A Note on Personal inequality in Post Keynesian Growth Models

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Abstract

Although Post-Keynesian growth models have been already extensively extended, the issue of personal inequality has only recently started to be dealt with. The strategy, however, has been the insertion of more functional classes or the observation of intra-class inequality. While empirically credible, these strategies are formally complex and hamper the spread of their conclusions, which are normally of important changes in the main results of traditional models. In this context, this paper proposes a simpler formulation of the problem, both as a didactic introduction and as a way of disseminating the discussion. More than displaying complex mathematical formulations, the paper aims to provide intuitive and graphical tools for understanding and reading the problem of personal distribution on post-Keynesian growth models. The objective will be pursued by construing the model from the tautological fact that the total income of the economy may be represented by the sum of the income of all individuals in that economy. The functional form representing this sum is the Pareto distribution. This strategy provides two different interpretations for the model: class-conflict and earnings-composition.

Keywords: wage-led; personal distribution; functional distribution; demand regimes

Resumo

Embora os modelos de crescimento pós-keynesianos já tenham sido extensivamente ampliados, a questão da desigualdade pessoal só recentemente começou a ser resolvida. A estratégia, no entanto, tem sido a inserção de mais classes funcionais ou a observação da desigualdade intra-classe. Ainda que empiricamente críveis, essas estratégias são formalmente complexas e dificultam a disseminação de suas conclusões, que normalmente são de mudanças importantes nos principais resultados dos modelos tradicionais. Neste contexto, este trabalho propõe uma formulação mais simples do problema, tanto como introdução didática quanto como forma de disseminar a discussão. Mais do que exibir fórmulas matemáticas complexas, o artigo visa fornecer ferramentas intuitivas e gráficas para entender e ler o problema da distribuição pessoal em modelos de crescimento pós-keynesianos. O objetivo será buscado construindo o modelo a partir do fato tautológico de que a renda total da economia pode ser representada pela soma da renda de todos os indivíduos naquela economia. A forma funcional que representa essa soma é a distribuição de Pareto. Essa estratégia fornece duas interpretações diferentes para o modelo: conflitos de classe e composição de lucros.

Palavras-chave: wage-led; distribuição pessoaç; distribuição funcional; regimes de demanda.

Classificação JEL: D31; E25.

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Introduction

Since the initial formulations of the Cambridge model of economic growth - normally ascribed to the works of Robinson (1962), Kaldor (1957) and Kalecki (1954) - post-Keynesian models have traveled a long way through numerous different routes. One of the longest roads traveled has been on the "growth regimes" route: after Rowthorn's (1981) and Dutt's (1984) exclusive wage-led results, Bhaduri and Marglin (1990) demonstrated that demand-led models might present at least two possible growth regimes: wage-led and profit-led. From then on, post Keynesians heartily faced the task of testing empirically Bhaduri and Marglin's ideas[2], as well as the burden of extending the original formulation theoretically[3].

This paper joins the task of developing the model theoretically. However, instead of joining the community at the edge of the route, the present formulation intends to demonstrate a sideway that may have been condoned when previous travelers took the road: inserting personal inequality on the model. Leaving the analogy behind, the paper rather than reformulate based on sophisticated current formulations of post-Keynesian models, will review the most basic presentation with the minor inclusion of individuality. As a result, two different perspectives for interpreting the model will be analyzed and, most importantly, personal inequality will be inserted without unnecessary complications.

Concerns have been rising recently around the issue of personal inequality. Piketty and Saez (2003, 2006), for instance, discuss the increasing inequality of income in English speaking countries. More recently, Piketty's (2014) extensive work has received worldwide attention for its disturbing findings, suggesting a spread of inequality. Although his theoretical proposals may be controversial, the wide analysis of data suggests at least that the topic must be urgently faced.

The fact is that, although at this point post-Keynesians may be far on the road, only recently attention has started to be paid towards personal inequality in "growth regime" models. Palley (2014, 2016), Carvalho and Rezai (2014) and Tavani and Vasudevan (2014) present some interesting contributions to the debate. Still, while refined on their theoretical formulations and empirically trustworthy, none of the current models intends to fill the gap of a comprehensive introduction to the issue of personal inequality on post-Keynesian growth models. Consequently, the formulations miss simpler presentations of the problem, what may be hampering the burgeoning of the discussion. The point is that most of these models have important conclusions concerning the reliability of canonical "growth regime" models and they still do not reach a wider audience.

In this context, this paper proposes a simpler formulation of the problem, both as a didactic introduction and as a way of disseminating the discussion. The proposed formulation, as it will be demonstrated, is versatile and is a viable mode of rereading more complex formulations. More than displaying complex mathematical formulations, the paper aims to provide intuitive and graphical tools for understanding and reading the problem of personal distribution on post-Keynesian growth models.

This objective will be pursued in the following manner. First, the next section will present the canonical kaleckian model, as found on Post-Keynesian textbooks. Next, the inclusion of individuality will be discussed, presenting two different perspectives for reading the model with individuals: earnings-composition and class-conflict. Finally, personal inequality will be inserted into the model through a Pareto distribution and two closures of the model will be presented. In the last section, concluding remarks will be declared.

The canonical model

To review how post-Keynesian models may include personal distribution in their formalizations, it is interesting to remember the canonical version of the kaleckian model for a closed economy. The canonical

[2] Bowles and Boyer (1995) presented the seminal econometric work of the literature. Interesting posterior implementations have been Hein and Vogel's (2007), Naastepad and Storm's (2006, 2007) and Onaram and Galanis's (2012).

[3] Some of the most commonly cited extensions are: open economy (Blecker 1989, 2011), Harrodian (Skott 1989, 2010), Marxian (Shaikh 2007, 2009) and Sraffian (Serrano 1995). For theoretical reconstructions of the history of post Keynesian models see: Hein (2014), Lavoie (2016), Taylor (2004) and Stockhammer (1999)

formalization of post-Keynesian growth models may be presented in numerous forms and is didactically exposed in textbooks such as Hein's (2014) and Lavoie's (2016). Here, a simple version based on such expositions will be presented.

