Macrodynamic Implications of Employee Profit Sharing as Effort Elicitation Device

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Abstract: We elaborate a macrodynamic model where the distribution of factor income is affected by the possibility of profit sharing with workers. Firms choose periodically to compensate workers with only a real base wage or a share of profits in addition to this real base wage as alternative effort-elicitation strategies. As validated by empirical evidence, labor productivity is higher in profit-sharing firms than in non-sharing firms. The frequency distribution of effort-elicitation strategies and labor productivity across firms are co-evolutionarily time-varying, which then affects the dynamics of the distribution of income between profits and wages and therefore the savings-determined growth rate. Heterogeneity in effort-elicitation strategies across firms (and hence earnings inequality across workers) can be a stable long-run equilibrium. In such a polymorphic equilibrium, the frequency of profit-sharing firms varies positively (negatively) with the real base wage (profit-sharing coefficient). As shown analytically and with numerical simulations, the micro- and macrodynamics of the economy are crucially affected by the profit-sharing coefficient and the real base wage as bifurcation parameters.

Keywords: Macrodynamics; profit sharing, coevolutionary dynamics, income distribution, economic growth.

Resumo: Elaboramos um modelo macrodinâmico no qual a distribuição da renda dos fatores de produção é afetada pela possibilidade de compartilhamento de lucros com os trabalhadores. As firmas escolhem periodicamente entre duas estratégias alternativas de elicitação de esforço: compensar seus trabalhadores somente com um salário real base ou com uma parcela salarial adicionalmente a esse salário real base. Como validado pela evidência empírica, a produtividade do trabalho é maior nas firmas que compartilham lucros que naquelas que não compartilham. A distribuição de frequência das estratégias de elicitação de esforço e da produtividade do trabalho entre as firmas variam no tempo de maneira coevolucionária, afetando, assim, a dinâmica da distribuição de renda entre salários e lucros e, portanto, do crescimento econômico, a qual é determinado pela poupança. A heterogeneidade nas estratégias de elicitação de esforço do trabalho entre as firmas (e, logo, a desigualdade de rendimentos entre os trabalhadores) pode ser um equilíbrio de longo prazo estável. Nesse equilíbrio polimórfico, a frequência de firmas que compartilham lucros varia positivamente (negativamente) com o salário real base (coeficiente de compartilhamento de lucros). Como mostrado analiticamente e com simulações numéricas, a micro e a macrodinâmica da economia são crucialmente afetadas pelo salário real base e o coeficiente de compartilhamento de lucros como parâmetros de bifurcação.

Palavras-chave: Macrodinâmica, compartilhamento de lucro, dinâmica coevolucionária, distribuição de renda, crescimento econômico.

JEL Codes: E11, E25, J33, O41.

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1. Introduction

Employee profit-sharing arrangements have experienced rising (albeit fluctuating) incidence in formal labor contracts in advanced economies in the last few decades (D'Art and Turner, 2004, Kruse, Blasi and Park, 2010). From a long-term standpoint, however, the presence of employee profit sharing in (in)formal labor contracts has gone through several waves since profit sharing was first conceived and introduced during the industrial revolution (Mitchell et al., 1990, D'Art and Turner, 2006, Blasi et al., 2013).

On theoretical grounds, connecting workers' earnings to the profit performance of the firm can be seen as potentially stimulating workers to increase commitment and effort, which would result in a rise in their productivity. On empirical grounds, meanwhile, surveys find that both employers and employees interpret profit sharing arrangements as contributing to enhance firm performance along several dimensions (Weitzman and Kruse, 1990, and Blasi et al., 2010). Arguably, profit sharing as a worker incentive device is subject to the free-rider problem, given that the rewards from individual effort are usually shared with other workers. Yet, there is evidence that such free-riding problem often appears to be overcome to some extent by worker co-monitoring and reciprocity (Freeman, Kruse and Blasi, 2010). In fact, Carpenter et al. (2016) provide experimental evidence that, although profit sharing may not be an effective enough device to elicit more labor effort by itself, it may nonetheless suffice to stimulate reciprocity-minded peers to report free riders given the cost imposed on them when others shirk. As a result, teams with peer reporting and profit sharing may exert more effort at work than those with profit sharing (or peer reporting) alone.

In practice, profit-sharing plans vary a great deal, and the major ways in which they differ usually involve what is shared (e.g., total profits or profits above some threshold level), when and how sharing happens (e.g., in a deferred or non-deferred way, in cash or company stocks) and with whom sharing is implemented (e.g., directly to workers or to some workers' retirement or pension plan). Nevertheless, as there is survey evidence that the combination of non-deferred and in-cash profit sharing is ranked first by employees as a motivation device (Blasi et al. 2010), we draw on this evidence to specify the model set forth in this paper.

There is considerable empirical evidence that profit sharing raises labor productivity. Albeit the estimated size of the respective productivity gain varies from study to study, it is often non-negligible. Weitzman and Kruse (1990) apply meta-analysis to sixteen studies and discover that the productivity gain associated with profit sharing is positive at infinitesimal significance levels. Doucouliagos (1995) applies meta-analysis to forty-three studies and also find that profit sharing is positively associated with productivity. Cahuc and Dormont (1997) use French data and find that profit sharing firms outperform other firms in both productivity and profitability. Conyon and Freeman (2004) use U.K. data and discover that profit-sharing firms usually outperform other firms in both productivity and financial performance. D'Art and Turner (2004) use data for 11 European countries and find that profit sharing is positively associated with productivity and profitability. Magnan and St-Onge (2005), using data for Canada, find that firms adopting profit sharing enhance their profitability in comparison to their own prior performance and to firms that are not adopting profit sharing. Meanwhile, Robinson and Wilson (2006), employing U.K. data, and Kato et al. (2010), using Korean data, find that profit sharing has a positive effect on productivity. Kraft and Ugarkovic (2006), using data for Germany, find that profit sharing has a positive effect on profitability, although Dube and Freeman (2010) use U.S. data and discover that profit sharing has a positive effect on productivity only when accompanied by shared modes of decision-making. In a field, quasi-experimental investigation, Peterson and Luthans (2006) could randomly assign profit sharing at three of twenty-one establishments within a U.S. company, having detected that productivity and profits rose in the profit-sharing establishments relative to the control group.

Motivated by all these suggestive pieces of empirical evidence, this paper sets forth a macrodynamic model of capital accumulation and economic growth, in which the distribution of income between wages and profits can feature profit sharing with workers. Firms are (and remain) technologically heterogeneous with regard to the output to capital ratio. In attempting to elicit effort (and hence productivity) from labor more effectively and profitably, firms can behave heterogeneously regarding the adoption of employee compensation strategy. Workers are (and remain) homogeneous skill- and qualification-wise, and firms periodically choose to compensate workers with either a base wage or a share of profits on top of this base wage. Also, consistent with the empirical evidence reported above, workers in sharing firms exhibit higher productivity than workers in non-sharing firms.

Even though firms can behave differently as to employee compensation strategy, any existing heterogeneity in effort-elicitation strategies across firms (as well as the productivity differential gained by sharing firms and the ensuing earnings inequality across workers), are not time invariant, but instead co-evolve endogenously towards the long run as driven by an evolutionary protocol. Therefore, our suggested evolutionary approach provides an alternative framework for addressing what has been dubbed in the literature 'the fixed wage puzzle', according to which, basically, the potential benefits of profit sharing in theory suggest that it should be adopted in practice more frequently than it is normally found in empirical studies (Jerger and Michaelis, 2011). In fact, our evolutionary framework seems appropriate to explore potentially testable explanations for evidence such as that U.K. firms often switch strategies of employee compensation, with the respective gross changes (which include profit sharing) being far more numerous than the net changes (Bryson and Freeman, 2010). On a general note, although our suggested evolutionary learning framework is applied in this paper to an exploration of dynamics of choice of employee compensation strategy, we would invite readers to contemplate the possibility that other instances of strategy (or, more broadly still, behavior) switching are also subject to evolutionary learning.[1]

From a macrodynamic perspective, we explore the implications of the evolutionarily coupled dynamics of the frequency distribution of effort-elicitation strategies across firms and the labor productivity differential obtained by profit-sharing firms for the distribution of income between profits and wages and thereby the savings-determined capital accumulation and economic growth.[2] Hence, this paper joins a recent revival of interest on the determinants of changes in factor (especially capital and labor) shares in aggregate income and the ensuing implications of such changes for macroeconomic dynamics.[3] In fact, although at least since Kaldor (1961) the constancy of the labor share in income has been often regarded as one of

[1] For instance, Lima and Silveira (2015) offer evolutionary foundations to the nominal adjustment of the price level to a monetary shock by introducing the notion of 'boundedly rational inattentiveness'. A firm either pays a cost (featuring a random component) to update its information to establish the optimal price or use costless non-updated information to set a lagged optimal price. Instead of the standard replicator dynamic used herein, Lima and Silveira (2015) use an asymmetric evolutionary protocol (only firms that choose not to pay the information-updating cost rely on random pairwise comparisons of losses).

