The Trouble with Harrod: the fundamental instability of the warranted rate in the light of the Sraffian Supermultiplier

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ABSTRACT

The paper argues that Harrodian instability is an instance of what Hicks (1965) called *static instability*, related to the *direction* (and not to the intensity) of the disequilibrium adjustment process. We show *why* such instability obtains in demand-led growth models in which the ratio of capacity creating private investment to output ratio is given exogenously by the aggregate marginal propensity to save. We also show that Sraffian Supermultiplier model overcomes the Harrodian instability and that its demand-led equilibrium is *statically stable*. It is explained that the latter results do not follow from the presence of autonomous non-capacity creating expenditure component *as such* but from its presence within a model in which investment is driven by the capital stock adjustment principle (i.e., the flexible accelerator). Finally, we argue that, although being statically stable, the equilibrium growth path of the Sraffian Supermultiplier model can be *dynamically* stable or unstable depending on the *intensity* of the reaction of investment to demand. We then provide a discrete time sufficient condition for the dynamic stability of such equilibrium that implies that the marginal propensity to invest remains lower than the marginal propensity to save during the adjustment process, a modified *Keynesian stability condition*.

RESUMO

O artigo argumenta que a instabilidade de Harrodiana é um exemplo do que Hicks (1965) chamou de instabilidade estática, relacionada à direção (e não à intensidade) do processo de ajuste em desequilíbrio. Nós mostramos a razão pela qual este tipo de instabilidade é obtida em modelos de crescimento liderados pela demanda nos quais a taxa de investimento é determinada exogenamente pela propensão marginal a poupar. Também mostramos que o modelo do Supermultiplicador Sraffiano permite superar o problema da instabilidade de Harrodiana e que seu *steady-state* é estaticamente estável. Explicamos que estes últimos resultados não decorrem apenas da presença de um componente autônomo da demanda agregada que não cria capacidade, mas sim da sua presença num modelo em que o investimento é impulsionado pelo princípio do ajustamento do estoque de capital (isto é, o acelerador flexível). Finalmente, argumentamos que, embora seja estaticamente estável, a trajetória de crescimento de equilíbrio do modelo Supermultiplicador Sraffiano pode ser dinamicamente estável ou instável dependendo da intensidade da reação do investimento à demanda. Em seguida, fornecemos uma condição suficiente em tempo discreto para a estabilidade dinâmica de tal equilíbrio. Esta condição implica que a propensão marginal a investir permanece menor que a propensão marginal à poupar durante o processo de ajustamento ao equilíbrio, uma condição Keynesiana de estabilidade modificada.

Palavras-chave: Demanda Efetiva; Crescimento Econômico; Instabilidade Harrodiana.
Key words: Effective Demand; Economic Growth; Harrodian Instability.
JELCode: E11; E12; O41

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1 INTRODUCTION

Harrod's (1939, 1948, 1973) concept of the "Fundamental Instability of the warranted rate of growth", now known as "Harrodian instability", has for long been seen as an obstacle to the development of satisfactory demand-led growth models based on the "marriage between the multiplier and the accelerator". The paper uses the results of the Sraffian Supermultiplier model[1] to *explain* under which conditions and, more importantly, *why* this "fundamental instability" characterizes the equilibrium path of the Harrodian model. From our analysis, it follows that the possible incompatibility between demand-led growth models and capacity creating private investment being driven by the (flexible) accelerator is only apparent. The Sraffian supermultiplier model overcomes such apparent incompatibility by combining the investment function based on the capital stock adjustment principle with the presence of an autonomous non-capacity creating component in aggregate demand.

Our analysis proceeds in two successive steps. First, we clarify the economic meaning and some important theoretical implications of the Harrodian notion of warranted rate of growth and the related "principle of fundamental instability". For this purpose, we use the theoretical results provided by the Sraffian Supermultiplier model (Serrano 1995a, 1995b), together with Hicks's notions of "static" and "dynamic" stability (Hicks, 1965), the latter being related to the distinction between the *direction* versus the *intensity* of a disequilibrium adjustment. We then explain why the Harrodian warranted rate of growth path is indeed fundamentally or *statically unstable* under very general conditions.[2] This step in our argument is important because sometimes Harrodian instability has been misinterpreted as being the consequence of the intensity of the investment reaction to the disequilibrium between capacity and aggregate demand. In contrast, we argue that real problem behind the Harrodian instability is that the adjustment of capacity towards normal utilization requires changes in the investment to output ratio that the Harrodian model cannot produce since the saving ratio (the marginal propensity to save) is exogenously determined. Thus, the investment function based on the capital stock adjustment principle should not be blamed for the Harrodian instability. The latter is characterized by the wrong direction of the disequilibrium adjustment process, being, thus, a case of static instability in the Hicksian sense.

In the second step of our analysis, we explain why this type of Harrodian instability does *not* occur in the Sraffian Supermultiplier model. In this connection, we show that the presence of autonomous non-capacity creating demand implies that the reaction of induced investment to an initial imbalance between capacity and demand has, at some point during the disequilibrium adjustment process, a greater impact on the rate of growth of productive capacity than on the rate of growth of demand. The latter result reverts the tendency towards an increasing deviation of the actual degree of capacity utilization from its normal level that characterizes Harrodian instability. Thus, we argue that the demand-led growth equilibrium path of the Sraffian Supermultiplier model is *statically stable*, in direct contrast with the static instability that characterizes the Harrodian equilibrium. As we explain, it is important to note that the latter result does not follow from the presence of a non-capacity creating expenditure component *as such*, but from its presence within a model in which investment is driven by the capital stock adjustment principle. Nonetheless, we argue that, although being statically stable, the equilibrium growth path of the Sraffian Supermultiplier model can be *dynamically* stable or unstable depending on the intensity of the reaction of investment to demand. We then provide a discrete time sufficient condition for the dynamic stability of such equilibrium that implies that the marginal propensity to invest remains lower than the marginal propensity

[1] This model is characterized by distribution being exogenous (and determined along Sraffian lines), investment being totally induced by the adjustment of capacity to demand and the importance of autonomous demand components that do not create capacity for the private sector of the economy. These hypotheses were largely inspired by the work of Garegnani (1962), which explains why the model was called Sraffian Supermultiplier in Serrano (1995a, 1995b). Recently, Cesaratto (2016) has discovered that the idea of adapting Hicks's (1950) trade cycle Supermultiplier for the analysis of the trend of demand-led growth driven by autonomous demand was first introduced by Ackley (1963) in an econometric model developed for the Italian economy and published only in Italian. The latter work was probably influenced by discussions relating to Garegnani (1962). Recently, chapters III and IV of the latter work have been published in English as Garegnani (2015).

[2] In this paper we are not concerned with the other problem pointed out by Harrod, that is, the reconciliation of the warranted rate of growth and a "natural rate" of growth given by the sum of exogenous rates of growth the labor force and its productivity.

to save during the adjustment process.[3] Finally, it is argued that the latter condition is a modified "Keynesian stability" condition in the sense that it can deal with the variability of the propensity to invest required by the operation of the flexible accelerator.

It should be stressed that our purpose here is not to present an exegetical analysis Harrod's writings. What matters to us here is the general problem posed by the Harrodian model to modern heterodox growth theory.[4]

The rest of the paper is organized as follows. In section 2 we present, very briefly, Hicks's notions of static and dynamic stability. In section 3 we discuss the meaning of Harrod's warranted rate and show that growth at this rate is indeed fundamentally or statically unstable, as the adjustment always goes in the wrong *direction*. In section 4 we discuss the Sraffian Supermultiplier and show that while the model is statically stable, it may be dynamically stable or unstable depending on the *intensity* of the reaction of investment to demand. The next section (section 5), situates our contribution in relation to the recent literature on the Harrodian and Supermultipler growth models. Finally, in the last section, we present some final remarks. Appendices A and B contain discrete time formal proofs of Harrodian fundamental or static instability, and a set of sufficient conditions for the dynamic stability of the Sraffian Supermultiplier, respectively.

