The Evolutionary Brexit Game: Uncertainty and Location Decision

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ABSTRACT: A new methodology is proposed to analyze the strategic decision about firms' location choice faced with the uncertainty surrounding Brexit. For this, we combine Evolutionary Game Theory (EGT) and Agent Based Simulation (ABS) approaches with input-output analysis. A study of case is presented to empirically evaluate our model. Firms are competing in two different sectors: (a) crop and animal production, hunting and related service activities; (b) financial service activities, except insurance and pension funding. We desegregate the European Union in manifold regions. To decide where to locate, firms consider the following exogenous factors: (i) potential market; (ii) local productive interdependence; (iii) labor costs and (iv) displacement cost. To generate the results, we create hypothetical scenarios, in which firms can assign specific weights to each of these factors. The results suggest that in traditional sectors firms tend to seek unsaturated markets. Otherwise, in sectors related to services, the greater the uncertainty, the greater the likelihood that firms will move.
Key-words: Location Decision; Brexit; Input-Output Analysis.
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RESUMO: Propõe-se uma nova metodologia para analisar a decisão de localização das firmas em ambiente de incerteza diante do *Brexit*. Uma abordagem de Jogos Evolucionários e Simulação por Agente em conjunto com a análise de insumo-produto foi utilizada. Para avaliar empiricamente nosso modelo, consideramos que as firmas competem em dois setores diferentes: (a) produção agrícola e animal, caça e atividades de serviços relacionados; (b) atividades de serviços financeiros, exceto seguros e fundos de pensão. Ao desagregar a União Europeia em países, a escolha de localização das firmas considera os fatores exógenos: (i) mercado potencial; (ii) interdependência produtiva local; (iii) custos de mão de obra e (iv) custo de deslocamento, os quais assumem pesos distintos. Os resultados sugerem que nos setores tradicionais, as firmas tendem a buscar mercados não saturados. Nos setores relacionados a serviços, quanto maior a incerteza, maior a probabilidade das firmas se relocalizarem.
Palavras-chave: Decisão de Localização; *Brexit*; Insumo-Produto.

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1 Introduction

Firm's strategic location decision is discussed in many fields of Economic literature. From the Regional Economics perspective, the location decision is based on the balance between a complex relation of attraction and repulsion of consumers and firms (1, 2). In this way, the economic activity to be developed in a given region can be determined by minimizing costs [1] or by maximizing market potential [2]. Recently, through the New Economic Geography approach, firms' location decision has been handled jointly by these two perspectives, as can be seen in 12, 13.

By it turns, the Industrial Organization (IO) framework is mostly concerned with the theoretical understanding of the competition nature in markets when firms strategically decide where to locate. According to 14, 15, by introducing interregional labor mobility as one of the central aspects in locational decision, it is possible to establish a dialogue between the regional economy, industrial organization, international trade and theories of growth and economic development.

According to 16, despite the vast literature[3] that has been established about location decision in these fields of study, there are few studies about spatial competition[4] lying on the interface of evolutionary game theory (EGT), Agent Based Simulation (ABS) models and regional science, since the EGT and ABS models have been applied mainly in IO researches. Although it may still be in its infancy, this theme should attract more interest from regional analysis, because the competitive locational problem emerges as a prototype of many economic situations involving dynamically interacting decisions in which firms can learn with their own choices over time.

A fact that illustrates the circumstances presented above is the issue involving Brexit. As presented in 23, the possible withdrawal of the United Kingdom (UK) from the European Union (EU) may generate an environment of uncertainty and economic instability. To be more precise, Brexit may imply uncertainties about firms' locational decision. 24 argue that the end of common markets could harm the customs union and reduce competition and the scale of production. Another possible consequence is an increase in legal and economic insecurity.

In this way, the United Kingdom's exit from the European Union may imply in a reconsideration of the optimal location by firms according to productivity differentials, factor prices (25), regional characteristics, size of internal markets and regulatory issues (26). Thus, under such conditions, EGT and ABS models may bring some insight into the behavioral pattern of firms' location decision. While the traditional theory of games requires that players have a very high level of rationality, the EGT model has been used to successfully explain a number of aspects of agents' behavior. More specifically, EGT and ABS may accomplish better success in describing and predicting the choices of locational decisions, since it is better equipped to handle the weaker rationality assumptions[5].

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1 See (3, 4, 5, 6, 7).

2 See (8, 9, 10, 11).

3 To illustrate the interface of these fields, we can mention 17, who considered a slightly modified version of Hotelling's model, in which exists a tendency for both sellers to maximize their differentiation. This constitutes a counterexample to the conclusions originally presented by 18. By its turn, 19 provided the framework for a spatial competition model and the location of firms

4 In 20, 21and 22 there is a useful compendium of spatial analysis techniques which points out the commonalities among models used to locate facilities one at a time and to forecast the economic development pattern in an entire region.

5 A small sampling of topics that have been analyzed from the evolutionary perspective includes altruism (27) and behavior in public goods game (28).

Considering the existent literature and the lack of contributions of EGT and ABS approaches to spatial theories as mentioned before, in this work we develop a new model that considers the projection made from the Regional Economic analysis for dynamically guiding firms to the Evolutionary Stable Strategy (ESS), i.e., to the optimal strategy location decision in the long term, considering the possible occurrence of Brexit. The idea behind an ESS is to ensure that a so called mutant strategy will not be able to dominate a competitive environment that embraces the incumbent strategy. Furthermore, as explained in 29, the EGT provides a refinement of the dynamic approach applied in the traditional game theory, allowing an inference about which Nash Equilibrium (NE) corresponds to an ESS. By it turns, the ABS method implements a stochastic learning rule in dynamic equilibrium analysis and, in the face of more complex systems analysis, have provided a robust way to find the ESS[6]. Accordingly, using this dialogue among the metologies, as evidenced by 33, a learning model based on the premise that agents have bounded rationality is introduced to evaluate firms' strategic locational decision.

Therefore, we present a study of case with the objective of offering a reasonable explanation about firms' location patterns in the European Union countries due to the possible occurrence of Brexit. In order to aim this goal, we evaluate the behavior of firms under a new methodology. We consider that firms are competing in two different sectors: (a) crop and animal production, hunting and related service activities (Sector 1); (b) financial service activities, except insurance and pension funding (Sector 41). In the evolutionary game presented here, we built the payoffs based on information received from the input-output methods using the WIOD database.

With the purpose of capturing the effects of exogenous variables that can affect strategic location decisions, the payoffs incorporate weighted factors, whose weights vary with each scenario we built. Since there are multiple possible outcomes that vary with each weight we consider, our results show that the scenario and its inherent uncertainty may affect the strategic decisions when the outputs cannot be predicted.

On this matter, starting from an evolutionary perspective, the main objective of this paper is to offer a reasonable contribution to the comprehension of the strategic behavior pattern of firms' spatial competition. Thus, we assume in our analysis representative multinational firms that have to decide between United Kingdom (UK) and rest of European Union (EU) whether to locate. To make that decision, firms assign probability to the Brexit event and the following factors are considered: (i) potential market; (ii) local productive interdependence; (iii) labor costs and (iv) Displacement Costs. In order to reach our purpose and develop the discussion proposed in this introduction, this article is subdivided into 4 more sections. Section 2 brings our methodology. The Evolutionary Brexit Game is presented in section 3. The results will be analyzed and discussed in section 4. Finally, section 5 will bring a brief conclusion and some important remarks to be addressed in future research.

