Optimal Composition of the Public Spending and Economic Growth*

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Abstract

The objective of this paper is to investigate the relationship among size of the government, composition of public spending, and economic growth. We expand the theoretical model due to Devarajan et al (1996) by including technological progress in a more general constant elasticity of substitution (CES) production function. In addition, we use a balanced panel data for the Brazilian states to estimate the model's structural parameters and compute optimal ratios derived from the theoretical modeling. We found that private capital and government spending are substitute inputs in the production, while productive and unproductive expenditures are combined in a fixed ratio in the aggregate government spending. The public spending in investment is considerable lower than in costing, as occurs in developing countries with low economic dynamism. Finally, the estimated average tax burden is below the optimal level implied by the model, meaning that there is space for increasing taxation without hurting economic growth for some Brazilian states.

Keywords: Public spending; Optimal taxation; Economic growth.
JEL Codes: O41, H50.
Area 5 - Economy of the Public Sector.

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Resumo

O objetivo desse trabalho é investigar a relação entre o tamanho do governo, composição do gasto público e crescimento econômico. Nós expandimos o modelo teórico feito por Devarajan et al (1996), incluindo progresso tecnológico em uma função de produção mais geral do tipo CES (constant elasticity of substitution). Adicionalmente, nós usamos um painel balanceado dos estados do Brasil para estimar os parâmetros estruturais do modelo e com isso calcular relações ótimas derivadas da modelagem teórica. Nós encontramos que o capital privado e o gasto público são insumos substitutos na produção, enquanto os gastos produtivos e improdutivos são combinados em uma proporção fixa no gasto agregado do governo. O gasto público em investimento é consideravelmente menor do que no custeio, como ocorre nos países em desenvolvimento com baixo dinamismo econômico. Finalmente, a carga tributária média estimada está abaixo do nível ótimo implícito no modelo, o que significa que há espaço para aumentar a taxação sem prejudicar o crescimento econômico de alguns estados brasileiros.

Keywords: Gasto público; Taxa ótima; Crescimento econômico.
JEL Codes: O41, H50.
Área 5 - Economia do Setor Público.

1 Introduction

Several countries around the world have recently faced episodes of fiscal crises due to the incapacity of their governments to bridge a deficit between public expenditures and tax revenues.[1] These crises share some common features, given that they are usually accompanied by economic, social, and political distresses and the recovery is painful to the society as hole because it simultaneously requires to cut government spending and to increase tax on individuals and firms. Due to the relevance of the fiscal policy to a country's economic performance, it is important to keep an eye on both the relationship between the size of government and economic growth and the effects of the composition of the public expenditure on the country's growth rate. The latter issue rests on the fact that some public expenditures are seen as productive while others are considered unproductive in terms of their impacts on the economic activity. Thus, under this perspective, a country would be able to improve its economic performance by changing the mix between these two kinds of public expenditures.

[1]The cases of Greece, Portugal, Italy, Spain, and more recently Brazil, for instance, are well documented by the general media.

The empirical and theoretical literatures have devoted a considerable amount of work to analyze the relationship among the size of the government, composition of the public expenditure, and economic growth. Aschauer (1989), Lindauer and Velenchik (1992), and Barro (1990, 1991), for instance, investigated the impacts of aggregate government spending on economic growth and productivity. In a pioneer study, Devarajan et al (1996) analyzed the relationship between composition of public expenditure and economic growth using both theoretical and empirical frameworks. Davoodi and Zou (1998), Xie et al (1999), and Zhang and Zou (1998) examined the growth effects of aggregate public expenditure by different levels of government in a fiscal-federalism environment. Finally, Zhang and Zou (2001) unified the previous literature by focusing on the growth impacts of the allocation of public expenditure among multiple sectors (such as health, education, transportation, among others) with multiple levels of government (such as local, state, and federal). It is still missing in the literature, however, studies on the optimal size of the government and on the optimal composition of the public expenditure.

For the Brazilian case, Rocha and Giuberti (2007) and Divino and Silva Jr. (2012) provide empirical evidence on the optimal composition of the public expenditure for states and municipal districts, respectively. Both authors do not impose any a priori restriction on the productivity of the public expenditure and find that the optimal share of the current spending should range from 61 to 81% of the total public spending. However, no attempt was made to model the relationship among optimal size of the government, composition of the public spending and economic growth.

The objective of this paper is to fill this gap by investigating both theoretical and empirically the relationship among optimal size of the government in the economy, optimal shares of productive and unproductive public expenditures in the aggregate government spending, and economic growth. We extend the framework proposed by Devarajan et al (1996) by including an exogenous technological progress in a more general constant elasticity of substitution (CES) production function and we show how those optimal shares and government size depend on the structural parameters of the economy. We focus on economic growth instead of other measure of welfare for comparison purposes with the results by Devarajan et al (1996) and because it is important to identify the contribution of different components of the public expenditure to the economic growth.