2 HICKS ON "STATIC" AND "DYNAMIC" INSTABILITY

For Hicks, the distinction between "static" and "dynamic" instability relates to the *direction* and *intensity* of the disequilibrium adjustment process, respectively. Thus, an equilibrium is statically unstable if the disequilibrium adjustment process leads the economic system in the "wrong" direction, away from its equilibrium state, independently of the intensity (or speed) of the adjustment process. However, even an equilibrium that is "statically" stable can still be "dynamically" unstable, if the adjustment process is too intense so as to lead to a chronic overshooting of the equilibrium position through undamped cycles. "Static" stability is thus a necessary, but not sufficient condition for "dynamic" stability. On the other hand, a "statically" unstable model is thus inherently unstable.

In a long footnote Hicks (1965, p. 18, fn. 2)[5] clarifies these concepts with the simple example of a Neoclassical partial equilibrium analysis of a market with given supply and demand curves. In this context, if the resulting excess demand function is negatively sloped the equilibrium is statically stable. On the other hand, if for some reason the excess demand function is positively sloped, the equilibrium of the model is statically, and thus inherently, unstable. However, assuming that the excess demand function is well behaved and negatively sloped is not sufficient to ensure dynamic stability. Indeed, if the reaction of the market price to excess demand is not continuous but happens in discrete or lumpy jumps, for sufficiently high values of the parameters one may find that the equilibrium is dynamically unstable. In this case, although static conditions point the adjustment process in the right direction as defined by economic theory, there can be an overshooting of the equilibrium point, as in the well-known undamped cycles of the cobweb theorem. Conversely, if the reaction parameters are sufficiently small, there will be a tendency towards equilibrium whether monotonic or through dampened cycles. Thus, static stability does not depend on the intensity or magnitude of the reaction to disequilibria but only on its *direction*. Dynamic stability, on the other hand, depends on the magnitude of the adjustment parameters and, therefore, on the intensity of the adjustment process.

Hicks (1965) concludes that static stability conditions are more basic, in the sense that if they are not met, the equilibrium of the model in question will be dynamically unstable for any value of the adjustment parameters compatible with the subjacent economic theory.[6] We believe that Harrod's principle of the "fundamental instability" of growth at the "warranted rate" can be fruitfully interpreted as an example of "static" instability in the sense of Hicks.

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[3] We develop the model in a discrete time framework because we believe that it provides a more intuitive description of the economic meaning of the dynamic adjustment processes between capacity and demand.

[4] We readily acknowledge that in our analysis we left out some specific characteristics of Harrod's own analytical framework such as: the use of instantaneous rates of growth; the discussion of the short-term disequilibria between production and demand; the assumption of a large set of available techniques with only one chosen at the exogenously given rate of interest; some nonlinearities in the behavior of the saving ratio and the technical capital-output ratio; the integrated analysis of both the trend and the cycle, among others. On these matters see Besomi (2001).

[5] To the best of our knowledge, Hicks never used these concepts to interpret either Harrod or his own version of Supermultiplier model. The present paper is not about Hicks's own work. We are taking from him just the crucial distinction between the direction and the intensity of disequilibrium adjustments.

[6] For a fuller discussion of the importance of these Hicksian concepts, illustrated by the debate between Sraffian and Neoclassical theories of distribution and relative prices, see Serrano (2011).

3 HARROD'S WARRANTED RATE AND THE PRINCIPLE OF "FUNDAMENTAL INSTABILITY"

3.1 The actual, capacity and warranted growth rates

Harrod (1939, 1948 and 1973) presented a growth model that was to be based on the "marriage between the 'principle of acceleration' with the 'theory of the multiplier'." The combination of the multiplier and the accelerator should allow him to deal with the dual character of investment. The multiplier treats investment[7] as a source of aggregate demand, while the accelerator deals with the capacity creating role of investment and its possible impact on further investment decisions. Harrod investigates the conditions for steady growth in a simple model, i.e., under which conditions the demand and capacity effects of investment can be reconciled, allowing the existence of a growth path in which productive capacity and demand are balanced with a continuous utilization of productive capacity at its normal or planned level.[8]

These conditions are expressed using Harrod's "fundamental equation", derived from the equality between investment and saving when output is equal to demand, divided by the capital stock. The right-hand side component of such equality can be tautologically decomposed as follows:

$$g_{Kt+1} = \frac{I_t}{K_t} = \frac{S_t}{Y_t}\frac{Y_t^*}{K_t}\frac{Y_t}{Y_t^*} = \frac{s}{v}u_t \quad 1)$$

The above expression tells us that the rate of growth of the capital stock is identical to the product of the *average* propensity to save (S_t/Y_t), the reciprocal of the normal capital-output ratio ($v = 1/(Y_t^*/K_t)$) and the actual degree of capacity utilization ($u_t = Y_t/Y_t^*$). In Harrod's model, the *average* propensity to save is equal to and determined by the *marginal* propensity to save (s), taken here as exogenously determined by consumption habits and income distribution. In his analysis of the fundamental instability of the warranted rate, Harrod did not consider the existence of an autonomous and independently growing level of autonomous consumption.

Thus, with all consumption being induced, the actual level of output determined by effective demand is given by:

$$Y_t = \frac{I_t}{s} \quad 2)$$

Thus, in this model, for a given value of the marginal propensity to save, the actual rate of growth of the economy (g_t) is equal to, and determined by, the rate of growth of investment g_{It} (since consumption expenditures always grows at the same rate as investment). Moreover, the rate of growth of the capital stock (and capacity output) (g_{Kt}) also always follows, with a certain lag, the rate of growth of investment. This happens because the relation between the rate of growth of (net) investment and the rate of growth of the capital stock is given by:[9]

$$g_{Kt+1} = g_{Kt}\left(\frac{1+g_{It}}{1+g_{Kt}}\right) \quad 3)$$

Which is always tending to $g_{Kt} = g_{It}$. Thus, it follows that (1) will tend to:

$$g_{It} = \frac{s}{v}u_t \quad 4)$$

From (4) we obtain Harrod's (1939, p. 17) "fundamental equation" by setting the actual degree of capacity utilization equal to its planned value ($u_t = u_n = 1$):

$$g_W = \frac{s}{v}$$

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[7] In this paper by investment we mean only those expenditures that can create productive capacity for the private business sector of the economy. We thus leave out of our analysis private residential investment and investment by government and state owned enterprises.

[8] In what follows we are making a number of standard simplifying assumptions. The economy is closed and there is no distinct government sector. We take all magnitudes net of depreciation. We also assume that the economy produces a single product using only homogenous labor and itself as fixed capital, by means of single method of production with constant returns to scale. We also assume that labor is always abundant and thus capacity output is given by the size and efficiency of the capital stock. We take the real wage and labor productivity (no technical progress) as exogenously given. We further assume that firms have planned spare capacity so that normal or potential output Y_t^* is below maximum capacity output Y_t^{max}. We normalize the normal or planned degree of capacity utilization as $u_n = 1$ so that $u^{max} = Y_t^{max}/Y_t^* = 1+\gamma$ (where γ is the percentage of planned spare capacity). Finally, we suppose that short-term expectations are always correct so that there is no involuntary accumulation of inventories.

[9] The capital available at the *beginning* of period $t+1$ is $K_{t+1} = K_t + I_t$. Hence, we have $g_{Kt+1} = I_t/K_t = (I_{t-1}/K_{t-1})\big((1+g_{It})/(1+g_{Kt})\big) = g_{Kt}\big((1+g_{It})/(1+g_{Kt})\big)$.

5)

Equation (5) shows the condition for the balance between the growth of capacity and demand *in* Harrod's model. Harrod called "warranted rate" this particular rate of growth (g_W). The warranted rate is a positive function of the marginal propensity to save and a negative function of the normal capital-output ratio.