2 Methodology

2.1 Strategic Elements Evaluated in the Location Decision

In order to develop the methodology, a game involving two types of firm is considered. The $TypeA$ firm is situated in the rest of the European Union countries (EU) and assesses the possibility of migrating to the United Kingdom (UK) in the face of the uncertainty

[6] See 30, 31 and 32.

surrounding Brexit. The $TypeB$ firm is situated in (UK) and assesses the possibility of migrating to (EU). The payoffs of the game that mimic firms' decision-making upon Brexit occurrence were constructed based on the results of the regional input-output matrix, made available by the WIOD for 2014, with sector opening of 43 countries and 56 sectors, as well as an adjustment account called "Rest of the World".

In addition, the 43 countries were aggregated so that the interregional input-output matrix used in this paper contains the same 56 sectors for three regions: United Kingdom (GBR), Rest of European Union (RoEU) and Rest of the World (RoW).

2.2 Market Potential

Firms see the maximization of the market area as an important factor in the location decision (34, 8, 11). It can be measured by the input-output model, expressed as in 35 by:

$$X = (I - A)^{-1}F \tag{1}$$

where $X_{n\times 1}$ is a column vector with n rows, $I_{n\times n}$ is a dimension identity matrix n by n, $SA_{n\times n}$ is a matrix of technical coefficients, whose size is $n \times n$, $F_{n\times 1}$ is the column vector of final demand and $(I - A)^{-1}$ is known in the economic literature as the inverse matrix of Leontief, here denoted by B. Thus, we can rewrite equation (1) as follows:

$$X = BF \tag{2}$$

From 2 we can represent the model in its interregional form, as (36), by:

$$B^D X + F^D = X \tag{3}$$

$$B^M X + F^M = X \tag{4}$$

$$\mu B^D + \mu B^M + B_v = \mu \tag{5}$$

where B represents a $n \times n$ matrix of the coefficients for domestic production. F is a final demand vector, of size $1 \times n$, which includes the gross fixed capital formation, public and private consumption, and exports. M is a import vector $n \times 1$. B_v is a vector of size $1 \times n$ which indicates the rate of value added on the total production for the sector i from the country j. μ is a unit vector of size $1 \times n$. The overwritten D indicates that the variables are domestic and the overwritten M indicates that the variables are exported. Subscribers i and j indicates the sector and the country, respectively.

Equations (3) and (4) represents the equilibrium conditions for the production of domestic amd imported goods, respectively. Equation (5) is the equilibrium condition that adds a constraint in the input-output coefficients. The sum of the elements on the rows of the sector i in equation (3) must be equal to the sum of the sales for all domestic and intermediate use in the economy for this same sector i.

Similarly, in equation (4), the sum of the column elements j indicate the total imports of the sector i, which must be equal to the sum of the sales of the product of country j in the same sector for all users of the economy, including intermediate inputs for all sectors, final household consumption and gross fixed capital formation. Finally, the elements of equation (5) imply that the total production, X, in each sector i must be equal to the sum of the value added directly in the sector i and equal to the cost of intermediate inputs for all domestic and imported production.

In fact, the market potential for non-Brexit cases will be expressed as the sum of the technical production coefficients for all i sectors of j countries of the European Union. In the case of Brexit, the market potential will be measured by means of the hypothetical extraction of the United Kingdom from the European Union [7]. The United Kingdom's market potential will be the sum of the United Kingdom's own technical production coefficients weighted by the intersectoral reduction of total production as a result of the extraction of the sector j in the United Kingdom. The market potential of the Rest of the European Union will be the sum of the technical coefficients of all i sectors of j countries of the European Union and the complement of the United Kingdom's market potential.

2.3 Productive Integration

To measure the degree of productive integration, defined as the degree of productive interdependence among i sectors of j countries, we made use of the value-added indicator on gross exports (VAX rate), which is a traditional measure of the Global Value Chains. The higher the coefficient, the lower the degree of productive integration of the country in this segment (36, 39). Following 36, 40, 41, 42, we can obtain the domestic content from an interregional model of input-output, as shown in equations (3) a (5), by:

$$VAX = A_v'(I - A)^{-1}F^{DM} \tag{6}$$

A_v' represents the row vector A_v transposed share of total value added for the sector i from the country j, of size $1 \times n$, $(I - A)^{-1}$ is the inverse of Leontief and F^{DM} is a column vector of final demand for domestic D and imported M products. Each element of the column vector of equation (6), with size $n \times 1$, can be interpreted as the externally share of value added in the production of exported domestic goods. According to 39, 40, it can be considered as a measure of productive integration.

2.4 Labor Cost

Firms consider that the cost of the labor force is a key variable for the location decision process (43, 44, 3, 4, 5). Therefore, this variable should reflect the costs absorbed by firms taking into account a skilled $\Psi 1$ and unskilled $\Psi 2$ labor force for the same amount of production X in each sector i and in each country j. The cost of the high skilled labor and low skilled labor can be obtained by the product between the participation of each type of labor in the total remunerations [8] and the gross value of the production of each sector i and for each country j, X_{ij}.

2.5 Displacement Costs

The displacement costs impact the relocation decision of firms (6, 46, 44). In this sense, the higher the displacement costs, the lower the firm's i availability to locate in another j country. As an result, the displacement costs should mimic the firm's ability to obtain productive advantages associated with its relocation in the new region. For the purposes of this paper, displacement costs are associated with differences in the share of

[7] about hypothetical extraction see 37 and 38

[8] For further details on the construction of labor force participation in total remuneration see 45

high skilled labor and low skilled labor between different countries for the same quantity produced:

$$\mu = (\psi 1_{ij} - \psi 2_{ij}) X_{ij}, \forall s \tag{7}$$

$$\mathbf{M} = (\Psi 1_{ij} - \Psi 2_{ij}) X_{ij}, \forall (1-s) \tag{8}$$

$\Psi 1_{ij}$ represents the share of the remuneration of high skilled labor over total remuneration in the sector i of country j, $\Psi 2_{ij}$ represents the share of the remuneration of low skilled labor over total remuneration in the sector i of country j and X_{ij} is the total production in each sector i of country j. The intuition is that when the participation of the high skilled labor in the sector i of country j is higher than the participation of the low skilled labor, the cost of displacement is positive for the same amount of production X_{ij}. It indicates that the cost will be the same per unit of production, but with more participation of the high skilled labor, which may represent, for example, productivity gains (42, 24, 39).

3 The Evolutionary Brexit Game Model

In an evolutionary game it is assumed bounded rationality, a large population, n, of players ($n \to \infty$) and an implicit recognition that agents learn. Every period, a player is randomly matched with another player and they play a two-player game. Each agent is randomly assigned a strategy at the initial step ($t = 0$), which can be updated over time via the systematic interaction with other agents. Thus, one player can imitate other players' strategies.

Following this intuition, now we intend to evaluate how robust is the strategic behavior of firms that will decide whether to locate in the United Kingdom (UK) or the rest of the European Union (EU). Initially, the strategies available to each player are UK and EU. Now, consider two representative firms tagged as firm A and firm B. Firm A belongs to a population of firms that have a manufacturing plant in one of the European Union member countries, excluding UK. Firm B belongs to a population of firms that have a production plant in the UK.