[2] Weitzman (1985) use a Keynesian model to claim that profit sharing can deliver full employment with low inflation. If part of workers' total compensation is shared profits, so that the base wage is lower, the marginal cost of labor is lower and firms will be willing to hire more workers. As the marked up price is lower, a real balance effect creates a higher aggregate demand and hence a higher desired output. In our model, in contrast, the fraction of sharing firms and the average productivity are endogenously time-varying. Also, we explore the implications of profit sharing for long-run economic growth in a supply-led dynamic model, in which the distribution of factor income plays a key role as a source of savings.

[3] For instance, Walsh (2016) develops a sticky-price, sticky-wage model with workers and capitalists where the distribution of factor income matters because wage income affects aggregate demand. Meanwhile, Broer et al. (2016) set forth a two-agent version of the standard New Keynesian model (featuring a worker who receives only labor income and a capitalist who receives only profit income) to explore how income inequality affects the monetary transmission mechanism.

the key stylized facts underlying macroeconomic models, there is compelling recent evidence on factor shares documenting otherwise. For instance, Piketty (2014), Piketty and Zucman (2014) and Karabarbounis and Neiman (2014) find a global decline in the labor share in the last decades. In this context, an interesting question that arises is whether (and to what extent) the broader sharing of profits by firms may eventually help reduce factor income inequality. In our model, a broader sharing of profits can be formally represented by a greater proportion of profit-sharing firms (for a given uniform profit-sharing coefficient) or a higher uniform profit-sharing coefficient (for a given proportion of profit-sharing firms), or both together. As a result, our model provides precise predictions about the implications of a broader sharing of profits so represented in terms of factor income distribution and economic growth, in both the short and the long run. One such prediction is that heterogeneity in effort-elicitation strategies across firms (and the resulting earnings inequality across workers) can be a stable long-run equilibrium. In such a polymorphic equilibrium, the frequency of profit-sharing firms varies positively (negatively) with the real base wage (profit-sharing coefficient). Also, when there is convergence to a long-run equilibrium, the average net profit rate and the growth rate are both at their highest possible long-run equilibrium values. As shown analytically and with numerical simulations, however, the long-run dynamics of the economy are crucially affected by the profit-sharing coefficient and the real base wage as bifurcation parameters.

In the model, one key feature of the evolutionarily coupled dynamics of the frequency distribution of effort-elicitation strategies across firms and the productivity differential gained by sharing firms is that there is strategic substitutability in the choice of compensation mode: a firm's decision to share profits has a negative payoff externality on potential sharing firms. The reason is that this reduces the effort-enhancing earnings differential represented by the excess of the compensation of workers in sharing firms over the compensation of workers in non-sharing firms, which lowers the labor productivity differential obtained by a sharing firm. Therefore, such externality differs considerably from the externality emphasized by Weitzman (1985) when speculating about the obstacles to the conversion of all (or almost all) firms to profit sharing. In Weitzman's model, as briefly described in footnote 2, if one firm alone converts to profit sharing it will hire new workers, but at the expense of driving down the pay of its original workers. Hence, such a positive externality of a tight labor market may require coordination to encourage people to convert to profit sharing. One possibility, in Weitzman's view, is to have the government reward profit-sharing workers, by preferential tax treatment of share income, for their part in creating the positive externality of a tight labor market.[4]

The remainder of this paper proceeds as follows. Section 2 lays out the structure of the model as it describes the behavior of the economy in the short run. Section 3 specifies the evolutionarily coupled dynamics of the distribution of effort-elicitation strategies across firms and the productivity differential accruing to sharing firms. This section also explores the implications of the specified evolutionary dynamics for the macrodynamics of the factor shares in income and so the savings-driven capital accumulation and economic growth. The final section summarizes the main conclusions reached along the way.

2. Structure of the model and its behavior in the short run

The economy is closed and without government activities, producing a single (homogeneous) good for both investment and consumption purposes. Production is carried out by a large (and fixed) population of firms, which use two (physically homogeneous) factors of production,

[4] Although the externality in our model differs from the one in Weitzman's (1985) model, one similar issue worth exploring is whether coordination is also needed in our model to induce the conversion to profit sharing. Analogously, one possibility would be to have the government reward sharing firms by preferential tax treatment. However worth exploring this possibility might be, especially in light of the evidence that several countries use tax incentives to stimulate the adoption of profit sharing, we leave it for future research.

capital and labor, by means of a fixed-coefficient technology. Firms produce (and hire labor) without facing any demand constraints, thereby being able to sell profitably all its output production at the prevailing price level. However, the model is cast in real terms.

In attempting to elicit effort from labor more effectively and profitably, an individual firm chooses periodically between two employee compensation strategies: it compensates workers with only a base real wage v (*non-sharing effort-elicitation compensation strategy*) or a base real wage and a share of total profits $\delta \in (0,1) \subset \mathbb{R}$ (*profit-sharing effort-elicitation compensation strategy*). In a given period there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of profit-sharing (or type *s*) firms, while the remaining proportion, $1-\lambda$, is composed by non-sharing (or type *n*) firms. Consistent with the empirical evidence on profit sharing reported in the preceding section, a profit-sharing firm is willing to play such effort-elicitation compensation strategy because the resulting labor productivity gain is strictly higher than otherwise. Meanwhile, productivity is homogeneous across workers in firms playing a given employee compensation strategy. Given that, for simplicity, the real base wage is taken to be the same under both effort-elicitation compensation strategies, a worker hired by a profit-sharing firm does receive a higher total compensation than a worker hired by a non-sharing firm along the economically meaningful domain given by strictly positive profits for sharing firms.[5]

To keep sharp focus on the dynamics of the frequency distribution of effort-elicitation compensation strategies and their implications for the distribution of factor income and hence the rates of capital accumulation and economic growth, we simplify matters by assuming that the real base wage, v, and the profit-sharing coefficient, δ, remain constant over time. The distribution of effort-elicitation compensation strategies across firms, $(\lambda, 1-\lambda)$, which is given in the short run as a result from previous dynamics, may change beyond the short run as driven by an evolutionary dynamic (the replicator dynamic). In the short run, for given values of the base wage, profit-sharing coefficient, productivity differential, distribution of effort-elicitation strategies and, then, individual and average profit shares, the short-run equilibrium value of the growth rate is determined. As the economy evolves towards the long run, the co-evolution of the distribution of effort-elicitation strategies and the productivity differential, by leading to changes in the average net profit share, causes changes in the short-run equilibrium value of the economic growth rate. Formally, we define the labor productivity differential as:

(1) $$\alpha \equiv \frac{a_s}{a_n},$$

where $a_i = X_i / L_i$ denotes labor productivity in firms of type $i = s, n$, X_i is the total output of firms of type $i = s, n$, and L_i is the total employment of firms type $i = s, n$. For simplicity, we normalize labor productivity in non-sharing firms, a_n, to one, so as to obtain $a_s = \alpha > 1$ in (1). Profit-sharing firms can be intuitively portrayed as firms willing to bet on the possibility of attaining a productivity differential, α, which is high enough to allow them to have a lower (higher) total unit labor cost (unit net profit) than non-sharing firms. In fact, a sharing firm envisages that such a productivity differential will be high enough so as to allow it to have a higher *net* profit rate (i.e., net of shared profits) than a non-sharing firm. However, as addressed in the next section, due to the presence of strategic substitutability in the choice of

[5] Empirical evidence shows that profit sharing has a meaningful effect on worker total compensation (Long and Fang, 2012). As found in Capelli and Neumark (2004), total labor costs exclusive of the sharing component do not fall significantly in pre/post comparisons of firms that adopt profit sharing, which suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

employee compensation strategy, the extra productivity gain accruing to profit-sharing firms may fall short of the minimum level needed for the profit-sharing bet to prove successful.

Using (1), the total profits of sharing and non-sharing firms are, respectively:

(2) $$R_s \equiv X_s - vL_s = \left(1-\frac{v}{\alpha}\right)X_s;$$

and:

(3) $$R_n \equiv X_n - vL_n = (1-v)X_n,$$

which requires further assuming (to ensure economically meaningful values for the two factor income shares) that $v < a_n = 1 < a_s$. Using (2) and (3), the shares of profit in output of sharing and non-sharing firms in the short run are given, respectively, by:

(4) $$\pi_s \equiv \frac{R_s}{X_s} = 1-\frac{v}{\alpha};$$

and:

(5) $$\pi_n \equiv \frac{R_n}{X_n} = 1-v.$$

The profit share expression in (4) denotes the proportion of gross profits in the output of sharing firms, since an exogenously given fraction of such profits, $\delta \in (0,1) \subset \mathbb{R}$, is shared with workers. Using (4), the unit net return of sharing firms in the short run is given by:

(6) $$\pi_s^c \equiv \frac{(1-\delta)R_s}{X_s} = (1-\delta)\left(1-\frac{v}{\alpha}\right),$$

Using (5) and (6), the short-run value of the average net profit share can be expressed as:

(7) $$\pi = \lambda\pi_s^c + (1-\lambda)\pi_n = \lambda(1-\delta)\left(1-\frac{v}{\alpha}\right) + (1-\lambda)(1-v).$$

As for capital productivity, to keep sharp focus on our main issue of interest of the effect of profit sharing on the rates of capital accumulation and economic growth, we simplify matters by assuming that the maximum output that can be produced by fully utilizing one unit of capital, k, is an exogenously given constant common to all firms:

(8) $$k_s = k_n = k,$$

where $k_i = X_i / K_i$ is the output to capital ratio (and K_i is the total capital stock) of firms of type $i = s, n$.