Although one of Harrod's aims was to extend to the longer run (when the capacity effects of investment matter) some of Keynes's arguments that were presented in a short run context, actual growth at the warranted rate has to be understood as a condition for the validity of *Say's Law* in the longer run. In fact, note that the warranted rate is *not* the actual rate of growth of aggregate demand and output, which, as we have seen above, is determined by the actual rate of growth of investment (g_{It}). The warranted rate is also *not* the actual rate of growth of the capital stock and potential output. The rate of growth of the capital stock, as we saw above, also tends to grow at the same rate as investment grows. Instead, Harrod's warranted rate represents the rate of growth of potential output that would only occur if investment happened to be *in every single period*, including the initial one, exactly equal to and determined by the saving obtained at normal or planned capacity utilization (capacity saving from now on). Growth at the warranted rate would only occur if demand adjusted itself to the level and growth rate of productive capacity. As Harrod follows Keynes in rejecting Say's Law, he concludes correctly that there is no reason for a market economy to grow at the warranted rate. If we take the rate of growth of investment as provisionally given, we know that the actual rate of growth of the economy will be determined by this rate of growth of investment ($g = g_I$). On the other hand, for a given technique, we know that the actual degree of capacity utilization will change according to the ratio between the rate of growth of demand g_t (and output) and the rate of growth of the capital stock g_{Kt}:

$$u_t = u_{t-1}\left(\frac{1+g_t}{1+g_{Kt}}\right) \qquad 6)$$

With an exogenously given rate of growth of investment g_I, both aggregate demand (and output) and the capital stock (with some lag) will tend to grow at this given rate and thus the actual degree of capacity utilization will tend to stabilize at the level:

$$u_t = \frac{g_I}{s/v} \qquad 7)$$

Hence, an actual rate of growth of investment and output above the warranted rate ($g_I > g_w$) would lead to persistent overutilization of productive capacity ($u > 1$) and that, conversely, a rate of growth of investment below the warranted rate ($g_I < g_w$) would make the economy tend to a situation of persistent underutilization of productive capacity ($u < 1$). Thus, with decisions to invest being independent from decisions to save, growth at Harrod's warranted rate would happen only as a fluke.

3.2 The fundamental instability of growth at the warranted rate

Harrod went beyond demonstrating that there was no reason for the economy to grow at the warranted rate (s/v) and that the actual degree of capacity utilization would tend to a level different from the planned level if investment grew at a given exogenous rate. Harrod showed that, if investment is taken to be induced, in the sense of being driven by what we now call the principle of capital stock adjustment (i.e., the accelerator), any rate of growth of investment different from the warranted rate would cause a cumulative disequilibrium process, illustrating what he called the "principle of fundamental instability" of growth at the warranted rate.

One way of representing the operation of the principle of fundamental instability is by taking the reaction of the rate of growth of investment to the deviation of the actual degree of capacity utilization from its planned level, according to:

$$g_{It} = g_{It-1} + \alpha(u_{t-1} - 1) \qquad 8),$$

where $\alpha > 0$, in accordance to the capital stock adjustment principle. Equation (8) shows that an overutilization of capacity ($u > 1$) will make firms increase the rate of growth of investment, while underutilization ($u < 1$) will make them reduce it. In both cases, this (reasonable) type of reaction, when firms are trying to adjust capacity to demand, will make the economy move further away from its warranted rate.[10]

While a given rate of growth of investment g_I leads to a stable level of the actual degree of capacity utilization, every time the growth rate of investment changes, the corresponding equilibrium level of the degree of capacity utilization also changes. This follows from the fact that the initial effect of a rise in g_I is the increase in the rate of growth of aggregate demand by more than the growth of capacity, because investment is always first an increase in demand and only later it leads to an increase in productive capacity (conversely a fall in g_I makes the growth of demand g fall before the fall of g_K according to equation (3)). Therefore, each round of increases (reductions) in the rate of growth of investment due to the actual degree of utilization being initially higher (lower) than the planned degree would lead to a new, even higher (lower), actual degree of capacity utilization and so on, as described by equation (6). This is the core of the principle of "fundamental instability" of Harrod's warranted rate. Any divergence between the actual and planned degree of utilization would, by the mechanism just described, be self-reinforcing.

This instability was considered fundamental by Harrod because the adjustment occurs in the wrong direction, independently of the (positive) magnitude of the reaction parameter α. Harrod himself noted that it was "independent of lags". It is also easy to note that introducing more lags in the connection between the growth rate of investment and deviations of capacity utilization from planned levels would not change the problem, as this would not change the direction of the adjustment process. The reason why growth at Harrod's warranted rate is fundamentally unstable has to do with the direction and is, in fact, independent of the intensity of the adjustment, being thus a case of Hicksian static instability (see Appendix A below for a formal proof).

This fundamental or static character of Harrod's instability principle has naturally given to many authors the impression that a demand-led growth model simply has to assume that, at least in the longer run, the growth rate of investment is either totally autonomous or, at most, that it reacts in a quite limited way to the deviation between actual and planned degrees of utilization. A limited reaction in the sense that the rate of growth of investment or the desired rate of capital accumulation must have an autonomous ("animal spirits") term and must reach a given stable value even if the actual degree of capacity utilization is still different from its planned degree (as in the Neo-Kaleckians models, Lavoie (2014)). Any other investment function, fully compatible with the capital stock adjustment principle (such as equation (8) above) would simply generate instability.

Note also that things would in fact be much worse if assumptions were made that, somehow, make growth at Harrod's warranted rate dynamically stable. First, because in this case, we would necessarily have to assume, directly or by some indirect route, that the rate of growth of investment in the economy, in fact, *increases* when the degree of capacity utilization is lower than the planned level and *decreases* when there is overutilization, something that is highly implausible. Secondly, and to make matters even worse, if somehow it is proven that growth at Harrod's warranted rate is stable, we would, at the same time, have also "proved" the validity of Say's Law in the longer run. For if, for instance, the warranted rate were stable then an increase in the marginal propensity to save would cause a permanent increase in the levels and rates of growth of investment, output and capacity output.

4 THE SRAFFIAN SUPERMULTIPLIER

However, the fact that growth at the warranted rate s/v is inherently unstable does not mean, as it may seem, that any demand-led growth model with investment induced according to the principle of capital stock adjustment will also be necessarily unstable. In fact, we may reach quite different results if we include an exogenous and independently growing component of autonomous expenditures that do not create capacity for the private business sector of the economy. The "marriage between the accelerator and the multiplier" can indeed succeed if this source of autonomous demand is present and its growth may lead to a stable process of truly demand-led growth.

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[10] Although we are here making the rate of growth of investment a function of the deviation between actual and the planned degrees of utilization, the same reasoning would apply if we use different specifications of the investment function based on the capital stock adjustment principle. For instance, the same results obtain if we make the growth rate of investment a function of an expected trend rate of growth, partially revised in the light of actually observed rates of growth of the economy, or if the rate of growth of investment depended both on the expected rate of growth and the deviation of the actual degree of utilization from its planned degree. As we shall explain shortly, the fundamental instability result follows from the fact that aggregate demand always rises and falls in the same proportion as investment, not on the specific form of the induced investment function based on the capital stock adjustment principle.

4.1 Autonomous consumption and the fraction

To see why let us assume that there is an autonomous component in consumption Z that grows at an exogenous rate z.[11] Now, differently from the Harrodian model from the previous section, the aggregate marginal and average propensities to save are not the same. Indeed, the average propensity to save (*S/Y*) is given by:

$$\frac{S_t}{Y_t} = s - \frac{Z_t}{Y_t} \quad 9)$$

The marginal propensity to save does not determine, but only imposes an upper bound, to the average propensity to save. Although the marginal propensity to save is exogenously given, the average propensity to save now depends on the actual level of output. An increase in the level of output in relation to autonomous consumption, caused by an increase in investment in relation to output, reduces the relative weight of the "dissaving" represented by the autonomous consumption component, increasing the ratio between average and the marginal propensities to save.

