Asymmetric Brazilian Phillips Curve: A Quantile Regression Approach

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Resumo

Este trabalho investiga o comportamento assimétrico da Curva de Phillips brasileira por meio de regressões quantílicas. Inicialmente, os resultados mostram mudanças nos coeficientes das variáveis dependentes ao longo dos quantis da inflação. Na mesma direção da literatura nacional, nossas estimativas OLS e GMM apontam para predominância do componente defasado sobre as expectativas. No entanto, o modelo quantílico fornece evidência adicional relevante de que essa dominância é apenas sustentável na cauda inferior da distribuição da variável dependente. Além disso, avançamos nas estimativas de funções densidade condicionais aos cenários de expansão e recessão. Encontramos diferenças nas dispersões das distribuições condicionais e probabilidades assimétricas de mudanças futuras na trajetória da inflação, indicando que a curva de Phillips pode capturar adequadamente a assimetria de ciclos descria por DeLong e Summers (1988) e Ball e Mankiw (1994).

Abstract

This paper presents an investigation about the asymmetric behavior of the Brazilian Phillips Curve, through the use of quantile regressions. First, our results show changes in slope of the dependent variables along the quantiles of the inflation. In the same direction of the national literature, our OLS and GMM estimates show the dominance of the backward-looking component over expactations. However the quantile model provides additional relevant evidence that this dominance is only sustainable for the lower tail of the current inflation distribution. Furthermore, advancing in the density estimations conditioned to the expansion and recession scenarios, we find differences in the dispersions of the inflation's conditional distributions and assymetric probabilities of future change in the trajectory of inflation, indicating that the Phillips curve adequately captures the asymmetry of cycles described by DeLong and Summers (1988) and Ball and Mankiw (1994).

Key-words: Phillips Curve; Quantile Regression; Asymmetry.
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1 Introduction

Asymmetry in economic cycles was empirically debated by authors such as Neftci (1984), Sichel (1993), Diebold and Rudebusch (1990), and Hamilton (1989). According to these works, most macroeconomic time series present different behavior across various phases of the cycles. When verified, the asymmetry should always be accounted for, since it allows macroeconomic models and forecasting tools to present more adequate performance. Intuitively, this means that policy proposals based on linear estimations can not be generalized in extreme periods. Recently, some studies such as Chauvet (2002), Ide and Moes (2003), Morley and Piger (2012) and Kiani (2016) reinforce the presence of asymmetry in Brazil, Euro Area and USA.

The classic example of this effect can be observed by the unemployment rate, which appears to increase sharply in recessions and to decline slowly in economic expansions, this asymmetric behavior can be considered a phenomenon of non-linearity and was verified in several OECD countries[1].

An important part of the literature has investigated whether asymmetric oscillations in real aggregates would also lead to similar effects on the monetary variables, such as the price level. This idea first appeared in DeLong and Summers (1988) and Ball and Mankiw (1994), who studied asymmetric price adjustments. These authors observed that when the product was above its potential level, prices increased rapidly, but when the product was below, the prices took longer to fall.

The relationship between real and monetary aggregates has led many authors to investigate the possibility of asymmetry in the Phillips curve. Buchmann (2009) and Balaban and Vintu (2010) argue that the Phillips curve itself has never intended to describe a symmetrical relationship. This question was verified using Smooth Transition Autoregressions (STAR), Markov Switching Models (MS) and Autoregressive Distributed Lag Structural Breaks (ARDL) methods for the following countries: Brazil by Correa and Minella (2010), South Africa by Nell (2006), Canada by Huh and Lee (2002) and United States, Sweden, Australia by Eliasson (2001). However, recently some studies have focused on the use of quantile regressions to investigate the slope variation in different regions of the response variable distribution, such as Euro Area by Chorteas and Panagiotidis (2012) and Turkey by Boz (2013).

Dealing with domestic price dynamics, the Brazilian economy experienced periods of hyperinflation in recent history, often reaching three digits in the 1980s. But since the 1990s, after the implementation of *Plano Real* in 1994, the price index returned to stability with an average value of 6.45 % p.p., from 1996 to 2014. However, with the external crisis, instability of public accounts, pressure on the exchange rate and growth of uncertainty, the deviation of the price level has returned to attention in recent quarters. The Extended Consumer Price Index (IPCA), for example, reached double digits in 2015, an unusual event since 2002.

We believe that recent events contradict the assumption of constant coefficients in the Brazilian Phillips Curve, which is in agreement with the evidence of asymmetry found by Correa and Minella (2010). Therefore, considering the hypothesis that the marginal effects of the explanatory variables on the inflation rate may differ, at various points of the conditional distribution, this study seeks to contribute to the understanding of asymmetry in the Phillips Curve by testing an approach not yet explored in the Brazilian case.

The main contribution of this paper, besides the use of Quantile Regressions to analyze Brazilian Phillips Curve, is to advance in the investigation of asymmetry through three sources: i-Testing the change in slope of the dependent variables along the quantiles of the response (inflation); ii-Verifying the existence of differences in the dispersion of the conditional distributions of inflation; iii-Calculating the probabilities of future change in the trajectory of inflation, according to the conditional value in which it is.

First, we found that wage mass was the best proxy for the real side of the economy, even with few quantiles of significance, and it seems that there is a gain in the estimates when we use such a variable. The results also provide evidence that the forward-looking and backward-looking

[1] See Wei (2016)

components appear to be significant in all mean estimates (OLS and GMM), with coefficients close to 0.5 and 0.6 respectively. In addition, the quantile model indicates that the behavior of these variables is increasing throughout the distribution. However, after performing the estimates of moving blocks bootstrap (MBB) confidence intervals, we found that only the forward-looking component presents a statistically relevant difference compared to the OLS and GMM values.

In the estimation of the density functions, we find evidence of asymmetry through changes in the dispersion of distributions (conditional standard deviation and conditional range) and through the estimated probabilities that inflation will shift z standard deviations above or below its previous value, conditioned to the expansion and recession scenarios.

2 Evolution of The Phillips Curve

The Phillips curve represents a famous macroeconomic identity based on the correlation between prices and real aggregates. The theoretical construction was originally proposed by Alban William Phillips in 1958, which suggested that low levels of unemployment led to higher wages. According to the original view, the Phillips Curve was interpreted as a stable long-term trade-off that provided a set of possibilities between inflation and unemployment for optimal policy choice. Its initial formulation follows:

$$\pi_t = \alpha - \gamma u_t + \sum_{i=1}^{N} \beta_i \pi_{t-i} \tag{1}$$

where π_t is the inflation rate, u_t is the unemployment rate and α, γ, β_i for $i = (1..., N)$ are parameters.

However, in the first part of the 1970s, inflation and unemployment increased together. This phenomenon, named "stagflation", weakened the view of an inverse and stable relation between variables. Authors of the New Classical school, such as Lucas (1972) and Sargent and Wallace (1973), explored more the phenomenon. They argued that demand-driven policies (fiscal and monetary) could not have an impact on output and employment, both in the short and long terms, due to the validity of rational expectations and market clearing assumptions. Thus, for these academics, the inflation's dynamic followed a mix between rational expectations and flexible prices.