Using (2)-(5) and (8), the (gross) profit rates of sharing and non-sharing firms in the short run can then be expressed as follows:

(9) $$r_s \equiv \frac{R_s}{K_s} = \left(1-\frac{v}{\alpha}\right)\frac{X_s}{K_s} = k\pi_s;$$

and:

(10) $$r_n \equiv \frac{R_n}{K_n} = (1-v)\frac{X_n}{K_n} = k\pi_n.$$

Using (6) and (9), the net profit rate of sharing firms in the short run is then given by:

(11) $$r_s^c \equiv \frac{(1-\delta)R_s}{K_s} = (1-\delta)\left(1-\frac{v}{\alpha}\right)\frac{X_s}{K_s} = k\pi_s^c.$$

Thus, using (10) and (11), the short-run value of the average net profit rate is given by:

(12) $$r^c = \lambda r_s^c + (1-\lambda) r_n = k\left[\lambda(1-\delta)\left(1-\frac{v}{\alpha}\right)+(1-\lambda)(1-v)\right].$$

Therefore, a comparison between (7) and (12) shows that the average net profit share and the average net profit rate always move in the same direction.

As assumed earlier, the population of firms, the base wage, v, the labor productivity in non-sharing firms, a_n, the profit-sharing coefficient, δ, and the output to capital ratio (which is common to all firms), k, all remain constant over time. The short run period t is defined as a time frame in which the aggregate capital stock, K_t, the labor supply, N_t, the productivity differential of profit-sharing firms, $a_{s,t} = \alpha_t$, the frequency distribution of effort-elicitation compensation strategies across firms, λ_t, and therefore the distribution of factor income as given by the average net profit share, π_t^c, can all be taken as predetermined by previous dynamics.[6]

The model economy is populated by firm-owner capitalists and workers. The latter inelastically provide labor and receive solely a base wage when they are hired by non-sharing firms. Meanwhile, workers in sharing firms also receive a share of the latter's profit income, which is the entire surplus over the respective base wage bill. We assume that workers' total compensation is entirely spent on consumption, and capitalists save a fraction, $\gamma \in (0,1) \subset \mathbb{R}$, of their net profit income. Capitalists save in order to finance capital accumulation, and all their net profit income not consumed is automatically and immediately invested. Thus, firms manage to sell all their output production at the current price, given that aggregate demand automatically and immediately adjust to remove any excess aggregate (and individual) supply in the product market. In the short-run product market equilibrium, aggregate investment, I_t, is then identically equal to net aggregate saving, S_t.

Using (8)-(11), the net aggregate saving as a proportion of the capital stock in period t can be expressed as follows:

(13) $$\frac{S_t}{K_t} = \gamma\left[\frac{(1-\delta)R_{s,t}+R_{n,t}}{K_t}\right] = \gamma\left[\lambda_t k_s \pi_{s,t}^c + (1-\lambda_t) k_n \pi_n\right] = \gamma k\left[\lambda_t \pi_{s,t}^c + (1-\lambda_t)\pi_n\right].$$

By substituting (13) in the product market equilibrium identity given by $I_t / K_t \equiv S_t / K_t$, and assuming that capital does not depreciate, we obtain the short-run equilibrium growth rate:

(14) $$g_t^*(\alpha_t, \lambda_t) = \gamma k\left[\lambda_t \pi_{s,t}^c(\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n\right].$$

In the short-run equilibrium, the growth rate depends on parametric constants in conjunction with the productivity differential and the frequency distribution of effort-elicitation strategies, which are predetermined in the short run and co-evolve evolutionarily towards the long run. Given that economic growth is driven by capital accumulation from capitalists' saving, the short-run equilibrium growth rate varies positively with the saving propensity and the average net profit share. Thus, given the frequency distribution of effort-elicitation strategies and the productivity differential, a higher profit-sharing coefficient, by virtue of lowering the average net profit share, reduces the growth rate in the short-run.

[6] As in the next section we explore the behavior of the economy in the transition from the short to the long run, thereafter we attach a subscript t to the short-run value of all variables (be they endogenous or predetermined).

3. Behavior of the model in the long run

In the long run we assume that the short-run equilibrium value of the rate of economic growth is always attained, with the economy moving towards the long run due to changes in the aggregate stock of capital, K, coming from changes in the individual stocks of capital, K_s and K_n, the supply of available labor, N, the labor productivity differential, α, and the distribution of effort-elicitation compensation strategies, λ. Given that the maximum output that can be produced by fully utilizing one unit of capital is an exogenously given constant k common to all firms, it follows from (8) that $K_{s,t} / K_{n,t} = k$. Meanwhile, to sharpen focus on the evolutionarily coupled dynamics of the distribution of effort-elicitation strategies and the productivity differential as state variables, and their resulting macrodynamic implications, we further assume, for convenience, that the supply of available labor grows endogenously at the same rate as the aggregate capital stock.[7]

Let us derive the dynamics of the labor productivity differential. In a given short-run period t there is a fraction $\lambda_t \in [0,1] \subset \mathbb{R}$ of firms, which may vary from one period to the next one, using the profit-sharing strategy. The complementary fraction $1-\lambda_t$ is made up of firms that compensate workers with solely the base wage. Let $\bar{y}_t \equiv \lambda_t y_{s,t} + (1-\lambda_t) y_n$ be the average earnings of workers in period t, where $y_{s,t} \equiv v + \delta R_{s,t} / L_{s,t}$ and $y_n \equiv v$ represent the earnings of a worker in a profit-sharing firm and a worker in a non-sharing firm in period t, respectively. Hence, the differential between the higher earnings and the average earnings can be written as $y_{s,t} - \bar{y}_t = (1-\lambda_t)\delta R_{s,t} / L_{s,t}$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$. As suggested by the empirical evidence recalled reported earlier, we assume that the extent to which labor productivity in profit-sharing firms is greater than labor productivity in non-sharing firms varies positively with the relative earnings differential represented by $y_{s,t} - \bar{y}_t$. In implicit form, we consider the following effort-elicitation differential function:

(15) $$\alpha_{t+1} = f(y_{s,t} - \bar{y}_t) = f\left((1-\lambda_t)\delta R_{s,t} / L_{s,t}\right),$$

where we assume that $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$ for all earnings differential $(y_{s,t} - \bar{y}_t) \subset \mathbb{R}$. We further assume that $\lim_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Thus, in accordance with the empirical evidence that the productivity gains arising from profit sharing are not unlimited, the effort-elicitation differential in (15) rises at a non-increasing rate and tends to become insignificant for very large values of the relative earnings differential.[8] We can use (2) to re-write (15) as follows:

(15-a) $$\alpha_{t+1} = f\left(\delta(1-\lambda_t)(\alpha_t - v)\right).$$

As re-written in (15-a), the effort-elicitation differential function has an intuitive interpretation. Given that α_t is the output per worker of a profit-sharing firm and v is the

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[7] Consequently, the constancy of the productivity differential and the distribution of effort-elicitation strategies in the long-run equilibrium guarantee the constancy of both the average labor productivity and the average employment rate. In fact, using (1) and (8)-(9), the average employment rate in the short-run equilibrium is $e_t^* \equiv \dfrac{L_{s,t}^* + L_{n,t}^*}{N_t} = \left[\dfrac{\lambda_t}{\alpha_t} + (1-\lambda_t)\right]\dfrac{K_t}{N_t} k$, where the expression in square brackets is the weighted average of the inverse of the individual productivities $a_{s,t} = \alpha_t$ and $a_{n,t} = 1$.

