currency concerns to daily trading volume in Bitcoin into Brazilian market; weighted prices is the ratio of volume currency and volume Bitcoin; daily returns is the Bitcoin log return solved by (1) and daily returns volatility is the estimate of volatiliy as detailed in (2)



4. Results

4.1. *Univariate Correlation*

Table 1 presents the descriptive statistics for Bitcoin returns, volatility, traded volume and speculation. We estimate Eq. (4) using 50-day rolling windows so that each day has a measure of speculative trading (*Speculation*). The estimate for β_2 from equation (5) is 0.5659, indicating that on the average day, trading activity in Bitcoin is speculative according to the definition in Llorente et al. (2002). We observe that volume (*Volume Currency*) is more volatile than returns

(*Return*) in the Brazilian Bitcoin market. On the average day, trading volume (*Volume Currency*) is more than 3,000,000, and turn (*Log Turn*) is approximately 2.6527%. The percent change for the average exchange rate on the average day (%Δ*Bitcoin*) is -1.12% while volatility (*Volatility*) is 0.0575%.

If we observe the results about volatility (*Volatility*) and the daily price variation of Bitcoin (%Δ*Bitcoin*), we can again observe the intensity of information asymmetry, since the price of Bitcoin in the Brazilian market fluctuates considerably. Even with a high peak in trading volume and appreciation in 2017, it has daily and negative price variation. That is, investors are led to buy and sell assets, guided by market trends, forming the speculative trading movement, so make the price of the asset fluctuate more than necessary.

Faced with this initial estimate, around the conclusion that on the average day, trading activity in Bitcoin is speculative according to the definition in Llorente et al. (2002), we give suport to EMH on strong evidence. This conclusion concerns that prices reflect instantly even hidden or privileged information. On the other hand, the long term behavior of returns and volatility are different. This kind of behavior could hide some complex underlying dynamics, which exceeds the aim of this analysis until this moment.

This conclusion becomes evident when we analyze on a more specific scale the behavior of daily return volatility (*Volatility*) and daily return (*Return*): is possible to observe that in bull period the returns do not follow the behavior of volatility. Still, in several moments the trajectories are divergent, reflecting the high degree of speculation of the market; as demonstrate in Figure 2.

Figure 2: **BTC/BRL: Daily Return x Daily Return Volatility**

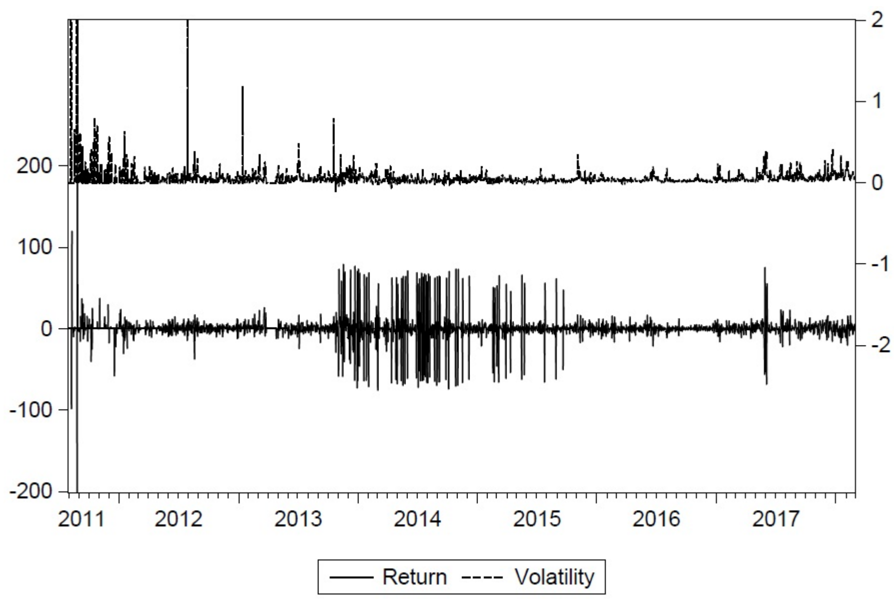


Table 1: **Summary Statistics**

The table reports summary statistics for a variety of different variables. Table reports the statistics for the Bitcoin: *Bitcoin* is the value of Bitcoin in R$. *%ΔBitcoin* is the daily percent change in the value of Bitcoin. *Volume Bitcoin* it the daily value on Bitcoin's contracts negotiated. *Volume Curreny* is the daily value on Bitcoins's on monetary terms (here, R$). *Log Turn* is the daily turnover ratio of the daily volume scalled. Volatility is the daily return volatility on Bitcoin's Brazilian Market. Return is the daily return on Bitcoin's Brazilian Market.