At each interaction, a firm tagged by type A randomly competes with a firm tagged by type B. According to the evolutionary game theory approach, this situation characterizes a two-dimensional[9] game, in which two populations of firms compete against each other. Players are invited to play multiple times and do not compete against their peers. In this sense, there will only be competition among rivals' population of firms. For simplicity, the intra-population's competition is not considered. The two population of firms (Type A; Type B) evaluate the possibility of relocation according to the probability of occurrence (s) or not ($1-s$) of Brexit. Let the row player be one representative firm of type A and the column player be one representative firm of type B. The payoff matrix of the stage game is given by (9):

$$\begin{array}{cc} & \begin{array}{cc} UK & EU \end{array} \\ \begin{array}{c} UK \\ EU \end{array} & \begin{pmatrix} \pi^A_{uk,uk} \ ; \ \pi^B_{uk,uk} & \pi^A_{uk,eu} \ ; \ \pi^B_{eu,uk} \\ \pi^A_{eu,uk} \ ; \ \pi^B_{eu,uk} & \pi^A_{eu,eu} \ ; \ \pi^A_{eu,eu} \end{pmatrix} \end{array} \tag{9}$$

[9] 29 for a detailed exposition and explanation.

The factors described in the previous section are contemplated in the game payoff matrix. Thus, given the (non) occurrence of Brexit with probability $(1-s)$ s, the market potential is given by $(\mathbf{P})$ ρ. The measure of the productive integration is provided by (Θ) θ. The factors (Ψ_1) ψ_1 and (Ψ_2) ψ_2 measures the cost of skilled and unskilled labor, respectively. The displacement cost is given by (M) μ. The weight assigned to each of the factors, conditioned to the (non) occurrence of Brexit with probability $(1-s)$ s is given by (W_I) w_i, where $(I = \{\mathbf{P}, \Theta, \Psi, \mathbf{M}\})$ $i = \{\rho, \theta, \psi, \mu\}$ and $(\Sigma W_I = 1)$ $\Sigma w_i = 1$. Generally, we can understand the structure of payoffs presented in(9) as follows.

If the Type A firm (row player) plays UK against a Type B firm (column player) that plays UK, both will split the market but only the Type A firm incurs in displacement costs, given that its production base is already established in EU. Thus, we have:

$$\pi^A_{uk,uk} = s[1/2\, w_\rho \rho_{uk,uk} + 1/2\, w_\theta \theta_{uk,uk} + 1/2\, w_\psi \left(\psi_{1uk} + \psi_{2uk}\right) - w_\mu \mu_{uk,uk}] + \\ (1-s)\left[1/2\, W_\mathbf{P} \mathbf{P}_{uk,uk} + 1/2\, W_\Theta \Theta_{uk,uk} + 1/2\, W_\Psi \left(\Psi_{1uk} + \Psi_{2uk}\right) - W_\mathbf{M} \mathbf{M}_{uk,uk}\right] \quad (10)$$

In contrast, Type B firm, which already has a production base in the UK, will not incur in displacement costs:

$$\pi^B_{uk,uk} = s[1/2\, w_\rho \rho_{uk,uk} + 1/2\, w_\theta \theta_{uk,uk} + 1/2\, w_\psi \left(\psi_{1,uk} + \psi_{2,uk}\right)] + \\ (1-s)\left[1/2\, W_\mathbf{P} \mathbf{P}_{uk,uk} + 1/2\, W_\Theta \Theta_{uk,uk} + 1/2\, W_\Psi \left(\Psi_{1,uk} + \Psi_{2,uk}\right)\right] \quad (11)$$

The other payoffs in the game follow this structure. And the argument is analogous to the case in which both firms are located in the EU. When firms adopt distinct localization strategies, there will be no market splitting. In addition, they will not incur in displacement costs if, and only if, they decide to remain located where they are already established.

For simplicity, the payoff matrix presented in (9) is normalized. This procedure will not affect the best response structure of the game and will simplify the analysis. Assume that $\eta_A = (\pi^A_{eu,uk} - \pi^A_{uk,uk})$, $\eta_B = (\pi^B_{eu,uk} - \pi^B_{uk,uk})$, $\upsilon_A = (\pi^A_{eu,uk} - \pi^A_{uk,eu})$, and $\upsilon_B = (\pi^B_{eu,eu} - \pi^B_{uk,eu})$. Players are randomly matched and compete against each other in a one-shot game. The level of aggregate strategies of populations do not change all at once. In fact, they continuously update their strategic behavior over time. The matrix (12) is the basis of the evolutionary dynamics of the game.

$$\begin{array}{cc} & \begin{array}{cc} UK & EU \end{array} \\ \begin{array}{c} UK \\ EU \end{array} & \begin{pmatrix} \eta_A; \eta_B & 0;0 \\ 0;0 & \upsilon_A; \upsilon_B \end{pmatrix} \end{array} \quad (12)$$

Let f_i be the fraction of firms using strategy UK in population i, so the fraction of firms adopting EU in population i is $1 - f_i, i = A, B$. The observed variations in the proportion of players who adopt each of the strategies reflects their evolutionary process within each population. Note that this relative frequency can be understood as the probability that a player will play a given strategy. Both the evolution of the game and the strategic behavior of the firms is conditioned to the fitness of their strategies.

The fitness, according to 47 and 48, depends on the player's payoff for a given strategy and on the relative frequency of the strategies observed in both populations. That is, players make decisions based on the expected utility of their payoffs. As we will see in the results section, it is possible that from the normalized matrix (12) we have a situation in which more than one Nash equilibrium compose the solution set. Therefore, the following question arises: departing from an initial game condition, which equilibrium will be reached?

Without loss of generality, to answer this question, we first analyze the evolution and the robustness of firm's strategic behavior with the analytic solution of the replicator dynamics (RD) system (49), which is a very general ordinary differential equation (ODE) system in evolutionary game theory. As presented in 50, in a dynamic system, the growth rate $\dot{f_A}/f_A$ equals the strategy UK's fitness $e^1 A(f_B, 1-f_B)^T$ less the average fitness $(f_A, 1-f_A)A(f_B, 1-f_B)^T$ of population A, where $\dot{f_A} = df_A/dt$ and $e^1 = (1,0)$ represents that all firms from population A chooses strategy UK.