Our major contribution is to show that the optimal size of the government in the economy, defined as the level of aggregate public expenditure over GDP that maximizes consumption growth, depends on the model's structural parameters. In addition, we find that the growth rate of consumption is inversely related with both the individuals' degree of impatience and the aggregate tax rate. For the Brazilian states, the average optimal size is around 20% under a general specification of the model. Considering the special case of a Cobb-Douglas production function, that optimal size negatively depends on the share of the capital in the production function.

The optimal share of productive public expenditure relatively to the unproductive spending, on its turn, depends on the share of the productive spending and the elasticity of substitution between productive and unproductive public spending in the CES production function of the government. Considering the case of a Cobb-Douglas, that optimal share depends exclusively on the share of the productive expenditure on the aggregate government spending. Thus, it is crucial to have good estimates for these parameters in order to calculate optimal ratios for any given country.

In the general case, however, the optimal taxation depends on the whole set of structural parameters. In particular, the technological progress has a direct effect on the optimal level of taxation while the share of private capital in total production has a negative effect on taxation. This theoretical finding coincides with the empirical results obtained for the Brazilian economy.

In order to estimate the structural parameters and find optimal ratios, we applied the model to a panel data for the Brazilian states in the recent period. A general CES production function, which combines private capital, composition of public expenditure, and technological progress, was estimated at the state level. We found that private capital and government spending are substitute inputs in the production, while government expenditures in investment and costing are combined in a fixed ratio due to the rigidity of the public budget imposed by legal aspects. We computed the composition of public spending, level of taxation, and economic growth implied by the estimated structural parameters for the Brazilian states. Then, we calculated the level of taxation and composition of the public expenditure that maximize the economic growth. We also performed a sensitivity analysis of the private capital productivity and average economic growth with respect to changes in the composition of public expenditure and tax rate.

The paper is organized as follows. The next section presents the model economy and derives the theoretical results. The empirical evidence for the Brazilian economy is reported and discussed in the third section. Finally, the fourth section is dedicated to the concluding remarks.

2 The model

The theoretical framework is based on Devarajan et al (1996), whose model is extended to consider a general CES (constant elasticity of substitution) function for both the government aggregate expenditure and the economy total production under a minimal set of restrictions on the parameter values. In addition, we add technological progress to the aggregate CES production function. Thus, differently from the original model, our production function depends on private capital stock, k, aggregate government government spending, x, and exogenous technological progress, A.[2] They are combined into a CES function expressed as:

$$y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha) x_t^{-\zeta}\right]^{-\frac{1}{\zeta}} \tag{1}$$

with $1 \geq \alpha > 0,\ \zeta \in (-1, 0) \cup (0, +\infty)$.

The aggregate government spending, x, is also given by a CES function which combines productive, g_1, and unproductive, g_2, expenditures. The government finances its spending by levying a flat-rate income tax, τ. The following equations represent these relationships:

$$x_t = \left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \tag{2}$$

and

$$\tau y_t = g_{1t} + g_{2t} \tag{3}$$

where $1 \geq \alpha_1 \geq 0, \zeta_1 \in (-1, 0) \cup (0, +\infty),\ 0 < \tau < 1$.

The private capital stock, k, follows a standard law of motion:

$$\dot{k}_t = (1-\tau) y_t - c_t \tag{4}$$

The representative agent chooses consumption, c, and capital, k, to maximize the expected discounted value of utility, which has the isoelastic form of a CRRA (constant relative risk aversion) function:

$$\int_0^{+\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{5}$$

with $\sigma > 0,\ \sigma \neq 1,\ \rho > 0$.

Taking all this into account, the representative agent's optimization problem might be written as:

[2] As in Devarajan et al (1996), labor does not enter directly in the production function.

$$(P)\begin{cases} Max & \int_0^{+\infty} \frac{c_t^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ s.t. & \dot{k}_t = (1-\tau)y_t - c_t \\ & y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha) x_t^{-\zeta}\right]^{-\frac{1}{\zeta}} \\ & x_t = \left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \\ & \tau y_t = g_{1t} + g_{2t} \\ & 1 \geq \alpha > 0,\ 1 \geq \alpha_1 \geq 0,\ \zeta, \zeta_1 \in (-1,0) \cup (0,+\infty),\ 0 < \tau < 1, \\ & 0 < \sigma,\ \sigma \neq 1,\ \rho > 0 \end{cases}$$

where the functions A, c, k, y, g_1 and g_2 are defined on $[0,+\infty)$ with positive values (i.e : $[0,+\infty) \to (0,+\infty)$).

By substituting equation (2) in (1), we have that:

$$y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha)\left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}}\right]^{-\frac{1}{\zeta}} \tag{6}$$

which is also a CES function.