[8] In the meta-analyses in Weitzman and Kruse (1990) and Doucouliagos (1995), the magnitude of the estimated effect of profit sharing on labor productivity is usually around 3 to 7 percent.

respective unit cost of labor, it follows that $\alpha_t - v$ is the profit per worker of a profit-sharing firm and $\delta(\alpha_t - v)$ is the amount of profit per worker of a profit-sharing firm which is shared with its workers. Thus, given α_t and λ_t, the next-period effort-elicitation differential varies positively with the profit-sharing coefficient and negatively with the base wage. Meanwhile, given α_t, v and δ, the next-period effort-elicitation differential varies positively with $1-\lambda_t$, the proportion of non-sharing firms in a given period, which is an indicator of the prospects of not receiving any shared profits in the next period. Since an increase in $1-\lambda_t$ so acts as an incentive on workers in profit-sharing firms in the next period to exhibit a higher productivity differential, it follows that $\delta(1-\lambda_t)(\alpha_t - v)$ can be interpreted as reflecting how valuable it is to work for a profit-sharing firm.

In fact, it follows from (15-a) that $\partial\alpha_{t+1}/\partial\lambda_t = -\delta(\alpha_t - v) f'\left(\delta(1-\lambda_t)(\alpha_t - v)\right) < 0$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$, where the latter condition was assumed earlier to ensure strictly positive gross profits for profit-sharing firms in (2). The greater the proportion of sharing firms in a given period, the smaller the effort-elicitation differential between sharing and non-sharing firms in the next period. The substance of this result is that one individual firm's decision to follow the profit-sharing effort-elicitation strategy in a given period, by reducing $\delta(1-\lambda_t)(\alpha_t - v)$ for a given α_t and consequently making it less valuable to workers to be employed by a sharing firm in the next period, has a negative payoff externality on all other sharing firms. Therefore, there is strategic substitutability in the firms' choice of effort-elicitation compensation strategy. Meanwhile, if all firms choose to adopt the profit-sharing effort-elicitation strategy $(\lambda_t = 1)$, it follows that the relative earnings differential represented by $y_{s,t} - \bar{y}_t = \delta(1-\lambda_t)(\alpha_t - v)$ vanishes. In this case with all firms sharing profits, given that productivity is uniform across firms, and should be higher than the average productivity when all firms pay only the base wage, we assume that $f(0) > 1$ (recall that productivity in non-sharing firms was normalized to one). When all firms play the non-sharing strategy $(\lambda_t = 0)$, the potential relative earnings differential given by $y_{s,t} - \bar{y}_t = \delta(\alpha_t - v)$ takes its maximum value. In this case, a non-sharing firm switching strategy to the profit-sharing mode reaps the largest possible extra effort-elicitation gain, since $f\left(\delta(\alpha_t - v)\right) > f\left(\delta(1-\lambda_t)(\alpha_t - v)\right)$ for all $\lambda_t \in (0,1] \subset \mathbb{R}$ and for any $\alpha_t > v$.

To facilitate the derivation of analytical results on the existence and stability of long-run equilibrium solutions and to implement numerical simulations, in what follows we assume a specific functional form for (15-a) given by:

(16) $$\alpha_{t+1} = \beta + \delta(1-\lambda_t)(\alpha_t - v),$$

where $\beta > 1$ is a parameter ensuring that $f(0) > 1$ in (15-a).

Some intuitive rationales can be suggested for the extra effort-elicitation gain in (15)-(16). First, the average earnings can be reasonably interpreted by workers as a rule-of-thumb, conventional estimate of their fallback position or outside option under uncertainty. It follows that workers who are shared profits in addition to receiving a base wage generate an extra effort-elicitation gain (comparatively to the effort-elicitation gain they would generate if paid solely a base wage) which varies positively with the excess of the higher earnings over their fallback position or outside option. Besides, the average earnings can be plausibly construed by workers as a reasonable reference point against which a compensation package featuring a base wage and shared profits should be compared when they are resolving how much above-normal productivity to fairly generate in return. Consequently, above-average earnings can be plausibly interpreted by workers as fairly warranting the generation of above-normal levels of productivity. Interestingly, Blasi, Kruse and Freeman (2010) suggest an intuitive rationale for profit-sharing compensation based on reciprocity and gift exchange (as these notions are articulated in Akerlof (1982)): a "gift" of higher worker compensation through profit sharing increases worker morale, and workers reciprocate with a "gift" of greater productivity. More broadly, a "gift" of profit-sharing compensation over and above a standard base wage can be intuitively seen as assisting to create and reinforce a sense of shared interests and the value of a reciprocal relationship.

While the frequency distribution of effort-elicitation compensation strategies is given in the short run, an individual firm revises periodically its choice of compensation strategy in a way described by the following replicator dynamic:[9]

(17) $$\lambda_{t+1} - \lambda_1 = \lambda_t (r_{s,t}^c - r_t^c) = \lambda_t (1-\lambda_t)(\pi_{s,t}^c - \pi_n) k\,,$$

where $r_t^c \equiv \lambda_t r_{s,t}^c + (1-\lambda_t) r_n$ is the average net profit rate, which is given by (12), and the latter equality is obtained using (10) and (11), so that $\pi_{s,t}^c$ and π_n are given by (5) and (6), respectively. According to the replicator dynamic represented in (17), the frequency of the profit-sharing effort-elicitation strategy across firms increases (decreases) exactly when it has above-average (below-average) payoff.

Using (5) and (6), the replicator dynamic in (16) becomes:

(17-a) $$\lambda_{t+1} = \lambda_t \left\{ 1 + (1-\lambda_t) \left[(1-\delta)\left(1 - \frac{v}{\alpha_t}\right) - (1-v) \right] k \right\}.$$

Thus, the state transition of the economy is determined by the system of difference equations (16) and (17-a), whose state space is $\Theta = \{(\alpha_t, \lambda_t) \in \mathbb{R}_+^2 : 0 \le \lambda_t \le 1, \alpha_t > v\}$.

We can show that the dynamic system represented by (16) and (17-a) has two long-run equilibria featuring survival of only one effort-elicitation strategy in each. These are monomorphic (pure-strategy) equilibria which we denote by E_1 and E_2. We can also show the possible existence of a third long-run equilibrium, now featuring the survival of both effort-elicitation strategies. This is a polymorphic (mixed-strategy) equilibrium which we denote by E_3.

Note that $\lambda_{t+1} = \lambda_t = 0$ for any $t \in \{0,1,2,...\}$ satisfies (17-a) for any state $(\alpha_t, 0) \in \Theta$. In this case, it follows that $\alpha_{t+1} = \alpha_t = \bar{\alpha}$ for all $t \in \{0,1,2,...\}$ in (16) if, and only if, the (potential) effort-elicitation differential is given by:

(18) $$\alpha_{t+1} = \alpha_t = \frac{\beta - \delta v}{1-\delta}\,,$$

where $\bar{\alpha} > 1$ is implied by our assumptions about the magnitude of the respective parameters. Therefore, one of the two monomorphic long-run equilibria of the system, E_1, is given by the state $(\bar{\alpha}, 0) \subset \Theta$, which nonetheless features the non-sharing effort-elicitation compensation strategy as the only survivor.

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[9] This replicator dynamic can be derived from a model of (social or individual) learning as in Weibull (1995, sec. 4.4).

Meanwhile, if $\lambda_{t+1} = \lambda_t = 1$ for any $t \in \{0,1,2,...\}$, the difference equation in (17-a) is satisfied for any state $(\alpha_t, 1) \in \Theta$ and, given (16), it follows that $\alpha_{t+1} = \alpha_t = \tilde{\alpha}$ for all $t \in \{0,1,2,...\}$ if, and only if, the value of the effort-elicitation differential is given by:

(19) $$\tilde{\alpha} = \beta > 1.$$

Therefore, the other monomorphic long-run equilibrium of the system, E_2, is represented by the state $(\tilde{\alpha}, 1) \subset \Theta$, which features the profit-sharing effort-elicitation compensation strategy as the only survivor. However, due to the existence of strategic substitutability in the choice of employee compensation strategy, it follows that $\tilde{\alpha} < \bar{\alpha}$.