To begin with, the tautological definition that describes the income of the economy (Y) as the sum of the total income of its classes, capitalists (P) and workers (W)[4], represents the most basic foundation of the model and is specified in the following manner:

$$Y = P + W$$

$$\frac{Y}{Y} = 1 = \frac{P}{Y} + \frac{W}{Y} = \pi + (1 - \pi)$$

As usual in post-Keynesian frameworks, π formalizes the share of profits of the economy. Normally, kaleckian models opt for a presentation that defines the functional shares of income as a function of the exogenous mark-up rate of the economy. While offering an interesting economic interpretation to the model, mathematically this means endogenizing the profit share as a function of the exogenous mark-up parameter. In fact, there is no mathematical difference in setting exogenously the mark up rate or directly the profit-share. Consequently, the reasons for opting for one or another are mostly related to interpretation. Here, however, the objective is to endogenize π as a function of personal distribution. Therefore, it seems preferable to condone the mark-up determination of functional distribution in order to maintain the clarity of the exposition. Thus, an exogenous direct determination of the profit-share stands as the base of the current modeling.

Following the tautological definition, to represent the supply side of the economy, the model assumes that each class has a unique propensity to save. Therefore, the savings of the economy (S) are the sum of the saved share of the income of each class given the defined propensities to save. As a result, the weighted sum of the propensities to save of each class - where the weights are the shares of profits and wages of the economy - describe the economy's propensity to save (s). These definitions are demonstrated according to the following equations:

$$S = s_p P + s_w \mathrm{W}$$

$$S/Y = s_p(\tfrac{P}{Y}) + s_w(\tfrac{\mathrm{W}}{\mathrm{Y}})$$

$$s = s_p \pi + s_w(1 - \pi)$$

Usually, the propensity to save of the capitalist class (s_p) is higher than the propensity to save of workers (s_w). It may even be assumed that the workers do not save, resulting in a model where the propensity to save of the capitalist class totally characterizes the propensity to save of the economy. Although the model becomes didactically more manageable on the presence of this assumption, there is no unnecessary complication in assuming two distinct propensities to save. Moreover, when personal inequality take part in the model, distinguishable propensities between classes will be demonstrated to be an important part of the post-Keynesian framework. Hence, this shall be the assumption henceforward.

The equations above presented are presented in level. To solve for the equilibrium of the economy, growth rates must be determined. The procedure is simple. The model is normalized, applying the usual ruse of dividing the savings equation by the stock of capital of the economy (K), in order to present the growth rate of the savings. Thus, the following equation expounds the long run growth of the supply side of the economy (g_s):

[4] Evidently, any number of classes could be included in the model. Canonical Kaleckian models, however, assume only two, following Marxian ideas.

$$S/K = (s_p P + s_w W)\left(\frac{1}{K}\right)\left(\frac{Y}{Y}\right)$$

$$g_s = \frac{S}{K} = s_p \pi + s_w (1 - \pi)\left(\frac{Y}{K}\right) = \left[s_p \pi + s_w (1 - \pi)\right] u = su$$

The growth rate is a function of the propensity to save of the economy and the utilization rate of the economy. Formally, the ratio between the actual outcome and the potential outcome of the economy ($u = \frac{Y}{Y_p}$) should define the utilization rate. However, it is implicitly assumed for simplification that the potential productivity of capital is constant. Consequently, the actual productivity of capital ($u = \frac{YY_p}{Y_p K} = \frac{YY_p}{KY_p} = \frac{Y}{K}$) may describe the potential outcome of the economy.

Considering that in a post-Keynesian framework supply and demand are individually determined, an investment function must be described as well. For this, kaleckian models assumed through time at least three different formulations (Stockhammer 1999). Here, the basic investment function that is denoted by the independent *animal spirit* of individuals (a), the rate of capacity of utilization (u) and the profit rate ($r = \frac{P}{K} = \frac{PY}{YK} = \pi u$) will be assumed[5]:

$$g_i = a + bu + c\pi u$$

The solution for the model, assuming a closed economy, occurs when supply and demand meet in the long run. In other words, the model is solved for the equilibrium when the growth rates of savings and investment equalize.

$$g_s = g_i$$

Therefore, the following result delineates the economy's capacity utilization that settles the equilibrium of supply and demand:

$$u^* = \frac{a}{s - b - c\pi} = \frac{a}{\left[\pi(s_p + \mathrm{c} - s_w)\right] - b + s_w}$$

This simple presentation neglects the existence of personal distributions completely. However, this does not imply that the model has not implicitly assumptions about the personal distribution of income. The following section will demonstrate how the model may be reinterpreted with the inclusion of individuality without altering any of the results of the model.

Individuality in Post-Keynesian Growth Models

By definition, the inclusion of *personal* distribution in the kaleckian model of growth entails the necessity of modeling individual behaviors. The canonical post-Keynesian model, however, is an aggregate model where individuality plays no significant role. As a result, the inclusion of an individual point of view alters the interpretation of the model, even though, as is going to be observed, the results may remain the same.

[5] Given the theoretical aspect of the paper, there is not the necessity of advocating in favor of this functional forms. In fact, an interesting exercise would be to apply the subsequent formulations to different investment functions.

Two interpretations will be presented in the next subsections[6]. However, in order to stick only to the presentation of the interpretations, the individual perspectives will be presented assuming a homogeneous behavior among individuals, which paradoxically means that the individuals behave as a group.

First, the inclusion of an individual perspective in which the profit share is formalized through the number of individuals in each class of the economy will be delineated. In this perspective, class-conflict is maintained and occurs through increasing or decreasing the numbers of individuals in the classes of the economy.

Second, the individual perspective allows an unusual interpretation of the model, where instead of distinct classes, different types of income are introduced. In this framework, class-conflict is renounced since all individuals pertain to the same class. The results remain the same, although the interpretation may be altered in a somewhat high degree.