2.1 Theoretical Results

The solution of problem (P) under the previous parametrization, and in the special case of a Cobb-Douglas production function, allows us to derive our major findings. They are reported in the following sequence of Lemmas and Theorems.

Lemma 1. *There exists $\phi : [0,+\infty) \to [0,1]$ such that $g_{1t} = \phi_t \tau y_t$ and $g_{2t} = (1-\phi_t)\tau y_t$ $\forall t \in [0,+\infty)$.*

Proof. Follows from the fact that $\tau y_t = g_{1t} + g_{2t}$ $\forall t \in [0,+\infty)$. □

Theorem 1. *If $Im(\phi) \subset (0,1)$, then there exists $\delta : [0,+\infty) \to (0,1) \cup (1,+\infty)$ defined by*

$$\delta_t = \left[\alpha_1 \phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}} \tag{7}$$

such that:

1. $x_t = \tau \delta_t^{-\frac{1}{\zeta}} y_t$.
2. $\delta_t \in (0,1)$ *when* $\zeta \in (-1,0)$.

3. $\delta_t > 1$ *when* $\zeta > 0$.

Proof. 1. By definition of the function x:

$$
\begin{aligned}
x_t &= \left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \\
x_t^{-\zeta_1} &= \alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1} \\
x_t^{-\zeta_1} &= \alpha_1 \phi_t^{-\zeta_1} \tau^{-\zeta_1} y_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} \tau^{-\zeta_1} y_t^{-\zeta_1} \\
x_t &= \left[\alpha_1 \phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \tau y_t \\
x_t &= \tau \delta_t^{-\frac{1}{\zeta}} y_t
\end{aligned}
$$

The aggregate public spending, x_t, is generated by $\tau\delta_t^{-\frac{1}{\zeta}}$ multiplied by the output of the economy. A special case emerges when $\zeta_1 \to 0$, which implies that $\delta = 1$, $x_t = \tau y_t$, and $x_t = g_{1t} + g_{2t}$. The third equation follows from the previous Lema.

2. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1-\phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1\phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\zeta}{\zeta_1} > 0$. If not, $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1-\phi_t)^{-\zeta_1} > 1$. So, $\alpha_1\phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\zeta}{\zeta_1} < 0$.

3. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1-\phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1\phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\zeta}{\zeta_1} < 0$. If not, $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1-\phi_t)^{-\zeta_1} > 1$. So, $\alpha_1\phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\zeta}{\zeta_1} > 0$.

□

Theorem 2. *If* $Im(\phi) \subset (0,1)$, *then there exists* $\theta : [0, +\infty) \to (0, +\infty)$ *defined by*

$$
\theta_t = \left[\frac{(A_t\tau)^\zeta - (1-\alpha)\delta_t}{\tau^\zeta \alpha}\right]^{-\frac{1}{\zeta}} \tag{8}
$$

such that $k_t = \theta_t y_t$ $\forall t \in [0, +\infty)$. *If* $A_t = 1$, *then* $\theta_t > 1$.

Proof. By definition of function y:

$$
\begin{array}{rcl}
y_t & = & A_t\left[\alpha k_t^{-\zeta}+(1-\alpha)x_t^{-\zeta}\right]^{-\frac{1}{\zeta}} \\
y_t^{-\zeta} & = & \alpha(A_t k_t)^{-\zeta}+(1-\alpha)(A_t x_t)^{-\zeta} \\
y_t^{-\zeta} & = & \alpha(A_t k_t)^{-\zeta}+(1-\alpha)(A_t\tau)^{-\zeta}\delta_t y_t^{-\zeta} \\
\alpha A_t^{-\zeta}k_t^{-\zeta} & = & \left[1-\frac{1-\alpha}{(A_t\tau)^{\zeta}}\delta_t\right]y_t^{-\zeta} \\
k_t & = & \left[\frac{(A_t\tau)^{\zeta}-(1-\alpha)\delta_t}{\alpha\tau^{\zeta}}\right]^{-\frac{1}{\zeta}}y_t \\
k_t & = & \theta_t y_t
\end{array}
$$

Thus, each unit of private capital generates $\frac{1}{\theta}$ units of output in the economy. In other words, $\frac{1}{\theta}$ represents the private-capital productivity.

The third equation follows from the substitution of x_t. Notice that $(A_t\tau)^{\zeta}-(1-\alpha)\delta_t>0$, because functions k and y are positive.

Now, we consider the particular case in which $A_t=1$.