This proposition of monetary policy neutrality, in the adjustment process of the real variables, came into conflict with the data analyzed in subsequent periods and created a crisis in the New Classical ideas. But it was only in the 1990s that the New Keynesian models, based on rational expectations and price rigidity, revived the discussion about the Phillips Curve, showing new theoretical arguments with empirical evidence. The attention given to New Keynesian Phillips Curve (NKPC) emerges due to the possible relation between real and monetary variables, proposed in the Phillip's equation. According to Gali (2008), inflation represents a level of economic inefficiency that is measured in terms of gap component and nominal rigidity. In other words, if monetary policy seeks to stabilize prices, then it contributes significantly to increasing the efficiency of economic activity.

To reaffirm this effect of monetary variables on real economic cycles, the formulation of the New Keynesian Phillips Curve (NKPC) was based mainly on three contributions: Taylor (1980), Rotemberg (1982) and Calvo (1983). The NKPC has been widely used in dynamic inflation models for monetary policy analysis. In short, Christiano and Evans (2005), Blanchard and Galí (2007) and Gali (2011) argue that the construction of NKPC is a combination of real business cycles theory with central aspects of Keynesian theory. This specification is usually obtained by rigorous mathematical derivations that take into account the interaction of agents in a micro-based economic system, with firms and consumers optimizing their choices.

The mathematical derivations focus on the effects of price and wage rigidity on the cyclical fluctuations of output and unemployment, which the authors above understand as fundamental components of Keynesian theory. When we assume an economy with imperfect competition and information asymmetry, the effects of rigidity pass through the channel of adjustment between aggregate supply and demand. Thus, if agents form their expectations in a "rational way" [2], rigidity affects the dynamics of the economy on both real and monetary sides.

The new equation was obtained by a Dynamic Stochastic General Equilibrium (DSGE) model [3], which considers nominal rigidity following Calvo (1983). In this formulation, a fraction θ of the firms does not readjust their prices and a fraction $(1-\theta)$ chooses new prices. Therefore, the level of aggregate prices can be written as:

$$p_t = \theta p_{t-1} + (1-\theta) p_t^* \tag{2}$$

p_t represents the level of aggregate prices in the current period, p_{t-1} the level of aggregate prices in the previous period and p_t^* the optimal price. But if we consider mc_t the marginal cost, μ the natural value of this cost, and β the subjective discount rate, the optimal price choice p_t^* can be written as:

$$p_t^* = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[mc_{t+k|t} + p_{t+k} \right] \tag{3}$$

We can notice that the decision of optimal price in t takes into account the variation of the expected future marginal cost, bringing this to present values. In the threshold situation, when all firms readjust the price $(\theta = 0)$, the price deviation equals the marginal cost deviation and the price decision occurs according to the expected marginal cost.

If we write inflation as the price difference in two periods $\pi_t = p_t - p_{t-1}$, then combination of (2) and (3) allows us to find the following Phillips Curve:

$$\pi_t = \beta E_t [\pi_{t+1}] + \lambda \hat{mc}_t \tag{4}$$

Where $\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\left(\frac{1-\alpha}{1-\alpha+\epsilon\alpha}\right)$, considering the following parameters: ϵ the elasticity of substitution, α the weight parameter of the production function and $\hat{mc}_t$ the deviation of marginal costs from the natural value. Basically, according to this approach developed by New Keynesians, current inflation became a function of future inflation expectations and real marginal costs.

Now let y_t be the logarithm of the output and y_t^* the logarithm of the potential output, we can write the gap as $x_t = y_t - y_t^*$. Gali (2008) shows that using this transformation we can assume a linear log approximation between the output gap and the marginal cost of the companies:

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) x_t \tag{5}$$

σ represents the consumer's risk aversion and φ the labor supply elasticity. From (5), we can rewrite NKPC with forward-looking component, such as:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t \tag{6}$$

where $\kappa = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\lambda$.

But the representation (6) was severely criticized for omitting the inflationary inertia verified in data. This persistence problem was solved by constructing a model that included the term backward-looking and led to better empirical performance. Gali and Gertler (1999) constructed this variant from Calvo's price structure, presented above, and complemented the analysis by assuming that only a part ω of the optimizing firms uses all the available information to determine the price p_t^f, the

[2] For rational way, we mean that, on average, the expectations of economic agents about variables are correctly formed when there is a coincidence between their particular expectations and the mathematical conditional expectation of the stochastic process, as emphasized by Muth (1961).

[3] See Gali (2008) for more details of derivations.

remaining $(1-\omega)$ choose p_t^b following a simple rule based on the past behavior of the aggregate prices. The specification (2) holds, but now p_t^* follows a new predetermined equation:

$$p_t^* = \omega p_t^b + (1-\omega) p_t^f \tag{7}$$

With:

$$p_t^b = \pi_{t-1} + p_{t-1}^f \tag{8}$$

Then, starting from this change in the dynamics of price rigidity, Gali and Gertler (1999) classify the "New-Keynesian Hybrid Phillips Curve" (NKPC) as:

$$\pi_t = \beta_b \pi_{t-1} + \beta_f E_t[\pi_{t+1}] + \Lambda x_t \tag{9}$$

The curve parameters can be recovered structurally as: $\beta_b = \frac{\omega}{\theta+\omega(1-\theta(1-\beta))}$, $\beta_f = \frac{\beta\theta}{\theta+\omega(1-\theta(1-\beta))}$ and $\Lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega(1-\theta(1-\beta))}$.

Finally, Blanchard and Galí (2007) demonstrate that we can rewrite the NKPC in a third way. Starting from a model based on the second best, they introduce exchange rate shocks and economic cycles in the Phillips Curve. The derivation follows the same steps as the standard New-Keynesian model, but now the stabilization of the gap is no longer desirable, since the gap between the first and second best output levels is not constant, reacting to the shocks.

In other words, including the unemployment rate (u_t) and changes in the price of non-produced inputs (Δv_t), we have:

$$\pi_t = \Psi_1 \pi_{t-1} + \Psi_2 E_t[\pi_{t+1}] - \Psi_3 u_t + \Psi_4(\Delta v_t) \tag{10}$$

where $\Psi_1 = \frac{1}{1+\beta}$, $\Psi_2 = \frac{\beta}{1+\beta}$, $\Psi_3 = \frac{\lambda(1-\alpha)(1-\gamma)\varphi}{\gamma(1+\beta)}$ and $\Psi_4 = \frac{\alpha\gamma}{(1+\beta)}$. Most of the parameters have already been presented, except for the real rigidity index (γ).

However, before reviewing the papers that try to estimate the Phillips Curve in the Brazilian case, it is important to highlight a current problem reported in the literature. For this, we first present two determinant equations of the standard New Keynesian model:

$$x_t = -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[x_{t+1}] \tag{11}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_x x_t + v_t \tag{12}$$

Equation (11) is known as *dynamic IS* and (12) as Taylor's rule. Where r_t^n is the natural level of the real interest rate, i_t is the nominal interest rate and v_t is an exogenous component with zero mean. Finally, ϕ_π and ϕ_y are non-negative coefficients, chosen by the monetary authority, and ρ is the intercept.

These equations make clear the main problem in the estimation of New Keynesian Phillips Curve: endogeneity. Substituting (12) into (11) we can write the gap as a function of current inflation, which in NKPC is the dependent variable. If we manipulate the equations, it is possible that inflation expectations also affect current inflation. However, there are feasible solutions that allow more consistent estimates of this dynamic, such as instrumentalisation strategies and robust methods, which will be discussed in the next topic.

3 What About the Data?

Recently, with the possible effects of the nominal variables on the real ones, there was a growing interest in the use of the NKPC for monetary policy analysis. But the empirical results diverge considerably.