The latter result becomes more clear when we express the average propensity to save in terms of the independent variables that determine it. As $S_t/Y_t = I_t/Y_t$ and $Y_t = (I_t + Z_t)/s$, then:

$$\frac{S_t}{Y_t} = s\left(\frac{I_t}{I_t + Z_t}\right) = s\left(\frac{1}{1 + Z_t/I_t}\right) \quad 10)$$

$$\frac{S_t}{Y_t} = sf_t \quad 11)$$

The endogenous variable f is what Serrano (1995b) called "the fraction". It corresponds to the ratio between average and marginal propensities to save. Equation (10) shows that the average propensity to save depends both on the marginal propensity to save and on the level of autonomous consumption relative to the level of investment. An increase in the latter increases now both the level and the *share* of saving in output. Below the upper limit given by the exogenous marginal propensity to save s, it is the (relative) level of investment that determines (through changes in the fraction *f*) the share of investment and saving in aggregate output.

4.2 The average propensity to save and the Supermultiplier

Let us now add the assumption that investment is an induced expenditure. In a first step, let us assume it is determined as a share of output:

$$I_t = hY_t \quad 12),$$

where h is the propensity to invest that is exogenously given. Now the level of output is determined by the level of autonomous consumption and a Supermultiplier that takes into account both induced consumption and induced investment:

$$Y_t = \frac{Z_t}{s - h} \quad 13)$$

Given the marginal propensity to save and the propensity to invest, effective demand and output will grow at the rate z at which autonomous consumption grows. In this case, the average propensity to save is entirely determined (for any value strictly below the marginal propensity to save s) by the propensity to invest. Indeed, from equation (13) we get the share of autonomous consumption in output:

$$\frac{Z_t}{Y_t} = s - h \quad 14)$$

Replacing this latter result in equation (9) we obtain:

$$\frac{S_t}{Y_t} = h \quad 15)$$

[11] For a discussion of the theoretical significance and empirical relevance of the autonomous components of demand that do not create capacity, see Fiebiger & Lavoie (2017), who call such expenditures "semi-autonomous".

4.3 The Static or fundamental stability of the adjustment of capacity to demand

Our Supermultiplier model allows us to rewrite equation (4) above as:

$$z = \frac{h}{v} u \qquad 16)$$

Due to the presence of autonomous expenditures that do not create capacity and grow at an exogenous rate z, the fact that investment is induced in the sense discussed above does not lead to fundamental or static instability as in the Harrodian model. In fact, contrary to what happens in the latter model, the autonomous consumption Supermultiplier presented here is fundamentally or statically stable in Hicksian terms, since the reaction of investors put the economy in the direction of the equilibrium point. In Harrod's case, as seen above, if initially the growth rate of investment happens to be above the warranted rate s/v, the actual degree of capacity utilization will be above its planned level and, conversely, if the rate of growth of investment happens to be lower than the warranted rate this will lead to a situation of underutilization of capacity. If investment follows the capital stock adjustment principle, the disequilibrium process drives the economy away from the equilibrium point, because overutilization (underutilization) leads to a higher (lower) rate of growth of investment and this will, by its turn, make the actual degree of utilization increase (decrease) even further.[12]

In the case of the Sraffian Supermultiplier model, growth at Harrod's warranted rate is still unstable because, as will see, that rate only determines an upper limit to feasible demand-led rates of growth. But in this model, where the rate of growth of the trend of demand is given by the rate of growth of autonomous expenditures z, growth at this rate is fundamentally or statically stable. Indeed, starting from a situation in which utilization is equal to its planned degree, if the rate of growth of investment g_{It} happens to be initially above z, the rate of growth of aggregate demand will be lower than the growth of capacity, which will lead to an underutilization of capacity. On the other hand, if the rate of growth of investment happens to be below the rate of growth of autonomous demand, demand will grow by more than investment, and this will lead to an overutilization of productive capacity.

If induced investment behaves according to the capital stock adjustment principle, either directly by the deviation of the actual degree of utilization from its planned degree, or by the effect of actually observed growth rates on the expected growth rate of the trend of demand (or both) this gives the signals in the right direction for the change in investment. In the case of underutilization (overutilization), there will be a tendency for investment to grow less (more) than demand, i.e., towards a lower (higher) investment share h, which will tend to make capacity grow by less (more) than demand.

As an example, let us assume that starting from a situation in which capacity and demand are balanced, there is a reduction in the rate of growth z of autonomous consumption. This reduction will provoke a reduction to the same extent of the rate of growth of demand and output g for given values of the marginal propensity to consume and of the investment share. The actual degree of capacity utilization will be reduced (leading to a value $u < 1$), as initially aggregate demand will start to grow less and only later the rate of growth of productive capacity and the capital stock will tend to expand at this same lower rate according to equation (3). The lower growth of capacity will prevail when the capacity effect of the lower absolute rate of growth of investment, for the given propensity to invest, h, materializes. Investment will grow at the same lower rate of growth as autonomous expenditures, reducing also the rate of growth of the stock of capital. When the rate of growth of the stock of capital adapts itself to this lower rate of growth of demand and output, the actual degree of capacity utilization will stabilize at a level lower than the planned or normal degree, according to equation (16) above (i.e., $u = vz/h$).

However, it seems to be reasonable to assume that, over time, the investment share h will itself be reduced to some extent as a response to the underutilization of capacity and/or to a reduction of the actual rate of growth

[12] Some authors (but not Harrod) have extended this idea of the warranted rate of growth for the case in which there are autonomous components in demand (Z). In this case the modified warranted rate would be equal to $g_w = (s - Z/Y^*)/v$, the ratio between the *average* propensity to save at a position in which capacity is utilized at its planned degree and the normal capital-output ratio. This modified warranted rate would measure the potential rates of growth of capacity saving, and, in general, is not constant over time, as Z/Y could only remain constant in case the rate of growth of autonomous consumption by chance happened to be equal to the rate of growth of capacity output. For references and a detailed analysis of this modified warranted rate and the confusion between this supply side rate of growth with the demand led Sraffian Supermultiplier see Freitas & Serrano (2015).

of demand. This gradual reduction of the propensity to invest[13] will have two effects. First, it will further reduce the growth of aggregate demand and output, lowering, even more, the actual degree of capacity utilization. Nevertheless, later on, the lower investment share will reduce the rates of growth of the capital stock and productive capacity. The presence of autonomous consumption demand growing at an exogenous rate z implies that the rate of growth of aggregate demand and output will fall proportionately *less* than the rate of growth of investment (for otherwise the investment share could not have fallen). The ensuing reduction in the rate of growth of capacity and of capital stock will be equal to the fall in the rate of growth of investment. This means that the actual degree of capacity utilization will eventually start to rise, because, while aggregate demand is growing at a slower pace than before, the final reduction of the rate of growth of the capital stock is even greater (something that would be impossible without the presence of autonomous consumption).

The process described above will continue to work as long as the actual degree of capacity utilization is below the planned degree. It will only stop when the investment share h has been sufficiently reduced to a level that would allow that, at the planned degree of capacity utilization, the rate of growth of the capital stock is fully adapted to the lower rate of growth of autonomous consumption. However, depending on the value of the parameters defining the intensity of the reaction of investment to demand growth and/or the gap in capacity utilization, this adjustment process may overshoot and cause a cyclical adjustment. If the resulting cycle were dampened, this would not cause any problem. But that will depend, as we shall see below, on the conditions for dynamic, not static, stability.

Symmetrically, the same process of adjustment of the propensity to invest occurs in the case of a permanent increase in the rate of growth of autonomous consumption z. We would then have an initial overutilization of capacity and, gradual increases in the investment share h that first would increase further the degree of overutilization. However, the higher investment share will eventually make the capital stock and the productive capacity grow at a faster pace than the aggregate demand and output. As a result, the actual degree of capacity utilization would gradually fall back to its planned degree either monotonically or through dampened cyclical oscillations, and the level and rate of growth of the productive capacity of the economy will adapt itself to the permanently higher rate of growth of autonomous demand z.