Class-conflict perspective

The inclusion of personal distribution in the model follows the same tautological definition that bolsters the canonical model with a minor reinterpretation. As seen, the fact that the economy's income may be described as the sum of the total income of each class - which were ddesignated as two - sustains the canonical model. The same idea may be applied to individuals. The total income of the economy is the sum of the income of all individuals in the economy. Analogously, the total income of a class is the sum of the income of the individuals pertaining to the class. The subsequent mathematical expositions demonstrate these definitions:

$$Y = \sum_{n=1}^{n} L_n = P + W$$

$$P = \sum_{n=1}^{n\gamma} E_{pi}$$

$$W = \sum_{i=n\gamma}^{n} E_{wi}$$

In this class-personal perspective, the sum of the total income (E_{pi}) of capitalist individuals (which in an ordered manner goes from the first individual to the $n\gamma$, where n stands for the total amount of individuals and γ for the exogenous parameter defined between 0 and 1 establishing the quantity of individuals in each class) denotes the profits, while wages are the sum of the total income (E_{wi}) of workers individuals (which following the order, are the individuals indexed from the $(n\gamma)$th individual to the (n)th individual). The definitions may be normalized dividing by the total income of the economy to reveal how the shares of profits and wages are defined in the class-individual view:

$$\frac{P}{Y} = \pi_c = \sum_{i=1}^{n\gamma} \frac{E_{pi}}{Y} = \sum_{i=1}^{n\gamma} C_{pi}$$

[6] These are the two most basic interpretations. There are possibilities of endogenizing the parameters defining the profit-share, for instance, that will not be analyzed.

$$\frac{W}{Y} = 1 - \pi_c = \sum_{i=n\gamma}^{n} \frac{E_{pi}}{Y} = \sum_{i=n\gamma}^{n} C_{wi}$$

Consequently, the shares of income from the class-personal point of view are delineated as the following tautology:

$$\frac{Y}{Y} = 1 = \frac{W}{Y} + \frac{P}{Y} = \sum_{i=1}^{n\gamma} C_{pi} + \sum_{i=n\gamma}^{n} C_{wi} = (1 - \pi_c) + \pi_c$$

The sum of the individual shares of income of all individuals must account for the total income of the economy. Once class incomes have been defined as the sum of individual incomes, the personal distribution may be disclosed. The first step towards unmasking personal distribution in this kaleckian framework is understanding the fact that each individual may receive a different share of the income. Thus, mathematically, a specific functional form may describe the personal shares. In other words, each individual may receive different amounts of income according to specified parameters.

In the canonical model, given the absence of such parameters, the personal distribution is negligible. It does not matter whether most of the total earnings of each class pertains to a few individuals. The only relevant distribution is the functional distribution. As a result, the model implicitly assumes that all individuals have the same personal share of the output, such that $C_{pi} = C_{wi} = \beta \ \forall \ i$. If a continuous function is assumed, the shares of income may be described as follows:

$$\sum_{i=1}^{n\gamma} \beta + \sum_{i=n\gamma}^{n} \beta = \sum_{i=1}^{n} \beta = \int_{1}^{n} \beta dn = \beta(n-1) = 1$$

$$\beta = \frac{1}{n-1}$$

The result is obvious: income is divided equally among all individuals and, thus, the share of income of each individual corresponds exactly to the ratio given by the division of one individual by the total quantity of individuals. Following the same tactic applied above and considering the presented personal shares, the class shares may also be disclosed. Therefore, solving for π_c gives:

$$\pi_c = \sum_{i=1}^{n\gamma} \beta = \int_{1}^{n\gamma} \beta dn = \frac{n\gamma - 1}{n-1}$$

Since a continuous function has been assumed, it is necessary to observe the behavior of the function when the number of individuals tends to increase. Applying the limit when n tends to infinity to the function describing the capitalist share of income demonstrates that:

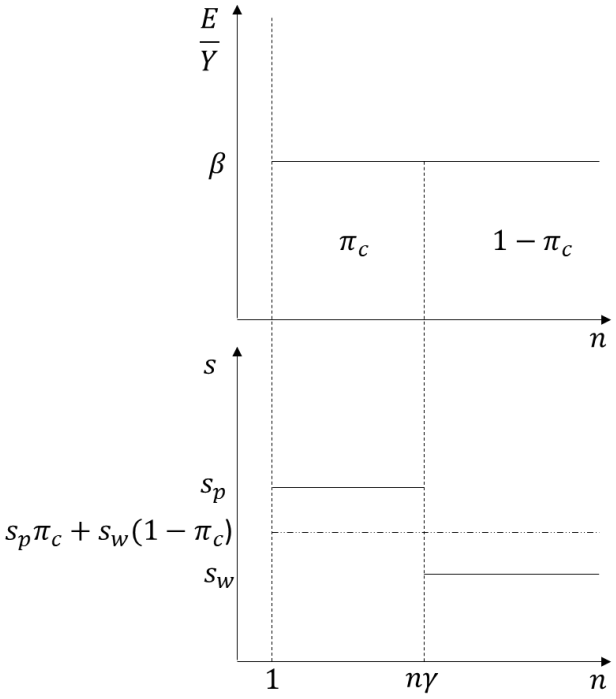
$$\lim_{n \to \infty} \pi_c = \gamma$$

The class-individual model, hence, endogenizes class shares as a function of the parameter γ. What this means is that the parameter which divides who is a capitalist and who is a worker determines the functional shares. An exogenous π has been substituted by an exogenous γ. Although both converge in the model and are mathematically indistinguishable, they offer different interpretations. π in the canonical model without personal distribution directly determined the capitalist share of income. γ determines the number of

individuals in the capitalist class. It is the equal personal distribution that allows both parameters to be interchangeable. Therefore, the solution of the canonical model does not change with the inclusion of homogeneous personal distribution. The only variation occurs in the interpretation of the model. When γ and homogeneous distribution are assumed, the variations on the class shares are due to variations in the quantity of individuals in the classes.