Gali and Gertler (1999) estimate NKPC for the US economy and find that real marginal cost and inflation expectations are important in determining current inflation. In this same approach, Gali and Lopez-Salido (2005) observe similar results for the Euro area. The authors note that even the simplest version of the NKPC, without the backward-looking component, represents a good approximation of the inflation dynamics in the United States and Europe.

Other studies such as Rudd and Whelan (2005) Rudd and Whelan (2007) and Stock and Watson (2007) provide a counterpoint to previous results. For these researchers, estimates of the New Keynesian Phillips Curve that consider the forward-looking term but omit the inertial component, verified in the data, can not be a good approximation of reality.

However, even the work focused on the hybrid analysis diverges about the validity and significance of the hypotheses: Roberts (2001) and Estrella and Fuhrer (2002) present evidences that the backward-looking component appears to have a large significant effect on US inflation, but the results of Roeger and Herz (2012) show the prevalence of the forward-looking models after testing traditional and New Keynesian Phillips curves specifications.

In the Brazilian case, some papers try to estimate the parameters of the Phillips curve. Minella and Muinhos (2003) develop a research for the period from 1995 to 2002 and provide important results about Brazilian price dynamics and monetary policy. The authors focus on the Taylor Rule and Phillips Curve estimates. Using IPCA for price variation and the unemployment measured by IBGE, they find that the Hybrid Phillips Curve, without expectations, has parameters of 0.56 to 0.62 for lagged inflation and -0.09 to -0.08 for unemployment, these results vary according to the dummies specifications and lag inclusions.

Mendonca and Santos (2006) investigate the effects of monetary credibility on the Phillips curve in the period after the implementation of the target regime. The variables used are the open unemployment rate and the inflation expectations, published monthly between 2000 and 2005 by IBGE and Brazilian Central Bank, respectively. Estimates indicate that the use of credibility improves the predictive power of regression. The inflation expectations present a parameter between 0.43 and 0.96 and the unemployment gap between -0.09 and -0.16[4].

Areosa and Medeiros (2007) test a variation of the NKPC in a structural model. The variables used as proxies for gap are: wage mass and industrial output. For inflation, the IPCA is adopted as a measure of prices. The Generalized Method of Moments (GMM) estimates indicate that lagged inflation is significant and has a coefficient close to 0.45 in the closed economy, but in the case of the open economy the range is 0.1 to 0.37. In addition, expectations are dominant in both models, with a coefficient of 0.53 in the closed model and between 0.63 to 0.81 in the open one. The impact of the real side, measured by marginal costs, is not significant in the first case and negligible in the second, for both proxies. Finally, the authors conclude that the introduction of the exchange rate, as a variable of commercial openness, seems to be important in the Brazilian case, positively affecting the forward-looking component estimates, from 1995 to 2003.

In the other hand, Mazali and Divino (2010) emphasize the importance of the backward-looking component in the Brazilian data adjustment. They advance estimating the Phillips Curve, from 1995 to 2008, using a similar version presented by Blanchard and Galí (2007). After application of the GMM method, the parameters found were 0.59 for lagged inflation, 0.44 for inflation expectations and about -0.13 for unemployment.

Based on more recent data, Mendonca and Medrano (2012) suggest that the modified version of the NKPCH has difficulties in representing the Brazilian price dynamics. For these researchers, only the effects of inflation expectations and lagged inflation remained robust on inflation dynamics. In other words, econometric divergences also occur in the Brazilian case, often through the use of different methods, proxies and instruments.

Currently, following the debate about the real business cycles, there is a growing interest in the possibility of asymmetries in the Phillips curve. Although the traditional theory suggests a linear relationship, authors such as Buchmann (2009) and Balaban and Vintu (2010) argue that the Phillips Curve itself has never intended to describe a symmetrical relationship. This asymmetry was verified

[4] Again, the results differ according to the specifications

in some countries: United States, Sweden and Australia by Eliasson (2001), Canada by Huh and Lee (2002), South Africa by Nell (2006), Brazil by Correa and Minella (2010), Euro Area by Chorteas and Panagiotidis (2012) and Turkey by Boz (2013).

An approach that has gained some attention in the studies of asymmetry is the quantile regression. Chorteas and Panagiotidis (2012) examine the asymmetry in distribution of Euro Area inflation at various quantiles. They estimate NKPC using two-stage quantile regressions to solve the problem of endogeneity. The results suggest that the inflation response, over the years, is asymmetric at various quantiles. But when inflation is high, the forward-looking component is significant and dominates the lagged component. Boz (2013) performs the same estimation procedure for Turkey, finding relevant differences in inflation response to changes in explanatory variables at various points of the distribution.

As we saw, in Brazil the Generalized Method of Moments (GMM) has been widely used in the estimates of the Phillips Curve components. The main argument, of this application, is usually the robustness for the treatment of the endogeneity bias, which in NKPC is caused by inflation expectations and output gap. However, starting from the strong internal and external oscillations that Brazil experienced in the last decade, according to Aragon and Medeiros (2015), we can expect some asymmetry in the effect of Phillips Curve components on inflation. Then, these mean estimations presenting an incomplete picture of the data distribution and ignore asymmetry.

Some papers find evidence of this asymmetry using the following formulations: i) Estimation of nonlinear forms and functional variants of the Brazilian Phillips Curve, by Correa and Minella (2010), Carvalho (2010) and Arruda and Castelar (2011) and ii) Investigation of structural breaks over the years, by Medeiros and Aragon (2015).

We propose a different approach, that allows us to study the asymmetric behavior of the Brazilian cycles through the Phillips curve. For this, we will use Two Stage Quantile Regressions (TSQR) that provide a larger picture of the data distribution and allow us to solve endogeneity problems from the New Keynesian Phillips Curve. In addition, we advance in the investigation of asymmetry by estimating the conditional density functions.

4 How to Check the Asymmetry?

4.1 Quantile Regression

We can define a quantile τ as the value q, such that $100\tau\%$ of the sample values are less than q, with $0 < \tau < 1$. This definition can be stated using the cumulative distribution of a random variable X:

$$F(x) = P(X \leq x) \tag{13}$$

If we use the inverse function of the cumulative distribution above, at point τ, we have that the quantil τ of the random variable X is:

$$F^{-1}(\tau) = inf\{x : F(x) \geq \tau\} \tag{14}$$

Then, the inverse function gives us the quantile value for the probability chosen from the *infimum* of those with a probabilistic sum greater than or equal to the quantile.

The Quantile Regressions (QR) method was introduced by Koenker and Bassett (1978). From this analysis, the researcher can estimate the relationship between a set of explanatory variables x and the τ quantile of the dependent variable y. Unlike the OLS models that are estimated on the mean of the response variable distribution, this approach is a useful technique because it allows us to study the effect of an explanatory variable at various quantiles of the dependent y_t. In other words, the QR models are able to incorporate heteroscedasticity, since they allow us to verify if the coefficients of the explanatory variables change significantly (statistically) at different points of the dependent variable distribution.

Considering a vector of continuous response variables $y = (y_1, y_2, ...y_t)$ and another vector of explanatory $x = (x_{1i}, x_{2i}, ..., x_{ki})$, with subscript i representing the series within the same variable, $i = (1, 2, ...t)$. A standard linear regression model can be written as $E(y|x) = x^{'}\beta$, such that β is a vector of k parameters.