If $\zeta\in(-1,0)$, then

$$
\begin{array}{ccccccc}
0 & < & \delta_t & < & 1 & < & \tau^{\zeta}\ \ \forall t\geq 0 \\
0 & < & (1-\alpha)\delta_t & < & 1-\alpha & < & (1-\alpha)\tau^{\zeta}\ \ \forall t\geq 0
\end{array}
$$

This implies that

$$
\begin{array}{ccccl}
0 & < & \alpha\tau^{\zeta} & < & \tau^{\zeta}-(1-\alpha)\delta_t\ \ \forall t\geq 0 \\
0 & < & 1 & < & \left[\frac{\tau^{\zeta}-(1-\alpha)\delta_t}{\alpha\tau^{\zeta}}\right]\ \ \forall t\geq 0 \\
0 & < & 1 & < & \theta_t\ \ \forall t\geq 0
\end{array}
$$

If not ($\zeta>0$), then

$$
\begin{array}{ccccccc}
0 & < & \tau^{\zeta} & < & 1 & < & \delta_t\ \ \forall t\geq 0 \\
0 & < & (1-\alpha)\tau^{\zeta} & < & 1-\alpha & < & (1-\alpha)\delta_t\ \ \forall t\geq 0
\end{array}
$$

This implies that

$$
\begin{array}{ccccl}
0 & < & \tau^{\zeta}-(1-\alpha)\delta_t & < & \alpha\tau^{\zeta}\ \ \forall t\geq 0 \\
0 & < & \left[\frac{\tau^{\zeta}-(1-\alpha)\delta_t}{\alpha\tau^{\zeta}}\right] & < & 1\ \ \forall t\geq 0 \\
1 & < & \theta_t\ \ \forall t\geq 0 & &
\end{array}
$$

Notice that all quantities are positive. The last inequality follows because $-\frac{1}{\zeta}<0$.

$\square$

This means that this model collapses to a traditional Ak growth model. In the special case of a Cobb-Douglas production function, where $\zeta = 0$ and $\zeta_1 = 0$, we have that both $\delta_t = 1$ and $\theta_t = 1$. From now on, we no longer assume that $A_t = 1$.

Theorem 3. *If $Im(\phi) \subset (0,1)$ and $c : [0, +\infty) \to (0, +\infty)$ is a continuously differentiable function, then the the growth rate of consumption is:*

$$\lambda_t = \frac{\dot{c}_t}{c_t} = \frac{1-\tau}{\sigma\theta_t} - \frac{\rho}{\sigma} \tag{9}$$

Proof. Following Lemma 2, we have that the dynamic equation for k might be rewritten as $\dot{k}_t = \left[\frac{1-\tau}{\theta_t}\right] k_t - c_t$. Thus, problem (P) is nothing else than a variational calculus problem, where:

$$L(t, k_t, \dot{k}_t) = \left[\frac{\left[\frac{1-\tau}{\theta_t} k_t - \dot{k}_t\right]^{1-\sigma} - 1}{1-\sigma}\right] e^{-\rho t}. \tag{10}$$

Here, $\frac{\partial L}{\partial k} = \left[\frac{1-\tau}{\theta_t}\right] c_t^{-\sigma} e^{-\rho t}$ and $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{k}}\right] = [\sigma c_t^{-1-\sigma}\dot{c}_t + \rho c_t^{-\sigma}]e^{-\rho t}$. The statement follows from the Euler equation. □

Corollary 1. *If $\frac{1-\tau}{\rho} \leq \theta_t$, then $\lambda_t \leq 0$.*

Corollary 2. *If $\frac{1-\tau}{\rho} > \theta_t$, then $\lambda_t > 0$.*

Theorem 3 states that the growth rate of consumption in this economy might be positive or negative, depending on $\frac{1-\tau}{\rho}$ and θ_t according to Corollaries 1 and 2. In order to keep a positive growth rate of consumption, $\lambda_t > 0$, the economy should be characterized by low degree of impatience and low tax rate, such that $\frac{1-\tau}{\rho} > \theta_t$. In this environment, the representative agent would be willing to transfer consumption across time and the after-tax income would be sufficient to allow for this transference. In the special case of a Cobb-Douglas, we have that $\lambda_t > 0$ only if $\tau + \rho < 1$, given that $\lambda_t = \frac{(1-\tau-\rho)}{\sigma}$ in this case. Next, in Lemma 2, we define the optimal share for productive (and unproductive) government spending also as a function of the economy structural parameters.

Lemma 2. *If $Im(\phi) \subset (0,1)$, then θ as a function of ϕ attains its minimum value at $\phi^* = \frac{\alpha_1^{\frac{1}{\zeta_1+1}}}{(1-\alpha_1)^{\frac{1}{\zeta_1+1}} + \alpha_1^{\frac{1}{\zeta_1+1}}}$.*

Proof. Since,

$$\theta_t = \left[\frac{(A_t \tau)^\zeta - (1-\alpha)\delta_t}{\alpha \tau^\zeta} \right]^{-\frac{1}{\zeta}} \tag{11}$$

then

$$\frac{\partial \theta}{\partial \phi} = -\frac{1-\alpha}{\alpha \tau^\zeta} \theta_t^{\zeta+1} \delta_t^{1-\frac{\zeta_1}{\zeta}} \left[\alpha_1 \phi_t^{-\zeta_1 - 1} - (1-\alpha_1)(1-\phi_t)^{-\zeta_1-1} \right] \tag{12}$$

So, $\frac{\partial \theta}{\partial \phi} = 0$ at $\phi^* = \frac{\alpha_1^{\frac{1}{\zeta_1+1}}}{(1-\alpha_1)^{\frac{1}{\zeta_1+1}} + \alpha_1^{\frac{1}{\zeta_1+1}}}$.