It is interesting to notice that the canonical model, even with the inclusion of personal distribution, assumes that each individual may pertain to a single class. Individuals cannot be capitalists and workers at the same time. This assumption assigns each individual a unique propensity to save according to the class that he pertains. Thus, when some authors use interchangeably the terms "propensity to save out of profits" and "propensity to save of the capitalist class", they may induce the erroneous understanding that individuals may receive both wages and profits in the model. However, the canonical model has a class perspective which entails class-conflicts. Graphically, the model with personal distribution and class conflict may be represented as follows:

Fig 1. - Homogeneous Personal Distribution with Class-Conflict



Source: prepared by the author

The horizontal axis represents the individuals indexed in an orderly manner, from the wealthiest to the poorest individual - even though in this case all individuals earn the same. The graph is defined starting on one, given that, although a continuous function is assumed, it is senseless to assume less than one individual. The first graph delineates the personal distribution of income, with the vertical axis representing the earnings of each individual divided by the total income of the economy. The horizontal line β outlines that each individual receives the same share of income. The profit share and the wage share are the areas of the graph determined by γ and β. The second graph depicts the propensities to save of each individual. Their classes, for which the delimiting parameter is γ, determine the propensities. As discussed, the propensity to save of the capitalist class is higher than the propensity to save of workers. Class conflict is preserved since individuals clearly pertain to single classes without overlaps.

Graphically, it becomes clearer the direct relationship between γ and π. Increasing the number of capitalists without changes in the total quantity of individuals increases the share of profits. Moreover, the graph evinces the role of a homogeneous distribution in the model. If the personal distribution in the first graph was not a simple division by the total quantity of individuals - what would depict a curve instead of a horizontal line - the parameters of the curve would determine the shares of profits and wages. Thus, the path

to inserting personal inequality is straightforward: defining the distributive curve of the economy. However, before that, a distinct interpretation of the parameter γ will be presented.

Earnings-composition perspective

While class-conflict supports the usual presentation of the canonical model, the model may also maintain its results when observed from a different perspective. Instead of assuming two different classes in a way that individuals cannot pertain to both due to the class-conflict, it is also tautological to state that the sum of all the different types of earnings the individuals receive represents the total income of the economy. Thus, if all individuals receive profits and wages, the total income of the economy is the sum of the individual profit shares and the individual wage shares. Class conflict does not exist in this scenario once all individuals earn both types of income. Mathematically, this point of view is formalized as follows:

$$P = \sum_{i=1}^{n} \gamma_i E_i$$

$$W = \sum_{i=1}^{n} (1 - \gamma_i) E_i$$

The individual share of profits γ_i of the total individual earnings E_i measure the individual profits. As a result, the total profits of the economy are the sum of the individual profits. Wages are analogously represented. Given that only two types of earnings are assumed, the individual wage share is formalized as $(1 - \gamma_i)$. The difference with the previous exposition is that now γ_i is a parameter that divides the earnings of an individual in two parts, whereas in the class conflict the parameter was responsible for dividing individuals in two different conflicting classes. The normalization of the total profits and total wages of the economy in order to delineate the profit share and wage share of the economy on this "income composition" perspective results in:

$$\frac{P}{Y} = \pi_e = \sum_{i=1}^{n} \frac{\gamma_i E_i}{Y} = \sum_{i=1}^{n} Z_{pi}$$

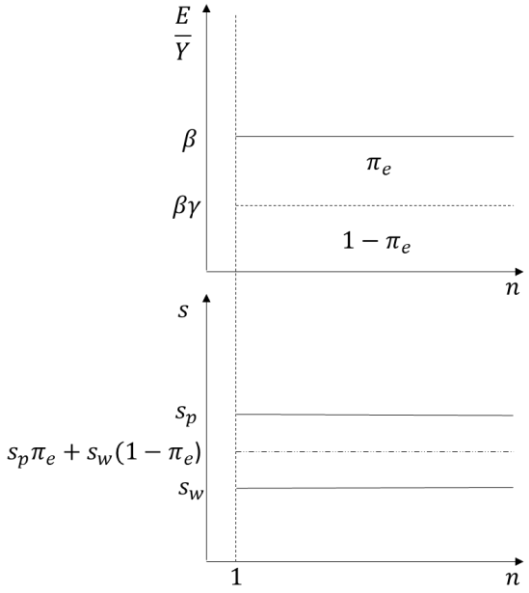
$$\frac{W}{Y} = (1 - \pi_e) = \sum_{i=1}^{n} \frac{(1 - \gamma_i) E_i}{Y} = \sum_{i=1}^{n} Z_{wi}$$

Once again, as in the class conflict point of view, the personal distribution is not defined, which means that π is an exogenous parameter that does not depend on the personal distribution. Thus, Z_{pi} and Z_{wi} do not have assigned functional forms. As follows, the parameter β representing the equal division of income among all individuals of the economy describes the personal distribution, which is assumed to be homogenous. Moreover, the canonical model has no explicit assumption about the individual composition of income. This means that, since all individuals earn the same amount of income, it may also be assumed that all individuals receive their earnings in exact same amounts of profits and wages and $\gamma = \gamma_i \; \forall \; i$. Therefore, the aggregate profit share of the economy may be established mathematically as:

$$\pi_e = \sum_{i=1}^{n} \gamma\beta = \int_{1}^{n} \beta\gamma dn = \gamma$$

Therefore, either in the earnings point of view or the class perspective, an exogenous parameter γ will define (and be the same as) the share of profits. The economic interpretation, however, differs according to the adopted perspective. In the class perspective, the share γ of individuals earning profits - usually called capitalists - delineates the share of profits. Thus, instead of interpreting the model with a direct exogenous value for the share of profits in the outcome, the model is reinterpreted with an exogenous parameter defining the share of individuals which are capitalist in the total quantity of individuals in the economy. Given the equal distribution, the share of capitalist individuals is the same as the share of profits. On the other hand, according to the "earnings composition" sight, the parameter defines how much is the share of profits of each individual in their total earnings. According to this view, the parameter outlines the individual shares of profits, which on its turn is the same as the economy's share of profits, once the distribution is equal among individuals. The following graph helps in the understanding of how the above perspective differs from the class conflict perspective:

Fig. 2 - Homogeneous Personal Distribution with Earnings Composition



Source: prepared by the author

The differences between the class-conflict and the earnings-composition are straightforward. First, the γ becomes a horizontal line instead of a vertical line. This means that all individuals pertain to the same class, renouncing the class-conflict usual interpretation. Furthermore, all individuals receive the two types of income, profits and wages, in equal shares. As a result, all individuals must have their own propensities to save out of wages and profits. Thus, the differences in the propensities to save are not a result of a difference in classes, but a result of the sources of income. In the image, the absence of a discontinuity in the lines defining the propensities to save for each kind of income depict the class distinction. The mean propensity to save of the economy is calculated as before: the profit and wage shares of the economy serve as weights for a weighted mean.