Now a quantile regression model can be understood as $Q_y(\tau|x) = x^{'}\beta(\tau)$, such that $\beta(\tau)$ is a matrix with dimensions of k parameters by τ quantiles, representing the effects of explanatory variables at various points of y. The regression parameters $\beta(\tau)$ are conditioned to the τ-quantile and estimates can be obtained by the solution below:

$$\min_{\forall\beta\in R}\sum_{i=1}^{t}\rho_\tau(y - x^{'}\beta(\tau)) \tag{15}$$

Given ρ_τ as a linear loss function:

$$\rho_\tau(u) = \begin{cases} \tau u, u \geq 0 \\ (\tau - 1)u, u < 0 \end{cases} \tag{16}$$

Replacing:

$$Q(\beta(\tau)) = min\left[\tau\sum_{i:y\geq x^{'}\beta(\tau)}^{t}|y - x^{'}\beta(\tau)| + (1-\tau)\sum_{i:y<x^{'}\beta(\tau)}^{t}|y - x^{'}\beta(\tau)|\right] \tag{17}$$

This non-differentiable function requires linear programming methods for their minimization. This problem can be summarized in:

$$\min_{\beta(\tau)\in R}\tau U + (1-\tau)V \tag{18}$$

$$\boldsymbol{s.a.}\ Y = \beta(\tau)X + U - V \tag{19}$$

The error vector u is composed of U and V. These terms represent the positive and negative parts of the regression residuals, respectively. Two approaches are commonly used in the solution of this problem: the *Simplex* method for moderate-size samples or the *Interior Point* method for larger databases, both guarantee a solution with finite number of iterations.

Finally, the construction of the confidence intervals is performed by the moving blocks bootstrap standard errors, which are more commonly used than the standard analytical errors [5]. The moving blocks bootstrap methodology is preferable since it makes no assumption about the distribution of the response variable, being able to generalize the (QR) results and estimate the intervals in any case of residual distribution and provides heteroscedasticity and autocorrelation robust standard errors.

4.2 Density Estimation

In order to investigate the possible asymmetry in response variable distributions, we estimate the conditional quantile density function $f(F^{-1}(\tau))$ according to Koenker and Xiao (2004):

$$f_n(F_n^{-1}(t)) = \frac{2h_n}{F_n^{-1}(t+h_n) - F_n^{-1}(t-h_n)} \tag{20}$$

where the function $F_n^{-1}(s)$ is an estimate of $F^{-1}(s)$ and h_n is a bandwidth. Here, we obtain $F^{-1}(s)$ using the empirical quantile function for the linear model proposed by Bassett and Koenker and Bassett (1978):

[5] Even when the residual errors are asymptotically distributed according to a normal

$$\hat{Q}(\tau|\overline{x}) = \overline{x}^T\hat{\alpha}(\tau) \tag{21}$$

Taking this into account, we can estimate $f(F^{-1}(t))$:

$$f_n(F_n^{-1}(t)) = \frac{2h_n}{x^T(\hat{\alpha}(t+h_n) - \hat{\alpha}(t-h_n))} \tag{22}$$

4.3 Dissecting the Cycle

To control the periods of expansion and recession applied to the density functions, we first need to date the Brazilian economic cycles. For this we use the methodology proposed by Harding and Pagan (2002), for quarterly data, which theoretically follows the seminal concepts developed by Prescott (1986), Plosser (1989) and Kydland and Prescott (1982) in the Real Business Cycles approach.

For detection of the cycles, Harding and Pagan (2002) expand the Bry and Boschan (1971) algorithm. From some rules imposed to the behavior of the series it is possible to classify peaks and troughs and, consequently, phases of expansion and recession. The essence of the algorithm consists:

1. Determination of a potential set of turning points, i.e. the peaks and troughs in a series.

$$\textit{There is a } \boldsymbol{peak} \textit{ in } t \textit{ if } \{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\}$$
$$\textit{There is a } \boldsymbol{trough} \textit{ in } t \textit{ if } \{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\}$$

2. A procedure for ensuring that peaks and troughs alternate.

3. A set of rules that re-combine the turning points established after steps one and two in order to satisfy pre-determined criteria concerning the duration and amplitudes of phases and complete cycles; what we will refer to as "censoring rules".

Thereafter, a minimum period is required for the duration of a phase cycle, i.e., the time elapsed between a peak (troughs) and a trough (peak), and also a minimum duration for the cycle, from peak to peak or from troughs to troughs. These restrictions eliminate oscillations and noise that are not related to the business cycle.

In this work, the specifications used for the quarterly data were: i) Minimum length of a cycle = 5; and ii) Minimum length of a phase of a cycle = 2. Thus, using software R, we try to calculate the cycles so that the recessions are the most compatible possible with those dated by CODACE-FGV, for a longer time series.

4.4 Our Proposal

Given the empirical evidence discussed in the previous section, we estimate two versions of the New Keynesian Phillips Curve, equations 9 and 10, which take into account the presence of the backward-looking component that seems to be relevant in Brazil:

$$\pi_t = \beta_b \pi_{t-1} + \beta_f E_t[\pi_{t+1}] + \Lambda x_t \tag{23}$$

$$\pi_t = \Psi_1 \pi_{t-1} + \Psi_2 E_t[\pi_{t+1}] - \Psi_3 u_t + \Psi_4(\Delta v_t) \tag{24}$$

But these versions of the Phillips curve present an endogeneity problem. Therefore, the simple Quantile Regression estimation is inconsistent and must be replaced by the Two Stage Quantile Regression (TSQR) method, which is robust in cases of endogenous variables. We will follow Kim and Muller

(2004) to perform this procedure. The authors argue, through Monte Carlo simulations, that two-stage instrumental quantile estimations lead to biased coefficients when the first stage is done by OLS regressions.

Then, in the first stage we estimated τ quantile regressions for each endogenous variable as a function of a set of instruments, saving the adjusted values. In the second step, we replaced these endogenous variables by the adjusted values of the previous regressions and estimated the Phillips Curves using the QR method for the same quantiles. Furthermore, to deal with heteroskedasticity and autocorrelation we use the *Moving Block Bootstrap* (MBB) method by Fitzenberger (1998).

After the estimations, we focused on the investigation of the asymmetry present in the New Keynesian Phillips Curve. For this, we estimate the conditional density functions for the response variable (inflation) and we measure how the economic cycles affect the dynamics of this variable by two additional sources: i) Verifying the existence of differences in the dispersions of the conditional distributions of inflation; ii) Calculating the probabilities of future change in the trajectory of inflation, according to the conditional value in which it is. The variables used and the data sources are presented in the table below.

Table 1 – NKPC - Variables

Name	**Time**	**Source**	**(9.1)**	**(9.2)**	**(10)**
Current Inflation	2002:03-2016:02	IBGE	π_t	π_t	π_t
Lagged Inflation	2002:03-2016:02	IBGE	π_{t-1}	π_{t-1}	π_{t-1}
Inflation Expectations	2002:03-2016:02	BACEN	$E_t[\pi_{t+1}]$	$E_t[\pi_{t+1}]$	$E_t[\pi_{t+1}]$
Output Gap	2002:03-2016:02	IBGE	x_t	-	-
Wage Mass	2002:03-2016:02	IBGE	-	x_t^{wm}	-
Unemployment Rate	2002:03-2016:02	IBGE	-	-	u_t
Nominal Exchange Rate	2002:03-2016:02	BACEN	-	-	v_t

In the specification 9 all the data are analyzed in monthly frequency. The inflation rate (π_t) is measured by the IPCA and seasonally adjusted using the X13- ARIMA. For inflation expectations ($E_t[\pi_{t+1}]$) we use the Central Bank reports based on the FOCUS estimates for next month's inflation, but since these data have daily frequency we select the median for each month.

