

On the Evolution of Income Elasticities of Imports and Thirlwall's Law: A Bayesian Analysis for Latin America (1970 – 2015)

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Abstract

This paper analyzes the evolution of income elasticities of imports and verifies the validity of Thirlwall's Law for eight Latin American countries over the period 1970-2015. Given the econometric issues inherent to classical time-varying parameter regressions, a Bayesian estimation procedure is implemented in order to provide more robust parameter estimates. A stochastic volatility specification is also included to take into account the potential presence of conditional heteroskedasticity. While some Latin American economies showed a rather stable behavior, the income elasticities of imports for Brazil, Colombia and Mexico presented an upward trend from 1970 to 2015. Regarding the validity of Thirlwall's Law, there were evidences of the balance-of-payments (BOP) posing a restriction to the growth performance in six of the eight selected Latin American countries. Estimates for the theoretical rate of economic growth with BOP equilibrium supported the idea of an overall catching-up trajectory for Latin America in the period.

Keywords: Import Elasticity; Thirlwall's Law; Balance-of-Payments-Constrained Growth; Latin America; Bayesian Estimation; Time-Varying Parameter

JEL Classification: C11; E12; F43; O47; O54

Resumo

O objetivo do presente estudo é analisar a evolução das elasticidades-renda das importações e verificar a validade da lei de Thirlwall para oito países da América Latina durante o período de 1970-2015. Dada as questões econométricas inerentes às regressões com parâmetros tempo-variante, utiliza-se métodos bayesianos a fim de fornecer estimativas mais robustas. A fim de levar em consideração a presença de heterocedasticidade condicional, é incluída a especificação com volatilidade estocástica no modelo. Enquanto algumas economias da América Latina apresentaram um comportamento estável em relação às elasticidades-renda das importações no período estudado, o Brasil, Colômbia e México mostraram uma tendência crescente. Em relação à validade da lei de Thirlwall, há evidências de que o balanço de pagamentos (BOP) coloca uma restrição à performance de crescimento em seis das oito economias selecionadas. As estimativas teóricas da taxa de crescimento com equilíbrio no balanço de pagamentos corrobora a ideia uma trajetória geral de convergência para a América Latina no período.

Palavras-chave: Elasticidade de Importação; Lei de Thirlwall; Crescimento Restrito pelo Balanço de Pagamentos; América Latina; Estimação Bayesiana; Parâmetros Tempo-Variantes

Classificação JEL: C11; E12; F43; O47; O54

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1 Introduction

Latin America has been the stage of both prosperity and recession periods. From its economic collapse in the 1980s and, consequently, the later commitment to Neoliberal macroeconomic reforms, the extensive historical disequilibria in the balance-of-payments (BOP) faced by many developing countries have reinforced the need of evaluating the argument of whether international trade is *de facto* a constraint to economic performance.

Since the seminal paper by Thirlwall (1979), which introduced the BOP-constrained growth framework, there has been a vast debate on its potential empirical applicability to economies throughout the world. While growth models of Neoclassical tradition rely on supply side issues such as factor accumulation, productivity increments or technological progress as driving forces of economic growth (Solow, 1956, 1957; Swan, 1956; Romer, 1986; Lucas, 1988; Mankiw et al., 1992; Aghion and Howitt, 1992), the former Post Keynesian approach postulates that output growth is demand-determined, with foreign exchange dynamics being crucial on explaining its sustainability. By embodying the assumption of trade deficits not being automatically corrected via adjustment of the real exchange rate, the presence of continuously increasing current account deficits would then hamper the growth of domestic output given the lack of also ever-growing financing sources. Hence, the insufficiency of foreign currency from trade imbalances would then set an upper limit to the growth rate of aggregate demand and, thus, to economic growth (Thirlwall and Hussain, 1982; McCombie, 1997).

Moreover, one should notice that the BOP-constrained growth approach has been steadily criticized by disregarding the biases and inconsistent parameter estimates due to structural breaks. Since the study of Bairam (1993), cointegration analysis has been the predominant econometric method adopted by the recent literature to evaluate the potential long-run relationship between economic growth and the BOP dynamics. However, given that this latter procedure requires a large sample size and that macroeconomic relations are likely to change over time, structural break problems have compromised the reliability of cointegration-based evidences (Bagnai, 2010).

Despite some recent attempts on dealing with these issues (Hieke, 1997; Atesoglu, 1997; Bairam and Ng, 2001; Bértola et al., 2002; Holland et al., 2004; Pacheco-López and Thirlwall, 2006; Cimoli et al., 2010; Bagnai, 2010), the Post Keynesian empirical literature has yet to tackle parametric instability considering the whole sample period. According to McCombie (1997), the parameters associated to trade equations should evolve slowly over time. In general, classical recursive and rolling regression methods often incur statistical issues such as *ad hoc* window selection, loss of inference efficacy (mainly due to degrees of freedom restrictions) and biased parameter estimates. Therefore, the application of nonlinear Bayesian estimation methods is an appealing alternative. By amending the univariate regression framework to allow for parameters following a first-order random walk process, it is possible to capture smooth parametric shifts, thus being a more suitable procedure for describing time-varying economic dynamics.

This paper focuses on the evolution of income elasticities of imports in Latin America between 1970 and 2015 and verifies whether the BOP was indeed a constraint to the region's economic growth. To this end, we first estimate time-varying parameter (TVP) univariate regressions with stochastic volatility for the Latin economies' import functions, following closely the methodology procedure proposed by Nakajima (2011). The time-varying parameter specification provides time-varying income elasticity of imports estimates while the stochastic volatility takes into account the potential conditional heteroskedasticity that might arise from potential stochastic shocks over the volatility of the considered macroeconomic aggregates (Hamilton, 2010). Based on these elasticity estimates, we then check the validity of Thirlwall's Law for the region.

As to the contribution to the debate, to the best of our knowledge, this paper is the first application of a Bayesian approach to deal with potential parametric instability while estimating the income elasticities of imports in Latin America. This would allow for more robust estimates and a better description of these elasticities' movements over time. In fact, the results show that the volatility of imports has, in general, decreased from 1970 to 2015 in Latin America. Also, from the estimates for the time-varying income elasticities of imports, it is possible to divide the Latin American economies into two different groups, with the first one presenting a rather stable behavior over time, while the second one showed a clear upward trend. This increase is a repercussion of the region's commitment to Neoliberal trade reforms. Regarding the validity of Thirlwall's Law, the results were inconclusive only for Colombia and Peru. Last, but not least, the BOP-constrained growth rates for the selected Latin American economies provided evidence of a catching-up trajectory relative to the world economy. However, when considering the period 2000-2015, the

results showed that only Bolivia and Brazil sustained such trajectory.

Besides this introduction and the concluding remarks, the paper is organized in four sections. First, Section 2 focuses on a historical overview of the trade developments in Latin America. Section 3 introduces the Post Keynesian theory of BOP-constrained growth. Section 4 describes the model structure and the estimation procedure of the TVP regression framework. Last, but not least, Section 5 presents the estimates for the time-varying income elasticities of imports, discussing their repercussions for the validity of Thirlwall’s Law in its seminal version and whether the selected economies were on a catching-up or falling behind trajectory in the period.

2 Trade Developments and Economic Growth in Latin America

From the 1950s, the development strategy in Latin America, mainly based on the adoption of import substitution policies, was responsible for reducing the openness and efficiency of the regional economies, increasing their external vulnerability, due to higher dependency on a narrow range of export products, and consequently hampering their ability to absorb external shocks (Rajagopal, 2005). Following the onset of the the oil crises in the 1970s, which considerably affected the developing economies that relied on the product, the external indebtedness faced in the 1980s produced large disparities of GDP per capita (Bleaney, 1999). In particular, the subsequent emergence of trade imbalances was a crucial element in deepening the economic growth slowdown and in the later commitment to Neoliberal trade reforms.

Recently, empirical research has provided evidences of a direct correlation between the Latin America’s external vulnerability and its rather volatile economic growth (Garret and Lange, 1986; Guillaumont, 2001; Cordina, 2004; Loayza and Raddatz, 2007; Briguglio et al., 2008; Seth and Ragab, 2012). Indeed, as shown in Table 1, the period 1974-1986 presented a pronounced degree of GDP growth volatility in comparison to the period 1986-2007. In addition, the increase on the coefficient of variation in the last decade could be explained by the adverse effects of the 2008 financial crisis. Hence, these results suggest that the stability of Latin American growth rates has been intrinsically related to external financing fluctuations and commodity price cycles.