Finally, if $\lambda_{t+1} = \lambda_t = \lambda^* \in (0,1) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$, the difference equation in (17-a) is satisfied if, and only if, the individual profit shares in (5) and (6) (and therefore the profit rates in (10) and (11)) are equalized. Given that the effort-elicitation differential (which is equal to the level of effort-elicitation in profit-sharing firms) is the only adjusting variable among the determinants of the profit shares in (5) and (6), the latter become equalized if, and only if, the value of the effort-elicitation differential is given by:

(20) $$\alpha^* = \frac{(1-\delta)v}{v-\delta},$$

where we assume that $v > \delta$, so that $\alpha^* > 1$. Furthermore, due to the existence of strategic substitutability in the choice of employee compensation strategy, it follows that $\tilde{\alpha} < \alpha^* < \bar{\alpha}$. Meanwhile, given that $\alpha_{t+1} = \alpha_t = \alpha^*$ for all $t \in \{0,1,2,...\}$, it follows that $\lambda_{t+1} = \lambda_t$ for all $t \in \{0,1,2,...\}$ in (17-a) if, and only if:

(21) $$\lambda^* = \frac{\beta - v\delta - (1-\delta)\alpha^*}{\delta(\alpha^* - v)}.$$

Given that $\alpha^* > 1 > v$, the denominator in (20) is strictly positive. Hence, in order to ensure that $\lambda^* > 0$, we assume that $\beta - v\delta - (1-\delta)\alpha^* > 0$. The latter inequality, upon substitution of (20) in it, is satisfied when the following inequality holds:

(22) $$\delta > \frac{(\beta-1)v}{\beta - v} \equiv \underline{\delta}.$$

Note that our assumptions on the magnitude of the respective parameters imply that $\underline{\delta} \in (0,1) \subset \mathbb{R}$. Now, in order to ensure that $\lambda^* < 1$, we assume that $\beta - v\delta - (1-\delta)\alpha^* < \delta(\alpha^* - v)$. The latter inequality, upon substitution of (20) in it, is satisfied when the following inequality holds:

(23) $$\delta < \frac{(\beta-1)v}{v^2 - 2v + \beta} \equiv \bar{\delta}.$$

Note that our assumptions on the magnitude of the respective parameters imply that $\bar{\delta} \in (0,1) \subset \mathbb{R}$ and $\bar{\delta} > \underline{\delta}$. Therefore, if the conditions in (22)-(23) are satisfied, there exists a third long-run equilibrium represented by the state $(\alpha^*, \lambda^*) \subset \Theta$, denoted by E_3, which features the survival of both effort-elicitation compensation strategies in the long run. The existence of a long-run equilibrium of the dynamic system given by (16) and (17-a) is summarized in the proposition that follows.

Proposition 1: *For a given vector of parameters (β, δ, v, k), the dynamic system represented by (16) and (17-a) has two monomorphic long-run equilibria given by*

$E_1 = (\bar{\alpha}, 0) = \left(\frac{\beta - v\delta}{1-\delta}, 0 \right) \in \Theta$ *and* $E_2 = (\tilde{\alpha}, 1) = (\beta, 1) \in \Theta$. *And if* $\underline{\delta} < \delta < \bar{\delta}$, *there is also a polymorphic long-run equilibrium given by* $E_3 = (\alpha^*, \lambda^*) = \left(\alpha^*, \frac{\beta - v\delta - (1-\delta)\alpha^*}{\delta(\alpha^* - v)} \right) \in \Theta$, *where* $\alpha^* = \frac{(1-\delta)v}{v-\delta} \in (\tilde{\alpha}, \bar{\alpha}) \subset \mathbb{R}$.

It can be checked that the productivity differential in the polymorphic equilibrium varies negatively (positively) with the profit-sharing coefficient (real base wage). In the same equilibrium, the proportion of sharing firms varies with these parameters as follows:

$$\frac{\partial \lambda^*}{\partial \delta} = \frac{\beta - 1}{\delta^2 (v-1)} < 0 \quad \text{and} \quad \frac{\partial \lambda^*}{\partial v} = \frac{\beta\delta - 2\beta\delta v + (\beta - 1 + \delta)v^2}{\delta(v-1)^2 v^2} > 0,$$

where it can be checked that the numerator of the second expression is positive for any $\beta > 1$ and $\delta \in (0, v) \subset \mathbb{R}$.

Meanwhile, the stability properties of the evolutionary long-run equilibria identified in Proposition 1 are established in the following proposition.

Proposition 2: *For a given vector of parameters* (β, δ, v, k), *the long-run equilibria of the dynamic system represented by (16) and (17-a) exhibit the following stability properties:*

i. *If* $0 < \delta < \underline{\delta}$, *the monomorphic equilibrium given by* $E_1 = (\bar{\alpha}, 0) = \left(\frac{\beta - v\delta}{1-\delta}, 0 \right) \in \Theta$ *is a repulsor and the monomorphic equilibrium given by* $E_2 = (\tilde{\alpha}, 1) = (\beta, 1) \in \Theta$ *is an attractor;*

ii. *If* $\underline{\delta} < \delta < \bar{\delta}$, *the monomorphic equilibria* E_1 *and* E_2 *are repulsors, whereas the polymorphic equilibrium given by* $E_3 = (\alpha^*, \lambda^*) = \left(\alpha^*, \frac{\beta - v\delta - (1-\delta)\alpha^*}{\delta(\alpha^* - v)} \right) \in \Theta$, *where* $\alpha^* = \frac{(1-\delta)v}{v-\delta}$, *is an attractor; and*

iii. *If* $\bar{\delta} < \delta < v$, *the monomorphic equilibrium given by* $E_1 = \left(\frac{\beta + v\delta}{1-\delta}, 0 \right) \in \Theta$ *is an attractor and the monomorphic equilibrium* $E_2 = (\beta, 1) \in \Theta$ *is a repulsor.*

<u>Proof</u>: See Appendix.

Interestingly, the dynamic system given by (16) and (17-a) also undergoes a bifurcation at $\delta = \underline{\delta}$. This bifurcation features a change in both the stability property of the monomorphic equilibrium given by $(\tilde{\alpha}, 1) \subset \Theta$ and the existence status of the polymorphic equilibrium given by $(\alpha^*, \lambda^*) \subset \Theta$. If the threshold given by $\delta = \underline{\delta}$ is crossed from below (above), the bifurcation features the monomorphic equilibrium given by $(\tilde{\alpha}, 1) \subset \Theta$ ceasing to be an attractor (a repulsor) to become a repulsor (an attractor), and by the polymorphic equilibrium given by $(\alpha^*, \lambda^*) \subset \Theta$, which is an attractor, coming (ceasing) to exist. The dynamic system given by (16) and (17-a) also undergoes a bifurcation at $\delta = \bar{\delta}$. This bifurcation features a change in both the stability property of the monomorphic equilibrium given by $(\bar{\alpha},0) \subset \Theta$ and the existence status of the polymorphic equilibrium given by $(\alpha^*,\lambda^*) \subset \Theta$. If the threshold represented by $\delta = \bar{\delta}$ is crossed from below (above), the bifurcation is characterized by the monomorphic equilibrium given by $(\bar{\alpha},0) \subset \Theta$ ceasing to be a repulsor (an attractor) to become an attractor (a repulsor), and by the polymorphic equilibrium represented by $(\alpha^*,\lambda^*) \subset \Theta$, which is an attractor, ceasing (coming) to exist.

In order to appropriately complement the analytical results derived above about the dynamics of the system, we now perform some illustrative numerical simulations. In Figure 1, the parameters of the model are given by the following empirically plausible values: capitalists' propensity to save, $\gamma = 0.4$; minimum labor productivity differential, $\beta = 1.05$; and output to capital ratio, $k = 0.3$. We typically set the profit-sharing coefficient at $\delta = 0.05$ and the real base wage at 0.6.[10]

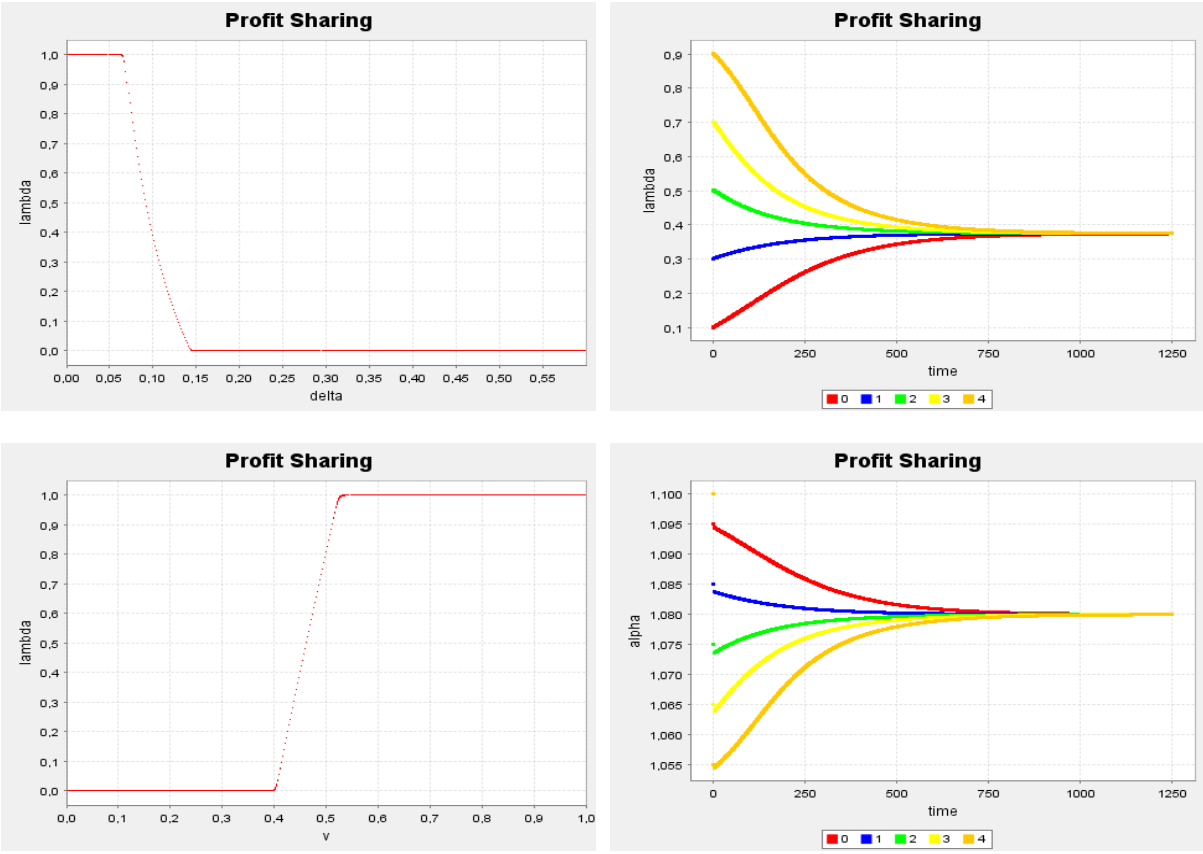

Figure 1. Single bifurcation and trajectory diagrams.