The crucial point is that the process of growth led by the expansion of autonomous consumption is thus fundamentally or statically stable. This is so because the reaction of induced investment to the initial imbalance between capacity and demand has, at some point during the disequilibrium process, a greater impact on the rate of growth of productive capacity than on the rate of growth of demand. Thus, in the case of an initial underutilization (overutilization) of capacity, the consequent reduction (increase) in the rate of growth of investment in relation to the growth rate of demand and output eventually leads to a situation in which the rate of growth of the capital stock (and capacity) is lower (higher) than the rate of growth of demand/output. The operation of the capital stock adjustment principle combined with the existence of an autonomously growing non-capacity creating expenditure reverts the initial tendency towards an increasing deviation between actual and planned degrees of capacity utilization. In this sense, we may say that disequilibrium process in the Sraffian Supermultiplier model goes in the correct *direction*.

In the Harrodian model this reaction *always* causes instability because, without the autonomous consumption component ($Z = 0$), the rate of growth of demand and output always changes by exactly the *same* amount as the rate of growth of investment. Given income distribution, the lack of autonomous consumption demand ensures that no matter how much the levels of investment change, the investment share cannot change, since it is uniquely determined by the marginal propensity to save s in the Harrodian model. In contrast, in the Sraffian Supermultiplier, the average propensity to save is entirely determined by the propensity to invest decided by firms. If the latter increases (decreases) with overutilization (underutilization) and/or with increases (decreases) in the rate of growth of aggregate demand, the same occurs with the average propensity to save (S/Y), that adjusts itself to the investment share that is required to adjust the level and growth rate of capacity to that of demand. In equation (11) above, given s and v, changes in the propensity to invest h modify the “fraction” $f = [I/(I + Z)]$,

[13] The central idea is that investors attempt to adjust the size of the capital stock to the trend level of demand. This implies that the investment share will respond to changes in the expected demand trend to situations of over/underutilization of capacity or to both. There are many forms to represent this process in simple terms in Supermultiplier models. One option is to assume that the investment share reacts linearly to discrepancies between the actual and the planned degree of capacity utilization, as done in Freitas & Serrano (2015) and Serrano & Freitas (2017). Here we adopt a different specification in which the investment share reacts linearly to the revisions on the expected trend rate of growth of demand, as suggested in Cesaratto, Serrano & Stirati (2003).

to the extent that is necessary for the economy to endogenously generate the saving ratio required by the expansion of aggregate demand, leading to the tendency of the degree of capacity utilization towards its planned degree ($u = 1$). In this sense, if we were to reinterpret the "warranted rate of growth" as the ratio between *average* propensity to save and normal capital-output ratio (see note 14 above), in the Supermultiplier model, it is the "warranted rate" that would adjust itself to the actual rate of growth through changes in the *average* (but not the marginal) propensity to save, triggered by induced variations in the investment share. The upshot of our analysis is that the "marriage" between the "accelerator" and the "multiplier" can in principle indeed be consummated, but only if a third element, autonomous expenditures that do not create capacity, is present.

4.4 Dynamic Stability and the limits to demand-led growth

In the discussion of the adjustment of capacity to demand above we have alluded to the idea of a gradual adjustment of the propensity to invest in relation to discrepancies between the actual (u) and the planned degree ($u = 1$) of capacity utilization. The reason for this is that the fundamental or static stability of the adjustment of capacity to demand is certainly a *necessary* but not a *sufficient* condition for the viability of a demand-led growth regime described by the the Sraffian Supermultiplier. The *partial* or *gradual* adjustment of the investment share is what is required to provide a set of sufficient conditions for the *dynamic* stability of the whole process.

If, for instance, given an increase in the growth rate of autonomous consumption z and the consequent increase in the rate of growth of aggregate demand and of the ensuing overutilization of capacity, the marginal propensity to invest reacts too intensely and increases too much, it is possible that the whole process of adjustment of capacity to demand becomes dynamically unstable. This is so, because, although the process is going in the right direction, its *intensity* may be excessive if induced investment increases too much. In fact, if the increase in the investment share is sufficiently large, the consequent growth of aggregate demand may become so high that it may be impossible to increase the supply of output at such a rate. Formally, it is easy to see that if the propensity to invest h when added to the marginal propensity to consume $1 - s$ becomes greater than one, then any positive level of autonomous consumption demand will induce an infinite total level of aggregate demand, which is, of course, impossible to meet with increases in output. The dynamic stability of the related to the model requires that this situation does not occur. This is why the model requires the additional assumption that the changes of the propensity to invest induced by the changes in the actual growth rates of demand and/or in the deviations from the planned degree of utilization should be gradual.

This idea of a partial or gradual adjustment can be illustrated by a version of the model in which the investment share depends only on the technical normal capital-output ratio and the expected rate of growth of the trend of aggregate demand g^e. The central point is that the investment share does not depend only on the actual rate of growth g_{t-1} observed in the most recent period (as in the so called "rigid" accelerator) but on the expected trend of demand growth over the life of the new capital equipment. When the actual rate of growth of aggregate demand g changes, the expected trend rate of growth of demand g^e will be revised, but only partially and gradually. This is so because firms understand both that demand is subject to fluctuations that may not be permanent and that in an economy that uses fixed capital equipment, the purpose of firms is to adjust capacity to demand over the lifetime of the equipment and not at each moment in time. This gradual or partial adjustment of demand expectations is known as the "flexible accelerator" as opposed to the "rigid accelerator" in which firms try to adapt capacity to demand immediately and treat all changes in demand as permanent. Thus, the gradual adjustment of the propensity to invest can be represented as follows:

$$h_t = v g_t^e \qquad 17),$$

$$g_t^e = g_{t-1}^e + \beta(g_{t-1} - g_{t-1}^e) \qquad 18),$$

where $0 \leq \beta \leq 1$ is an adjustment parameter in the equation of (adaptive) expectation formation. Of course, $\beta = 0$ would mean that the investment share is exogenous, in which case we would obtain the Supermultiplier model with a given propensity to invest analyzed above. On the other hand, $\beta = 1$ would represent the case of the "rigid accelerator". Finally, a positive and small β being the more realistic case of the "flexible accelerator".

Mathematically (see Appendix B below for details), a sufficient condition for the dynamic stability of the discrete time Sraffian Supermultiplier is that the aggregate marginal propensity to spend, both in consumption and investment, has to remain lower than one during the adjustment process, in which the marginal propensity to invest will naturally be changing. In the analysis of this condition we must take into account the investment share permanently induced or required by the trend rate of growth of the economy vz, the marginal change in the investment share induced by the revision of the expected trend of growth $v\beta$ out of equilibrium and the interaction term involving the two previous terms, $v\beta z$. For the stability condition to be met, the sum of these components must be lower than the marginal propensity to save s:[14]

$$vz + v\beta + v\beta z < s \tag{19}$$

From the above condition, we can show that, for a given value of β, there is a well-defined upper bound to what can be characterized as a demand-led growth regime. This limit shows that the economy is in a proper demand-led regime only if the growth rate of autonomous demand z is not "too high", namely if:

$$z < \left(\frac{s}{v} - \beta\right)\frac{1}{1+\beta} \tag{20}$$

If condition (19) is met and the equilibrium of the Sraffian Supermultiplier is dynamically stable, there will be a tendency for the investment share to adjust itself to the value required by the trend rate of growth of demand, which of course will be equal to and determined by the rate of growth of autonomous consumption z ($g^e = g^* = z$):

$$h^* = vz \tag{21}$$

and thus:

$$Y_t^* = \frac{Z_t}{s - vz} \tag{22}$$

As under these assumptions, the level of productive capacity and the capital stock tends to adjust itself to the trend levels of demand and output we also have that:

$$Y_t^* = \frac{1}{v}K_t = \frac{Z_t}{s - vz} \tag{23}$$

Therefore, there is a tendency for the levels of capacity output to follow the evolution of the trend of effective demand and for the rate of growth of demand to be led by the expansion of the autonomous expenditures that do not create capacity, Z.