Despite the existence of distinct patterns, the overall Latin American trade volume has presented an upward trend since the mid 1970s, with an average growth rate of nearly 4.8% per year (Table 1). More specifically, the percentage of trade growth in the Latin American economies slightly increased from an average of 5% during 1974-1986 to an average rate of 5.6% during 1986-2007. On the other hand, the coefficient of variation declined sharply, posting a contraction of 2.28 points (from 2.88 to 0.60). Yet, in regard to the last decade, Latin American trade grew at a slower pace. With less than half the average rate observed historically – roughly 2% per year –, the growth of trade followed the economic downturn in the aftermath of recent global crisis. According to the International Monetary Fund (2016), the recent slowdown in global economic activity, mostly in investment, is responsible for up to three-fourths of the trade deceleration. Furthermore, the end of the 1986-2007 boom also exposed many primary-exporting countries in the region to sizable terms of trade shocks, with the correction in commodity prices resulting in an acute decrease in the value of Latin America’s trade.

Table 1: Latin America GDP and Trade – Average Growth and Volatility (1970 - 2015)

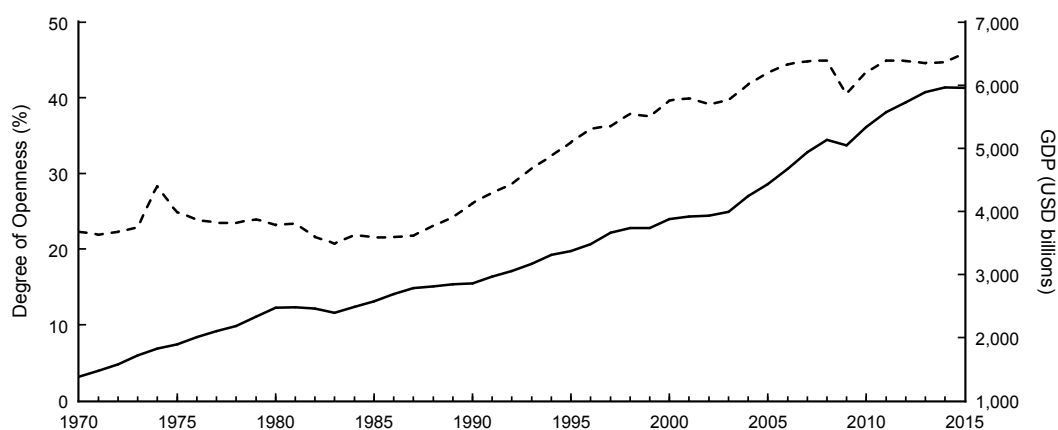
Period	GDP			Trade		
	Growth	Std. Dev.	C.V.	Growth	Std. Dev.	C.V.
1974 - 1986	3.58%	2.94	0.82	5.00%	14.37	2.88
1986 - 2007	3.02%	1.89	0.63	5.63%	3.36	0.60
2007 - 2015	2.77%	2.57	0.93	2.05%	4.74	2.31
1974 - 2015	3.04%	2.35	0.77	4.77%	4.26	0.89

Notes: The terms “Std. Dev.” and “C.V.” refer to the standard deviation and the coefficient of variation, respectively. Trade is defined as the region’s export of goods and services (at constant 2010 prices, in USD). Source: Compiled by the author based on data extracted from the World Development Indicators (WDI) database (World Bank, 2017).

In terms of development strategy, the shift from the import substitution paradigm toward the adoption

of the export-led growth framework in the late 1970s placed trade integration as the cornerstone of the Latin American industrialization process. Trade had become the catalyst of economic growth. The structural adjustment under the neoclassical prescription essentially relied on liberalization reforms, financial deregulation and dismantling of state intervention. While Latin America benefited from increased trade openness in the mid 1980s and early 1990s, its level remained rather stable thereafter. For instance, the region’s degree of openness increased from 21% in 1983 to 45% in 2007 (Figure 1). This result suggests that trade reforms implemented in Latin America broadened the integration of the region in the international market. Also, from visual inspection of Figure 1, one can conclude that the degree of openness seems to have mirrored the behavior of Latin America’s GDP, particularly after the early 1980s. However, in comparison to other emerging and developed economies, the relatively low average trade integration in Latin America could be partially explained by unrelated trade factors, such as geography, high transport costs and the weak quality of institutions (De Ferranti et al., 2003).

Figure 1: Degree of Openness and GDP – Latin America (1970-2015)



Notes: The Degree of Openness corresponds to the ratio of exports plus imports to GDP. The Latin America imports and exports of goods and services as well as GDP are at constant 2010 prices, in USD. Data series were extracted from the WDI database (World Bank, 2017). Degree of Openness (dashed line) and GDP (solid line).

Even though several theoretical and empirical studies have propelled the positive impacts of openness on economic performance (Dollar, 1992; Ben-David, 1993; Krugman, 1994; Sachs et al., 1995; Edwards, 1998; Frankel and Romer, 1999; Lucas, 2009; Kim et al., 2012), the debate has yet to be settled. From a structural Keynesian viewpoint, the export-led development would be ineffective in generating stable economic linkages and promoting genuine prosperity (Palley, 2002, 2004). Moreover, the restructuring of the productive system toward an export-oriented industrialization would be more subject to adverse external shocks in contrast to development strategies reliant on domestic demand (Foxley, 2009). Therefore, despite the upward path of Latin America’s trade performance engendered by its fairly commitment to the export-led growth paradigm, the resulting increase in its external vulnerability poses an additional predicament to the region. Consequently, in light of this pronounced sensitivity to international conditions, the recent end of the high commodity prices cycle *pari passu* the Chinese economic slowdown are likely to restrain even further its already stagnant economic performance. Ergo, as discussed by Ocampo (2013), rethinking Latin America’s development patterns and strategies is of major importance in order to promote social and economic welfare.

3 The Balance-of-Payments-Constrained Growth Model Revisited

Unless a country can finance ever-growing current account deficits, the economic performance of a given economy cannot grow faster than the rate consistent with BOP equilibrium (Thirlwall, 1979). In the presence of recurrent trade imbalances and the relative scarcity of financing sources, the resulting insufficiency of foreign currency would then set an upper limit to the growth rate of aggregate demand and, thus, to economic growth.

Let the current account equilibrium condition be given by

$$P_d X_t = P_f M_t E_t \quad (1)$$

where P_d is the domestic price of exports; X is exports; P_f is the foreign price of imports; M is imports; and E is the nominal exchange rate measured as the domestic price of foreign currency. The export and import demand functions are defined as

$$X_t = a \left(\frac{P_d}{P_f E_t} \right)^\delta Z_t^\lambda \quad (2)$$

and

$$M_t = b \left(\frac{P_f E_t}{P_d} \right)^\psi Y_t^\pi \quad (3)$$

where a and b are constants; Z is world income; Y is domestic income; δ (< 0) is the price elasticity of exports; λ (> 0) is the income elasticity of exports; ψ (< 0) is the price elasticity of imports; and π (> 0) is the income elasticity of imports. Note that this seminal model structure formalized by [Thirlwall \(1979\)](#) specified elasticities as constant quantities over time.

After transforming equations 1 to 3 into their respective growth rates and solving the subsequent system of linear equations for the domestic income growth rate (y), it is possible to define the economic growth compatible with BOP equilibrium (y_B), that is,

$$y_B = \frac{(1 + \delta + \psi)(p_d - p_f - e) + \lambda z}{\pi} \quad (4)$$

with lower-case letters corresponding to the growth rates of the variables. The time subscripts were dropped for notational convenience. From the relation depicted by equation 4, an improvement in the real terms of trade, *ceteris paribus*, would increase the growth compatible with BOP equilibrium if the sum of the price elasticities of exports and imports is less than -1 . Yet, in the presence of the Marshall-Lerner condition, $\delta + \psi > -1$, the deterioration in competitiveness due to an improvement in the real terms of trade would then decrease the growth compatible with BOP equilibrium. Also, only persistent depreciations of the exchange rate (i.e. $e > 0$) would lead to a permanently higher growth trajectory for a given country. Last, but not least, there exists an inverse relation between the income elasticity of imports and the growth compatible with BOP equilibrium.

Following [McCombie and Thirlwall \(1994\)](#), if relative prices in international trade are neutral in the long-run (i.e. $p_d - p_f - e = 0$), thus not presenting significant effects on the growth compatible with BOP equilibrium path of a given country, and if capital flows cannot finance current account deficits, equation 4 reduces to

$$y_t^* = \frac{\lambda}{\pi} z_t = \frac{x_t}{\pi} \quad (5)$$

This last equation has come to be known as Thirlwall's Law. However, as discussed by [Thirlwall \(2011\)](#) himself, this relationship and its implications were built on the previous theoretical macroeconomic developments of Harrod's foreign trade multiplier ([Harrod, 1933](#)), Hicks' super-multiplier ([Hicks, 1950](#)) and the Prebisch-Singer hypothesis ([Prebisch, 1959, 1962](#); [Singer, 1950](#)).