Let A $= \begin{bmatrix} \eta_A & 0 \\ 0 & \upsilon_A \end{bmatrix}$ be the payoff matrix of a representative firm A. After some trivial matrix algebra, the replicator dynamics equation for population A is $\dot{f_A} = f_A((e^1 - (f_A, 1-f_A))A(f_B, 1-f_B)^T$. Substituting the values of A (payoffs earned by the representative firm A), in order to derive the replicator dynamics system for population A and B:

$$\dot{f_A} = f_A(1-f_A)[\eta_A f_B - \upsilon_A(1-f_B)] \tag{13}$$

$$\dot{f_B} = f_B(1-f_B)[\eta_B f_A - \upsilon_B(1-f_A)] \tag{14}$$

$\dot{f_A}$ and $\dot{f_B}$ represent the growth rate of the proportion of firms that adopt the first pure strategy (UK) within each population (A and B). The stability of the system is a coordinate $(f_A, f_B) \in [0,1] \times [0,1]$, in which $\dot{f_A} = \dot{f_B} = 0$ is a necessary condition for the stationarity of (13) and (14). To check the stability of the points candidates for an ESS, i.e., an asymptotically stable steady state for the two-population game, we must use the Jacobian matrix (Ω). Calculating the eigenvalues: $\Omega(f_A, f_B) = \begin{bmatrix} \frac{d\dot{f_A}}{df_A} & \frac{d\dot{f_A}}{df_B} \\ \frac{d\dot{f_B}}{df_A} & \frac{d\dot{f_B}}{df_B} \end{bmatrix}$; doing the determinant $det(\Omega - \lambda_j) = \begin{vmatrix} \frac{d\dot{f_A}}{df_A} - \lambda & \frac{d\dot{f_A}}{df_B} \\ \frac{d\dot{f_B}}{df_A} & \frac{d\dot{f_B}}{df_B} - \lambda \end{vmatrix} = 0$. We finally have that $\lambda_{1,2} = tr\Omega \pm \sqrt{tr\Omega^2 - 4det\Omega}$.

For the stationary point to be asymptotically stable, the eigenvalues $\lambda_{1,2}$ of the matrix (Ω) evaluated at points that hold the condition $\dot{f_A} = 0$ and $\dot{f_B} = 0$ must have negative real parts. The analytical solution and the phase diagrams, responsible for the evolutionary game dynamics, were made with the open source software Dynamo[10]

3.1 Agent Based Simulation Algorithm

In this subsection, an algorithm to complement the evolutionary game and to guide the dynamic interaction among firms is presented. In this way, a stochastic component is implemented[11] in the analysis of evolutionary equilibria using the ABS method, which has

[10] See 51.

[11] To implement the algorithm, we made use of the Java programming language.

been largely used in the understanding of the evolution of cooperative behavior in social dilemmas[12], as can be seen in (31) and (32).

To implement the computational simulation, the intuition presented by (32) is followed. In this sense, at a time $t = 0$, we establish an initial proportion of firms' A and B in each population, (f_A, f_B), that play each one of the pure strategies available. Evolutionary dynamics are introduced in sequence and, at each Monte Carlo time Step (MCS), a Focal Agent i, which can update[13] its strategy, is chosen randomly. This occurs simultaneously in both populations. Focal Agent i, in turn, plays against a random opponent from the rival population and starts the game presented in matrix (9). Thus, Focal Agent i will obtain a payoff V_i. At the same time, an agent j is randomly chosen as a Reference Agent, which randomly plays against an opponent from the rival population and obtains a payoff V_j. Notice that focal and reference players belong to the same population. The Focal Agent i compares V_i and V_j to analyze the possibility of updating his strategy in two stages as stated ahead: (i) If $V_i \geq V_j$, focal player keeps his strategy; (ii) If $V_i < V_j$, focal player might update his strategy to the one adopted by reference player with a probability given by the variable w:

$$w = \frac{V_j - V_i}{max.payoff - min.payoff} \tag{15}$$

The maximum and minimum payoffs are obtained from the game matrix. By doing this procedure, we guarantee that $w \in (0, 1)$. To establish a decision criterion whether Focal Agent i updates his strategy or not, a random number generator is used, and is conveniently named $rnd \in (0, 1)$. In this way, a stochastic component on the dynamics of the game is implemented. Focal Agent i compares rnd with the probability w, so that: (iii) If $w \geq rnd$, Focal Agent i updates its strategy and imitates the Reference Agent j; (iv) If $w \leq rnd$, Focal Agent i does not update its strategy.

At every MCS, randomly selected individuals from both populations have the opportunity to, on average, change strategy at least once, comparing their payoffs with the Reference Agent j. We say that, on average, individuals can update strategies once, because within a MCS the same player may be invited to play many times, and other players may not, since the process of players' selection is random. Thus, when all players in both populations, on average, have the opportunity to update their strategies, a MCS is completed and a new MCS starts in order to repeat the dynamics of the game. This procedure characterizes the ABS model.

3.2 Spatial-structured Two-population

When applying this procedure in regular lattices, the number of opponents with which individuals interact depends on the spatial arrangement of the game and directly impacts the value of w. It is important to notice that, in a spatial-structured network, the concept of local neighborhood emerges, and there only will be competition among players that belong to certain positions in the $N \times N$ dimensional matrix.

[12] In (30), the study of social dilemmas is defined as the study of the conflict between individual and collective rationality. In a social dilemma, individually reasonable behavior leads to a situation in which everyone is worse off.

[13] Two mechanisms to update the population regarding the strategic interactions are largely used in agent based simulation methods: synchronous and asynchronous. Here, we use the second, since it allows the overlapping generations interactions. See (52) and (31).

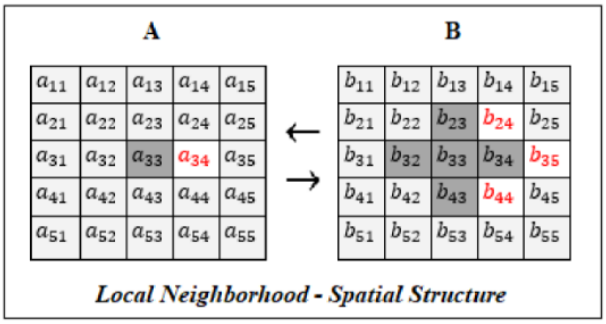

Figure 1 – Interaction among players in a spatial- structure population.

Figure 1 presents an illustration of the two-dimensional game dynamics in a spatial-structure format. It is possible to identify the neighborhood of each focal and reference agent according to their position in the matrix. In this paper we consider the von-Neuman[14] neighborhood. Suppose that the element $\{a_{33}\}$, ilustred on the right side of Figure 1, is the focal agent that is programmed to play the strategy UK. According to the von-Neuman neighborhood, the reference agent of $\{a_{33}\}$, i.e., the one which he will compare his payoff, to decide whether he updates his strategy or not, could be randomly selected between the elements $\{a_{34}, a_{32}, a_{23}, a_{43}\}$. Suppose, for example, that $\{a_{34}\}$, who is programmed to play EU, is selected as the reference agent.

The elements of the rival population (B) with which the focal agent $\{a_{33}\}$ competes are $\{b_{33}, b_{23}, b_{43}, b_{34}, b_{32}\}$. The reference agent $\{a_{34}\}$ competes with the individuals located at $\{b_{34}, b_{33}, b_{35}, b_{24}, b_{44}\}$. The same interaction happens simultaneously in the opposite direction, that is, there is competition between players of population B with the population A and vice verse. Attention should be paid in considering the strategic interactions of those players located on the border of the spatial structure. The local neighbor with which element $\{a_{11}\}$ randomly selects to compare his payoff is one of the elements $\{a_{51}, a_{21}, a_{15}, a_{12}\}$ and the set of players from the rival population that $\{a_{11}\}$ compete are located in the cells $\{b_{11}, b_{12}, b_{15}, b_{21}, b_{51}\}$. In this matter, we can see the spatial structure similar to a toroid[15].