In Figure 1, the left panels picture limit sets for the frequency of profit-sharing firms by means of parameter basins with respect to the profit-sharing coefficient, with $\delta \in (0, 0.6) \in \mathbb{R}$ (top) and the real base wage, with $v \in (0,1) \in \mathbb{R}$. The right panels, meanwhile, describe the trajectory over time of the frequency of profit-sharing firms (top) and productivity differential (bottom), with an emphasis on their transitional dynamics. We avoid detecting only local dynamics by specifying initial conditions at some distance from the long-run equilibria. In the left panels, we set initial values given by $\alpha = 1.1$ and $\lambda = 0.01$, and set

[10] All these numerical simulations (along with the respective graphical representations) were obtained with the open-source software iDMC, version 2.0.10, available at https://code.google.com/archive/p/idmc/downloads. Meanwhile, the source code used to generate the numerical results presented in this section is obtainable upon request to the authors.

5000 transient iterations (i.e. the number of iterations omitted before plotting begins) and 5000 iterations to be plotted.

In Figure 1, two single parameter bifurcation diagrams and two trajectory diagrams are shown to illustrate the dynamics of the system given by (16) and (17-a) with numerical simulations. In the top left panel, the bifurcation undergone by the system at $\delta = \underline{\delta} = 0.07$ and $\delta = \overline{\delta} = 0.14$ is shown. The phase transition from the monomorphic equilibrium given by all firms playing the sharing strategy to the monomorphic equilibrium with no firm playing the sharing strategy occurs as the profit-sharing coefficient rises from zero. The transition as such takes place along the range of values of the profit-sharing coefficient for which the polymorphic equilibrium exists and is an attractor. In the bottom left panel, meanwhile, it is shown the bifurcation undergone by system at two different values of the real base wage, which are associated with the respective bifurcation values of the profit-sharing coefficient according to (22) and (23). In any case, as the real base wage rises from zero, the phase transition now occurs from the monomorphic equilibrium given by no firm following the sharing strategy to the monomorphic equilibrium with all firms following the sharing strategy. The transition as such occurs along the corresponding range of values of the real base wage for which the polymorphic equilibrium exists and is an attractor.

In the right panels, multiple trajectories of the state variables are displayed. In the top panel, the value of the initial condition for the frequency of profit-sharing firms is increased from $\lambda = 0.1$ to $\lambda = 0.9$ with a change of 0.2 in each variation. Simulations are color-indicated and the number next to each color identifies which simulation is represented by that color. In the bottom panel, the labor productivity differential varies from its assumed initial condition also in response to the respective variation in the profit-sharing coefficient from each of its assumed initial conditions, as these two state variables co-evolve over time. The convergence of the dynamic system to the polymorphic long-run equilibrium (we set the profit-sharing coefficient at $\delta = 0.1$ so as to ensure it; see top left panel) takes place close to the end of the 1250 iterations that we set to be plotted.

Let us now compute the long-run equilibrium values of the distribution of factor income and the growth rate in each one of the three economically relevant ranges of the profit-sharing coefficient specified in Proposition 2. We can use (7) to establish:

(24) $$\pi(\overline{\alpha}, 0) = \pi(\alpha^*, \lambda^*) = \pi_n = 1 - v,$$

and:

(25) $$\pi(\beta, 1) = \pi_s^c(\beta) = (1-\delta)\left[1 - \frac{v}{\beta}\right],$$

where the first equality in (24) follows from the fact that $E_3 = (\alpha^*, \lambda^*)$ is defined implicitly by the condition given by $\pi_s^c(\alpha^*, \lambda^*) - \pi_n = 0$ (recall from (5) that the profit share of non-sharing firms is exogenously given). Hence, the value of the average net profit share is the same in the monomorphic equilibrium given by $E_1 = (\overline{\alpha}, 0)$ and in the polymorphic equilibrium given by $E_3 = (\alpha^*, \lambda^*)$. When $\delta > \overline{\delta}$, given that $\overline{\alpha} > \beta$, it follows that $\pi(\overline{\alpha}, 0) > \pi(\beta, 1)$, as can be verified using the Appendix, so that, using (14), we then obtain that $g(\overline{\alpha}, 0) > g(\beta, 1)$. When $\underline{\delta} < \delta < \overline{\delta}$, since $\alpha^* > \beta$, we have $\pi(\overline{\alpha}, 0) = \pi(\alpha^*, \lambda^*) > \pi(\beta, 1)$, so that (14) yields $g(\overline{\alpha}, 0) = g(\alpha^*, \lambda^*) > g(\beta, 1)$. Meanwhile, when $\delta < \underline{\delta}$, since $\alpha^* < \beta$, it follows that $\pi(\overline{\alpha}, 0) < \pi(\beta, 1)$, as can again be verified using the Appendix, so that, using (14), we then obtain that $g(\overline{\alpha}, 0) < g(\beta, 1)$. Therefore, if the economy converges to a long-run equilibrium (which may not occur when $\underline{\delta} < \delta < \overline{\delta}$, given that the condition in (A.8) in the Appendix may not be satisfied), the coupled evolutionary dynamics of profit-sharing adoption and productivity gain takes the economy to a position in which the average net profit share, the net profit rate and the rate of economic growth are all at their highest possible long-run equilibrium levels.

Finally, although it is possible the emergence of a stable long-run equilibrium with strategic dualism, the evolutionary dynamic in (17) ensures that the (net) rates of profit of the coexisting effort-elicitation strategies become equalized. However, in our one-sector model, net profit rate equalization is brought about in the long run not by market competition with capital mobility across sectors, but by evolutionary competition with firms (along with their capital stock) mobility across alternative effort-elicitation compensation strategies.

4. Conclusions

This paper is motivated by various pieces of suggestive empirical evidence. First, from a long-term historical perspective, there has been persistent heterogeneity in worker compensation strategies across firms, and profit-sharing arrangements have experienced an oscillating popularity. Second, profit sharing arrangements are productivity-enhancing, and attitude surveys find that employers and employees perceive profit sharing as contributing to improve firm performance in several dimensions. Third, there is survey evidence that non-deferred and in cash profit sharing is ranked first by workers as motivation device. Fourth, firms switch modes of worker compensation (which include profit sharing) with reasonable frequency, with the gross changes in modes being more numerous than the net changes. Fifth, earnings inequality across observationally homogeneous workers does exist, is of a non-negligible magnitude and is persistent over longer time spans.

In this paper, firms are modeled as periodically choosing to compensate workers with solely a base wage or a share of profits in addition to this base wage. The frequency distribution of effort-elicitation compensation modes and labor productivity in the population of firms are modeled as co-evolutionarily time-varying. The paper also investigates how such co-evolution at the micro level affects and the macrodynamics of the distribution of factor income and economic growth.

When the profit-sharing coefficient is relatively low, there are two long-run equilibria, one featuring all firms sharing profits and the other with no firm sharing profits. The former is an attractor, while the latter is a repulsor. When the profit-sharing coefficient is relatively high, the economy has the same two evolutionary long-run equilibria. However, the long-run equilibrium with all firms sharing profits is a repulsor, while the long-run equilibrium with no firm sharing profits is an attractor. When the profit-sharing coefficient is at a relatively intermediate level, there is a third long-run equilibrium, one that features heterogeneity in employee compensation strategies across firms, and therefore earnings inequality across workers. In this case, the long-run equilibria featuring survival of only one strategy are repulsors, while the long-run equilibrium with coexistence of both compensation modes can be an attractor. In fact, as shown analytically and with numerical simulations, the long-run dynamics are crucially affected by the profit-sharing coefficient and the real base wage as bifurcation parameters.