Several empirical studies have been conducted in order to validate the applicability of Thirlwall's Law for both developed and emerging economies ([Thirlwall and Hussain, 1982](#); [Bairam, 1988](#); [Atesoglu, 1993, 1997](#); [McCombie, 1997](#); [Hieke, 1997](#); [Alonso, 1999](#); [Moreno-Brid, 1998, 1999](#); [Bértola et al., 2002](#); [Razmi, 2005](#); [Kvedaras, 2005](#); [Garcimartín et al., 2008](#); [Britto and McCombie, 2009](#); [Felipe et al., 2010](#); [Bagnai, 2010](#)). In general, the obtained results have confirmed the validity of the BOP-constrained growth framework.

For the specific case of Latin America, [López and Cruz \(2000\)](#) incorporated the real exchange rate in the model in order to analyze its effect on the long-term economic growth of Argentina, Brazil, Colombia

and Mexico. Their results supported the idea that, in the presence of the Marshall-Lerner condition, faster rate of output growth could be achieved by adopting competitive exchange rate levels.

Additionally, with a sample of ten Latin American economies, [Holland et al. \(2004\)](#) tested the adherence of the predicted growth rate compatible with BOP equilibrium to the actual long-term growth rates. Despite the presence of some deviation, the results corroborated the argument of the BOP posing a restriction to the region's long-term economic growth.

[Pacheco-López and Thirlwall \(2006\)](#) investigated whether the adoption of trade liberalization by the Latin American economies had increased their income elasticity of imports. The authors implemented rolling regression models in order to capture a potential upward trend over time. Besides proving the validity of Thirlwall's Law, their results also confirmed this trend for the country sample.

Similarly, using rolling regressions with panel data for a broad sample of developed and emerging economies, [Cimoli et al. \(2010\)](#) also showed that the Latin American income elasticities of imports have increased since 1970. Along with those results, the authors provided further evidences that the decrease in the income gap of some developing countries was intrinsically related to the transformation of their economic structures.

Moreover, based on a multi-sectoral perspective, [Gouvea and Lima \(2010\)](#) evaluated the implications of changes in the sectoral composition of exports and imports on the external constraint for a sample of Latin American and Asian countries. Both the original and the multisectoral versions of Thirlwall's law were validated for Latin America.

4 The Time-Varying Parameter (TVP) Regression Model

A basic TVP regression model can be defined as

$$y_t = x_t' \beta + z_t' \alpha_t + \epsilon_t \quad (6)$$

with $\epsilon_t \sim N(0, \sigma_t^2)$ and $t = 1, \dots, n$. Note that y_t is a scalar of response; x_t and z_t are $(k \times 1)$ and $(p \times 1)$ vectors of covariates, respectively; β is a $(k \times 1)$ vector of time-invariant coefficients; and α_t is a $(p \times 1)$ vector of time-varying coefficients. In this paper, the regression model is assumed to have only time-varying coefficients (i.e. $x_t' \beta \equiv 0$).

As to the evolution process of α_t , the parameters are assumed to follow a driftless first-order random walk specification, given by

$$\alpha_{t+1} = \alpha_t + u_t \quad (7)$$

with $u_t \sim N(0, \Sigma)$ and $t = 1, \dots, n - 1$. We assume that $\alpha_0 = 0$ and $u_0 \sim N(0, \Sigma_0)$. This specification, in contrast to Markov-switching models, allows for both temporary and permanent shifts instead of discrete breaks, thus being a more suitable characterization of potential changes in private sector behavior or the learning dynamics of economic agents. More specifically, the time-varying coefficient is imposed as to capture a possible nonlinearity, such as a gradual change or a structural break ([Primiceri, 2005](#); [Nakajima, 2011](#)).

The stochastic volatility specification is included by imposing a log-volatility process for the time-varying variance σ_t^2 , that is,

$$\sigma_t^2 = \gamma \exp(h_t) \quad (8)$$

with $h_{t+1} = \phi h_t + \eta_t$; $\eta_t \sim N(0, \sigma_\eta^2)$; and $t = 1, \dots, n - 1$. Note that h_t is the stochastic volatility. Moreover, we assume that $\gamma > 0$; $|\phi| < 0$; $h_0 = 0$; and the initial condition is set based on the stationary distribution as $\eta_0 \sim N(0, \sigma_\eta^2 / (1 - \phi^2))$.

Consider α_t and h_t as state variables. Then, the TVP regression model consists of a nonlinear non-Gaussian state space representation. [Primiceri \(2005\)](#) argues that the Maximum Likelihood (ML) estimation cannot provide reliable estimates for this class of models due to intractability of the likelihood function. Therefore, the estimation procedure is mainly based on a Bayesian approach using the Markov Chain Monte Carlo (MCMC) method for the numerical evaluation of the posterior of the parameters.

By splitting up the original problem into a number of smaller steps, the Bayesian inference is able to deal with high-dimensional parameter space and potential nonlinearities in the likelihood function. Under the assumption of a certain prior probability density, $p(\theta)$, the MCMC algorithm is able to generate the joint posterior distribution of the parameters, $p(\theta|y)$, given by

$$p(\theta|y) = \frac{p(\theta)L(y|\theta)}{\int_{\theta} p(\theta)L(y|\theta)d\theta} \propto p(\theta)L(y|\theta) \quad (9)$$

where $y = \{y_t\}_{t=1}^n$ is the data; $m(y) = \int_{\theta} p(\theta)L(y|\theta)d\theta$ is the marginal distribution (also known as the normalizing constant); and $L(y|\theta)$ is the likelihood function. In other words, the Bayesian inference consists of updating the prior information on θ by observing y . However, since the posterior distribution usually does not have a closed form due to the intractability of the likelihood function and to the impossibility of an analytical assessment for the marginal distribution, the MCMC sampling methods are fairly robust algorithms capable of recursively sampling from the posterior distribution without computing them (Nakajima, 2011).

Regarding the TVP regression model, after defining the prior density $p(\theta)$, one can obtain the joint posterior distribution $p(\theta, \alpha, h|y)$, that is,

$$\begin{aligned} p(\theta, \alpha, h|y) &\propto p(\theta) \times \prod_{t=1}^n \frac{1}{\sqrt{2\pi\gamma}e^{h_t/2}} \exp\left\{-\frac{(y_t - x_t'\beta - z_t'\alpha_t)^2}{2\gamma e^{h_t}}\right\} \times \\ &\times \prod_{t=1}^{n-1} \frac{1}{(2\pi)^{k/2}|\Sigma_0|^{1/2}} \exp\left\{-\frac{1}{2}(\alpha_{t+1} - \alpha_t)'\Sigma^{-1}(\alpha_{t+1} - \alpha_t)\right\} \times \\ &\times \frac{1}{(2\pi)^{k/2}|\Sigma_0|^{1/2}} \exp\left\{-\frac{1}{2}(\alpha_1'\Sigma_0^{-1}\alpha_1)\right\} \times \\ &\times \prod_{t=1}^{n-1} \frac{1}{\sqrt{2\pi}\sigma_{\eta}} \exp\left\{-\frac{(h_{t+1} - \phi h_t)^2}{2\sigma_{\eta}^2}\right\} \times \frac{\sqrt{1-\phi^2}}{\sqrt{2\pi}\sigma_{\eta}} \exp\left\{-\frac{(1-\phi^2)h_1^2}{2\sigma_{\eta}^2}\right\} \end{aligned} \quad (10)$$

In this paper, the implementation of the MCMC algorithm to explore this posterior distribution follows the procedure developed by Nakajima (2011), which is constructed with the following sampling procedures:

• **Sample β** The prior for β is specified as $\beta \sim N(\beta_0, B_0)$, with the conditional posterior density given by

$$\begin{aligned} p(\beta|\gamma, \alpha, h, y) &\propto \exp\left\{-\frac{1}{2}(\beta - \beta_0)'B_0^{-1}(\beta - \beta_0)\right\} \times \left\{-\frac{\sum_{t=1}^n (y_t - x_t'\beta - z_t'\alpha_t)^2}{2\gamma e^{h_t}}\right\} \\ &\propto \exp\left\{-\frac{1}{2}(\beta - \hat{\beta})'\hat{B}^{-1}(\beta - \hat{\beta})\right\} \end{aligned} \quad (11)$$

where

$$\hat{B} = \left(B_0^{-1} + \sum_{t=1}^n \frac{x_t x_t'}{\gamma e^{h_t}}\right)^{-1}, \quad \hat{\beta} = \hat{B} \left(B_0^{-1}\beta_0 + \sum_{t=1}^n \frac{x_t \hat{y}_t}{\gamma e^{h_t}}\right),$$

and $\hat{y}_t = y_t - z_t'\alpha_t$, for $t = 1, \dots, n$. As shown by Nakajima (2011), note that the conditional posterior density is proportional to the kernel of the normal distribution whose mean and variance are $\hat{\beta}$ and \hat{B} , respectively. Finally, a sample is drawn as $\beta | \gamma, \alpha, h, y \sim N(\hat{\beta}, \hat{B})$.