For the gap variable (x_t), which is often treated as the marginal cost of the economy [6], we chose two proxies based on the works of Gali and Gertler (1999) and Sims (2008): 1) The Brazilian industrial output and 2) The share of total wages in output. We calculate (1) as the difference between the industrial production index, seasonally adjusted using the X13- ARIMA, and its potential value obtained through the Hodrick-Prescott filter. Finally, (2) is constructed as the ratio between the effective wage mass of the economically active population and the nominal GDP, seasonally adjusted using the X13-ARIMA method.

For the specification 10 we have inflation and expectations following the same source as the previous specification. For unemployment (u_t) proxy, we use the monthly open unemployment rate, calculated by IBGE for the metropolitan regions, seasonally adjusted by the X13-ARIMA method and submitted to the Hodrick-Prescott filter. For changes in the prices of the non-produced input, (Δv_t), we follow Mazali and Divino (2010) and calculate the percentage change in the nominal exchange rate between real and dollar for a three-period interval, according to the following formula $\Delta v_t = 100ln\left(\frac{v_t}{v_{t-3}}\right)$.[7]

The following variables are endogenous: Output Gap, Wage Mass, Inflation Expectations, Unemployment and Exchange Rate. In this context, following Blanchard and Galí (2007), Mazali and

[6] See the derivations of the New Keynesian model, Gali (2008)

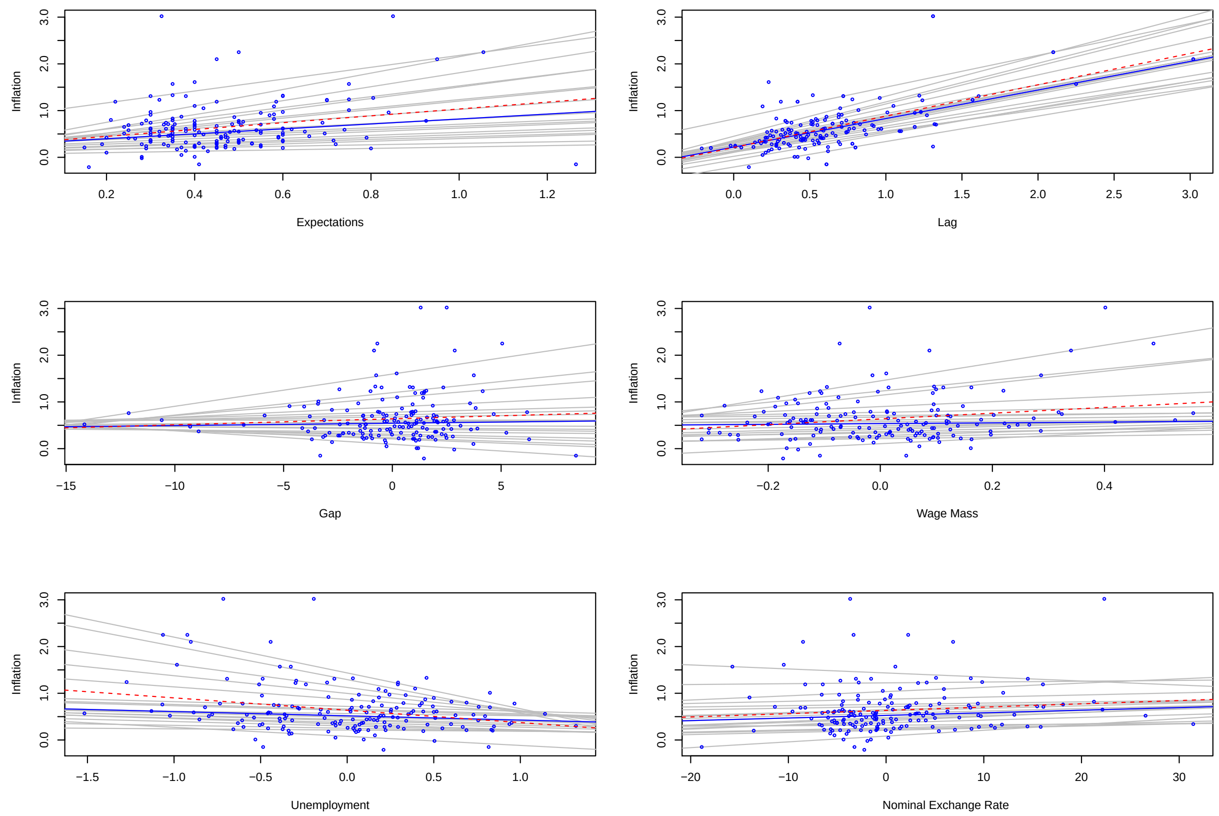
[7] All variables, after treatments, are stationary. The tests used were ADF, ADF-GLS, PP and KPSS.

Divino (2010), Chorteas and Panagiotidis (2012) and Boz (2013), all these variables were instrumentalized using two inflation lags and two lags of the variable itself, except in the case of inflation expectations that the literature suggests the use of only one lag.[8]

To motivate the investigation, we estimate simple quantile regressions to illustrate the relationship between inflation and the other variables used in the proposed specifications: expectations, lagged inflation, output gap, wage mass, unemployment and nominal exchange rate.

We consider the quantiles $\tau = \{0.05, 0.10, 0.15, ...0.90, 0.95\}$. The blue line represents the OLS estimation, the red line the QR estimation in the 0.50 quantile and the gray lines represent the other estimated quantiles.

Figure 1 – Fitted Lines



Initially, we observed that for all variables the estimates in the mean and median already differ from each other. Expectations, the lagged component, and the wage mass appear to be positively related to current inflation, especially in the higher quantiles of the inflation distribution.

On the other hand, as expected, unemployment appears to negatively affect inflation. Finally, the gap and the nominal exchange rate, initially, appear to have ambiguous behavior (change from positive to negative and negative to positive respectively) depending directly on the quantile analyzed.

5 Results

5.1 Two Stage Quantile Regression

The estimated coefficients of the New Keynesian Phillips Curve follow in the tables below. The comparison values are the OLS and GMM instrument estimates, which are in the first and second

[8] From the Cragg-Donald and Kleibergen-Paap tests, we reject the null hypothesis that the chosen instruments are weak for endogenous variables.

lines. Asterisks indicate statistical significance: one asterisk is relative to 1 %, two asterisks 5% and three 10%. In addition, all standard error estimates and the covariance matrix of the regressions were based on the moving blocks bootstrap method with 10,000 replications.