Meanwhile, when there is convergence to a long-run equilibrium, the average net profit rate and the growth rate are both at their highest possible long-run equilibrium values. Moreover, there is net profit rate equalization even in the long-run equilibrium with coexisting effort-elicitation strategies across firms and the resulting earnings inequality across workers. This equalization is an outcome of the evolutionary competition involving the alternative effort-elicitation employee compensation modes between which firms periodically choose.

References

Akerlof, G. (1982) ‘Labor contracts as partial gift exchange’, Quarterly Journal of Economics, 87, 543-69.

Blasi, J., R. Freeman, C. Mackin and D. Kruse (2010) ‘Creating a bigger pie? The effects of employee ownership, profit sharing, and stock options on workplace performance’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.

Blasi, J., D. Kruse and R. Freeman (2010) ‘Epilogue (and Prologue)’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.

Blasi, J., R. Freeman and D. Kruse (2013) The Citizen’s Share: Putting Ownership Back into Democracy, New Haven: Yale University Press.

Broer, T., N.-J. H. Hansen, P. Krusell and E. Öberg (2016) ‘The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective’, NBER Working Paper No. 22418, July.

Bryson, A. and R. Freeman (2010) ‘How does shared capitalism affect economic performance in the United Kingdom?’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds), Shared Capitalism at Work, Chicago: University of Chicago Press.

Cahuc, P. and B. Dormont (1997) ‘Profit-sharing: does it increase productivity and employment? A theoretical model and empirical evidence on French micro data’, Labour Economics, 4, 293–319.

Capelli, P. and D. Neumark (2004) ‘External churning and internal flexibility: Evidence on the functional flexibility and core-periphery hypotheses’, Industrial Relations, 43(1), 148-82.

Carpenter, J. P., A. Andrea and P. Akbar (2016) ‘Profit sharing and peer reporting,’ IZA Discussion Papers 9946, Institute for the Study of Labor (IZA).

Caselli, F. (2005) ‘Accounting for cross-country income Differences,’ in Phillipe Aghion and Steven Durlauf (eds.), Handbook of Economic Growth, Amsterdam: North-Holland Press, 679–741.

Caselli, F. and J. Feyrer (2007) ‘The marginal product of capital’, Quarterly Journal of Economics, 122(2), 535–568.

Conyon, M. and R. Freeman (2004) ‘Shared modes of compensation and firm performance: U.K. evidence’, in Richard Blundell, David Card and Richard Freeman (eds.), Seeking a Premier League Economy, Chicago: University of Chicago Press.

D’Art, D. and T. Turner (2004) ‘Profit sharing, firm performance and union influence in selected European countries’, Personnel Review, 33(3), 335–350.

D’Art, D. and T. Turner (2006) ‘Profit sharing and employee share ownership in Ireland: A new departure?’, Economic and Industrial Democracy, 27(4), 543–564.

Doucouliagos, C. (1995) ‘Worker participation and productivity in labor-managed and participatory capitalist firms: A meta-analysis’, Industrial and Labor Relations Review, 49(1), 58–77.

Dube, A. and R. Freeman (2010) ‘Complementarity of shared compensation and decision-making systems: evidence from the American labor market’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.

Farebrother (1973) ‘Simplified Samuelson conditions for cubic and quartic equations’, The

Manchester School, 41(4), 396–400.
Freeman, R., D. Kruse and J. Blasi (2010) ‘Worker responses to shirking under shared capitalism,’ in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.
Jerger, J. and J. Michaelis (2011) ‘The fixed wage puzzle: why profit sharing is hard to implement?’, Economics Letters, 110(2), 104-106.
Kaldor, N (1961) ‘Capital accumulation and economic growth,’ in F. A. Lutz and D. C. Hague (eds.), The Theory of Capital, New York: St. Martin’s Press, pp. 177–222.
Karabarbounis, L. and B. Neiman (2014) ‘The global decline of the labor share’, Quarterly Journal of Economics, 129(1), 61–103.
Kato, T., J.H. Lee and J.-S. Ryu (2010) ‘The productivity effects of profit sharing, employee ownership, stock option and team incentive plans: Evidence from Korean panel data’, in T. Eriksson (ed.), Advances in the Economic Analysis of Participatory and Labor-Managed Firms, Volume 11, Bingley: Emerald, pp. 111–135.
Kraft, K. and M. Ugarkovic (2006) ‘Profit sharing and the financial performance of firms: evidence from Germany’, Economics Letters, 92, 333–338.
Kruse, D., J. Blasi and R. Park (2010) ‘Shared capitalism in the U.S. economy: Prevalence, characteristics, and employee views of financial participation in enterprise’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.
Kruse, D., R. Freeman and J. Blasi (2010) ‘Do workers gain by sharing? Employee outcomes under employee ownership, profit sharing, and broad-based stock options’, in Douglas Kruse, Richard Freeman and Joseph Blasi (eds.), Shared Capitalism at Work, Chicago: University of Chicago Press.
Lima, G. T. and J. J. Silveira (2015) ‘Monetary neutrality under evolutionary dominance of bounded rationality,’ Economic Inquiry, 53(2), 1108–1131.
Long, R. J. and T. Fang (2012) ‘Do employees profit from profit sharing? Evidence from Canadian panel data’, Industrial and Labor Relations Review, 65(4), 899–927.
Magnan, M. and S. St-Onge (2005) ‘The impact of profit sharing on the performance of financial services firms,’ Journal of Management Studies, 42(4), 761–791.
Mitchell, D. B., D. Lewin and E. E. Lawler (1990) ‘Alternative pay systems, firm performance, and productivity’, in Alan S. Blinder (ed.), Paying for Productivity, Washington, D.C.: Brookings Institution, 15–88.
Peterson, S. and F. Luthans (2006) ‘The impact of financial and nonfinancial incentives on business-unit outcomes over time’, Journal of Applied Psychology, 91(1), 156–165.
Piketty, T. (2014) Capital in the Twenty-First Century, Cambridge, MA: Harvard University Press.
Piketty, T. and G. Zucman (2014) ‘Capital is back: Wealth-income ratios in rich countries 1700–2010’, Quarterly Journal of Economics, 129(3), 1255–1310.
Robinson, A. M. and N. Wilson (2006) ‘Employee financial participation and productivity: An empirical reappraisal’, British Journal of Industrial Relations, 44(1), 31–50.
Walsh, C. E. (2016) ‘Workers, capitalists, wages, employment and welfare’, Department of Economics, University of California, Santa Cruz, mimeo.
Weibull, J. W. (1995) Evolutionary Game Theory, Cambridge, MA: The MIT Press.
Weitzman, M. (1985) ‘The simple macroeconomics of profit sharing’, American Economic Review, 95, 937–953.
Weitzman, M. and D. Kruse (1990) ‘Profit sharing and productivity’, in Alan S. Blinder (ed.),

Paying for Productivity, Washington, D.C.: Brookings Institution, 95–140.

Appendix – Proof of Proposition 2 on the stability properties of the long-run equilibrium

The Jacobian matrix evaluated around the monomorphic equilibrium $(\bar{\alpha},0)\subset\Theta$ is:

(A-1) $$J(\bar{\alpha},0)=\left[\begin{array}{c|c}\delta & -\delta(\bar{\alpha}-v)\\ \hline 0 & 1+\left(\pi_s^c(\bar{\alpha})-\pi_n\right)k\end{array}\right],$$

where $\pi_s^c(\bar{\alpha})=(1-\delta)\left(1-\dfrac{v}{\bar{\alpha}}\right)$ and $\pi_n=1-v$.

Let ξ be an eigenvalue of the Jacobian matrix in (A.1). The eigenvalues of the Jacobian matrix in (A.1) are given by:

(A.2) $\xi_1=\delta$ and $\xi_2=1+[\pi_s^c(\bar{\alpha})-\pi_n]k$.

Given that $0<\delta<1$, it follows that $|\xi_1|<1$.

Let us investigate the absolute value of ξ_2. We want to find out under what condition(s) it follows that $-1<\xi_2<1$. Per (A.2), it follows that $-1<\xi_2<1$ obtains if, and only if, $-2<[\pi_s^c(\bar{\alpha})-\pi_n]k<0$.