• **Sample α** By considering α as the state variable, the model given by equations (6) and (7) consists of a linear Gaussian state space model. As in Nakajima (2011), α is sampled simultaneously from the

conditional posterior distribution $p(\alpha|\beta, \Sigma, \gamma, h, y)$ through the algorithm of simulation smoother on the state space model

$$\begin{aligned} y_t &= X_t\beta + Z_t\alpha_t + G_tu_t, & t = 1, \dots, n, \\ \alpha_{t+1} &= T_t\alpha_t + H_tu_t, & t = 0, \dots, n-1 \end{aligned} \quad (12)$$

where $\alpha_0 = 0$, $u_t \sim N(0, I)$ and $G_tH_t' = 0$. Then, the simulation smoother algorithm mainly draws $\eta = (\eta_0, \dots, \eta_n) \sim p(\eta|\omega, y)$, with $\eta_t = H_tu_t$, for $t = 0, \dots, n$, and ω consisting of all model parameters. In order to initialize the Kalman filter approach, we set $a_1 = 0$ and $P_1 = H_0H_0'$. Then, the Kalman filter is recursively run:

$$\begin{aligned} e_t &= y_t - X_t\beta - Z_t a_t, & D_t &= Z_t P_t Z_t' + G_t G_t', & K_t &= T_t P_t Z_t' D_t^{-1}, \\ L_t &= T_t - K_t Z_t, & a_{t+1} &= T_t a_t + K_t e_t, & P_{t+1} &= T_t P_t L_t' + H_t H_t', \end{aligned}$$

for $t = 1, \dots, n$. Subsequently, with $r_n = U_n = 0$ and $\Lambda_t = H_t H_t'$, the simulation smoother algorithm is performed, such that:

$$\begin{aligned} C_t &= \Lambda_t - \Lambda_t U_t \Lambda_t, & \eta_t &= \Lambda_t r_t + \varepsilon_t, & \varepsilon &\sim N(0, C_t), & V_t &= \Lambda_t U_t L_t, \\ r_{t-1} &= Z_t' D_t^{-1} e_t + L_t' r_t - V_t' C_t^{-1} \varepsilon_t, & U_{t-1} &= Z_t' D_t^{-1} Z_t + L_t' U_t L_t + V_t' C_t^{-1} V_t, \end{aligned}$$

for $t = n, n-1, \dots, 1$. Regarding the initial state, η_0 is drawn as $\eta_0 = \Lambda_0 r_0 + \varepsilon_0$, $\varepsilon_0 \sim N(0, C_0)$ with $C_0 = \Lambda_0 - \Lambda_0 U_0 \Lambda_0$. After drawing η , α_t is computed by using the state equation (12), replacing $H_t u_t$ by η_t .

One should note that, for the TVP regression model, we have:

$$X_t\beta = x_t'\beta, \quad Z_t = z_t', \quad G_t = \left(\sqrt{\gamma}e^{h_t/2}, 0_p'\right), T_t = I_p, \quad H_t = \left(0_p, \Sigma^{1/2}\right), \quad H_0 = \left(0_p, \Sigma_0^{1/2}\right)$$

where 0_p is a $(p \times 1)$ zero vector and I_p is a $(p \times p)$ identity matrix.

• **Sample Σ** For the conditional posterior density of Σ , the prior distribution for Σ is specified as $\Sigma \sim IW(\nu_0, \Omega_0^{-1})$, where IW is the inverse-Wishart distribution. The conditional posterior distribution for Σ is given by

$$\begin{aligned} p(\Sigma|\alpha) &\propto |\Sigma|^{-\frac{\nu_0+p+1}{2}} \exp\left\{-\frac{1}{2}\text{tr}(\Omega_0\Sigma^{-1})\right\} \times \\ &\quad \times \prod_{t=1}^{n-1} \frac{1}{|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\alpha_{t+1} - \alpha_t)' \Sigma^{-1}(\alpha_{t+1} - \alpha_t)\right\} \\ &\propto |\Sigma|^{-\frac{\hat{\nu}+p+1}{2}} \exp\left\{-\frac{1}{2}\text{tr}(\hat{\Omega}\Sigma^{-1})\right\} \end{aligned} \quad (13)$$

where $\hat{\nu} = \nu_0 + n - 1$ and $\hat{\Omega} = \Omega_0 + \sum_{t=1}^{n-1} (\alpha_{t+1} - \alpha_t)(\alpha_{t+1} - \alpha_t)'$. Equation (13) forms the kernel of the inverse-Wishart distribution, which depends only on α . Last, but not least, we draw the sample as $\Sigma | \alpha \sim IW(\hat{\nu}, \hat{\Omega}^{-1})$.

• **Sample h** First, note that equations (6) and (8) consist of a nonlinear and non-Gaussian state space model. Following Nakajima (2011), a multi-move sampler algorithm is implemented to draw samples from the exact posterior distribution of the original model. Consider the model as

$$y_t^* = \exp(h_t/2)e_t \quad (14)$$

with $t = 1, \dots, n$ and $y_t^* = (y_t - x_t' \beta - z_t' \alpha_t) / \sqrt{\gamma}$. Also, let the stochastic volatility h follow a driftless random walk process, that is, $h_{t+1} = \phi h_t + \eta_t$, where $t = 0, \dots, n-1$, $h_0 = 0$ and $\eta_0 \sim N(0, \sigma_\eta^2 / (1 - \phi^2))$. Note that the disturbances e_t and η_t are specified as

$$\begin{pmatrix} e_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right)$$

with $t = 1, \dots, n$.

In order to sample a block (h_r, \dots, h_{r+d}) (where $r \geq 1$, $d \geq 1$ and $r + d \leq n$) from its joint posterior density, one should consider the draw of

$$\begin{aligned} (\eta_{r-1}, \dots, \eta_{r+d-1}) &\sim p(\eta_{r-1}, \dots, \eta_{r+d-1} | \omega) \propto \\ &\propto \prod_{t=r}^{r+d} \frac{1}{e^{h_t/2}} \exp\left(-\frac{y_t^{*2}}{2e^{h_t}}\right) \times \prod_{t=r-1}^{r+d-1} f(\eta_t) \times f(h_{r+d}) \end{aligned} \quad (15)$$

where

$$f(\eta_t) = \begin{cases} \exp\left\{-\frac{(1-\phi^2)\eta_0^2}{2\sigma_\eta^2}\right\} & (\text{if } t = 0) \\ \exp\left\{-\frac{\eta_t^2}{2\sigma_\eta^2}\right\} & (\text{if } t \geq 1) \end{cases}$$

and

$$f(h_{r+d}) = \begin{cases} \exp\left\{-\frac{(h_{r+d+1} - \phi h_{r+d})^2}{2\sigma_\eta^2}\right\} & (\text{if } r + d < n) \\ 1 & (\text{if } r + d = n) \end{cases}$$

One should note that $\omega = (h_{r-1}, h_{r+d+1}, \beta, \gamma, \phi, \sigma_\eta, \alpha, y)$. The posterior draw of (h_r, \dots, h_{r+d}) is obtained by running the state equation along with the draw of $(\eta_{r-1}, \dots, \eta_{r+d-1})$ given h_{r-1} .

In order to sample $(\eta_{r-1}, \dots, \eta_{r+d-1})$ from the density (15), [Nakajima \(2011\)](#) advocates in favor of combining the classic accept-reject (AR) sampling¹ and the Metropolis-Hastings (MH) algorithm² with a proposal distribution given by

$$q(\eta_{r-1}, \dots, \eta_{r+d-1} | \omega) \propto \prod_{t=r}^{r+d} \exp\left\{-\frac{(h_t^* - h_t)^2}{2\sigma_t^{*2}}\right\} \times \prod_{t=r-1}^{r+d-1} f(\eta_t) \quad (16)$$

where

¹The objective of the accept-reject (AR) sampling, also known as the von Neumann sampling, is to sample from a target density $\pi(x) = f(x)/K$, with $f(x)$ being an unnormalized target density and K an unknown normalizing constant. However, this target distribution might be impossible to simulate by an inverse transformation. Hence, suppose it is possible to sample from the density $h(x)$ and that there exists a constant c for which $f(x) \leq ch(x)$ for all x . In order to obtain a draw from $\pi(x)$, the AR sampling proceeds to draw a candidate z from h and u from the uniform distribution $U(0, 1)$. If $u \leq f(z)/ch(z)$, z is accepted. Otherwise, the process is repeated.