Thus, after observing these two different interpretations of the canonical model with a homogeneous distribution of income among individuals, it is possible to advance to the next step: transforming homogeneity in heterogeneity. The next sections will include heterogeneity as personal inequality in the model. This inclusion will, as already have happened above, endogenize the profit share, transforming it on a function of parameters defining the personal distribution (absent in the above canonical formulations, given the homogeneous distribution assumption) and the division of class or types of earnings (γ).

Personal inequality

Finally, the present exposition reaches the point where it may be asked: how is it possible to include personal inequality in the model? How can it be included individual heterogeneity in the model? A quick look at the graphs may produce the automatic understanding that heterogeneity could be included both through distinct personal propensities to save and through distinct personal shares of income (or even both ways). In other words, a horizontal line in the personal distribution graph could be maintained, while curves in the propensity to save graph could be included or the other way around. This is a biased cue. Personal distribution is the only route through which individual heterogeneity in kaleckian models can be included. This results from a simple mathematical fact as is going to be observed.

The sum of the earnings of each capitalist times his individual propensity to save describes the total savings of the capitalist class in the following mode:

$$S_p = \sum_{i=1}^{n\gamma} E_{pi}\, s_i$$

The division of the total savings of the capitalist class by the total income of the economy represents the mean propensity to save of the class:

$$\overline{s_p} = \frac{S_p}{Y} = \sum_{i=1}^{n\gamma} \frac{E_{pi}}{Y} s_i = \sum_{i=1}^{n\gamma} C_{pi}\, s_i$$

Thus, a weighted sum, in which the shares of income of each capitalist represent the weights, formalize the mean propensity to save of the capitalist class. The weighted sum could be normalized with a division by the weights to define the weighted mean of capitalists' propensity to save in the following manner:

$$s_p = \frac{\sum_{i=1}^{n\gamma} C_{pi} s_i}{\sum_{i=1}^{n\gamma} C_{pi}} = \frac{\sum_{i=1}^{n\gamma} C_{pi} s_i}{\pi} = \frac{\overline{s_p}}{\pi}$$

The weighted mean, thus, is the mean propensity to save of the capitalist class divided by the share of income of this class. Consequently, the weighted mean times the share of profits is equal to the mean propensity to save of the capitalist class.

$$s_p \pi = \overline{s_p}$$

Therefore, the individual propensities to save converge to the weighted mean times the income share of a class. This result can be easily demonstrated for different perspectives (maintaining or not class conflict, for instance), for different classes and the whole economy. According to this result, assuming heterogeneity in the savings function is innocuous. Mathematically, the simple determination of a single value that represents the weighted mean (or the simple mean) of the propensity to save of the targeted class achieves the same result as defining individual propensities to save. This result may be called class-propensity correspondence, given that propensities only correspond to classes. Personal propensities do not exist in the post-Keynesian framework. Thus, *individuals save as class*. To clear up, in post-Keynesian growth models of the kind presented above, there is no reason for determining functional forms for personal savings, unless one is inclined to assume the propensities to save as the weights of the shares of income - an assumption that would make no obvious sense.

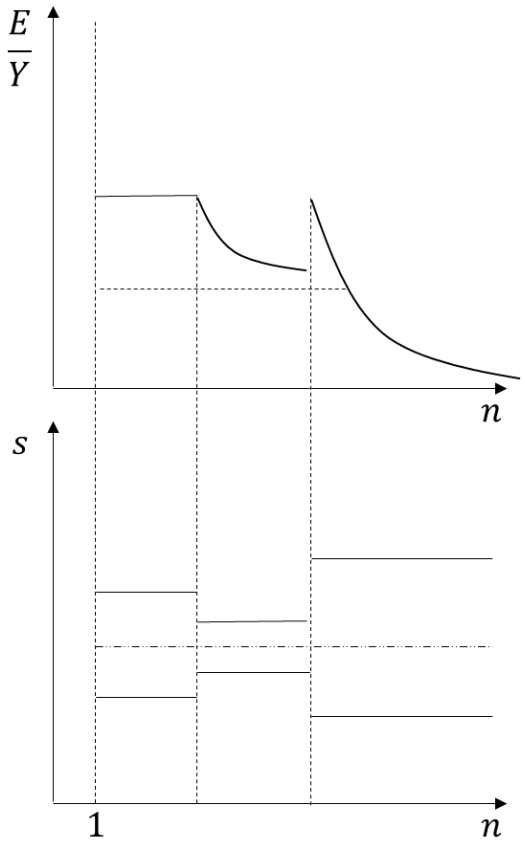
Thus, answering the initial question of the section, heterogeneity has to be included only through a heterogeneous personal distribution of income. Hence, The addition of a functional form in the place of the β that represented the homogeneous personal distribution in the previous interpretations of the canonical model formalizes heterogeneity.

However, before mathematically formalizing the extended model with personal inequality, it is interesting to outline some intuitive possibilities of extending the model - given the two interpretations presented and the constraint above explained. As seen, three new aspects can be inserted in the model: the parameter γ in a class-conflict; the parameter γ in an earnings-composition interpretation; and a functional form defining personal distribution. The model, thus, can be extended with the inclusion of numerous classes and types of earnings either through the inclusion of a functional form to the parameter γ or additional parameters (as horizontal or vertical lines in the graph). Thus, the model may combine both class-conflict and earnings composition points of view defining some areas of the graph as profits and others as wages.