Table 2 – Results NKPH (9)

		Industrial Output Gap			**Wage Mass**		
Estimation	**Quantile**	β_b	β_f	Λ	β_b	β_f	Λ
OLS		$0,636^{*}$	$0,501^{*}$	0,003	$0,626^{*}$	$0,513^{*}$	$0,293^{***}$
GMM		$0,639^{*}$	$0,490^{*}$	0,002	$0,609^{*}$	$0,536^{*}$	$0,285^{***}$
	0,05	0,227	0,141	-0,026	0,305	-0,006	$0,689^{***}$
	0,10	$0,459^{*}$	0,047	-0,017	$0,411^{*}$	0,131	0,371
	0,15	$0,426^{*}$	$0,220^{**}$	$-0,014^{***}$	$0,470^{*}$	$0,189^{***}$	$0,301^{**}$
	0,20	$0,467^{*}$	$0,233^{*}$	-0,011	$0,490^{*}$	$0,226^{*}$	$0,257^{**}$
	0,25	$0,493^{*}$	$0,262^{*}$	-0,010	$0,525^{*}$	$0,229^{*}$	$0,271^{**}$
	0,30	$0,551^{*}$	$0,260^{*}$	-0,006	$0,561^{*}$	$0,240^{*}$	$0,317^{**}$
	0,35	$0,578^{*}$	$0,295^{*}$	-0,002	$0,549^{*}$	$0,296^{*}$	0,234
	0,40	$0,616^{*}$	$0,318^{*}$	0,001	$0,581^{*}$	$0,340^{*}$	0,162
	0,45	$0,594^{*}$	$0,405^{*}$	0,004	$0,581^{*}$	$0,406^{*}$	0,12
QR	**0,50**	$0,582^{*}$	$0,459^{*}$	0,005	$0,591^{*}$	$0,449^{*}$	0,052
	0,55	$0,562^{*}$	$0,577^{*}$	0,002	$0,569^{*}$	$0,564^{*}$	0,08
	0,60	$0,590^{*}$	$0,606^{*}$	0,003	$0,616^{*}$	$0,594^{*}$	0,158
	0,65	$0,614^{*}$	$0,638^{*}$	0,001	$0,606^{*}$	$0,671^{*}$	0,172
	0,70	$0,604^{*}$	$0,745^{*}$	-0,003	$0,605^{*}$	$0,720^{*}$	0,148
	0,75	$0,584^{*}$	$0,894^{*}$	0,006	$0,579^{*}$	$0,878^{*}$	0,274
	0,80	$0,619^{*}$	$0,973^{*}$	0,005	$0,674^{*}$	$0,855^{*}$	0,197
	0,85	$0,660^{*}$	$1,027^{*}$	0,009	$0,671^{*}$	$0,968^{*}$	0,051
	0,90	$0,698^{**}$	$1,104^{*}$	0,014	$0,677^{*}$	$1,139^{*}$	0,037
	0,95	$1,038^{***}$	$1,467^{*}$	$0,044^{***}$	0,723	$1,616^{*}$	0,918

Table 2 presents the version 9 of the NKPCH, with the two *proxies*. In the left side, for the Industrial Output Gap, according to the OLS and GMM instrument estimates, we observed that on average the coefficients of inflation expectations and lagged inflation are statistically significant, with values close to 0.50 and 0.60 respectively. However, the *proxy* used to representing the real side of the economy was not significant.

The QR results show that the marginal effects of the explanatory variables on the inflation rate oscillate at various quantiles. When the inflation rate is low, between the 10% and 25% quantiles, the backward-looking component is significant and less than the estimated mean value (OLS and GMM). In upper quantiles, 75 % to 95 %, we also observed significance of this variable, but weaker in the last quantiles and without large differences of the mean models (except for the 95% quantile).

The *forward-looking* component is significance from the third quantile (15 %) onwards. This variable follows behavior similar to the lagged component, increasing, but it presents larger deviations from the OLS and GMM values.

Finally, the estimated coefficients for the gap also vary, diverging about the positive or negative impact at different quantiles, but are not statistically significant in most of distribution, indicating a possible absence of effects on inflation behavior.

The right side of Table 2 considers the wage mass as proxy for the gap. The effects of the backward-looking and forward-looking components on inflation are similar to the previous simulation, for both the estimates: instrumental (OLS and GMM) and quantile. In addition, it seems that the expectations component also fluctuates significantly among the extremes of the inflation distribution.

The second proxy for the gap is significant at the 10% level in the OLS and GMM estimates. In quantile regressions, the variable presents significance only in five quantiles of the lower tail, with small deviations from the mean value. Thus, even with little expressive variations, it seems that there is an advance in the estimates using this proxy.

Table 3 – Results NKPH 10

Estimation	Quantile	Ψ_1	Ψ_2	Ψ_3	Ψ_4
OLS		$0,624^{*}$	$0,508^{*}$	$-0,102^{**}$	0,004
GMM		$0,658^{*}$	$0,468^{*}$	-0,087	0,005
	0,05	0,200	0,035	0,024	$0,008^{***}$
	0,10	$0,440^{*}$	0,092	-0,122	0,003
	0,15	$0,455^{*}$	0,170	$-0,146^{*}$	0,001
	0,20	$0,531^{*}$	0,176	$-0,100^{**}$	0,002
	0,25	$0,515^{*}$	$0,225^{***}$	$-0,089^{**}$	0,001
	0,30	$0,483^{*}$	$0,371^{*}$	$-0,096^{**}$	0,002
	0,35	$0,530^{*}$	$0,366^{*}$	$-0,086^{***}$	0,002
	0,40	$0,565^{*}$	$0,390^{*}$	$-0,087^{***}$	0,003
	0,45	$0,572^{*}$	$0,430^{*}$	$-0,084^{***}$	0,002
QR	**0,50**	$0,574^{*}$	$0,474^{*}$	$-0,092^{***}$	0,003
	0,55	$0,606^{*}$	$0,505^{*}$	$-0,092^{***}$	0,004
	0,60	$0,631^{*}$	$0,518^{*}$	-0,088	0,005
	0,65	$0,626^{*}$	$0,588^{*}$	-0,048	0,004
	0,70	$0,622^{*}$	$0,675^{*}$	-0,005	0,004
	0,75	$0,637^{*}$	$0,768^{*}$	0,029	0,004
	0,80	$0,633^{*}$	$0,971^{*}$	0,005	-0,0008
	0,85	$0,674^{*}$	$1,000^{*}$	0,003	-0,002
	0,90	$0,658^{*}$	$1,310^{*}$	-0,098	-0,004
	0,95	$0,940^{***}$	$1,521^{*}$	-0,255	-0,009

Table 3 presents the second version of the New-Keynesian Phillips Curve, suggested by Blanchard and Galí (2007). In the Brazilian case, again, we can observe that inflation expectations and lagged inflation are fundamental. These terms behave similarly to the curve previously estimated, but the significance of the expectations component in QR estimation only occurs from 25% quantile onwards. We observed evidence that for QR estimation both components of inflation appear to have an increasing impact, with more expressive oscillations in the forward-looking term.

Unemployment is significant at the 10% level only in the OLS instrumental estimation and the exchange variation is not significant in the mean (GMM and OLS) estimates. For quantile analysis, unemployment has a weak significance from 15 % to 55 % quantiles, but without large fluctuations (compared to the mean estimates). On the other hand, the exchange variation only appears to be significant at the 10% level in the first estimated quantile, weakening the evidence that this variable is a determinant of current inflation in Brazil, for the analyzed period.

Briefly, the first part of the analysis provided evidence that the backward-looking and forward-looking components appear strongly significant in the Phillips curve estimates, with few differences between estimated versions. The OLS and GMM estimates indicate dominance of the backward-looking component, but the quantile model shows that this dominance is only sustained at the lower tail of the current inflation distribution. During periods of high inflation the forward-looking component is expressively dominant, reinforcing that the oscillation in this variable is more representative.

However, in a complementary way, we need to prove the statistical difference between the quantile estimates and the instrumentalized OLS/GMM values, for this we consider the moving blocks bootstrap confidence intervals.