²Given the impossibility of direct sampling from the posterior distribution, the Metropolis-Hastings (MH) algorithm is a MCMC method implemented in order to simulate samples by combining the use of the full joint density function and a proposal distribution. This proposal distribution is usually chosen as to correspond as close as possible to the target conditional posterior distribution. More specifically, define $q(\theta^* | \theta^{(i)})$ as the probability density function of the proposal given the current point $\theta^{(i)}$ and $\alpha(\theta_0, \theta^*)$ as the acceptance rate from the current point θ_0 to the proposal θ^* , which is given by

$$\alpha(\theta_0, \theta^*) = \min\left\{1, \frac{p(\theta^* | y) q(\theta_0 | \theta^*)}{p(\theta_0 | y) q(\theta^* | \theta_0)}\right\}$$

where $p(\theta | y)$ is the target posterior distribution and θ^* is a candidate from the proposal $q(\theta^* | \theta^{(i)})$. The MH algorithm consists of choosing an arbitrary starting point $\theta^{(0)}$ and setting $i = 0$. Then, it generates a candidate θ^* and evaluates this candidate according to the probability $\alpha(\theta^{(i)}, \theta^*)$. If θ^* is accepted, it sets $\theta^{(i+1)} = \theta^*$; otherwise, it sets $\theta^{(i+1)} = \theta^{(i)}$. Finally, we set $i = i + 1$ and repeat the process. According to [Nakajima \(2011\)](#), this iteration process produces the sample from the target conditional posterior distribution.

$$\sigma_t^{*2} = -\frac{1}{g''(\hat{h}_t)} \quad (17)$$

and

$$h_t^* = \hat{h}_t + \sigma_t^{*2} g'(\hat{h}_t) \quad (18)$$

for $t = r, \dots, r + d - 1$ and $t = r + d$ (when $r + d = n$). One should note that

$$g'(\hat{h}_t) = -\frac{1}{2} + \frac{y_t^{*2}}{2e^{\hat{h}_t}}$$

and

$$g''(\hat{h}_t) = -\frac{y_t^{*2}}{2e^{\hat{h}_t}}$$

which are obtained from a second-order Taylor expansion of

$$g(h_t) \equiv -\frac{h_t}{2} - \frac{y_t^{*2}}{2e^{h_t}}$$

around a certain point \hat{h}_t , namely,

$$\begin{aligned} g(h_t) &\approx g(\hat{h}_t) + g'(\hat{h}_t)(h_t - \hat{h}_t) + \frac{1}{2}g''(\hat{h}_t)(h_t - \hat{h}_t)^2 \\ &\propto \frac{1}{2}g''(\hat{h}_t) \left\{ h_t - \left(\hat{h}_t - \frac{g'(\hat{h}_t)}{g''(\hat{h}_t)} \right) \right\}^2 \end{aligned}$$

For $t = r + d$ (when $r + d < n$), we have

$$\sigma_{r+d}^* = \frac{1}{-g''(\hat{h}_{r+d}) + \phi^2/\sigma_\eta^2} \quad (19)$$

and

$$h_{r+d}^* = \sigma_{r+d}^* \left\{ g'(\hat{h}_{r+d}) - g''(\hat{h}_{r+d})\hat{h}_{r+d} + \phi h_{r+d+1}/\sigma_\eta^2 \right\} \quad (20)$$

[Nakajima \(2011\)](#) argues that this proposal density has been chosen given its direct correspondence to the state space model

$$\begin{aligned} h_t^* &= h_t + \zeta_t, \quad t = r, \dots, r + d \\ h_{t+1} &= \phi h_t + \eta_t, \quad t = r - 1, \dots, r + d - 1 \\ \begin{pmatrix} \zeta_t \\ \eta_t \end{pmatrix} &\sim N \left(0, \begin{pmatrix} \sigma_t^{*2} & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \right), \quad t = r, \dots, r + d \end{aligned} \quad (21)$$

with $\eta_{r-1} \sim N(0, \sigma_\eta^2)$, when $r \geq 2$, and $\eta_0 \sim N(0, \sigma_\eta^2/(1 - \phi^2))$. Given ω , a candidate point of $(\eta_{r-1}, \dots, \eta_{r+d-1})$ for the AR-MH algorithm is drawn by implementing the simulation smoother procedure over the state space model characterized by (21).

Then, one must find $(\hat{h}_r, \dots, \hat{h}_{r+d})$ as near as possible to the mode of the posterior density. This is performed as to guarantee efficiency during the sampling process. For this, [Nakajima \(2011\)](#) proposes the following steps:

1. Initialize $(\hat{h}_r, \dots, \hat{h}_{r+d})$;
2. Compute (h_r^*, \dots, h_{r+d}) and $(\sigma_r^{*2}, \dots, \sigma_{r+d}^{*2})$ by equations (17), (18) and (20);
3. Run the moment smoother using the current (h_r^*, \dots, h_{r+d}) and $(\sigma_r^{*2}, \dots, \sigma_{r+d}^{*2})$ on (21) and obtain $\hat{h}_t^* \equiv E(h_t|\omega)$ for $t = r, \dots, r+d$;
4. Replace $(\hat{h}_t, \dots, \hat{h}_{r+d})$ by $(\hat{h}_r^*, \dots, \hat{h}_{r+d}^*)$;
5. Go back to (2).

These steps are looped several times until $(\hat{h}_t, \dots, \hat{h}_{r+d})$ reach near the mode of the posterior density. In addition, one should note that $E(h_t|\omega)$ is the product in the simulation smoother as $\Lambda_t r_t$ with $\varepsilon_t = 0$. Following Nakajima (2011), (h_1, \dots, h_n) is divided into $K+1$ blocks, that is, $(h_{k_{i-1}+1}, \dots, h_{k_i})$ for $i = 1, \dots, K+1$ with $k_0 = 0$ and $k_{K+1} = n$, then sampling each block recursively. The author also highlights that the determination of (k_1, \dots, k_K) is done by implementing the stochastic knots method, which proposes $k_i = \text{int}[n(i + U_i)/(K+2)]$, for $i = 1, \dots, K$, with U_i being a random sample from the uniform distribution $U(0, 1)$. The (k_1, \dots, k_K) is randomly chosen for each iteration of the MCMC sampling in order to guarantee a flexible draw of (h_1, \dots, h_n) .

• **Sample ϕ** First, the prior distribution for ϕ is defined as $p(\phi)$ and it is assumed that $(\phi + 1)/2 \sim \text{Beta}(\alpha_{\phi_0}, \beta_{\phi_0})$ in order to satisfy the restriction $|\phi| < 1$. Hence, the conditional posterior distribution of ϕ is

$$\begin{aligned}
p(\phi|\sigma_\eta, h) &\propto p(\phi) \times \sqrt{1-\phi^2} \exp\left\{-\frac{(1-\phi^2)h_1^2}{2\sigma_\eta^2}\right\} \times \exp\left\{-\frac{\sum_{t=1}^{n-1}(h_{t+1}-\phi h_t)^2}{2\sigma_\eta^2}\right\} \\
&\propto p(\phi) \times \sqrt{1-\phi^2} \times \exp\left\{-\frac{\sum_{t=2}^{n-1}h_t^2}{2\sigma_\eta^2}\left(\phi - \frac{\sum_{t=1}^{n-1}h_t h_{t+1}}{\sum_{t=2}^{n-1}h_t^2}\right)^2\right\}
\end{aligned} \tag{22}$$

As discussed by Nakajima (2011), by omitting the term $p(\phi) \times \sqrt{1-\phi^2}$, the posterior distribution in (22) would then correspond to a kernel of the normal distribution, which allows the implementation of the MH algorithm.

Regarding the TVP regression model, in order to sample ϕ , a candidate sample is drawn as $\phi^* \sim TN_{[-1,1]}(\mu_\phi, \sigma_\phi^2)$, where TN denotes the truncated normal distribution over the domain $-1 < \phi < 1$, $\mu_\phi = \sum_{t=1}^{n-1} h_t h_{t+1} / \sum_{t=2}^{n-1} h_t^2$ and $\sigma_\phi^2 = \sigma_\eta^2 / \sum_{t=2}^{n-1} h_t^2$. Note that this proposal density is similar to the target conditional posterior distribution and is also truncated for the same domain of the target. As to the acceptance rate for the candidate ϕ^* from the current point ϕ_0 , namely $\alpha(\phi_0, \phi^*)$, it is given by

$$\alpha(\phi_0, \phi^*) = \min\left\{1, \frac{p(\phi^*|\sigma_\eta, h)q(\phi_0)}{p(\phi_0|\sigma_\eta, h)q(\phi^*)}\right\} = \min\left\{1, \frac{p(\phi^*)\sqrt{1-\phi^{*2}}}{p(\phi_0)\sqrt{1-\phi_0^2}}\right\} \tag{23}$$

where $q(\phi)$ is the probability density function of the proposal and ϕ_0 is the old sample (current point) drawn in the previous iteration. One should note that the acceptance rate corresponds to the ratio of the omitted terms from the conditional posterior distribution of ϕ . As to the acceptance step, the candidate ϕ^* is accepted if $u < \alpha(\phi_0, \phi^*)$, where u is a random number drawn from the uniform distribution $U(0, 1)$.