The research based on the Sraffian Supermultiplier was set out to determine under which conditions growth could be unambiguously demand-led under exogenous distribution and with investors driven to adjust capacity to demand. Three such conditions were found to be required. The first is the existence of an autonomous component in aggregate demand that does not create capacity for the private business sector of the economy. The second was that investment must be induced by the capital stock adjustment principle. The third is that, in the adjustment of capacity to demand, the further amount of induced consumption and investment generated should not be excessive (i.e., infinite). This third condition has two elements. First, one structural element is that the rate of growth of autonomous demand must be lower than Harrod's warranted rate s/v as the share of required induced investment to meet the expansion of autonomous demand vz must be permanently lower than the marginal propensity to save s, which implies that $z < s/v$. The second element is due to the fact that room must be made also for the extra induced investment that is necessary to bring the economy back to the planned degree of capacity utilization when it deviates from it during the adjustment process (i.e., $v\beta$). Including this second element (and its interaction with the first one) we get a lower maximum rate of demand-led growth**,** described by equation (20) above (i.e., we have $(s/v - \beta)(1/[1+\beta]) < s/v$ for $\beta > 0$).

Note however that this more stringent condition (20), while sufficient, is not strictly necessary and could be relaxed to some extent if some of the model parameters were variable. This relaxation could be accomplished

[14] Existing proofs of the dynamic stability of the Sraffian Supermultiplier by Freitas & Serrano (2015), Pariboni (2015), Dutt (2015) and Allain (2015) use continuous time and are equivalent to $vz + v\beta < s$. In discrete time we see that we have to add the interaction term $v\beta z$ to the marginal propensity to invest. Note also that this condition shows that the usual stability conditions for trendless multiplier-accelerator models of business cycles in which autonomous demand remains constant, namely, $v\beta < s$ appear here as a special case of equation (19), by setting z equal to zero.

if we assume that the marginal propensity to spend happens to be higher than one in the vicinity of the position where capacity is fully adjusted to demand, but then becomes lower than one again when the economy is further away from that position, generating a limit cycle. There are many reasons for these type of assumptions, some reasonable, some quite arbitrary and implausible. Some of these possibilities will be the subject of further research. But it is important to note that the structural character of the third condition above related to the fact that the maximum rate of demand-led growth must be lower than Harrod's warranted rate[15] simply cannot be relaxed, for it is a necessary condition. In this sense, from the standpoint of the Sraffian Supermultiplier model, Harrod's warranted rate of growth has to be reinterpreted as an upper-limit for a demand-led growth process compatible with a normal degree of capacity utilization.

5 DISCUSSION OF THE LITERATURE

5.1 On Harrodian instability

We have shown above that the fundamental instability of Harrod's warranted rate is valid under very general conditions. We argued that Harrodian instability is a case of static instability, in the sense of Hicks (1965), as the adjustment goes in the *opposite direction* in relation to the equilibrium position, independently of the magnitude of the reaction parameter α.

These quite general results stand in sharp contrast with a recent contribution by Trezzini (2017), where he explicitly argues that the "the cornerstone of Harrodian instability" (Trezzini, 2017, p.2) is the intensity of the reaction of invest to demand: "the assumption of the elasticity of investment to any divergence between actual and planned utilization must be reconsidered. As the concept of Harrodian instability is based on this assumption, it appears to lose most if not all of its relevance" (Trezzini, 2017, pp. 21-22). As we saw in section 3 above and confirmed in appendix A below, Harrodian instability is fundamental or static precisely because it depends only on the sign but not on the magnitude of the reaction of investment to demand or to the deviation of capacity utilization from its planned level. Low reaction coefficients will certainly not prevent the instability of economic growth at Harrod's warranted rate.

Our results also make us somewhat skeptical of Setterfield (2016) claim that the canonical Neo-Kaleckian model may avoid Harrodian instability if investment only reacts to large deviations from the planned degree of utilization. The argument is developed as if investment is done by a single firm and it is not clear that it could be generalized to a number of different firms. To us it seems reasonable to think that if only a few firms experience underutilization (or overutilization) of its capacity large enough to trigger the proper capital stock adjustment principle, then a process of reduction (increase) in induced investment would quickly drag the degree of capacity utilization of other firms outside their tolerance bands and joint in the explosive contraction (expansion). In any case, even if aggregate investment does not react to discrepancies between capacity and demand, it would be growth at the rate determined by the pace of capital accumulation as given by the Neo-Kaleckian investment function and not growth at Harrod's warranted rate, that could be considered stable.

Our results on Harrodian instability also differ from a claim made by Dejuán (2016). The author uses his variant of the Sraffian Supermultiplier model that has an autonomous component that does not create capacity and thus, as we have seen above does not suffer from Harrod's fundamental or static instability. He does make further assumptions that firms immediately adjust the propensity to invest to the trend rate of growth of autonomous demand and perform temporary levels of investment to deal with initial under or overutilization of capacity. These latter "ancilliary" investments, according to him, have no further accelerator effects as they do not affect the expected trend rate of growth. The author is correct in attributing the strong (dynamic) stability of his own Supermultiplier model to these (in our view unrealistic) assumptions that firms know how to distinguish clearly permanent from temporary changes in demand and also know that the trend of the economy depends on the growth of autonomous demand component. However, he also argues that: "[t]he absence of proper autonomous demand in Harrod's model hindered the structural way out. But even in his simple model, stability would prevail if it were not for the bizarre reaction function bestowed on investors" that "links investment both to permanent and transient demand;" (DeJuán, 2016, p. 22). Here he claims that his assumptions on the ability of firms to distinguish between permanent and transient changes in demand would be sufficient to ensure that "stability

[15] This is the maximum rate of growth proposed initially by Serrano (1995a, 1995b).

would prevail" in the Harrodian model without autonomous demand. But what would be "stable" in such circumstances, and in the mere sense of being constant over time, would be the actual rate of growth of investment, which would not change given the assumed inelasticity of demand expectations. As he readily admits, this actual rate of growth would be different from Harrod's warranted rate and the actual degree of capacity utilization would correspondingly be permanently different from its planned level. Therefore, despite his claim, DeJuán (2016) analysis is not really capable to deny that growth at Harrod's warranted rate is fundamentally unstable, *if* investment is allowed to respond to demand or to the deviation of capacity utilization from its planned degree.

On the other hand, our results allows us to understand the analytical properties of Harrodian models in which growth at the warranted rate *s/v* is made to become dynamically stable. In such models static Harrodian instability is avoided by making explicit assumptions that either economic policy (Dumenil & Levy, 1999, Franke, 2017), or the reaction of private investment to labor market conditions (Skott, 2010), is such as to reverse the sign of the response of investment based on the capital stock adjustment principle. For instance, Franke (2017, p.10) recognizes that in order for one to be "Keynesian or Kaleckian in the short run but classical in the long run" one would need that the sign of the total reaction of the growth of investment to the deviation between the actual and the planned degree of utilization should be *negative*. In our terms this can occur only if $\alpha < 0$.[16]

5.2 On the dynamic stability of the supermultiplier

Franke (2017) has also argued that even a dynamically stable adjustment of capacity to demand via a flexible accelerator Supermultiplier can be made unstable, if combined with another adjustment process such as those used to stabilize growth at Harrod's warranted rate. But we know (see footnote 12 above) that the latter adjustment process implies that the rate of growth of investment increases when the degree of utilization falls below the planned level (and decreases with overutilization). Thus in the context of the Sraffian Supermultiplier if this added effect is sufficiently strong it could counteract the capital stock adjustment principle and would be equivalent to assuming a *negative* value for the reaction of investment to demand parameter β. We do not find that the assumptions leading to this result to be plausible, but, in any case, it should be noted that under such extreme circumstances it would be the Sraffian Supermultiplier that would become statically unstable as the adjustment process would be clearly going in the wrong direction.