We will use what was exposed previously, but now restricting the analysis only to the significant variables.

For the next figures, the black dotted line represents the estimated coefficients for each quantile and the shaded region shows the confidence interval for these estimates. In addition, the horizontal lines represent the OLS (black) and GMM (orange) values with their respective confidence intervals (dashed black/orange lines).

Figure 2 – Inflation Expectations

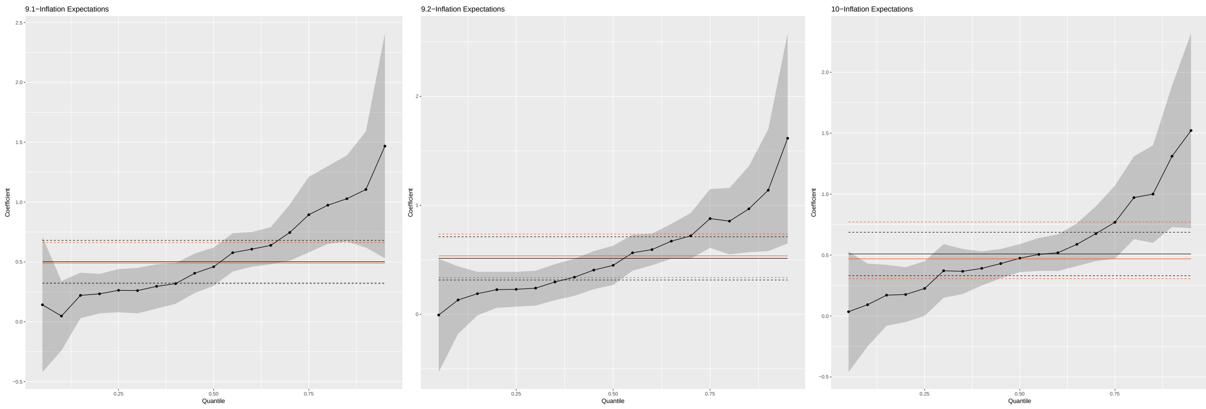


Figure 2 shows what we have already discussed, the forward-looking component grows gradually. But looking at the confidence intervals, it seems that for regions closer to the median we can not reject the equality hypothesis between the OLS/GMM and QR coefficients, because both OLS and GMM confidence intervals covers the QR estimates for these quantiles.

However, at the extreme quantiles, the effect of expectations is expressive. In both the lowest (less than 25%) and the highest (greater than 75%) quantiles, the QR values are outside of the OLS and GMM confidence intervals. Even with a small intersection between the OLS/GMM and QR confidence intervals, the confidence intervals of the mean estimations does not contain the quantile regression values and vice versa, indicating a first validity of the estimates.

Following the analysis, Figure 3 presents interesting results for the backward-looking component. According to the visual analysis of the three estimated Phillips Curve specifications, we can not reject the equality hypothesis between the OLS/GMM and QR coefficients for most quantiles. We can observe that the OLS/GMM confidence intervals cover the QR estimates, except at 5 %, 10 %, 15 % and 95 % quantiles.

Even at the exception quantiles the QR confidence intervals contain the OLS/GMM values, which gives us indications that the statistical difference between these estimates and the OLS and GMM coefficients is not significant.

5.2 Density Estimation

In this second part, we try to capture the asymmetry in the Phillips curve using conditional density functions. For this, we follow the intuition proposed by DeLong and Summers (1988) and Ball and Mankiw (1994), which affirm that when the product was above its potential, level prices increased rapidly, but when the product was below, the prices took longer to fall.

We construct the conditional distribution of inflation based in economic cycles, through the use of Harding-Pagan(2002) methodology, for Brazilian GDP quarterly data. First we delimit the periods of recession and expansion and then we select the points in time, which will be the conditioned values, based on our output gap series. That is, in each cycle we select the extreme points of the gap series and use the values of all variables at that moment.

Figure 3 – Lagged Inflation

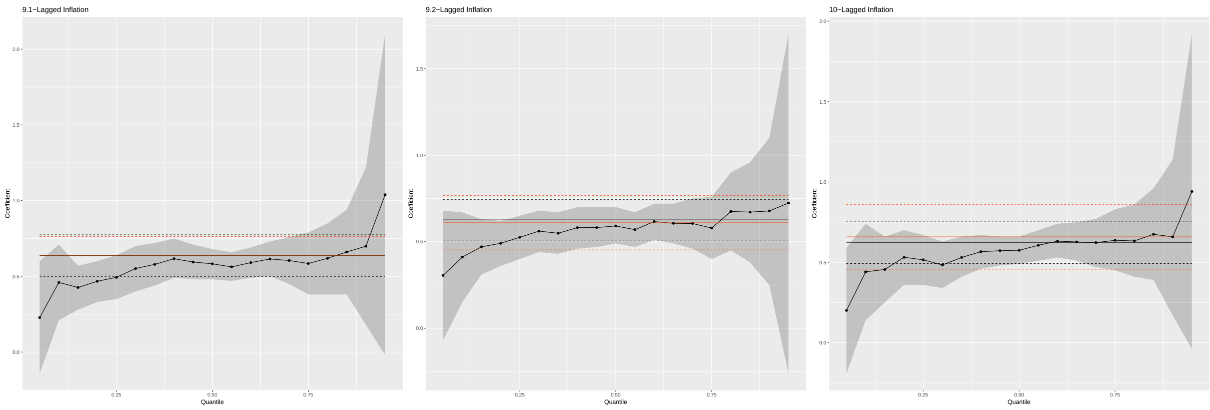


Table 4 – Cycles

Expansions			Recession		
Start	**End**	**Selected**	**Start**	**End**	**Selected**
2002:Q1	2002:Q4	E_1 = **Jun 2002**	2003:Q1	2003:Q2	R_1 = **Jun 2003**
2003:Q3	2008:Q3	E_2 = **Jun 2008**	2008:Q4	2009:Q1	R_2 = **Dec 2008**
2009:Q2	2014:Q1	E_3 = **Jun 2013**	2014:Q2	2016:Q2	R_3 = **Feb 2016**

The points selected represent, respectively, the period where the Brazilian product was more than its potential and the period where it was lower. As all variables presented at least one quantile of significance in all the estimations performed in the previous section, we chose to maintain the original specifications of the Phillips curve, trying to better filter the dynamics of the cycles within the regressions.

Note that when we select specific points in time we can analyze the asymmetry conditional distribution, but we can also infer about the probability of inflation rising or falling in the next period, since all the specifications used in the Phillips curve have the inflation component lagged. Thus, our main objective is to verify if the asymmetric movement reported by DeLong and Summers (1988) and Ball and Mankiw (1994) applies to Brazilian inflation. In other words, if inflation is more likely to rise in periods of expansion than to decline in period of recession.

Our analysis of the density functions will be performed using standard conditional deviation $\hat{\sigma}(\hat{\pi}_t|Q_x(\tau))$, and conditional range, $\hat{R}(\hat{\pi}_t|Q_x(\tau))$. Finally, we calculate the conditional probabilities $(Pr_L(z); Pr_U(z))$ of inflation to be above and below z standard deviations away from the previous conditioned value. Figure 4 and Table 5 summarize the estimates for the six cycles selected in each of the three NKPC specifications used.