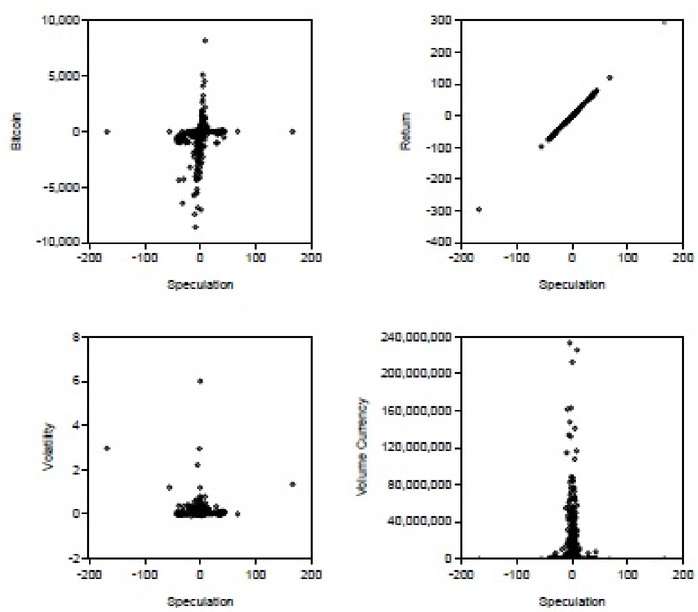
	Bitcoin	**%ΔBitcoin**	**Volume Bitcoin**	**Volume Currency**	**Log Turn**	**Volatility**	**Return**
Mean	3,777.44	-1.12	289.09	3,589,526.00	2.6527	0.0575	0.3351
Median	1,057.95	-0.05	134.54	133,587.20	3.0244	0.0336	0.0000
Maximum	68,610.76	81.94	5,401.37	2,330,000,000.00	4.8363	5.9914	294.2859
Minimum	0.00	-86.29	0.00	0.00	-5.5934	-0.1024	-296.7330
Std. Dev.	9097.38	6.54	428.53	14,764,957.00	1.2426	0.1727	17.3129
Skewness	4.04	-0.03	3.74	8.30	-1.4627	22.2259	0.01785
Kurtosis	20.53	0.64	26.71	94.40	8.4082	658.3414	80.6928
Jarque-Bera	37,357	3,801	61,944	864,559	37,87	43,216,737	604,624
Probability	0.00	0.00	0.00	0.00	0.0000	0.0000	0.0000

Therefore, when we analyze the relation between speculative trading and another variables, as shown in Figure (3), is possible to confirm the hypothesis around the assymetric information. *Speculation* has a perfect correlation with *Return*, for obviously second equation which estimated it. Thus, speculative trading does not respond positively or negatively to daily return volatility (*Volatility*) , with exception for some extreme cases (we can imagine they are news events). Even more, *Speculation* is indifferent to the daily monetary value in circulation (*Volume Currency*), since it does not present variations when it occurs in high peaks. Lastly, speculative trading responds in a way aligned to the daily variations in Bitcoin's price in Brazilian market (*Bitcoin*), when these have a high variance; that is, when there are significant increases or reductions.

Interestingly, Blau (2018) did not found that volatility (or the extreme indicator variables) is positively related to speculative trading. By this way, we observe a similarity between USA Bitcoin market and BRL Bitcoin market.