The strategy update criterion is calculated by the average payoff of each player, founded by the arithmetic mean of the payments (V) obtained in each interaction with the n players that compose the local neighborhood. The Focal Agent i and the Reference Agent j now receive a payoff given by, respectively:

$$V_i = \frac{\sum_{i=1}^{n} v_i}{n} \tag{16}$$

$$V_j = \frac{\sum_{j=1}^{n} v_j}{n} \tag{17}$$

As explained before, if $V_j > V_i$, the focal agent may imitate reference agents' strategic behavior with probability w. Note that w is based on the averages received by the focal and reference agents. The worse the focal players' performance in relation to the reference, the greater the probability that he imitates reference agents' strategy.

[14] Such a neighborhood consists of four cells arranged orthogonally around the so-called central cell. For more detail, see (53).

[15] In mathematics, a toroid is a surface of revolution with a hole in the middle, like a doughnut, forming a solid body. The axis of revolution passes through the hole and so does not intersect the surface.

4 Results

To generate the following results, we assign weights to the variables obtained from the input-output[16] analysis for sectors 1 and 41. Table 2 summarizes the values chosen for each of the evaluated cases. In the homogeneous scenario, firms are considering that the probability of Brexit occurrence is $s = 1/2$ and they are attributing homogeneous weights for each of the factors considered. In scenario II it is assigned a high probability of Brexit occurrence, ($s = 0.7$), with firms attributing more weight to the variables associated with market potential$(\theta, \mathbf{\Theta})$ and productive integration $(\rho, \mathbf{P})$. In scenario III it is also considered a high probability of occurrence of Brexit ($s = 0.7$), with firms assigning greater weight to the variables associated with labor cost $(\psi, \mathbf{\Psi})$ and displacement cost $(\mu, \mathbf{M})$.

Homogeneous Scenario		Scenario I		Scenario II	
(s = 0.5)	(1-s = 0.5)	(s = 0.7)	(1-s = 0.3)	(s = 0.7)	(1-s = 0.3)
$w_\rho = 0.25$	$W_{\mathbf{P}} = 0.25$	$w_\rho = 0.45$	$W_{\mathbf{P}} = 0.45$	$w_\rho = 0.05$	$W_{\mathbf{P}} = 0.05$
$w_\theta = 0.25$	$W_{\mathbf{\Theta}} = 0.25$	$w_\theta = 0.45$	$W_{\mathbf{\Theta}} = 0.45$	$w_\theta = 0.05$	$W_{\mathbf{\Theta}} = 0.05$
$w_\psi = 0.25$	$W_{\mathbf{\Psi}} = 0.25$	$w_\psi = 0.05$	$W_{\mathbf{\Psi}} = 0.05$	$w_\psi = 0.45$	$W_{\mathbf{\Psi}} = 0.45$
$w_\mu = 0.25$	$W_{\mathbf{M}} = 0.25$	$w_\mu = 0.05$	$W_{\mathbf{M}} = 0.05$	$w_\mu = 0.45$	$W_{\mathbf{M}} = 0.45$

Table 1 – Summary of the weighted factors considered in each Scenario.

4.1 Homogeneous Scenario

The homogeneous scenario features a unique balance for both Sector 1 and Sector 41. In figure 2 we present the phase diagrams. The black dots correspond to the ESS strategies and the white dots are unstable. The colors of the phase diagram can be interpreted as follows: the region in red (blue) corresponds to the maximum (lowest) speed of convergence of the system. In this way, let Φ^{ESS} be the set of evolutionary stable strategies. For firms operating in Sector 1 and in Sector 41, we have $\Phi^{ESS} = \{(UK, EU)\}$. Note that the ESS matches with the Nash equilibrium.

$$\text{Sector 1} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ 0.14; -0.1 & \mathbf{0;0} \\ 0;0 & -0.62; -0.38 \end{pmatrix} \qquad \text{Sector 41} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ 0.16; -0.07 & \mathbf{0;0} \\ 0;0 & -0.64; -0.41 \end{pmatrix}$$

We observed firms that were initially operating in the EU updating its locational decision to UK. On the other hand, firms established in UK, observing this movement, will do the opposite movement, that is, they will start to migrate to EU. The economic intuition of this equilibrium is that firms has some benefits when incurring at the displacement cost, since it relocates in an unsaturated market. Therefore, we observe that this result may reflect the characteristics of Sector 1, since costs of labor and costs of displacement are relatively low. Another argument may be due to the low product differentiation in this sector. The economic intuition of this result reflects that the benefits of exploring a new market outweigh the displacement costs in a scenario of high uncertainty and homogeneous weights for the factors considered in the location decision.

[16] In the appendix, the table 2 presents the values of each of the parameters referring to sectors 1 and 41, respectively. These values were extracted from the input-output matrix. Departing from the gross

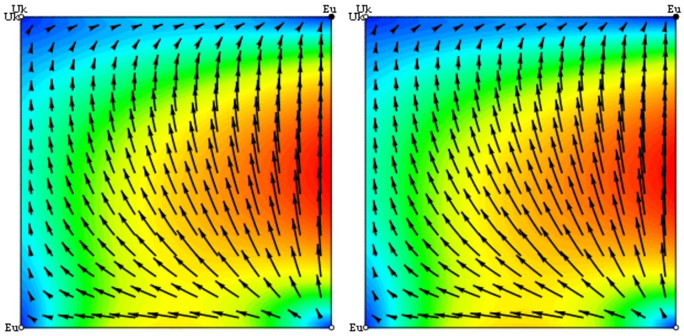

Figure 2 – Phase diagram for Sector 1 and for Sector 41

By considering a low level of spatial desegregation, in addition to not providing greater relevance to any of the factors considered in the analysis, there is a need to better evaluate the trade-off between the advantages of agglomeration (ρ and θ) and production costs (ψ and μ). In addition, another relevant issue is to know more precisely in which of the EU countries firms will decide to locate. In this sense, we apply the ABS model presented in subsections 3.1 and 3.2 in order to better evaluate and understand the dynamic equilibrium of the spatial competition among firms.

As previously described, there are results in the literature that emphasize a robustness between the analytical solution of the replicator dynamics (RD) and that obtained through numerical simulations via ABS. By comparing figure 2 with figures 3 and 4, we can observe the convergence between the methods of evaluation of the spatial game. In MCS 1, the initial distribution of the available strategies for the homogeneous scenario is obtained. For both sector 1 (top) and sector 41 (bottom), market shares were considered as the initial condition of the game, i.e., $(f_{UK1}; f_{EU1}) = (0.18; 0.82)$ and $(f_{UK41}; f_{EU41}) = (0.52; 0.48)$.

From then on, notice that the learning rule is linked to the spatial neighborhood competition. In other words, observing a higher relative frequency of firms playing EU in MCS 10, there is an increase in the relative frequency of firms adopting the UK strategy in both populations. As already stated, this graphically illustrates the search for unsaturated markets. This movement is interrupted as the frequency of firms playing UK in both populations rises concomitantly - which equals the cost-benefit ratio of relocation. In MCS 30, we observe a large clustering of type A firms playing UK, while type B firms choose to update their strategy towards UK. Finally, from figure 3, the balance occurs with type A firms migrating to UK and type B firms migrating to EU. From this homogeneous scenario, we might conclude that the benefit from the less exploited market outweighs the costs of moving towards this market. The rationale is analogous for sector 41.