Figure 4 – Conditional Density Cycles

(a) Cycle 1 (b) Cycle 2 (c) Cycle 3

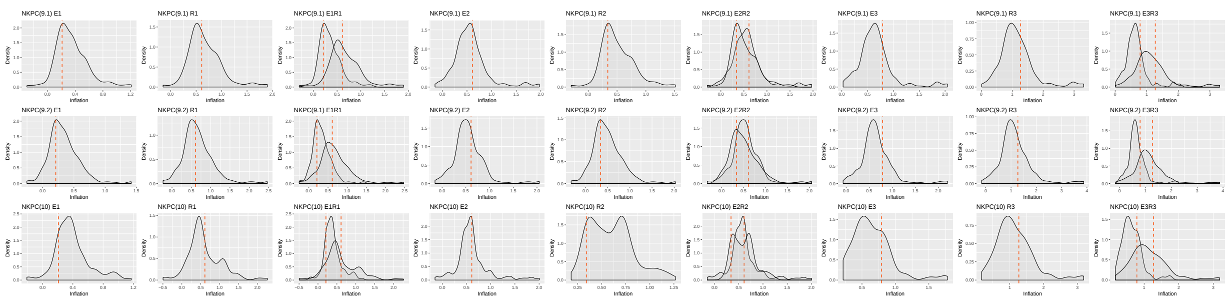


Table 5 – Probabilities

$Q_x(\tau)$	$\hat{\sigma}(\hat{\pi}_t \vert Q_x(\tau))$	$\hat{R}(\hat{\pi}_t \vert Q_x(\tau))$	$Pr_L(z=1)$	$Pr_U(z=1)$	$Pr_L(z=1.5)$	$Pr_U(z=1.5)$	$Pr_L(z=2)$	$Pr_U(z=2)$
NKPC (9.1)								
E_1	0.232	1.507	0.017	0.279	0.005	0.153	0.000	0.072
R_1	0.334	2.049	0.051	0.163	0.014	0.073	0.003	0.044
E_2	0.351	2.119	0.097	0.078	0.028	0.051	0.000	0.04
R_2	0.283	1.807	0.019	0.294	0.008	0.165	0.002	0.075
E_3	0.350	2.021	0.180	0.057	0.055	0.044	0.008	0.038
R_3	0.562	3.304	0.136	0.067	0.043	0.046	0.008	0.037
NKPC (9.2)								
$Q_x(\tau)$	$\hat{\sigma}(\hat{\pi}_t \vert Q_x(\tau))$	$\hat{R}(\hat{\pi}_t \vert Q_x(\tau))$	$Pr_L(z=1)$	$Pr_U(z=1)$	$Pr_L(z=1.5)$	$Pr_U(z=1.5)$	$Pr_L(z=2)$	$Pr_U(z=2)$
E_1	0.243	1.670	0.027	0.235	0.007	0.115	0.311	0.058
R_1	0.405	2.701	0.060	0.121	0.018	0.061	0.001	0.038
E_2	0.322	2.199	0.116	0.063	0.042	0.034	0.008	0.026
R_2	0.345	2.354	0.031	0.202	0.008	0.093	0.000	0.044
E_3	0.339	2.262	0.200	0.044	0.065	0.030	0.018	0.025
R_3	0.614	4.038	0.122	0.064	0.041	0.040	0.007	0.029
NKPC (10)								
$Q_x(\tau)$	$\hat{\sigma}(\hat{\pi}_t \vert Q_x(\tau))$	$\hat{R}(\hat{\pi}_t \vert Q_x(\tau))$	$Pr_L(z=1)$	$Pr_U(z=1)$	$Pr_L(z=1.5)$	$Pr_U(z=1.5)$	$Pr_L(z=2)$	$Pr_U(z=2)$
E_1	0.228	1.378	0.018	0.257	0.004	0.146	0.000	0.084
R_1	0.445	2.757	0.099	0.133	0.030	0.053	0.008	0.034
E_2	0.348	2.154	0.069	0.084	0.034	0.055	0.003	0.043
R_2	0.227	1.083	0.000	0.504	0.000	0.329	0.000	0.125
E_3	0.293	1.560	0.298	0.042	0.087	0.032	0.000	0.026
R_3	0.554	3.015	0.176	0.061	0.043	0.038	0.000	0.029

Initially, we analyzed the dispersion of distributions in terms of conditional standard deviation and conditional range. The results show that $\hat{\sigma}(\hat{\pi}_t|Q_x(\tau))$ and $\hat{R}(\hat{\pi}_t|Q_x(\tau))$ are different for periods of recession and periods of expansion. Moreover, they are larger in periods of recession. This means that we expect a greater variability in the values of conditional inflation, predicted by the NKPC, in cases of recession.

In sequence, we analyze the probabilities that inflation in t being at z standard deviations above and below its value in the previous period. These measures inform not only the asymmetric behavior in the adjustment of the cycle, but also quantify the probabilities of being in the tails. Again, asymmetry is captured when we notice that the probabilities change between states of recession and expansion and even between going up and down in the same cycle.

Furthermore, in most cases, especially at points $(E_1; R_1)$ and $(E_2; R_2)$, we note that the Phillips curve adequately captures the asymmetry of cycles described by DeLong and Summers (1988) and Ball and Mankiw (1994). In other words, the probability of inflation rising Pr_U in times of expansion E_i is usually greater than the probability of inflation falling Pr_L during periods of recession R_i.

6 Conclusions

The main objective of this paper, besides the use of Quantile Regressions to analyze Brazilian Phillips Curve, is to advance in the investigation of asymmetry. First, we test the change in the slope of dependent variables along the quantiles of the response (inflation). We found that the wage mass[9] was significant at the 10 % level in the OLS/GMM estimates and at the 5% level in the QR estimation between the 15 % and 30 % quantiles. However, even with a few quantiles of significance for the real variable, it seems that there is a gain in the estimates when we use such a proxy.

[9] Second proxy used for the real side of economy in specification 9

But the main results of the quantile analysis were about the asymmetric effects of the backward-looking and forward-looking components on current inflation, with few differences in the coefficients between the estimated versions. In the extreme tails of current inflation distribution, the lowest (less than 25 %) and the highest (greater than 75 %) quantiles, the forward-looking coefficient oscillate and fall outside the OLS confidence interval. On the other hand, for lagged inflation, we can not reject the hypothesis of equality between the QR and OLS estimates, from 25% to 95% quantiles.In the same direction of the national literature, our OLS and GMM estimates show the dominance of the backward-looking component. However, the quantile model provides additional relevant evidence that this dominance is only sustainable for the lower tail of the current inflation distribution, in periods of high inflation the forward-looking component presents expressive dominance, reinforcing that the oscillation in the latter variable is more representative.

Following, we try to verify the existence of differences in the dispersions of the conditional distributions of inflation. The results show that conditional standard deviation and conditional range are different for periods of recession and periods of expansion. Moreover, they are larger in periods of recessions, which means that we expect a greater variability in the values of conditional inflation, predicted by the NKPC, in cases of recession. Finally, we calculate the probabilities of future change in the trajectory of inflation, according to the previous conditional value. Again, asymmetry is captured when we notice that the probabilities change between states of recession and expansion and even between going up and down in the same cycle. Besides, we note that the Brazilian Phillips Curve adequately captures the asymmetry of Cycles described by DeLong and Summers (1988) and Ball and Mankiw (1994). These results provide evidence of some asymmetry in the Brazilian New-Keynesian Phillips Curve. Then, we believe that the Brazilian NKPC estimates should consider these effects, especially the impact of the forward-looking component.

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