• **Sample σ_η** The prior distribution for σ_η is set as $\sigma_\eta^2 \sim IG(\nu_0/2, V_0/2)$, with IG referring to the inverse gamma distribution. Hence, the conditional posterior distribution of σ_η is

$$\begin{aligned}
p(\sigma_\eta|\phi, h) &\propto \sigma_\eta^{-(\frac{\nu_0}{2}+1)} \exp\left\{-\frac{V_0}{2\sigma_\eta}\right\} \times \\
&\quad \times \frac{1}{\sigma_\eta} \exp\left\{-\frac{(1-\phi^2)h_1^2}{2\sigma_\eta^2}\right\} \times \prod_{t=1}^{n-1} \frac{1}{\sigma_\eta} \exp\left\{-\frac{(h_{t+1}-\phi h_t)^2}{2\sigma_\eta^2}\right\} \\
&\propto \sigma_\eta^{-(\frac{\nu_0}{2}+1)} \exp\left\{-\frac{V_0 + (1-\phi^2)h_1^2 + \sum_{t=1}^{n-1}(h_{t+1}-\phi h_t)^2}{2\sigma_\eta}\right\}
\end{aligned} \tag{24}$$

Since the posterior distribution (24) corresponds to the kernel of the inverse gamma distribution, the samples are simply drawn as $\sigma_\eta^2 \mid \phi, h \sim IG(\hat{\nu}/2, \hat{V}/2)$, with $\hat{\nu} = \nu_0 + n$ and $\hat{V} = V_0 + (1 - \phi^2)h_1^2 + \sum_{t=1}^{n-1} (h_{t+1} - \phi h_t)^2$.

• **Sample γ** As shown by Nakajima (2011), the process of sampling γ is similar to the sampling of σ_η . The prior distribution of γ is defined as $\gamma \sim IG(\gamma_0/2, W_0/2)$, with its conditional posterior distribution given by $\gamma \mid h \sim IG(\hat{\gamma}/2, \hat{W}/2)$, where $\hat{\gamma} = \gamma_0 + n$ and $\hat{W} = W_0 + \sum_{t=1}^n (y_t - x_t' \beta - z_t' \alpha_t)^2 / e^{h_t}$.

5 Time-Varying Income Elasticity of Imports and Economic Growth in Latin America

Following the import function defined by Thirlwall (1979), the TVP regression model is applied in order to obtain time-varying income elasticities of imports for eight Latin American countries³, namely, Bolivia, Brazil, Chile, Colombia, Ecuador, Mexico, Peru and Uruguay. In 2015, these countries represented together nearly 81% of Latin America GDP (at constant 2010 prices, in USD) (ECLAC, 2017). Hence, the time-varying econometric model for each country is defined as

$$m_t = \pi_t(y_t) + \psi_t(reer_t) + \epsilon_t \quad (25)$$

where m_t is the import growth rate; y_t is the domestic output growth rate; $reer_t$ is the real effective exchange rate (REER) growth rate; and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. This is a linear in growth rates model, whose parameters are interpreted as elasticities⁴. As discussed in Section 4, the time-varying parameters π_t and ψ_t follow a driftless random walk process, that is, $\alpha_{t+1} = \alpha_t + u_t$, with $\alpha_t \equiv (\pi_t, \psi_t)$ and $u_t \sim N(0, \Sigma)$. Also, the time-varying variance σ_t^2 is defined as a log-volatility process, so that $\sigma_t^2 = \gamma \exp(h_t)$, where $h_{t+1} = \phi h_t + \eta_t$ and $\eta_t \sim N(0, \sigma_\eta^2)$. Note that h_t is the stochastic volatility.

Annual data on Gross Domestic Product (GDP), imports and exports of goods and services were obtained from the World Development Indicators (WDI) database, maintained by the World Bank (2017). These data were extracted at constant 2010 prices, in USD. As to the annual REER index, the data were collected from the database developed by Darvas (2012), with its base changed to 2010. The growth rates were constructed as the natural logarithm of the ratio of the period t and $t - 1$ values. The growth rate dataset spans from 1970 to 2015.

With respect to the choice of prior distributions, this paper follows closely the default values proposed in Nakajima (2011):

$$\begin{aligned} \beta &\sim N(0, 10 \times I), & \Sigma &\sim IW(4, 40 \times I), & \alpha_1 &\sim N(0, 10 \times I) \\ \frac{\phi + 1}{2} &\sim \text{Beta}(20, 1.5), & \sigma_\eta^2 &\sim IG(2, 0.02), & \gamma &\sim IG(2, 0.02) \end{aligned}$$

These could be considered rather diffuse priors, which contribute to mitigate potential identification issues. Moreover, in order to compute the posterior estimates, the TVP regression model for each country is estimated by drawing 100,000 samples after the initial 10,000 samples were discarded in the burn-in period.

Autocorrelation and convergence are common issues associated to MCMC sampling methods. However, the estimation results showed that the sampling method efficiently produces uncorrelated samples, since the sample paths looked stable and the sample autocorrelations dropped stably for all countries. Also, we observed that the convergence diagnosis (CD) and the inefficiency factors (IF) of Geweke (1992) confirmed that, for the eight considered countries, the null hypothesis of convergence to the posterior distribution

³Regarding Argentina, due to the recent political manipulation of official economic statistics by its government, the country was excluded from the sample.

⁴In a study on measurement issues in estimating elasticity values, Casler (2015) found that the linear in growth rate regression specification provides more accurate estimates than the commonly used linear, logarithmic, logarithmic-linear and linear-logarithmic specifications. Furthermore, the results obtained by the author also advocates in favor of the natural logarithmic growth rate method in comparison to the standard base, the reverse base and the geometric-mean growth rate methods.

is not rejected for the parameters and state variables at the 10% significance level and that the sampling method is efficient, since the IF values are rather low.⁵

Figure 2 plots the posterior estimates for the stochastic volatility of the Latin American countries. In general, the volatility of imports has decreased from 1970 to 2015 for all eight countries. More specifically, the estimates for Brazil, Chile, Ecuador, Peru and Uruguay show that the variance of their imports decreased after the mid 1980s, however slightly increasing after 2008. These evidences are in accordance with the stabilization of international trade after the 1970s and early 1980s oil crises and the uncertainty engendered by the recent financial crisis, respectively. On the other hand, regarding Bolivia and Mexico, the instability of their imports from the 1970s to the early 1980s is relatively higher than the other considered Latin American economies. Yet, specially in Mexico, the stochastic volatility of imports has steadily decreased after the mid 1980s. In the case of Colombia, the increase on the variance of imports in the 1990s is in line with the Colombian trade liberalization policies enacted during the period.

As the variance of imports depicts a reasonably pronounced time-varying behavior from 1970 to 2015, one should also expect time-variation on the estimates for the Latin American income elasticities of imports. In fact, as shown in Figure 3, the estimates for all eight countries presented some degree of time-variation. With exception of Chile, it is possible to combine the considered Latin American economies into two different groups based on the evolution pattern of their income elasticities of imports. The first group, consisting of Bolivia, Ecuador, Peru and Uruguay, showed rather stable estimates over time, with small changes in the early 1980s and mid 1990s. As to the second group, consisting of Brazil, Colombia and Mexico, there has been an upward trend in their estimates. Following the tradition set out by [Thirlwall \(1979\)](#), these evidences would suggest that the rate of economic growth compatible with the BOP equilibrium has decreased for these countries. For instance, the recent studies by [Pacheco-López and Thirlwall \(2006\)](#) and [Cimoli et al. \(2010\)](#) support these results, with the authors arguing that the increase in the income elasticities of imports in Latin America is highly correlated to the region's commitment to Neoliberal trade liberalization policies, mainly in the form of tariff reductions, import quotas and licenses. Finally, in the case of Chile, despite its estimates being relatively volatile through the sample period, the country has recently shown income elasticity of imports levels similar to those from the beginning of the sample.

Even though these evidences are in favor of the Post Keynesian theory of BOP-constrained growth, the validity of the seminal model developed by [Thirlwall \(1979\)](#) has yet to be properly tested. Following the strategy suggested by [McCombie \(1997\)](#), an ordinary least squares (OLS) regression is implemented for each country in order to statistically test whether the observed growth rate (y) and the theoretical rate of economic growth with BOP equilibrium (y^*) are different. If so, the result would then advocate against the validity of Thirlwall's Law. Note that y^* is computed based on equation 5. Hence, the OLS specification to be tested for each country is

$$y_t^* = \rho y_t + \epsilon_t \quad (26)$$

where ϵ_t is a white noise disturbance. The Thirlwall's Law is validated if the parameter ρ is not statistically different from the unity.

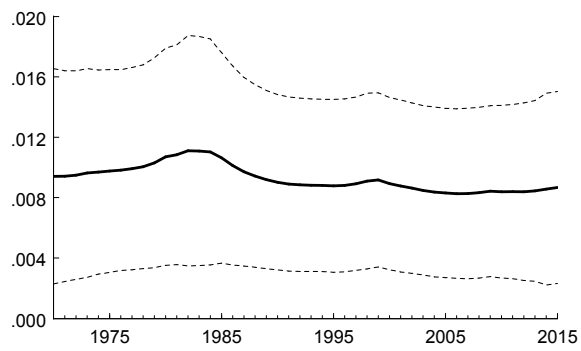
Table 2 presents the estimation results. With the exception of Peru, the null hypothesis of ρ being equal to zero is rejected by all countries at the 1% significance level. On the other hand, apart from Colombia and Peru, the null hypothesis of ρ being statistically equal to the unity is not rejected at the 10% significance level (note that, in the case of Mexico, the null hypothesis is not rejected at the 5% significance level). Hence, the obtained results confirm the validity of Thirlwall's Law for most of the country sample, which implies that the BOP has indeed posed a restriction to their economic performance from 1970 to 2015.