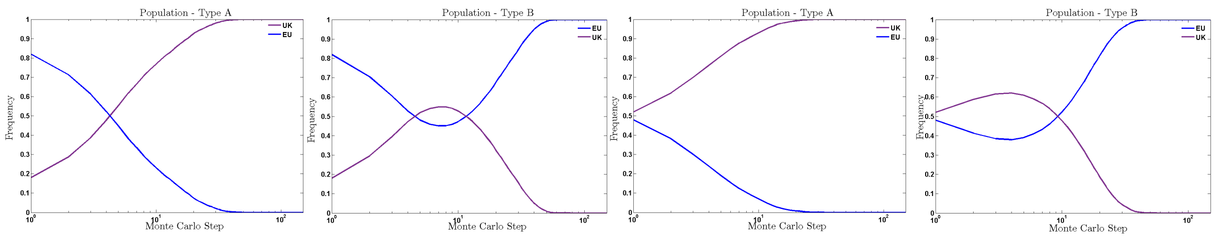

Figure 3 – **Left**: Game Dynamics - Sector 1. **Right**: Game Dynamics - Sector 41.

values, we performed a comparative static exercise, in which the values were scaled for the interval $(0, 1)$. The largest parameters observed assumed value equal to 1.

Using the convergence between RD and ABS obtained in this scenario, only the numerical simulation method will be applied in order to evaluate scenarios II and III. By increasing the number of available strategies for each player our analysis will be more complex, making the analytical treatment very costly and difficult to estimate.

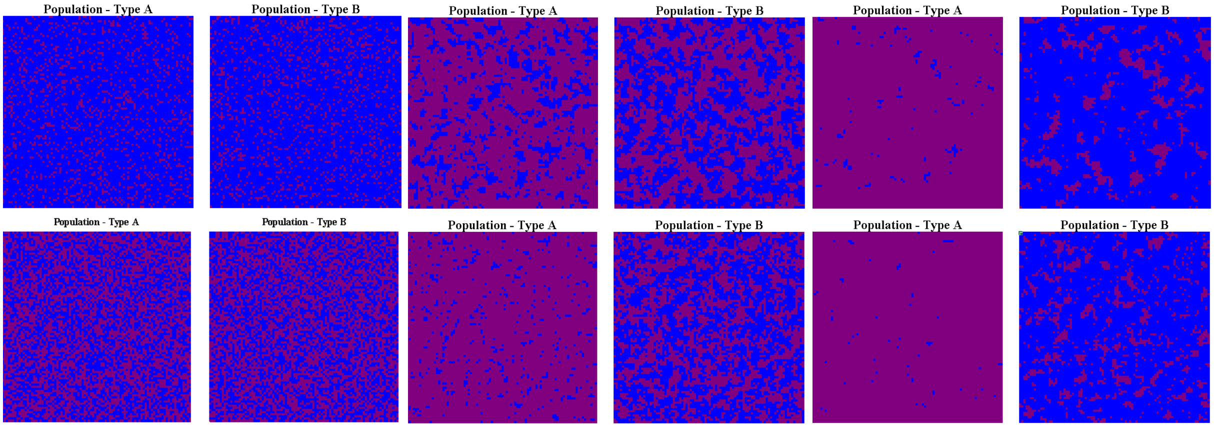

Figure 4 – **Top**: MCS 1, 10 and 30 - Sector 1. **Bottom**: MCS 1, 10, 20 - Sector 41.

4.2 Scenario II

Based on the homogeneous scenario, it is possible to observe that the equilibrium in the location decision depend on the sectoral characteristics and the dichotomy between the advantages associated to the productive integration and the displacement costs. These outcomes suggest that it is possible to evaluate location decisions due to a greater degree of country desegregation and by attributing different weights for each of the strategic variables for the location decision.

Following in this direction, scenario II assigns greater weights to the variables associated with the agglomeration advantages such as market potential, $(\rho, \mathbf{P})$, and productive integration, (θ, Θ), (8, 9, 10, 11). Therefore, the regions were desegregated in accordance to their relevance in terms of market potential and productive integration, as expressed in Table 2, available in the appendix. For sector I, in addition to the UK and EU, Luxembourg (LU) and Portugal (PO) countries were included as strategies.

Sector 1

	UK	EU	LU	PO
UK	$0.29; 0.32$	$0.60; 0.77$	$0.60; 0.90$	$0.60; 0.84$
EU	$0.76; 0.63$	$0.38; 0.39$	$0.76; 0.90$	$0.76; 0.84$
LU	$0.90; 0.63$	$0.90; 0.77$	$0.45; 0.45$	**0.90;0.84**
PO	$0.84; 0.63$	$0.84; 0.77$	**0.84;0.90**	$0.43; 0.43$

Sector 41

	UK	EU	FR	FI
UK	$0.32; 0.35$	$0.64; 0.67$	$0.64; 0.84$	$0.64; 0.90$
EU	$0.67; 0.67$	$0.33; 0.33$	$0.67; 0.84$	$0.67; 0.90$
FR	$0.88; 0.67$	$0.84; 0.67$	$0.43; 0.43$	**0.84;0.90**
FI	$0.90; 0.67$	$0.90; 0.67$	**0.90;0.84**	$0.45; 0.45$

From the payoff matrix, we can see that for both sector 1 and sector 41, the proposed desegregation was able to provide a more accurate analysis about firms' location decision. It is still possible to identify that UK and EU strategies are strictly dominated by LU and PO in sector 1, and by FR and FI in sector 41. Thus, considering $(f_{UK1}; f_{EU1}; f_{LU1}; f_{PO1}) = (0.16; 0.76; 0.04; 0.04)$, the dynamic balance of the game consists of the LU and PO strategies. The result suggests that, although initially few firms adopt the strategy of locating in Luxembourg or Portugal, to the extent that this behavior generates larger payoffs, neighboring firms tend to imitate it. This process is visible at the left side of figure

5 and at the bottom of figure 6, in which an evolutionary equilibrium is composed by a population of firms migrating to PO and by the other population migrating to LU[17].

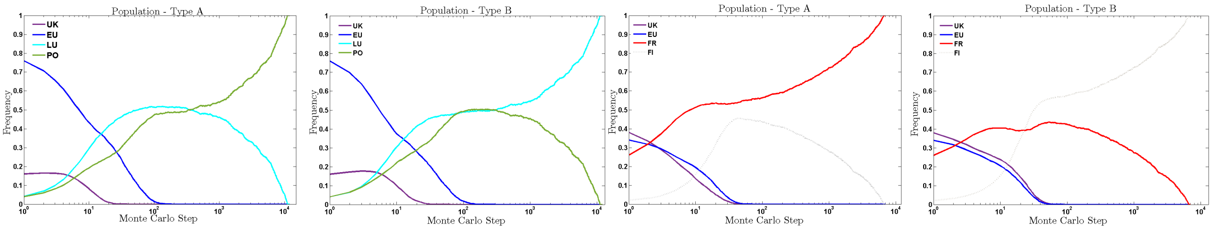

Figure 5 – **Left**: Game Dynamics - Sector 1. **Right**: Game Dynamics - Sector 41.