It is interesting to notice that the functional form of the personal distribution must not be continuous and may present numerous local maximums. Economically, this would make sense as intra-class distributions. For instance, the individuals in the working class could have their income distributed differently than the capitalists. Although mathematically possible, the inclusion of different intra-earnings distributions would be of difficult interpretation.

Depending on the number of classes, the number of different types of earnings, and the number of different intra-class distributions, the model can become quickly complex. The following image demonstrates a conceptual case with three classes, two types of earnings and intra-class inequality:

Fig. 3 - Conceptual Case



Source: prepared by the author

The example could be even more peculiar, given that there is not the necessity of using lines and well-behaved curves, but the illustration nonetheless achieves the point of demonstrating where this type of interpretation may go. Still, and more importantly, some insights may be acquired without complex

formulations. As follows, it seems necessary to present simpler cases in order to set the standards for the inclusion of more complex cases of personal distribution and disclose the main intuitions behind this perspective of modeling post-Keynesian models.

Therefore, from here on, the paper will be interested in formally presenting two cases of the model with personal distribution not yet present in the literature: class-conflict with overall inequality and earnings-composition with overall inequality. Consequently, the only addition to the two canonical perspectives above outlined will be the inclusion of a functional form defining the overall distribution of income in the economy. It is interesting to notice that any functional form may be assumed. The only necessary condition is that the sum of personal shares equals one. Still, for reasons to be discussed soon, in this paper the Pareto function of distribution, which the following density probability function (PDF) establishes, will be the chosen functional form:

$$\frac{\beta x_m}{x^{\beta+1}} \overset{x_m=1}{\Longrightarrow} \frac{\beta}{x^{\beta+1}}$$

The usual interpretation of the Pareto function, however, is through its cumulative density function, with which it is possible to observe the concentration of income by percentages of the population. In fact, it is quite unusual to interpret income distribution in such a disaggregated degree. Commonly, income shares are a preferable mode of observation, which allows interesting interpretations. Here, this interpretation, while useful, will not be utilized for technical reasons. As described in the above formalizations, the income is being represented as the sum of individual incomes and not as the sum of shares of the population. This demands a characterization of a functional form for such a sum. In this context, the Pareto density probability function, breaking away from more common presentations, will be utilized as a ruse for describing an ordered sum of personal incomes in the following manner:

$$Y = \sum_{n=1}^{n} L_n = \sum_{\mathrm{n}=1}^{n} \frac{\beta Y}{n^{\beta+1}} \approx \int_1^{\infty} \frac{\beta Y}{n^{\beta+1}}\, dn$$

$$\frac{Y}{Y} = \int_1^{\infty} \frac{\beta}{n^{\beta+1}}\, dn = 1$$

According to the equations, each individual receives a share $\frac{\beta}{n^{\beta+1}}$ of income, where n represents the index of the individual in the scale of wealth. A peculiarity occurs when describing the sum of individual incomes with the functional form of a pareto probability density function. In a continuous function where β is higher than zero, the PDF where x_m equals one is a convergent series with a convergence value of one. Still, there is no economy with an infinite number of individuals. As follows, when observing the function discretely, it seems paradoxical to assume a β higher than one, since the first individual would have a share of income higher than the total income of the economy. Consequently, for a sounder economic interpretation, it seems preferable to assume that β stands between zero and one. This way, β represents the share of income of the richest individual in the economy, which cannot surpass the total income of the economy. Still, given that a continuous function will be assumed henceforward[7], there is no impediment for assuming higher values of β.

Besides this idiosyncrasy of the proposed formulation, the Pareto distribution, as pointed by Yakovenko (2012), is a fair approximation of the personal distribution of the income of economies. Moreover, it has a simple interpretation of inequality. The higher the β parameter, the more unequal the economy becomes. Consequently, although any functional form could be assumed, the fair approximation and simple interpretation present the pareto function as an interesting option for further formalizations. This task will be

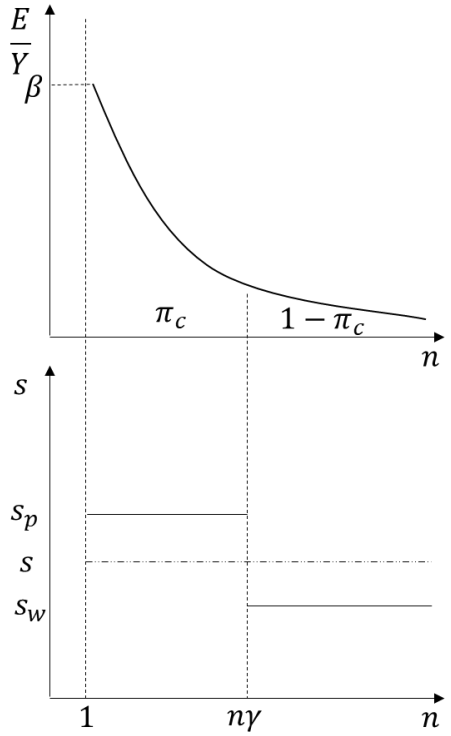
[7] Continuity is necessary for using calculus.

faced on the two sections in the sequence. The first will formalize overall personal distribution with the pareto sum of individual incomes in a context of class conflicts. Next, the context of class conflict will be substituted for observing the pareto function in a perspective of earnings-composition.