We point that if $\phi \in (0, \phi^*)$, then $\frac{\partial \theta}{\partial \phi} < 0$. Analogously, if $\phi \in (\phi^*, 1)$, then $\frac{\partial \theta}{\partial \phi} > 0$. Thus, the statement follows. □

Essentially, the optimal share of the productive public expenditure, ϕ^*, depends on α_1 and ζ_1. In the special case of a Cobb-Douglas production function, this optimal share simplifies to $\phi^* = \alpha_1$. This result is intuitive because the higher is the elasticity of the productive expenditure in the aggregate public spending, x_t, the higher will be the optimal share of the productive expenditure.

Theorem 4. *The maximizer of the growth rate consumption is the minimizer of θ as a function of ϕ.*

Proof. Since the growth rate consumption is

$$\lambda_t = \frac{\dot{c}_t}{c_t} = \frac{1-\tau}{\sigma \theta_t} - \frac{\rho}{\sigma} \tag{13}$$

The statement follows because the maximum value of λ is achieved at the same ϕ^* that θ achieves its minimum value with respect to ϕ. □

Next, we try to find the optimal τ, which is compatible with the maximum growth rate of consumption. Notice that, by problem (P), this optimal τ also corresponds to the optimal size of the government expenditure in the economy.

Theorem 5. *Function k increases when τ decreases.*

Proof. Since $k_t = \theta_t y_t$, the statement follows because

$$\frac{\partial \theta}{\partial \tau} = -\frac{1-\alpha}{\alpha} \left[\frac{\theta_t}{\tau} \right]^{\zeta+1} \delta_t \tag{14}$$

□

Theorem 6. *The maximizer of the growth rate consumption as a function of* τ *is* $\tau^* = \left[\frac{(1-\alpha)\delta_t}{A_t^\zeta}\right]^{\frac{1}{\zeta+1}}$.

Proof. Notice that

$$\frac{\partial\lambda}{\partial\tau} = \frac{-\sigma\theta_t - (1-\tau)\sigma\frac{\partial\theta}{\partial\tau}}{\sigma^2\theta_t^2} \tag{15}$$

But, from Theorem 5 we have that $\frac{\partial\theta}{\partial\tau} = -\frac{1-\alpha}{\alpha}\left[\frac{\theta_t}{\tau}\right]^{\zeta+1}\delta_t$.

So,

$$\frac{\partial\lambda}{\partial\tau} = \frac{-A_t^\zeta\tau^{\zeta+1} + \delta_t(1-\alpha)}{\sigma\tau\theta_t((A_t\tau)^\zeta - (1-\alpha)\delta_t)} \tag{16}$$

The statement follows, because for each $\tau < \tau^*$ the function λ increases. Analogously, for each $\tau > \tau^*$ the function λ decreases. In the special case where $\zeta \in (-1, 0)$, a positive variation of A_t leads to an increase in $\tau*$. This suggests that a higher technological progress allows for a higher level of optimal taxation, provided that all other variables are kept unchanged. □

Theorem 7. *The growth rate of consumption,* λ*, is an increase function of* A.

Proof. This follows from the fact that

$$\frac{\partial\lambda}{\partial A} = \left[\frac{1-\tau}{\alpha\sigma}\right](\theta_t A_t)^{\zeta-1} \tag{17}$$

□

3 Empirical Evidence

3.1 Econometric Model

We estimate, by nonlinear least squares, the aggregate CES function which emerges from the combination of the previous equations (1) and (2). In a panel data environment, considering the data set that will be used in the estimation, it might be written as:

$$y_{jt} = A_{jt}\left[\alpha k_{jt}^{-\zeta} + (1-\alpha)\left[(1-\eta_1)g_{1jt}^{-\zeta_1} + \eta_1 g_{2jt}^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}}\right]^{-\frac{1}{\zeta}} + \varepsilon_{jt} \tag{18}$$

This equation is equivalent to the one that comes from the theoretical model, but making $(1-\eta_1) = \alpha_1$ as the share public spending in investment.

The structural parameter α defines the share of private capital in the output while $(1-\alpha)$ represents the share of the aggregate government spending in the output. In addition, η_1 is the share of current spending and $(1-\eta_1)$ is the share of public investment in the aggregate government spending[3]. The parameters ζ and ζ_1 generate elasticities[4] and ε is an additive random error.