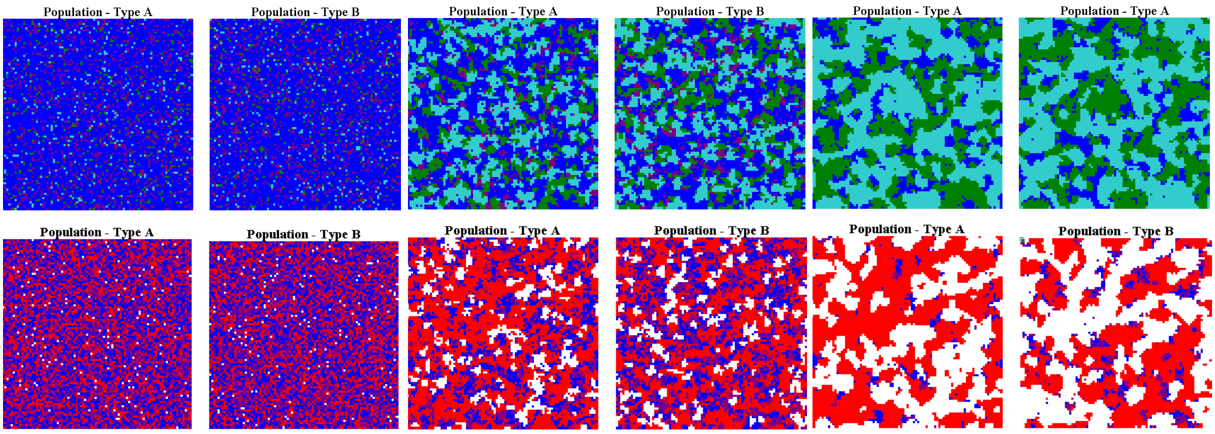

Figure 6 – **Top**: MCS 1, 10 and 40 - Sector 1. **Bottom**: MCS 1, 20, 40 - Sector 41.

Similarly, considering $(f_{UK41}; f_{EU41}; f_{FR41}; f_{FI41}) = (0.38; 0.34; 0.26; 0.02)$, UK and EU strategies are strictly dominated by FR and FI. So, the economic intuition suggests that, despite the peculiarities attached to each sector, a behavioral pattern is observed. Firms always decide to locate in countries where they observe greater advantages of agglomeration, reflected by ρ and θ. This result is maintained even when we assign higher probabilities to the occurrence of Brexit.

4.3 Scenario III

Sector 1

	UK	EU	LU	PO	FR	NL
UK	$-0.08; 0.21$	$-0.05; 0.20$	$-0.05; 0.10$	$-0.05; 0.20$	$-0.05; 0.97$	$-0.05; 0, 86$
EU	$0.13; 0.24$	$0.09; 0.16$	$0.13; 0.10$	$0.13; 0.20$	$0.13; 0.97$	$0.13; 0.86$
LU	$0.10; 0.24$	$0.10; 0.20$	$0.05; 0.05$	$0.10; 0.20$	$0.10; 0.97$	$0.10; .860$
PO	$0.20; 0.24$	$0.20; 0.20$	$0.20; 0.10$	$0.16; 0.16$	$0.20; 0.97$	$0.20; 0.86$
FR	$0.97; 0.24$	$0.97; 0.20$	$0.97; 0.10$	$0.97; 0.20$	**0.93;0.93**	$0.97; 0.86$
NL	$0.86; 0.24$	$0.86; 0.20$	$0.86; 0.10$	$0.86; 0.20$	$0.86; 0.97$	$0.83; 0.83$

In this latter scenario, a greater relevance is given to ψ and μ, which reflect the costs with labor and the displacement costs. We then break up the rest of the European

[17] As the simulation process is random, we observe equilibria composed of Type A firms playing PO and Type B firms playing LU. The opposite was also observed, i.e., firms of Type A playing LU and firms of Type B playing PO.

Union according to the lower costs observed, following (3, 4, 5, 6, 7). For sector 1, the countries listed as possible destinations in this case are $(f_{UK1}; f_{EU1}; f_{LU1}; f_{PO1}; f_{FR1}; f_{NL1}) = (0.10; 0.51; 0.01; 0.03; 0.26; 0.04)$.

Sector 41

	UK	EU	GR	FI	GE	AU
UK	0.04; 0.30	0.08; 0.13	0.08; 0.08	0.08; 0.08	0.08; 1.10	0.08; 0.22
EU	0.10; 0.33	0.06; 0.09	0.10; 0.08	0.10; 0.08	0.10; 1.10	0.08; 0.22
GR	0.09; 0.33	0.08; 0.13	0.04; 0.04	0.08; 0.09	0.08; 1.10	0.08; 0.22
FI	0.09; 0.33	0.09; 0.13	0.09; 0.08	0.04; 0.04	0.09; 1.10	0.09; 0.22
GE	1.10; 0.33	1.10; 0.13	1.10; 0.08	1.10; 0.09	**1.06;1.06**	1.10; 0.22
AU	0.22; 0.33	0.22; 0.13	0.22; 0.08	0.22; 0.09	0.22; 1.10	0.18; 0.18

By it turns, for sector 41 the following countries are listed as possible destinations: $(f_{UK1}; f_{EU1}; f_{GR1}; f_{FI1}; f_{DE1}; f_{AU1}) = (0.33; 0.29; 0.03; 0.02; 0.29; 0.04)$. In the same way as in the previous scenario, it is possible to observe a behavioral pattern in the locational decision of firms. The results show an equilibrium in dominant strategy for both sectors.

Thus, in the face of Brexit's uncertainty, if firms uphold their location decision on the factors associated with cost minimization, the observed net effect is that firms initially established in both the UK and the EU will re-establish themselves in the French and German markets for sectors 1 and 41, respectively. The fact that convergence has occurred more rapidly in sector 41 may be linked to greater mobility of the productive factors in this sector. On the other hand, sector 1, which is strongly associated with natural resources, showed a slower convergence. So, the result may suggest that in the case in which one firm bases its decision on the advantages of cost minimization, indirectly, it will take into account the degree of productive integration in order to maximize the payoff utility.

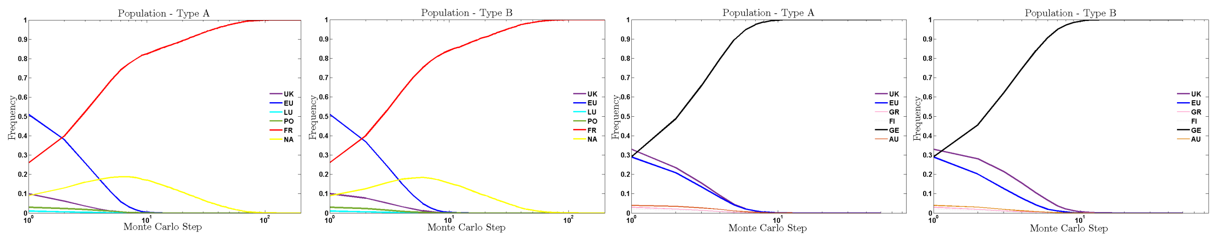

Figure 7 – **Left**: Game Dynamics - Sector 1. **Right**: Game Dynamics - Sector 41.