Figure 3: **Speculative Trading Behavior**

The graphics reports summary behavior for a variety of different variables. Figures reports the relation for speculative trading (*Speculation*) comparing to the daily percent change in the value of Bitcoin (*Bitcoin*), the daily return volatility on Bitcoin's Brazilian Market (*Volatility*), the daily return on Bitcoin's Brazilian Market (*Return*) and the daily value on Bitcoin in monetary terms, here R$, (*Volume Curreny*).



We estimate volatility as the long-run average standard deviation in a GARCH(1,1) model.

The model can be written as follows:

$$\sigma_t^2 = \gamma V_t + \alpha m_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{6}$$

We, however, estimate the following version of the model below:

$$\sigma_t^2 = \omega + \alpha \mu_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{7}$$

and obtain estimated parameters for ω, α and β. Once these parameters are obtained, we can estimated γ, where $\gamma = 1 - \alpha - \beta$. Since $\omega = \gamma V_l$ and we observe σ_t^2 , we solve for the long-run variance V_l. Our measure of volatility is the square root of this numeric solution for the long-run variance.

Table 2 reports the matrix of Correlation Coefficients. A few results are noteworthy. First, speculative trading (*Speculation*) and Bitcoin returns volatility (*GARCH(1,1)*) are positively related. Interestingly, we find that volatility (*GARCH(1,1)*) is positively related do speculative trading (*Speculation*), which confirms our proposition around Bitcoin and an opportunity of speculative investment.

Table 2: Correlation Matrix

Note: This table reports the Correlation Coefficients. The variables included in the matrix are our variables about speculative trading (*Speculation*), the Bitcoin daily return (*%Δ Bitcoin*), our estimate for volatility (*GARCH(1,1)*), indicator variables *High* and *Low*.

	Speculation	**%Δ Bitcoin**	**GARCH(1,1)**	**Volume Currency**	**Low**	**High**
Speculation	1	0.1906	0.0885	-0.0008	0.0039	0.0017
%Δ Bitcoin	0.1906	1	0.0465	-0.2793	-0.3097	-0.3267
GARCH(1,1)	0.0885	0.0465	1	-0.0883	-0.0490	-0.0585
Volume Currency	-0.0008	-0.2793	-0.0883	1	0.7864	0.8180
Low	0.0039	-0.3097	-0.0490	0.78644	1	0.9974
High	0.0017	-0.3267	-0.0585	0.8180	0.9974	1

4.2. ***Multivariate Tests***

Next, we provide a more robust multivariate test, estimating the following equation:

$$GARCH(1,1) = \beta_0 + \beta_1 Speculation_t + \beta_2 \%\Delta Bitcoin_{t-5,t-1} + \beta_3 VolumeCurrency + \epsilon_t \tag{8}$$

We estimate Equation (8) using GMM and report p-values that are obtained form Newey and West standad errors. Table 3 reports simple regressions in colums. We find that the daily percent change in the value of Bitcoin (*%ΔBitcoin*) has a positive impact on volatility (*GARCH(1,1)*), but is not statistically significant. Thus, daily value on Bitcoin (*Volume Currency*) produces negative estimates while speculative trading (*Speculation*) produces positive coefficients. These results indicate that while daily value on Bitcoin's on monetary terms do not affect directly volatility, speculative trading affects.

Table 3: Volatility Regressions

Note: The table reports the results from estimating the following equation using GMM.
$GARCH(1,1) = \beta_0 + \beta_1 Speculation_t + \beta_2 \%\Delta Bitcoin_{t-5,t-1} + \beta_3 VolumeCurrency + \epsilon_t$
The dependent variable is GARCH(1,1), which is our estimate for Bitcoin volatility. The independent variables include our measure of speculative trading (*Speculation*), the prior five-day Bitcoin return ($\%\Delta Bitcoin$), the daily value on Bitcoin's on monetary terms (*Volume Currency*). The equation is estimated using GMM and p-values, which are obtained form Newey and West standard errors are reported in parentheses. *,**,*** denote statistical significance at the 0.10, 0.05, 0.01 levels, respectively.