One should note that the present approach to the BOP-constrained growth model has only considered the seminal model of [Thirlwall \(1979\)](#). Therefore, these results should be considered as a preliminary investigation for the selected Latin American economies. Yet, the novelty of these results is associated to the use of a robust Bayesian econometric framework to estimate time-varying income elasticities of imports, controlling for stochastic volatility.

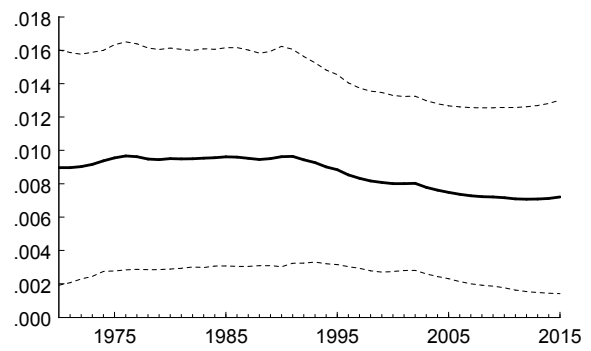
Furthermore, given the long-term nature of the theoretical rates of economic growth with BOP equilibrium, these estimates are also considered a measure for evaluating whether an economy has been in a

⁵The detailed results of the estimation procedure are available upon request from the authors.

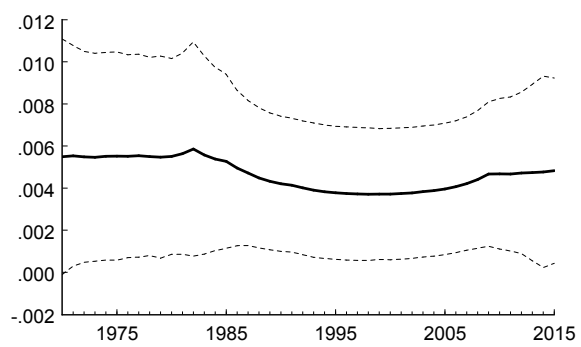
Figure 2: Posterior Estimates for Stochastic Volatility, $\sigma_t^2 = \gamma \exp(h_t)$ (1970 - 2015)



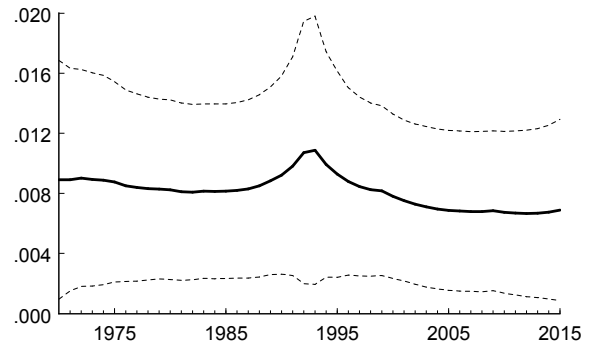
(a) Bolivia



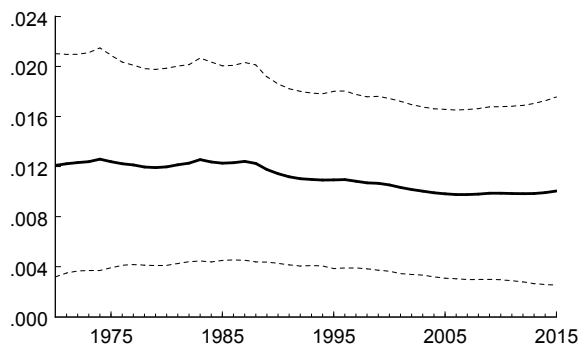
(b) Brazil



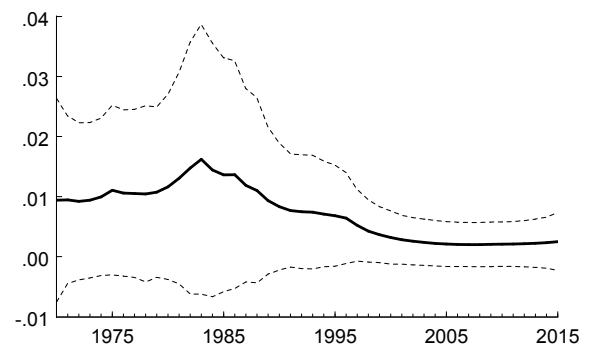
(c) Chile



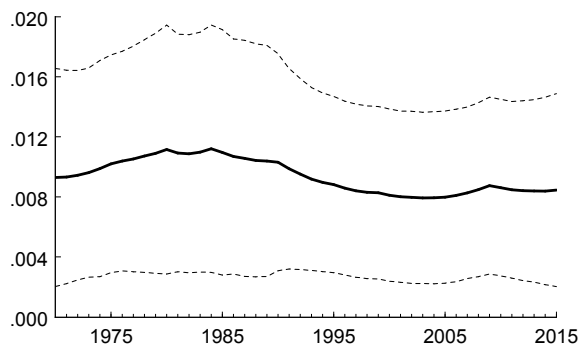
(d) Colombia



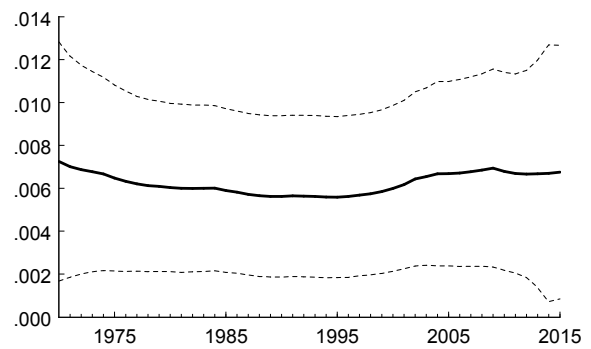
(e) Ecuador



(f) Mexico



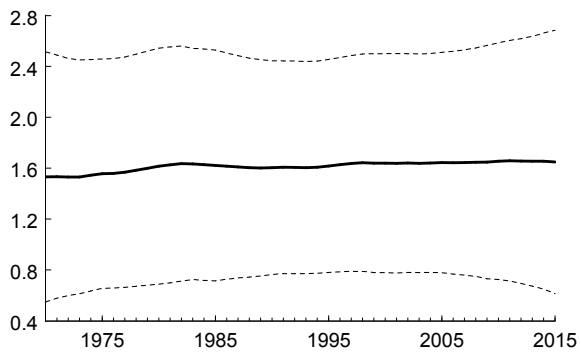
(g) Peru



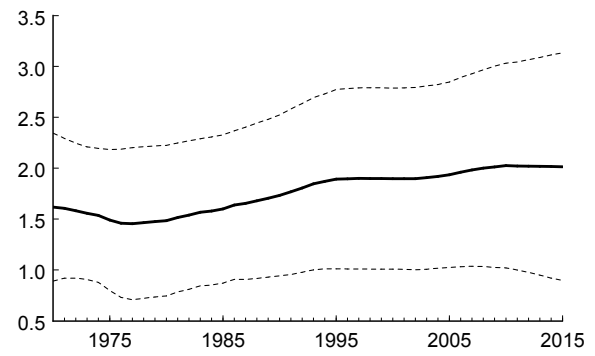
(h) Uruguay

Notes: Posterior mean (solid line) and 95% credible intervals (dotted line). Source: Compiled by the author.