The conditions for the static and dynamic stability of the autonomous demand-led supermultiplier growth model has been misunderstood by Dávila-Fernández *et all* (2017) who attempt to criticize the model arguing that the mere presence of an autonomous component of demand is not sufficient to ensure the stability of the process of adjustment of capacity to demand. As pointed out by Lavoie (2017) no such claim was ever made by the proponents of such models. The critique shows a lack of understanding that even the static stability of the supermultiplier requires both that there is an autonomous component in demand *and* capacity generating investment follows the capital stock adjustment principle. And that the sufficient condition for dynamic stability (as shown in section 4 and proved in appendix B) depends on a gradual or partial reaction of the investment share.

Recently, some Neo-Kaleckian authors have used the adjustment mechanism of the Sraffian Supermultiplier, with autonomous demand allowing the endogenous adjustment of the investment share to its required value through changes in the ratio between the average and marginal propensity to save, in order to tackle what they call the "Harrodian instability" of their original demand-led growth models (Allain, 2015 and Lavoie, 2016). These authors are correct in considering that Neo-Kaleckian models without autonomous non-capacity creating demand are subject to Harrodian instability if investment is allowed to follow the capital stock adjustment principle consistently. They are also correct in seeing that, in models that do include such an autonomous demand component, the equilibrium can be dynamically stable if the reaction of investment to demand is not excessive. The problem of the traditional Neo-Kaleckian models without non-capacity creating autonomous demand is indeed one of Harrod's fundamental or static instability. That is why instability depends only on the sign of the parameters and not on its magnitude. However, in the case of their Supermultiplier type of models, the notion of Harrodian (fundamental) instability does not apply. The long run (fully adjusted)

[16] In this respect, see also Dutt (2011) and Hein *et all* (2012).

equilibrium of these models can of course also be unstable if the reaction of investment is too strong. Nevertheless, if this happens to be the case, it is a matter of dynamic, not static or fundamental instability. Moreover, as the dynamic stability condition for the Supermultiplier is directly related to the requirement of the marginal propensity to spend to be lower than one during the adjustment process, it is clearly a variant of what the Neo-Kaleckians call "Keynesian instability" (marginal propensity to invest lower than marginal propensity to save) instead of "Harrodian instability". We thus think that the notion of Harrodian instability should be used only in demand-led models without autonomous demand, where the investment share is determined by a given marginal propensity to save.[17] It should not be applied to Supermultiplier models (whether of Sraffian or Neo-Kaleckian inspiration) to avoid confusing the necessary conditions of fundamental or static stability with those of the sufficient conditions of dynamic stability.[18]

6 FINAL REMARKS

The upshot of our analysis in this paper is that, after all, Harrod's principle of fundamental (or static) instability of his warranted rate is valid under very general conditions, but should not be seen as a "problem". Given the fact that the warranted rate s/v is, at best, an upper limit of feasible rates of demand-led growth of capacity output, there is indeed no reason for such a rate of growth to be stable. This is so, because there is no reason for investors following market signals in a decentralized monetary capitalist economy to make the economy expand along a path described by Harrod's warranted rate. So we do not think it is either theoretically fruitful or realistic (given that we would have had to assume a completely implausible positive reaction of investment to underutilization) to try to stabilize growth at Harrod's warranted rate.

The latter result, however, does not imply that the multiplier-accelerator interaction in the analysis of growing economies is, in general, fundamentally unstable. Quite the contrary, if there is an autonomous demand component that does not create capacity in the model, as shown by the Sraffian Supermultiplier, demand-led growth at the rate at which this component grows is fundamentally (or statically) stable. We also argue that the latter result follows from assumptions about the non-capacity creating expenditures and not from those about the investment function. However, although the adjustment is fundamentally stable, assumptions on the investment function, in particular, that of a gradual or flexible accelerator, are relevant because, if the accelerator effect is too strong (as measured by a high value for the reaction parameter β) the model may nevertheless be dynamically unstable. This does not reduce in our view the relevance of the Sraffian Supermultiplier model, as we do think that for both theoretical and empirical reasons, a flexible accelerator moderate reaction of the investment share to demand is a reasonable assumption.[19]

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[17] It is beyond our purpose here to discuss the Cambridge closure which endogenizes the warranted rate by means of changes in distribution. For a criticism of this closure from the perspective of the Sraffian Supermultiplier see Serrano(1995b) and Serrano & Freitas (2017).

[18] For a detailed analysis of this point in relation to these new Neo-Kaleckian growth models see Fagundes & Freitas (2017).

[19] For empirical evidence supporting the Sraffian Supermultiplier see Girardi & Pariboni (2016) and Avancini, D., Freitas, F. & Braga, J. (2016). More generally, as shown by Hillinger (1992), among many others, there is a lot of evidence in favor of dampened "flexible" accelerator business investment cycles. As for the objections raised by Skott (2016) concerning the plausibility of the stability condition in a context of a calibration exercise, we fully agree with Lavoie's (2016) reply.

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APPENDIX A: On the Fundamental or Static Instability of Growth at Harrod's Warranted Rate

We start from the equations for the determination of the level of output and the rate of growth of investment:

$$Y_t = \frac{I_t}{s} \tag{A.1}$$

and

$$g_{It} = g_{It-1} + \alpha(u_{t-1} - 1) \tag{A.2}$$

with the reaction parameter $\alpha > 0$ in accordance with the capital stock adjustment principle.
Next, we take the growth rate of the capital stock given by:

$$g_{Kt+1} = \frac{h_t}{v} u_t = \frac{s}{v} u_t \tag{A.3}$$

where $h_t = I_t/Y_t$ is the investment share, which is equal to and determined by the exogenous marginal propensity to save s.
The following difference equation gives the dynamics of the actual degree of capacity utilization:

$$u_t = u_{t-1}\left(\frac{1+g_t}{1+g_{Kt}}\right) \tag{A.4}$$

Since the marginal propensity to save is an exogenous variable, it follows from equation (A.1) that the rate of growth of output is equal to the growth rate of investment:

$$g_t = g_{It} \tag{A.5}$$

Therefore, from the equations above, we obtain the following system:

$$g_t = g_{t-1} + \alpha(u_{t-1} - 1) \tag{A.6}$$

$$u_t = u_{t-1}\left(\frac{1 + g_{t-1} + \alpha(u_{t-1} - 1)}{1 + \left(\frac{s}{v}\right) u_{t-1}}\right) \tag{A.7}$$

In equilibrium, we have $u_t = u_{t-1} = u^*$ and $g_t = g_{t-1} = g^*$. Therefore, the system yields:

$$u^* = 1 \tag{A.8}$$

$$g^* = g_I^* = g_K^* = \frac{s}{v} \tag{A.9}$$

Thus, along the equilibrium path, we have normal capacity utilization and growth at Harrod's warranted rate, which is a supply (capacity) constrained growth rate.
We shall now investigate the stability of the Harrodian equilibrium. Evaluated at the equilibrium point, the Jacobian matrix is:

$$\mathbf{J}^* = \begin{bmatrix} \frac{\partial g_t}{\partial g_{t-1}}(g^*, u^*) & \frac{\partial g_t}{\partial u_{t-1}}(g^*, u^*) \\ \frac{\partial u_t}{\partial g_{t-1}}(g^*, u^*) & \frac{\partial u_t}{\partial u_{t-1}}(g^*, u^*) \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ \frac{1}{1+\left(\frac{s}{v}\right)} & \frac{1+\alpha}{1+\left(\frac{s}{v}\right)} \end{bmatrix} \tag{A.10}$$

Its trace and determinant are:

$$\mathbf{Tr}(\mathbf{J}^*) = 1 + \frac{1+\alpha}{1+\left(\frac{s}{v}\right)} \tag{A.11}$$

$$\mathbf{Det}(\mathbf{J}^*) = \frac{1}{1+\left(\frac{s}{v}\right)} \tag{A.12}$$

The stability conditions are the following:

$$1 - \mathbf{Det}(\mathbf{J}^*) > 0$$

$$1 - \mathbf{Tr}(\mathbf{J}^*) + \mathbf{Det}(\mathbf{J}^*) > 0$$

$$1 + \mathbf{Tr}(\mathbf{J}^*) + \mathbf{Det}(\mathbf{J}^*) > 0$$

From the first condition, we obtain:

$$\frac{s}{v} > 0 \tag{A.13}$$

since the variables involved in Harrod's warranted rate have positive values. Hence, the first condition above is satisfied.
Next, from the third condition we have:

$$2 + \frac{1+\alpha}{1+\left(\frac{s}{v}\right)} + \frac{1}{1+\left(\frac{s}{v}\right)} > 0 \tag{A.14}$$

which is also satisfied since the three terms on the left-hand side of the above inequality have a positive value.
The second condition is *not* satisfied. To see why, from the condition under analysis we obtain:

$$\alpha < 0 \tag{A.15}$$

However, according to the capital stock adjustment principle, the value of α is clearly positive. Therefore, the positive sign of the α parameter is a sufficient condition for the instability of the Harrodian equilibrium. Moreover, the latter equilibrium is unstable in a strong sense, since the instability depends only on the sign of the reaction parameter and not on its magnitude. In this sense, the Harrodian equilibrium is indeed characterized by a fundamental, or static (in Hicksian terms) instability.