Class-Conflict with overall inequality

The model has only one change when compared to the canonical model with class conflict in an interpretation with individuality. Instead of a horizontal line determined by the equal share of income for all individuals in the economy, the Pareto distribution depicts the model's distribution of income. The following image intends to facilitate the interpretation of the following formalization of the model:

Fig. 4 - Class-Conflict with Overall Inequality



Source: prepared by the author

As it can be observed, the profit and wage shares are now dependent not only on the γ parameter that defines the individual class conflict, but also on the parameter defining the personal distribution of income, the parameter β. Given the property of integration that allows to separate any integral in the sum of its parts, the totality of the individual shares may be described as the sum of the wage and profit share as follows:

$$\int_1^{n\gamma} \frac{\beta}{n^{\beta+1}} dn + \int_{n\gamma}^{\infty} \frac{\beta}{n^{\beta+1}} dn = \pi_c + (1-\pi_c) = 1$$

Consequently, when solving the integrals, the following equations portray the profit share, π_c, and the respective wage-share, $(1-\pi_c)$:

$$\pi_c(\beta,\gamma) = \int_1^{n\gamma} \frac{\beta}{n^{\beta+1}} dn = \left(-\frac{1}{(n\gamma)^{\beta}} + 1\right)$$

$$(1-\pi_c) = \int_{n\gamma}^{\infty} \frac{\beta}{n^{\beta+1}} dn = 1 - \int_1^{n\gamma} \frac{\beta}{n^{\beta+1}} dn = \frac{1}{(n\gamma)^{\beta}}$$

As a result, now the functional shares of the economy are functions of two exogenous parameters. First, the already discussed parameter that sets the class conflict, γ. Second, the parameter that defines the degree of inequality in the economy, β. Differentiating the equation results in the marginal response of the functional shares for increases in these parameters:

$$\frac{d\pi_c}{d\beta} = \frac{\ln n\gamma}{(n\gamma)^{\beta}} > 0$$

$$\frac{d\pi_c}{d\gamma} = \frac{\beta}{(n\gamma)^{\beta}\gamma} > 0$$

The first derivative demonstrates that increases in the degree of inequality increase the profit share of the economy, whereas the second demonstrates that increasing the number of individuals in the capitalist class increases the profit share. These two results were expected. There is no paradox. The inclusion of these two results in the equilibrium presented by u^* in the canonical model reaffirms common expectations:

$$u^* = \frac{a}{\pi_c(s_p + \text{c} - s_w) - b + s_w}$$

$$\frac{du^*}{d\beta} < 0$$

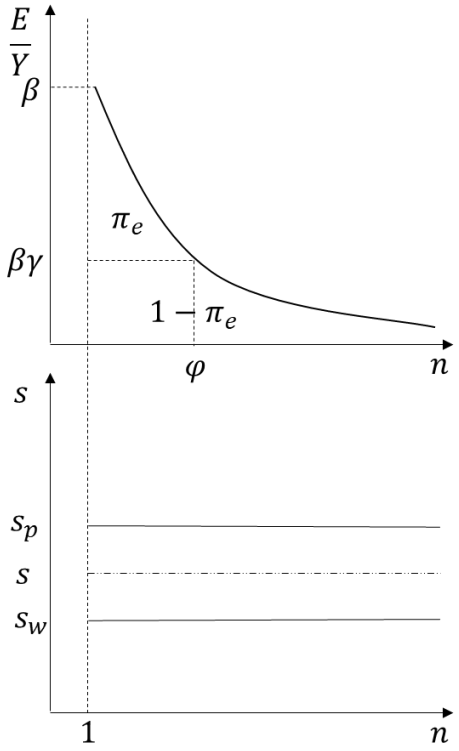
$$\frac{du^*}{d\gamma} < 0$$

Thus, higher inequality in the class-conflict with overall personal distribution (assuming a Pareto distribution) entails inferior degrees of utilization in the economy. Furthermore, the higher the number of capitalists, the less the degree of utilization of equilibrium. Do these results maintain in an earnings composition perspective? The following section formalizes this different point of view to answer this question.

Earnings-Composition with overall inequality

While the previous formalization has not changed much from the canonical version beyond the endogenization of the profit share as a function of personal inequality and individual class conflict, the earnings composition has a slightly different parameter interpretation in the presence of personal inequality. The γ in the canonical version with a composition perspective was interpreted as the share of profits in the income of all individuals in the economy, set to be equal among all of them. However, with the inclusion of personal inequality, the parameter will not affect all individuals. Consequently, the parameter now describes who are the individuals that receive two kinds of income. Knowing that β, beyond being the parameter that defines the degree of inequality in the economy, is also the highest share of income of an individual in the economy, allows to interpret γ as a parameter of comparison. More clearly, all individuals who earn a percentage γ or less of the wealthiest individual, do not earn profits. As follows, when an individual reaches the share $\beta\gamma$ of income, he starts to diversify its earnings, receiving a share as profits. Before that, the individual only receives wages. Consequently, γ will be called diversification parameter. The following image clarifies this interpretation:

Fig. 5 - Earnings-Composition with Overall Inequality



Source: prepared by the author

As it can be observed, not all individuals compound their income with profits and wages. The point φ in the horizontal axis, determined by the intersection between the distributional curve and $\beta\gamma$, describes the threshold where individuals earn two types of income or only one. The following equation specifies the point:

$$\varphi = \left(\frac{1}{\gamma}\right)^{\frac{1}{\beta+1}}$$

With the intersection point in the horizontal axis, it becomes simple to disclose the areas of the graph representing the profit and wage shares. The profit share is calculated as follows:

$$\pi_e(\beta,\gamma) = \int_1^{\varphi} \frac{\beta}{n^{\beta+1}}\,dn - \int_1^{\varphi} \beta\gamma\,dn$$

$$\pi_e(\beta,\gamma) = -\gamma^{\frac{\beta}{\beta+1}}(\beta+1) + \beta\gamma + 1$$

This result implies that the wage share may be represented in the following manner:

$$(1-\pi_e) = 1 - \int_1^{\varphi} \frac{\beta}{n^{\beta+1}}\,dn + \int_1^{\varphi} \beta\gamma\,dn = \int_1^{\varphi} \beta\gamma\,dn + \int_{\varphi}^{\infty} \frac{\beta}{n^{\beta+1}}\,dn$$

$$(1-\pi_e) = \gamma^{\frac{\beta}{\beta+1}}(\beta+1) - \beta\gamma$$

Once again, the profit and wage shares were endogenized as functions of the equality parameter, β, and the parameter γ, now called diversification parameter. Thus, the exogenous forces controlling the degree of inequality in the economy and the degree of diversification of income command the behavior of the functional shares. To see how modifications in these forces alter the profit and wage shares it is possible to observe the results of the following derivatives:

$$\frac{d\pi_e}{d\beta} = -\gamma^{\frac{\beta}{\beta+1}}\left(\frac{\ln\gamma}{\beta+1}+1\right)+\gamma <> 0$$

$$\frac{d\pi_e}{d\gamma} = \beta\left[1-\left(\frac{1}{\gamma}\right)^{\frac{1}{\beta+1}}\right] < 0$$