The heterogeneous technological progress in each state is A_{jt}. Following Duffy and Papageorgiou (2000), it is modeled by:

$$A_{jt} = exp(\gamma p_i + \nu t) \tag{19}$$

We use the cumulative distribution of patents across the Brazilian states to create a dummy variable for the $i\%$ group of the least productive ones in terms of number of registered patents. These states are considered as having low degree of technological progress. They will have an output smaller than the average whenever $\gamma < 0$.[5] Given that the technological progress might change over time, ν accounts for the estimation of this time effect.

Finally, we considered variables in both levels and per capita terms. Thus, we estimate four regression models, as described below:

1. Model (A): Without technological progress, such that $A_{jt} = 1$ for all states;
2. Model (B): With technological progress, such that $i = 5\%$ of the Brazilian states that least registered patents;
3. Model (C): Similar to the model (A), but with per capita variables;
4. Model (D): Similar to the model (B), but with per capita variables.

3.2 Data

We use a balanced panel composed by the 27 Brazilian states in the period from 2004 to 2010 with annual data, totalizing 189 observations for each variable.[6] The nominal variables were deflated by the wide consumer

[3] $(1-\eta_1)$ is the share of public investment because g_1 does not include expenses with interest rate and debt rollover.

[4] Because it is a CES, the constant elasticities are given by $\psi = \frac{1}{1+\zeta}$ and $\psi_1 = \frac{1}{1+\zeta_1}$.

[5] We considered $i = 1\%$ and $i = 5\%$ in the estimation, but the results were similar. We also used other variables, such as number of patents, average of registered patents and average of registered patents in the last 5 years. However, we had converge problems. Thus, we chose to work with $i = 5\%$.

[6] The sample is limited to this period due to the data availability for the computation of the private capital, k.

price index (Indice Nacional de Preos ao Consumidor Amplo - IPCA), which is calculated by the Brazilian Institute of Geography and Statistics (IBGE) and used by the Central Bank of Brazil in the inflation targeting regime. The variables are describe in sequence.

y: Gross domestic product (GDP) of the Brazilian states released by Ipeadata[7].

g_1: Government spending in investment by each Brazilian state, which consists of total capital spending minus payment of interest rate and debit amortization. The source also is Ipeadata.

g_2: Government spending on costing (or current spending) by each Brazilian state, also collected from the Ipeadata.

k: Stock of private capital of each Brazilian state, computed according to the procedure proposed by Sanches and Rocha (2010).

The technological progress of each Brazilian state was measured by the amount of registered patents provided by the National Institute of Intellectual Property (Instituto Nacional da Propriedade Industrial - INPI). We create a dummy variable, p_5, to represent the 5% of the Brazilian states that least registered patents in each time period. Thus, $p_5 = 1$ when state j is part of these 5% that least registered patents according to the cumulative distribution of patents and $p_5 = 0$ otherwise.

3.3 Empirical Results

Initially, we tested the panel data for the presence of unit root. We applied tests due to Levin-Lin-Chu (LLC), Im-Pesaran-Shin (IPS), Fisher-ADF, Hadri, and Pedroni (1999). The results indicated that the panel is stationary at the 5% significance level. The non-linear estimation of equation (18) was then carried out with all variables in levels. Initial values of the parameters were set at 0.0001 in order to allow for convergence to either positive or negative values.

Due to individual heterogeneity of the Brazilian states, which might lead to heteroscedasticity in the residuals, we performed a correction in the variance-covariance matrix for robust standard errors in clusters. However, for the results reported in Table 1, the coefficients that were statically significant at the 1% significance level maintained this significance with or without that correction.

The parameter ν in equation (19), which represents the deterministic component of the technological progress, was not statistically significant in

[7] www.ipeadata.gov.br.

any estimation. This might be due to short time horizon of the sample. Thus, the time effect was excluded from the estimations and only estimated values for γ are reported in Table 1. As mentioned before, the first two estimated models refer to variables in levels while the last two use per capita variables. In addition, models A and C assume no technological progress across the Brazilian states ($A_{jt} = 1$), while models B and D estimates technological progress according to equation (19) with $\nu = 0$.

Table 1: Non-linear estimation

Parameters	(A)	(B)	(C)	(D)
α	0.660***	0.659***	0.694***	0.689***
	(0.074)	(0.072)	(0.075)	(0.049)
ζ	-0.287***	-0.290***	-0.303***	-0.319***
	(0.066)	(0.065)	(0.068)	(0.049)
η_1	0.887***	0.853***	0.989***	0.861***
	(0.132)	(0.163)	(0.042)	(0.077)
ζ_1	0.356	0.210	1.270	0.203
	(0.730)	(0.708)	(1.528)	(0.340)
γ	NA	-0.634***	NA	-0.680***
		(0.086)		(0.101)
R-squared	0.965	0.972	0.968	0.990
Adj. R-squared	0.964	0.971	0.967	0.990

Source: Estimated by the authors. Notes: ***, **, and * indicate that the estimated coefficient is statistically significant at the 1, 5, and 10% levels, respectively.