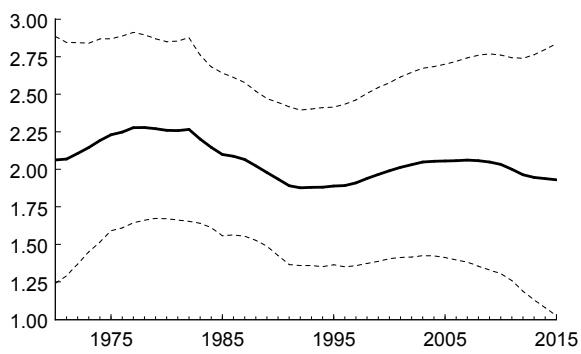
Figure 3: Posterior Estimates for Income Elasticity of Imports, π_t (1970 - 2015)



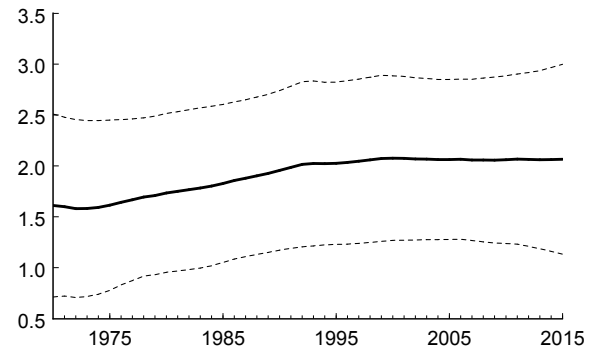
(a) Bolivia



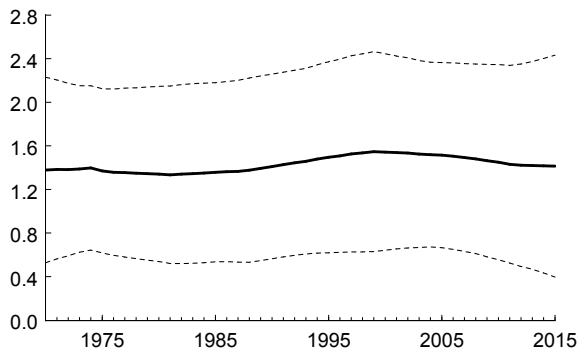
(b) Brazil



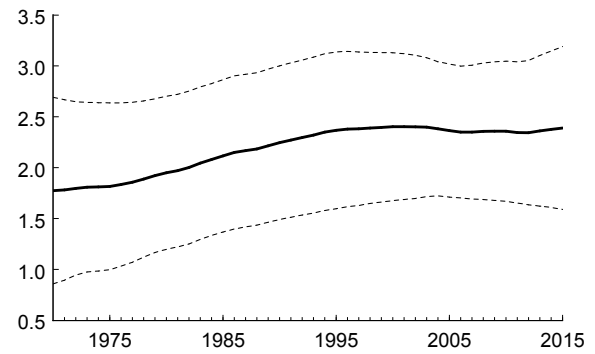
(c) Chile



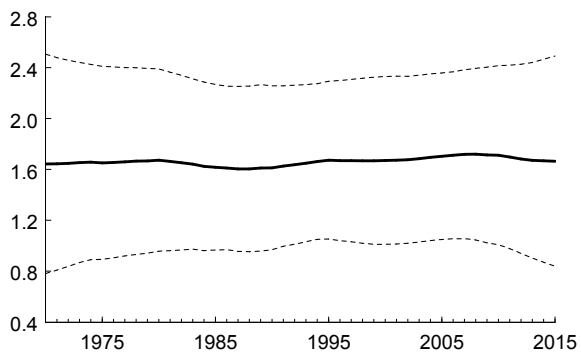
(d) Colombia



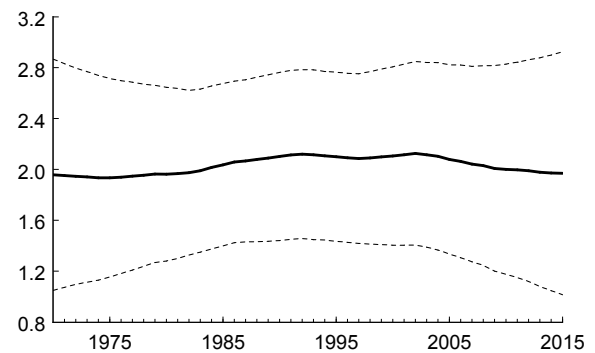
(e) Ecuador



(f) Mexico



(g) Peru



(h) Uruguay

Notes: Posterior mean (solid line) and 95% credible intervals (dotted line). Source: Compiled by the author.

Table 2: OLS Validity Test of Thirlwall's Law for Latin America (1970 - 2015)

	Bolivia	Brazil	Chile	Colombia	Ecuador	Mexico	Peru	Uruguay
ρ	0.789	0.790	0.892	0.603	1.310	0.750	0.232	0.788
Std. Dev.	0.177	0.148	0.092	0.118	0.297	0.124	0.131	0.131
t -Statistic ($H_0 = 0$)	4.440*	5.351*	9.699*	5.102*	4.403*	6.033*	1.773***	6.023*
t -Statistic ($H_0 = 1$)	-1.191	-1.423	-1.171	-3.352*	1.042	-2.005***	-5.854*	-1.616

Notes: $d.f. = 45$. *, ** and *** indicate that the null hypothesis is rejected at the 1%, 5% and 10% significance level. The term "Std. Dev." refers to the parameter standard deviation. Source: Compiled by the author.

catching-up trajectory relative to the world economy (Nassif et al., 2015, 2016). Table 3 reports the average BOP-constrained growth rate for the selected Latin American economies as well as the average world growth rate. The World GDP data (at constant 2010 prices, in USD) were retrieved from the WDI database (World Bank, 2017).

With the exception of Bolivia and Uruguay, there are evidences of a catching-up process by Brazil, Chile, Ecuador and Mexico from 1970 to 2015. Similar results are found for the period 1970-1985. Yet, between 1985 and 2000, the estimates show that the Bolivian economy embarked on a trajectory of catching up, which continued during the period 2000-2015. When considering the last fifteen years (2000-2015), this catching-up process is only observed in Bolivia and Brazil. Following Ocampo (2013), this cyclical process of catching-up and falling behind could be described as one of truncated convergence. Given the international trade slowdown and the recent economic crises, the observed trajectory shift by the Latin American countries could be the result of the fragility of their economies to external imbalances. However, one should note that the average BOP-constrained growth rate for Brazil in the period has relatively decreased in comparison to its past economic performance, reaching a level similar to the one shown by the world economy. Without effective measures to reverse this downward trend, this might indicate that Brazilian economy could potentially face a trajectory of falling behind in the near future. Last, but not least, the results also highlight the fact that Uruguay remained on a falling behind trajectory throughout the whole sample period.

Table 3: BOP-Constrained Growth Rate for Latin America (Annual Average)

	1970-2015	1970-1985	1985-2000	2000-2015
Bolivia	2.38%	-0.45%	3.17%	3.86%
Brazil	4.02%	6.11%	3.36%	2.85%
Chile	3.17%	3.04%	4.67%	1.76%
Ecuador	4.69%	7.27%	4.36%	2.28%
Mexico	3.84%	5.23%	4.07%	1.88%
Uruguay	2.51%	2.74%	2.62%	2.22%
World Economy	3.08%	3.46%	3.09%	2.58%

Notes: For the World Economy, the reported values correspond to the actual GDP growth rates, extracted from the WDI database (World Bank, 2017). Given the inconclusive results about the validity of the considered BOP-constrained growth model for Colombia and Peru, these countries were not included. Source: Compiled by the author.

6 Concluding Remarks

Economic growth in Latin America has been an intriguing phenomenon, driving a vast variety of theoretical and empirical research. Given the recurrence of trade imbalances during its economic history, the Post Keynesian theory of BOP-constrained growth has emerged as a robust theoretical alternative regarding the potential limits of the region's long-term economic performance. Several studies have tried to provide empirical evidences supporting these ideas (see e.g. Thirlwall and Hussain (1982), McCombie (1997), Holland et al. (2004), Pacheco-López and Thirlwall (2006) and Cimoli et al. (2010)).

In general, however, the Post Keynesian research has overlooked econometric issues related to structural breaks and parameter instability over the sample period, compromising the reliability of the estimates due

to biases and inconsistency. Despite some attempts on dealing with those issues (see e.g. [Atesoglu \(1997\)](#), [Bairam and Ng \(2001\)](#), [Bagnai \(2010\)](#)), the estimation of time-varying parameters considering the whole sample is a subject that has yet to be properly explored. This paper intended to contribute to the debate on this matter. Combining nonlinear Bayesian techniques and classical OLS estimations, TVP regressions with stochastic volatility for the import functions of eight Latin American economies were implemented and the respective validity of Thirlwall's Law was tested.

For some Latin American economies, namely, Brazil, Colombia and Mexico, the obtained results showed that there has been an upward trend in their estimates of the income elasticity of imports from 1970 to 2015. On the other, for the second group of countries, consisting of Bolivia, Ecuador, Peru and Uruguay, a rather stable behavior was observed. In the particular case of Chile, even though its estimates were relatively volatile over time, the country has recently presented income elasticity of import demand levels similar to those from beginning of the sample. Moreover, apart from Colombia and Peru, the BOP-constrained model developed by [Thirlwall \(1979\)](#) was validated for the period. This corroborates the idea of trade imbalances effectively limiting the Latin American economic performance for the last 50 years. As to a potential catching-up process in the region, the estimates for the theoretical rate of economic growth with BOP equilibrium supported the idea of an overall catching-up trajectory for Brazil, Chile, Ecuador and Mexico. Yet, one should note that the Bolivian economy has reversed its falling behind trajectory after 1985, while the Brazilian growth has relatively decreased in comparison to its past performance, leading to uncertainty about the future maintenance of its catching-up process. On a side note, the results also highlighted the persistence of a falling behind trajectory by the Uruguayan economy for the whole period.

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