Intercept	**Speculation**	$\%\Delta Bitcoin$	**Volume Currency**	R^2
-0.0053 ***	0.0001 *	0.0014	-0.0001 ***	0.1566
(0.0005)	(0.0001)	(0.0000)	(0.0000)	

4.3. ***Volatility Estimation: A Comparison of GARCH Models***

Bitcoin has therefore a place in the financial markets and in portfolio management, and examining its volatility is crucial. Moreover, the presence of long memory and persistent volatility (Bariviera, 2017) justifies the application of GARCH-type models (Kasiampa, 2017).

Autoregressive conditional heteroskedasticity (ARCH) was developed in order to forecast the variance of financial and economic time series during time. ARCH models have been generalized to become the generalized ARCH or GARCH models. These models have become common tools for dealing with time series heteroskedastic models; providing a volatility measure that can be utilized in portfolio selection, risk analysis and derivative pricing.

A GARCH (1,1) model is very common in financial time series data, but ARCH and GARCH models have been expended in the direction of returns, not just the magnitude. They include, for example, the IGARCH model which allows for volatility shocks to be permanent, the TARCH (threshold ARCH) and the EGARCH (exponential GARCH) which consists in asymmetric models that allow negative shocks to behave differently from positive shocks. An EGARCH overcomes the problem around the standard ARCH/GARCH models where symmetry is imposed on the conditional variance.

In 1993 was introduced a new variant called the power ARCH (PARCH) model. In this inovation to the ARCH family, the power term is estimated within the model rather than being imposed by the author. The advantage is that rather than imposing a structure on the data, the PARCH model allows a power transformation term inclusive of any positive value and so permits a virtually infinite range of transformations.

The power term is the means by which the data are transformed. Term captures volatility clustering by changing the influence of the outliers. However, when the data is non-normally distributed, or where it is not otherwise possible to characterize the distribution on mean and variance, the utilization of a squared power transformation is not appropriate and other power transformations are necessary in order to use higher moments to adequately describe the distribution.

Backing to our subject of interest, earlier studies have applied GARCH-type models, such as the linear GARCH (F Glaser, 2014), the Thresold GARCH (TGARCH) (Dyhrberg, 2016; E Bouri, 2017), the Exponential GARCH (EGARCH) (Dyhrberg, 2016; Kasiampa, 2017). However, as most of the previous studies have foccused on USA market, and the most of it used a single conditional model, a question that remains unanswered is which conditional heteroskedasticity model can better explain the Bitcoin Brazilian's data. Hence, the aim to finalize this research is to investigate which conditional heteroskedasticity model can describe the Bitcoin price volatility better over the whole period since its introduction on Brazilian's market.

Table 4 presents the different GARCH-type models used in this research, namely GARCH, EGARCH, TGARCH, Asymetric Power ARCH (APARCH). The optimal model is chosen according to three information criteria, namely Akaike (AIC), Bayesian (BIC) and Hannan-Quinn (HQ), all of which consider both how good the fitting of the model is and the number of parameters in the model, rewarding a better fitting and penalising an increased number of parameters for given datasets. The selected model is the one with the minimum criteria values.

Table 4: GARCH-type models used

GARCH	$h_t^2 = \omega + \alpha u_{t-1}^2 + \beta h_t^2$
EGARCH	$log(h_t^2) = \omega + \alpha[\lvert\frac{u_{t-1}}{h_{t-1}}\rvert - \sqrt{2/\pi}] + \beta log(h_{t-1}^2) + \delta\frac{u_{t-1}}{h_{t-1}}$
TGARCH	$h_t^2 = \omega + \alpha u_{t-1}^2 + \beta h_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$
APARCH	$h_t^\delta = \omega + \alpha(\lvert u_{t-1}\rvert - \gamma u_{t-1})^\delta + \beta h_{t-1}^\delta$ $h_t^2 = q_t + \alpha(u_{t-1}^2 - q_{t-1}) + \beta(h_{t-1}^2 - q_{t-1})$

Table 5 shows the estimation results of the GARCH-type models. It can be noticed that the log-likehood value is maximised under the AR-APARCH model. Interestingly, all the three information criteria also select the AR-APARCH model. Moreover, all the parameter estimates are statistically significant for the AR-APARCH model which indicate that the selected model is appropriate for the Bitcoin daily returns volatility. Finally, even though the residuals of the AR-APARCH model still depart from normality, the value of the Jarque-Bera test has considerably decrease.