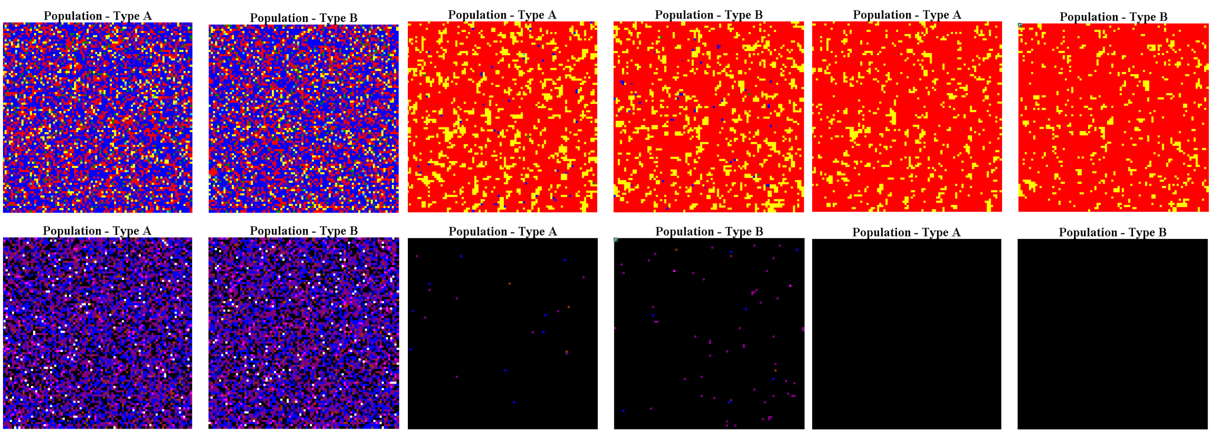

Figure 8 – **Top**: MCS 1, 10 and 20 - Sector 1. **Bottom**: MCS 1, 10, 20 - Sector 41.

5 Conclusion

The objective of this article was to analyze the influence of Brexit on firm's location decision. In order to achieve this goal, we proposed a model based on the bounded rationality assumptions. From then on, a stochastic learning rule was created, in which the agents could imitate the behavior of neighboring firms over time with a probability generated from the relative distance between the payoffs earned. Two sectors with very particular characteristics were chosen: (i) Sector 1, Crop and animal production, hunting and related service activities, characterized by high locational advantages, strong dependence on natural resources and largely subsidized by local governments; (ii) Sector 41, Financial services activities, except insurance and pension funding, characterized by being a service provider, with high integration and relevance in the United Kingdom economy.

Thus, the results indicated that the occurrence of Brexit can be determinant in the decision of firm location. It was possible to find evidence that it may modify the distribution of the relevant geographic market on the European continent. On this matter, the sensitivity of the firms due to the Brexit is conditioned to the sector of performance and the relevance given to each of the factors evaluated in the decision process. In this sense, in sectors associated with high locational advantages, strong dependence on natural resources and traditionally benefited by fiscal subsidies, the prospect of Brexit indicates that in an environment of uncertainty, firms tend to search for unsaturated markets.

Given the above, we can observe the following behavior pattern of firms for the proposed game: (a) if the greater motivation is tied to agglomerative advantages, the associated displacement costs are irrelevant, and firms always see greater benefits when moving from the host country; (b) on the other hand, when the strategic variable is cost minimization, firms weigh these factors (indirectly) with regions that have greater productive integration, so that they can maximize the utility of their payoff.

Therefore, when firms determine the degree of agglomeration as a key variable in their location decision making, we observe from the desegregation proposed in scenario II, that the most successful strategy for a firm competing in sector 1 (41) is to migrate to LU or PO (FR or FI). In the opposite direction, If they are concerned with cost minimization, firms in sector 1 (41) decide to locate in France (Germany). Note that under these conditions, they see greater benefits in dividing the same market. This suggests that the advantages found in such regions outweigh the costs linked to the displacement of the productive base. Therefore, the net effect is a migration of firms previously established in both the UK and the EU to France and Germany, respectively.

Concluding, there are many direction to extent this research. One could consider a different degree of desegregation of countries and other productive factors. Another possibility is to elaborate a payoff structure of the game in which it is possible to endogenize the sensitivity of the game balance according to the incentive of the "nth firm" to migrate, given the proportion of its rivals that have already made that decision.

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6 Appendix

Table 2 – Standardization of variables for Sectors 1 and 41

Countries	**P**	**Θ**	**Ψ**	**Δ**	**M**	ρ	θ	ψ	δ	μ
Sector 1 - No Brexit						Sector 41 - No Brexit				
UK	1	1	0,52	0,30	0,67	1	1	1,00	-0,05	-0,43
GE	1	1	0,68	-0,91	1,00	1	1	0,57	0,01	-1,79
BE	1	1	0,11	0,57	-0,26	1	1	0,11	0,01	-0,23
FR	1	1	0,96	0,37	-1,25	1	1	0,79	-0,20	0,61
IT	1	1	1,00	-0,31	-1,21	1	1	0,62	-0,02	0,99
LU	1	1	0,00	0,42	-0,01	1	1	0,40	0,06	1,00
NL	1	1	0,77	0,21	-1,31	1	1	0,31	-0,07	-0,08
DA	1	1	0,17	1,00	0,00	1	1	0,08	0,27	-0,19
EI	1	1	0,40	0,06	1,00	1	1	0,14	0,06	0,30
GR	1	1	0,24	0,00	-0,43	1	1	0,05	-0,06	0,03
SP	1	1	0,95	-0,42	-0,96	1	1	0,30	0,31	0,40
PO	1	1	0,04	-0,52	-0,21	1	1	0,07	-0,26	0,11
AU	1	1	0,07	0,27	-0,13	1	1	0,06	0,08	-0,28
FI	1	1	0,07	-0,78	0,00	1	1	0,04	1,00	0,08
SW	1	1	0,08	-0,02	-0,10	1	1	0,11	0,01	0,15
EU	1	1	0,20	0,02	-0,21	1	1	0,06	-0,01	0,01
Sector 1 - Brexit						**Sector 41 - Brexit**				
UK	0,65	0,43	0,48	0,30	0,64	0,65	0,54	0,41	-0,05	0,97
GE	1	0,47	0,66	-0,91	1,00	1	0,60	0,70	0,01	-1,69
BE	1	0,63	0,10	0,57	-0,26	1	0,25	0,12	0,01	-0,19
FR	1	0,43	0,93	0,37	-1,25	1	0,77	1,00	-0,20	0,59
IT	1	0,25	1,00	-0,31	-1,25	1	0,41	0,83	-0,02	1,00
LU	1	1,00	0,00	0,42	-0,01	1	0,30	0,27	0,06	0,51
NL	1	0,36	0,72	0,21	-1,26	1	0,35	0,37	-0,07	-0,07
DA	1	0,46	0,16	1,00	0,00	1	0,48	0,10	0,27	-0,19
EI	1	0,30	0,14	-0,09	-0,29	1	0,40	0,11	0,06	0,18
GR	1	0,40	0,19	0,00	-0,36	1	0,19	0,06	-0,06	0,03
SP	1	0,17	0,95	-0,42	-0,99	1	0,48	0,37	0,31	0,38
PO	1	0,77	0,04	-0,52	-0,22	1	0,46	0,09	-0,26	0,11
AU	1	0,33	0,06	0,27	-0,12	1	0,52	0,07	0,08	-0,26
FI	1	0,39	0,07	-0,78	0,00	1	1,00	0,06	1,00	0,08
SW	1	0,65	0,08	-0,02	-0,10	1	0,40	0,14	0,01	0,14
EU	1	0,54	0,15	0,02	-0,13	1	0,26	0,04	-0,01	0,01

Authors' own elaboration based on WIOD.