The share of private capital in the total output, α, was estimated above 65% in all models, indicating that private capital is more important than the government's compound spending for the aggregate production of the Brazilian states. This high estimated value might also capture the effect of labor in the output, which is not explicitly modeled in the production function (1) based in Devarajan et al (1996). The negative estimated value for ζ points out that private investment and government spending are substitute inputs in the production. This suggest that the ratio of these inputs utilization in the production changes more than proportionately to any change in the ratio of their marginal products, but we are not able to identify in which direction flows these changes.[8]

Considering the composition of the government spending, η_1 is always

[8]The elasticity of substitution, ψ, ranges from 1.403 in model A to 1.468 in model D.

above 85% meaning that the government spends a larger fraction of the public budget on current spending (g_2) than in public investment (g_1). This finding is in line with the fact that public investment is still a small fraction of public spending in the Brazilian economy. The coefficient ζ_1 was not statistically different from zero at the 5% significance level, suggesting that the public spending composition, x_t, might be represented by a Cobb-Douglas function, which is a special case of the general CES assumed in equation (2). This means that g_1 and g_2 are combined in a fixed ratio in the aggregate government spending, independently of the ratio of their marginal products. This results reflects the rigidity of the public budget of the Brazilian states, where the shares of several expenditures are fixed by law.

The Brazilian states classified as having lower technological progress according to the amount of registered patents presented a negative and statistically significant value for γ. This means that they have a smaller output than the other states, which are in the group that registered more than 5% of the patents in each time period. Thus, the states contained in p_5 are expected to have an output which is, on average, about 49 and 47% smaller than the states that are outside p_5 according to models (B) and (D), respectively.[9] This finding is in line with the literature, according to which the less technologically developed economies are also the ones with lower levels of production.

To find the growth rate of consumption λ and the other compound parameters implied by the theoretical model, we need to set values for the risk aversion coefficient σ and the intertemporal discount factor, ρ, in addition to use the estimated coefficients from Table 1. Notice that the preference of the government for investment spending is $\alpha_1 = (1 - \eta_1)$. The values for $\sigma = 4.89$ and $\rho = 0.123$ were obtained from from Issler and Piqueira (2000).[10] Using these values, we can find the average growth rate of consumption, $\overline{\lambda}$, the ratio of private capital to output, $\overline{\theta}$, and the deviations from the predicted $\overline{\lambda}$ to the observed growth rate from the data, as reported in Table 2.

The inverse of $\overline{\theta}$ measures the productivity of the private capital in the economy. Thus, the lower $\overline{\theta}$ the more productive is the private capital. Table 2 indicates that this productivity ranges from 0.52 to 0.29, depending on the

[9]These values were computed by making $A_{jt} = exp(\gamma p_5)$, where γ = -0.634 and -0.680 for the states that belong to p_5 in models B and D, respectively, according to Table 1.

[10]Issler and Piqueira (2000) reported the intertemporal discount rate as $\beta = 0.89$. To find the intertemporal discount factor, ρ, we use $\beta = \frac{1}{1+\rho}$, which yielded $\rho = 0.123$. Other authors, such as Catalo and Yoshino (2006), Costa and Carrasco (2015), and Faria and Ornelas (2015) have found similar values for σ and ρ.

Table 2: Compound structural parameters

Parameters	(A)	(B)	(C)	(D)
α_1	0.113	0.147	0.011	0.139
$\overline{\theta}$	2.151	3.453	1.920	3.148
$\overline{\lambda}$	0.060	0.033	0.070	0.040
$Mean\ deviation$	-0.010	0.017	-0.033	0.003
$Mean - square\ deviation$	0.000	0.001	0.001	0.001

Source: Calculated by the authors.

estimated model.

Comparing the estimated models with and without technological progress, model (A) yields the smallest mean-squared deviation from $\overline{\lambda}$ with respect to the observed consumption growth rate from the data. Except for model (C), the values of the estimated parameters are similar across the alternative models. This meas that the estimated structural parameters are quite stable across the models.

The optimal fraction of investment spending, ϕ^*, and optimal tax rate, τ^*, that maximize consumption growth, λ, are reported in Table 3. The short time horizon of the sample might have biased the impact of the government spending in investment over the economic growth of the Brazilian states, as measured by ϕ^*.[11] Given that the average total tax revenue is around 16% of the GDP for the Brazilian states, according to the data for the 2004 to 2010 period available from the Ipeadata, one might argue that $\overline{\tau} < \tau^*$ and some states have average taxation below the optimum level.

Table 3: Optimum levels of ϕ and τ.