Interestingly, different from the class-conflict perspective, modifications in the equality parameter do not have clear signals. It is interesting to notice that in economies where the pattern of diversification starts in lower levels of income (γ less than 0.3 for instance, given that for $\ln\gamma < -(\beta+1)$, $\frac{d\pi_e}{d\beta}$ is always positive), the profit-share may respond in a positive manner to increases in inequality. In these economies, more inequality results in a higher profit-share. However, for economies where the diversification occurs only at the top level of income shares, an increase in inequality decreases the profit share. This occurs because the increase in inequality reduces the number of individuals willing to (or capable of) diversify their income. An increase in the share of income of those individuals who keep diversifying does not totally compensate such reduction. Therefore, while inequality always increased the profit share in the class conflict model, this is not true from an earnings composition point of view.

Still, the effects of the diversification parameter are analogous to the effects of the class conflict parameter γ. Their signals, however, are opposed, because an increase in the number of individuals earning profits is now associated with a decrease in the threshold defining the individuals willing to diversify. Thus, even though the signals are opposed, in both models the exogenous increase in the quantity of individuals earning profits provides increases in the profit share of the economy. It is interesting to add the results to the equilibrium to see the effect in the equilibrium utilization rate of the economy. This is represented by the equations:

$$u^* = \frac{a}{\pi_c(s_p + \text{c} - s_w) - b + s_w}$$

$$\frac{du^*}{d\beta} <> 0$$

$$\frac{du^*}{d\gamma} > 0$$

As it can be analyzed, the higher the threshold of diversification the less the degree of utilization. This means that more individuals are opting solely for wages which is profitable for increases in the degree of utilization. As in any canonical version of Kaleckian models, high profit shares are associated with low outputs. Furthermore, inequality may paradoxically increase the degree of utilization depending on the value of the diversification parameter. In economies where to diversify an individual has to earn a share too close of the wealthiest individual, inequality decreases the profit share and increases the degree of utilization of the economy. As a result, inequality may be beneficial for the output of economies.

Concluding Remarks

While it is common in the post-Keynesian literature to organize demand-led models starting on the tautological fact that the total income of an economy is the sum of the income its classes, it is unusual to extend this tautology to affirm that the total income may also be represented as the sum of individual incomes. In fact, *both tautologies are the same when it is assumed that a single individual forms each class*. The lack of such extension suppresses the role of individuality in post-Keynesian growth models. As a result, personal distribution as an impersonation of individuality is commonly overlooked. Functional distribution suffices as a category of analysis.

Notwithstanding, the growing concern with personal inequality and its known impacts on the growth patterns of economies demands the attention of the post-Keynesian literature. Still, models have only recently started to cope with personal inequality in a demand-led context. Beyond multiple class models and intra-class formulations, few have been the didactic expositions of the problem. The complexity, unfortunately, may be hampering the progression of important discussions. Moreover, it conceals important implications for the usual interpretations and results of canonical models.

The present paper intended to fulfill this gap of the literature, presenting a didactic note on personal inequality in post-Keynesian growth models. The strategy was simple and direct: individuality was inserted through the tautology that affirms that the sum of a class income may be represented by the sum of the income of its individuals. Consequently, through the definition of how these shares are distributed the model could be closured with personal inequality.

Interestingly, the insertion of individuality allowed the model to be interpreted from two different perspectives: class-conflict and earnings-composition. The first did not flee from canonical interpretations and established class conflict as the increase or decrease in the number of individuals in each class. The second, however, presented a slight change on normal interpretations of post-Keynesian models. In this perspective, classes do not distinguish individuals. Furthermore, individual may receive more than one type of income. While this second aspect is clear for researchers, class-conflicts' frameworks usually impede this interpretation. With the earnings-composition presentation of the canonical models, it becomes clear that there is no problem in interpreting the model with multiple types of income rather than with multiple classes, given that in the canonical versions γ converges to π in both interpretations.

To formalize the model with overall inequality, the Pareto distribution was used as a ruse to represent the sum of individual shares of income. The effect was an endogenization of the profit-share as a function of the degree of inequality of the economy. Depending on the perspective adopted, the closure with personal inequality presented different results. On the class conflict side, the profit share increased both with increases in the inequality and with the increase in the number of individuals in the capitalist class. On the earning-composition side, the profit share increases when the number of individuals capable of diversifying their income increases. However, the increase in inequality not necessarily increases the profit-share of the economy. This occurs because the increase in inequality reduces the number of individuals capable of diversifying and, also, increases the share of income of the individuals who keep diversifying. Depending on the size of each one of these effects, the profit-share may increase or decrease.

As it can be seen, this simple reformulation combined with a different perspective already allowed the observation of a different result. Consequently, more important than the formal aspects of the paper, the presentation of a simple form of including personal distribution and a different point of view for interpreting post-Keynesian growth models opens the way for different formulations and may promote the dissemination of formal and informal discussions of personal inequality on the literature.

Finally, it is important to clarify the theoretical nature of the paper. The empirical validity of the presented models is not under scrutiny. The main intention is to propose a framework of observation, rather

than to ingress in the usual post-Keynesian discussion of closeness how close to reality models should be. It is of the nature of models to be simplifications. As simplifications, models are false. It is not always a matter of degree. A model more empirically valid is nonetheless false. The question that matters here is whether models offer an interesting tool for thought experiments. In this sense, the proposed framework is far from contending that previous aggregated models were invalid fo neglecting individuality. The point is to expand the toolbox, rather than to substitute the tools.

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