Given that $0<\pi_s^c(\bar{\alpha})<1$ and $0<\pi_n<1$, we find that $-1<\pi_s^c(\bar{\alpha})-\pi_n<1$. Since the output to capital ratio, k, typically satisfies the condition given by $0<k<1$ (see, e.g., Caselli, 2005, and Caselli and Feyrer, 2007), it follows that $-2<-k<[\pi_s^c(\bar{\alpha})-\pi_n]k$. Note that $\pi_s^c(\alpha)-\pi_n=(1-\delta)\left(1-\dfrac{v}{\alpha}\right)-(1-v)$ is increasing in α. Given (20), it then follows that $\pi_s^c(\alpha^*)-\pi_n=0$. Since $\bar{\alpha}-\alpha^*=0$ at $\delta=\bar{\delta}$ and $\left.\dfrac{\partial(\bar{\alpha}-\alpha^*)}{\partial\delta}\right|_{\delta=\bar{\delta}}=\dfrac{-(v^2-2v+\beta)^3}{(v-1)^3v(v-\beta)}<0$ for all $v\in(0,1)\subset\mathbb{R}$ and $\beta>1$, if $\delta>\bar{\delta}$, we then obtain $\bar{\alpha}<\alpha^*$, so that $\pi_s^c(\bar{\alpha})-\pi_n<0$ and hence $[\pi_s^c(\bar{\alpha})-\pi_n]k<0$. Thus, it follows from $\delta>\bar{\delta}$ that $|\xi_2|<1$. Meanwhile, if $\delta<\bar{\delta}$, so that $\bar{\alpha}>\alpha^*$, it follows that $\pi_s^c(\bar{\alpha})-\pi_n>0$ and hence $[\pi_s^c(\bar{\alpha})-\pi_n]k>0$. Thus, it follows from $\delta<\bar{\delta}$ that $|\xi_2|>1$. As it turns out, at $\delta=\bar{\delta}$ the system undergoes a bifurcation characterized by a change in both the stability property of the monomorphic equilibrium $(\bar{\alpha},0)\subset\Theta$ and the existence status of the polymorphic equilibrium $(\alpha^*,\lambda^*)\subset\Theta$. If the threshold given by $\delta=\bar{\delta}$ is crossed from below (above), the bifurcation is characterized by the monomorphic equilibrium $(\bar{\alpha},0)\subset\Theta$ ceasing to be a repulsor (an attractor) to become an attractor (a repulsor), and by the polymorphic equilibrium $(\alpha^*,\lambda^*)\subset\Theta$ ceasing (coming) to exist. In any case, this completes the demonstration that the long-run equilibrium with no firm playing the profit-sharing strategy, $(\bar{\alpha},0)\subset\Theta$, is an attractor if $\delta>\bar{\delta}$ and a repulsor if $\delta<\bar{\delta}$.

The Jacobian matrix evaluated around the monomorphic equilibrium $(\tilde{\alpha},1)\subset\Theta$ is:

(A.3) $$J(\tilde{\alpha},1)=\left[\begin{array}{c|c}0 & -\delta(\beta-v)\\ \hline 0 & 1-\left(\pi_s^c(\beta)-\pi_n\right)k\end{array}\right],$$

where $\pi_s^c(\beta) = (1-\delta)\left[1-(v/\beta)\right]$ and $\pi_n = 1-v$.

Let ξ be an eigenvalue of the Jacobian matrix in (A.3). In this case, the eigenvalues of the Jacobian matrix is (A.3) are also easily computed:

(A.4) $\quad \xi_1 = 0 \quad$ and $\quad \xi_2 = b = 1-\left[\pi_s^c(\beta)-\pi_n\right]k$.

Therefore, the local stability of $(\tilde{\alpha},1) \subset \Theta$ depends on the absolute value of ξ_2. Given (A.4), it follows that $-1 < \xi_2 < 1$ obtains if, and only if, $0 < \left[\pi_s^c\left(f(0)\right)-\pi_n\right]k < 2$.

Since $0 < \pi_s^c(\beta) < 1$ and $0 < \pi_n < 1$, it follows that $-1 < \pi_s^c(\beta)-\pi_n < 1$. Therefore, given that it is typically the case that $0 < k < 1$, we can infer that $\left[\pi_s^c(\beta)-\pi_n\right]k < 2$.

It can be seen that $\pi_s^c(\alpha)-\pi_n = (1-\delta)\left(1-\dfrac{v}{\alpha}\right)-(1-v)$ is increasing in α. Given (20), it follows that $\pi_s^c(\alpha^*)-\pi_n = 0$. Given that $\beta-\alpha^* = 0$ at $\delta = \underline{\delta}$ and $\dfrac{\partial(\beta-\alpha^*)}{\partial\delta} = \dfrac{-(1-v)v}{(v-\delta)^2} < 0$ for all $v \in (0,1) \subset \mathbb{R}$ and $\delta \neq v$, if $\delta > \underline{\delta}$, we then obtain $\beta < \alpha^*$, so that $\pi_s^c(\beta)-\pi_n < 0$ and therefore $\left[\pi_s^c(\beta)-\pi_n\right]k < 0$. Thus, it follows from $\delta > \underline{\delta}$ that $|\xi_2| > 1$. Meanwhile, if $\delta < \underline{\delta}$, so that $\beta > \alpha^*$, it follows that $\pi_s^c(\beta)-\pi_n > 0$ and hence $\left[\pi_s^c(\beta)-\pi_n\right]k > 0$. Thus, it follows from $\delta < \underline{\delta}$ that $|\xi_2| < 1$. As it turns out, at $\delta = \underline{\delta}$ the system undergoes a bifurcation characterized by a change in both the stability property of the monomorphic equilibrium $(\tilde{\alpha},1) \subset \Theta$ and the existence status of the polymorphic equilibrium $(\alpha^*,\lambda^*) \subset \Theta$. In fact, if the threshold given by $\delta = \underline{\delta}$ is crossed from below (above), the bifurcation is characterized by the monomorphic equilibrium $(\tilde{\alpha},1) \subset \Theta$ ceasing to be an attractor (a repulsor) to become a repulsor (an attractor), and by the polymorphic equilibrium $(\alpha^*,\lambda^*) \subset \Theta$ coming (ceasing) to exist. In any case, this completes the demonstration that the long-run equilibrium with all firms playing the profit-sharing strategy, $(\tilde{\alpha},1) \subset \Theta$, is an attractor if $\delta < \underline{\delta}$ and a repulsor if $\delta > \underline{\delta}$.

The Jacobian matrix evaluated around the polymorphic equilibrium $(\alpha^*,\lambda^*) \subset \Theta$ is:

(A.5) $$J(\alpha^*,\lambda^*) = \left[\begin{array}{c|c} \delta(1-\lambda^*) & -\delta(\alpha^*-v) \\ \hline \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}k & 1 \end{array}\right].$$

Let ξ be an eigenvalue of the Jacobian matrix in (A.5). We can set the characteristic equation of the linearization around the equilibrium as:

(A.6) $$|J-\xi I| = \left|\begin{array}{c|c} a-\xi & -b \\ \hline c & 1-\xi \end{array}\right| = \xi^2-(a+1)\xi+(a+bc) = 0,$$

where $a \equiv \delta(1-\lambda^*) > 0$, $b \equiv \delta(\alpha^*-v) > 0$, and $c \equiv \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}k > 0$.

We can use the Samuelson stability conditions for a second order characteristic equation to determine under what conditions the two eigenvalues are inside the unit circle. Based on Farebrother (1973, p. 396, inequalities 2.4 and 2.5), we can establish the following set of simplified Samuelson conditions for the quadratic polynomial in (A.6):

(A.7) $1+a+bc > |-(a+1)| = a+1$ and $a+bc<1$.

Let us prove that these conditions are satisfied if $a+bc<1$.

First, note that $1+a+bc > a+1$ simplifies to $bc>0$, which is trivially satisfied given that $b>0$ and $c>0$.

Meanwhile, the second inequality, $a+bc<1$, can be expressed as follows:

(A.8) $$a+bc=\delta(1-\lambda^*)\left\{1+\frac{v(1-\delta)\lambda^*(\alpha^*-v)k}{(\alpha^*)^2}\right\}<1.$$

Substitution of (20) and (21) in the expression $a+bc$ yields:

(A.8-a) $$a+bc=\frac{[-\beta\delta+v(\beta-1+\delta)]\{v(\delta-1)\delta+k(v-\delta)[v^2\delta+\beta\delta-v(\beta-1+2\delta)]\}}{(1-v)v^2(1-\delta)\delta}.$$

Note that at $\delta=v$ it follows that $a+bc=1$. Besides, we have that:

(A.9) $$\frac{\partial(a+bc)}{\partial\delta}=\frac{(v-\beta)\{[k(v-2)-1]v+\beta k\}+\dfrac{k(v-1)^3(v-\beta)\beta}{(\delta-1)^2}+\dfrac{kv^3(\beta-1)^2}{\delta^2}}{(1-v)v^2}<0$$

if we assume that the following condition is satisfied:

(A.10) $$\beta<\frac{v+kv(2-v)}{k}.$$

Thus, since $a+bc=1$ at $\delta=v$ and given (A.9), it follows that the inequality in (A.8) holds for all $\delta\in(0,v)\subset\mathbb{R}$. As a result, by assuming that (A.10) is satisfied, we can conclude that for all $\delta\in(\underline{\delta},\overline{\delta})\subset(0,v)\subset\mathbb{R}$ the stability condition in (A.8) is satisfied as well, so that the polymorphic equilibrium represented by $(\alpha^*,\lambda^*)\subset\Theta$ is an attractor.