Parameters	(A)	(B)	(C)	(D)
ϕ^*	0.113	0.147	0.011	0.139
τ^*	0.220	0.195	0.183	0.155

Source: Calculated by the authors.

Because ζ_1 is not statistically different from zero, we have that $\phi^* = \alpha_1$. In this case, only with the optimal levels of investment spending, ϕ^*, and tax rate, τ^*, it is not possible to access how the consumption growth rate depends on these parameters. To do so, we need to compute the partial derivatives reported in Tables 4 and 5.

[11] Divino and Silva Jr. (2012) found that the optimal share of public spending in capital is 32% for high-income, 23% for middle-income, and 19% for low-income Brazilian municipal districts.

Table 4: Sensitivity of θ with respect to ϕ and τ.

Parameters	(A)	(B)	(C)	(D)
$\partial\theta/\partial\phi$	0.168	0.086	0.061	0.102
$\partial\theta/\partial\tau$	-4.889	-6.338	-3.534	-4.613

Source: Calculated by the authors.

The ratio of public spending in investment, ϕ, according to Lemma 2 is above the optimal level because $\partial\theta/\partial\phi > 0$. This result implies that an increase in ϕ leads to a decrease in the productivity of the private capital and so to a decrease in in the growth rate of consumption, λ. According to Theorem 4, the maximization of λ as a function of ϕ occurs when θ is at the minimum. This might also mean that public spending in costing, g_2, is below the optimal level.

An increase in the tax rate, τ, decreases the private capital relatively to the output of the economy. This is a classic result given that an increase in taxation will reduce the amount of private capital available in the economy. However, the effect of a higher τ on λ is not obvious because the productivity of the private capital increases.

Table 5: Sensitivity of λ with respect to τ and A.

Parameters	(A)	(B)	(C)	(D)
$\partial\lambda/\partial\tau$	0.087	0.064	0.067	0.046
$\partial\lambda/\partial A$	0.104	0.094	0.112	0.103

Source: Computed by the authors.

In fact, as shown in Table 5, $\partial\lambda/\partial\tau > 0$ for the Brazilian states. The effective taxation is below the optimal level, suggesting that there is space for a tax increase without harming economic growth of the economy.

Another result from Table 5 is the positive effect of changes in technological progress to the consumption growth. This is expected once an increase in technology has a direct effect on output, which leads to a rise in consumption. This effect is very similar across the alternative models, as illustrated in Table 5.

4 Concluding Remarks

The objective of this paper was to investigate both theoretical and empirically the relationship among optimal size of the government in the economy, optimal shares of productive and unproductive public expenditures in the aggregate government spending, and economic growth. We expanded the model by Devarajan et al (1996) to include exogenous technological progress in a more general constant elasticity of substitution (CES) production function. We showed how those optimal shares depend on the structural parameters of the economy and provided empirical evidence by using a balanced panel data for the Brazilian states in the recent period.

The Cobb-Douglas and Ak production functions are obtained as special cases of the general CES specification. In the special case of Cobb-Douglas production function, economic growth requires that the sum of tax rate and intertemporal discount factor in the utility be strictly smaller than one. In addition, the optimal public spending that maximizes consumption growth is given by the fraction of public spending allocated on capital.

In the general case, the growth rate of consumption is inversely related with both the individuals' degree of impatience and the tax rate.The optimal taxation, defined as the one that maximizes the consumption growth, depends on the whole set of structural parameters. In particular, the technological progress has a direct effect on the optimal level of taxation while the share of private capital in the output has a negative effect on the optimal taxation. This theoretical finding coincided with the empirical results obtained for the Brazilian economy.

The estimated parameters for the Brazilian states suggest that, on average, the share of private capital in the total production is 0.66 while the share of total government spending is 0.34. The estimated elasticity of substitution between these two inputs ranged from 1.40 to 1.45, depending on the model specification. Thus, private capital and government spending are substitute inputs in the production and the latter has a significant share in the output of the Brazilian states.

Devarajan et al. (1996) argue that developing countries, due to the low economic dynamism, require a significant fraction of government spending allocated to costing. This finding was observed for the Brazilian economy, were about 85% of the total public spending was in costing and only 15% was in public investment. This result is in line with the widely spread consensus that public investment is still a small fraction of public spending in the Brazilian economy.

Taking as a whole, the government spending is below the optimal level. Thus, it is possible to increase the growth rate of consumption by rising the government spending under a balanced public budget. The estimated optimal taxation by the model that best fitted the data was 19.5%, while the average taxation observed from the data for the Brazilian states is around 16%. Thus, there is space for increasing taxation, and so for rising government spending, without hurting the economic growth.

For future research, it would be interesting to include public debt in the government budget constraint along with its dynamics in an intertemporal framework. It would be also interesting to expand the empirical analysis to a panel of countries with data for the pre and post international financial crisis. Some of these suggestions are object of our current research.

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