APPENDIX B: On the dynamic stability of a discrete time Sraffian Supermultiplier

The basic equations of the Sraffian Supermultiplier model in terms of discrete time are the following:

$$Y_t = \frac{Z_t}{s - vg_t^e} \tag{B.1}$$

$$I_t = vg_t^e Y_t \tag{B.2}$$

$$g_t^e = \beta g_{t-1} + (1-\beta) g_{t-1}^e \tag{B.3}$$

$$g_{Kt+1} = u_t g_t^e \tag{B.4}$$

$$u_t = \left(\frac{1+g_t}{1+g_{Kt}}\right)u_{t-1} \tag{B.5}$$

Based on the equations above, we can obtain a system of difference equations in g and g^e:

$$g_t = z + \frac{v(1+z)\beta(g_{t-1} - g_{t-1}^e)}{s - vg_t^e} \tag{B.6}$$

$$g_t^e = \beta g_{t-1} + (1-\beta)g_{t-1}^e \tag{B.7}$$

The equilibrium is given by $g_t = g_{t-1} = g^*$ and $g_t^e = g_{t-1}^e = g^e$, which from equations (B.6) and (B.7) implies that in equilibrium we have that the rate growth of autonomous consumption determines the equilibrium rate of growth of output and expected rate of of growth of demand (i.e., $g^e = g^* = z$). Further, from equation (B.5), in order to obtain a stationary value for the degree of capacity utilization (i.e., for $u_t = u_{t-1} = u^*$) we need the rate of growth of output to be equal to the rate of growth of the capital stock (i.e. $g^* = g_K^*$). Thus, the rate of growth of autonomous consumption also determines the rate of growth of the capital stock and productive capacity. As a result, we obtain:

$$g^e = g_K^* = g^* = z \tag{B.8}$$

Therefore, contrary to what happens in the Harrodian growth model, the Supermultiplier model exhibits a demand (consumption) led pattern of economic growth.

Finally, from equations (B.4) and (B.8) we can determine the equilibrium value of the degree of capacity utilization as:

$$u^* = 1 \tag{B.9}$$

The latter result shows that in equilibrium capacity fully adjusts to demand at the planned or normal degree of capacity utilization.

As for the analysis of the stability of the equilibrium, the Jacobian matrix of the dynamic system evaluated at the equilibrium point is:

$$\mathbf{J}^* = \begin{bmatrix} \frac{\partial g_t}{\partial g_{t-1}}(g^*, g^e) & \frac{\partial g_t}{\partial g_{t-1}^e}(g^*, g^e) \\ \frac{\partial g_t^e}{\partial g_{t-1}}(g^*, g^e) & \frac{\partial g_t^e}{\partial g_{t-1}^e}(g^*, g^e) \end{bmatrix} = \begin{bmatrix} \frac{v(1+z)\beta}{s-vz} & -\frac{v(1+z)\beta}{s-vz} \\ \beta & 1-\beta \end{bmatrix} \tag{B.10}$$

From (B.10), we can obtain the values of the trace and determinant of the Jacobian:

$$\mathbf{Tr}(\mathbf{J}^*) = \frac{v(1+z)\beta}{s-vz} + 1 - \beta \tag{B.11}$$

$$\mathbf{Det}(\mathbf{J}^*) = \frac{v(1+z)\beta}{s-vz} \tag{B.12}$$

Again, the stability conditions involving the trace and determinant of the Jacobian matrix are the following: [20]

[20] Note that these conditions are different from the trace and determinant conditions used so far in the analysis of the stability of the steady-state of Supermultiplier growth models (cf., Freitas & Serrano, 2015; Allain, 2015; Dutt, 2015; and Pariboni, 2015). The reason is that, instead of the continuous time, we are here in a discrete time setting. In a continuous time analysis, stability requires that the real characteristic roots are negative and that the complex roots have negative real parts, a condition on the *sign* of the eigenvalues of the Jacobian matrix of the dynamical system. On the other hand, in discrete time dynamical systems, the requirement is for characteristic roots with absolute values (moduli) smaller than one, a restriction on the *magnitude* of the eigenvalues of Jacobian matrix. The three inequalities below are derived from the Schur stability conditions in the case of a 2X2 system and guarantee that the (real or complex) eigenvalues obey the required conditions. In this respect, see Gandolfo (1997, pp. 58-59), Sydsaeter *et alli* (2005, pp. 404-405 and pp. 410-411), and Galor (2007, pp. 96-105).

$$1 - \mathbf{Det}(\mathbf{J}^*) > 0$$

$$1 - \mathbf{Tr}(\mathbf{J}^*) + \mathbf{Det}(\mathbf{J}^*) > 0$$

$$1 + \mathbf{Tr}(\mathbf{J}^*) + \mathbf{Det}(\mathbf{J}^*) > 0$$

From the last condition, we have that:

$$2 - \beta + \frac{v(1+z)\beta}{s - vz} + \frac{v(1+z)\beta}{s - vz} > 0 \quad \text{(B.13)}$$

Since from the assumptions of the Sraffian Supermultiplier model we have $0 < \beta < 1$ and $v, z, s > 0$, thus inequality (B.13) holds if $s - vz > 0$ or:

$$z < \frac{s}{v} \quad \text{(B.14).}$$

Inequality (B.14) shows that a *necessary* (but not sufficient) condition for the stability of the equilibrium of the model is that the rate of growth of autonomous consumption must be lower than the Harrodian warranted rate of growth and the latter is an upper-limit to a demand-led growth process with normal utilization of productive capacity.

Next, from the *second* condition we obtain:

$$\beta > 0 \quad \text{(B.15).}$$

This condition is met since, as we already pointed out, the adjustment parameter β assumes values within the interval $0 < \beta < 1$.

Finally, from the *first* stability condition above, we obtain:

$$vz + v\beta + v\beta z < s \quad \text{(B.16),}$$

or

$$z < \left(\frac{s}{v} - \beta\right)\frac{1}{1+\beta} \quad \text{(B.17).}$$

Inequalities (B.16) and (B.17) correspond, respectively, to inequalities (19) and (20) in the text. They represent, in two alternative ways, the *sufficient* condition for *dynamic* stability of the equilibrium of the Supermultiplier model that we analyze here. Inequality (B.16) represents the stability condition in the form of a generalized Keynesian stability condition that says that the equilibrium is stable whenever the disequilibrium marginal propensity to invest is smaller than the marginal propensity to save. On the other hand, inequality (B.17) says that a stable demand-led growth path is possible whenever the rate of growth of autonomous consumption is below a maximum rate of growth expressed by the term on the right-hand side of the inequality. Note also that, for positive values of β, we have the following set of inequalities that represent the necessary and sufficient conditions for the stability of the equilibrium:

$$z < \left(\frac{s}{v} - \beta\right)\frac{1}{1+\beta} < \frac{s}{v} \quad \text{(B.18).}$$