All in all, the AR-APARCH model appears to be an appropriate tool to describe the volatility of the Bitcoin daily returns volatility. This finding opens opportunities for application of other GARCH models related to the Brazilian market of Bitcoin, as reported in studies in the North American market, using the models CGARCH and ACCGARCH, as Kasiampa (2017).

Table 5: Estimation results of GARCH-type models for Bitcoin daily return volatility

Note: Standard errors os estimates are reported in parentheses. The p-values associated with the statistical tests are presented in brackets. * Represents the significance at the 10%. ** Represents the significance at the 5% level. *** Represents the significance at the 1% level.

	AR-GARCH	**AR-EGARCH**	**AR-TGARCH**	**AR-APARCH**
Const (c)	0.0256 ***	0.03085 ***	0.02188***	0.0285 ***
	(0.0003)	(0.0007)	(0.0000)	(0.0007)
AR(1) (ϕ_1)	0.6389 ***	0.6639 ***	0.6120***	0.6102 ***
	(0.0174)	(0.0126)	(0.0080)	(0.0066)
Const (ω)	- 0.0000 ***	-0,2072 ***	0.0000***	0.0004 ***
	(0.0000)	(0.0018)	(0.0000)	(0.0000)
ARCH (α)	0.4546 ***	0.1255 ***	0.2437***	0.1393 ***
	(0.0068)	(0.0045)	(0.0036)	(0.0036)
GARCH (β)	0.8397 ***	0.1422 ***	0.8562***	-0.5469 ***
	(0.0019)	(0.0028)	(0.0021)	(0.0021)
EGARCH (δ)	-	0.9780 ***	-	-
	-	(0.0005)	-	-
TGARCH (γ)	-	-	-1.1164***	-
	-	-	(0.2679)	-
APARCH (δ)	-	-	-	0.9129 ***
	-	-	-	(0.0021)
APARCH (γ)	-	-	-	0.0253 ***
	-	-	-	(0.0253)
LL	2882.862	2875.445	2450.179	2948.547
AIC	-2.3950	-2.3880	-2.0350	-2.4480
BIC	-2.3854	-2.3760	-2.0254	-2.4336
HQ	-2.3915	-2.3836	-2.0315	-2.4427
JB	3939601	2900663	4330588	2833245

5. Conclusion

Although a large amount of literature has focused on the role of traded volume in predicting movement in stock returns and volatility and inefficiency of Bitcoin into USA market, the predictability of speculative trading for the returns and volatility in the Bitcoin market remains few explored on a genereal context, and unexplored into Brazilian market. To address this literature gap, we examine daily data covering the period of 1st August 2011 to 28th February 2018, which interestingly show that Bitcoin returns, volatility and volume do not follow a same trajectory. Methodologically, behind provide an exploratory analysis about value and volatility of the Bitcoin on Brazilian market across time, we employ a speculative trading test based on Llorente et al. (2002) and explore the optimal heteroskedasticity model with regards to Bitcoin price data.

Our results are summarizes as follows: First, the Bitcoin's bull on Brazilian market presented a short duration, given the volatility exposed into period; showing signs of asymmetry in market information. Second, the estimated β_2 from Llorente et al. (2002) equation is positive, indicating that on the average day, trading activity in Bitcoin is speculative. Third, we observed the different trajectories between volatility and return, reflecting the high degree of speculation of the market. Fourth, we prove the ability of several competing GARCH-type models to explain the Bitcoin price volatility; and find evidence that the optimal model in terms of goodness-of-fit to the data is the AR-APARCH, a result which suggests the importance of having both component of conditional variance.

For intance, a viable explanation for the speculative trading may be the presence of high assymetric information degree on Brazilian market. By the way, it would be ideal to continue our tests exploring other possible explanations for speculative trading and volatility fluctuations.

On the other hand, Bitcoin is different from any other asset on the financial market and so creates new opportunities for stakeholders with regards to risk management and portfolio analysis. Hence, it can be an useful tool for managements, and our results can help investors make more informed decisions.

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