Trend-Cycle Decomposition of the Brazilian GDP: New Facts for the period between 1947 and 2012

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Abstract. We provide information about the Brazilian business cycle from 1947 to 2012, by suggesting a new method that averages over a variety of HP-filters and creates a set of facts which are more related to CODACE dates of expansions and recessions. The main findings are that Brazilian business cycle is asymmetric, with expansions lasting longer than recessions; the long-term trend presented a noticeable flatter slope after the 1980s, thus, real long-term growth rate decreased by 50%, from 8% per year, between 1947 and 1980, to 4% per year after that; and, output volatility decreased after 1996-1997, when a statistically significant structural break occurred.

Keywords: GDP decomposition; Business cycle; Stylized facts; Brazil.

Resumo. O presente artigo fornece fatos estilizados dos ciclos de negócios brasileiros para o período de 1947 a 2012, propondo um método de decomposição tendência-ciclo que toma uma média de vários resultados de filtragens HP e gera um conjunto de informações mais correlacionado com as datas de expansões e recessões do CODACE. Os principais resultados obtidos são que os ciclos brasileiros são assimétricos, com expansões mais longas que recessões; que a tendência de crescimento de longo-prazo apresentou uma significante redução em sua inclinação (cerca de 50%), fazendo a taxa de crescimento reduzir de 8% ao ano entre 1947 e 1980, para 4% ao ano em seguida; e, que a volatilidade do produto real se reduziu em 1996 ou 1997, quando uma quebra estrutural significativa foi encontrada.

Palavras-chave: Decomposição do PIB; Ciclos de negócios; Fatos estilizados; Brasil.

JEL codes: C22, E23, E32.

ÁREA ANPEC: 4 – Macroeconomia, Economia monetária e finanças.

1. Introduction

Economics has traditionally studied business cycle, but the contributions of Burns and Mitchell (1946) were a watershed, prompting a wave of interest in regularities and features of economic activity and its fluctuations in many countries. Such investigation assumes great relevance whenever it comes to provide grounds for policy-making (both public and private), forecasting, model calibration, and theories testing, to name a few. However, as it is stated in the recent literature, in order to obtain consistent business cycle information, one of the main issues is how to separate the input series into trend and cycle components. The answer for that is not trivial, and it is the object of this research work.

We will focus on depicting facts about the Brazilian business cycles, by decomposing a quarterly and seasonally adjusted GDP time series for the 1947-2012 period. Amongst all the currently available trend-cycle decomposition methods, the Hodrick-Prescott filter (HP-filter, Hodrick and Prescott, 1997) stands out most. The HP-filter has been largely employed. Examples include Kydland and Prescott (1990), Backus Kehoe (1992), Ravn and Uhlig (2002) and, more recently, Perron and Wada (2009) and Kodama (2013), who applied it to U.S. and international data. For the Brazilian case, this filter was utilized, inter alia, by Ellery-Jr., et al. (2002), Ellery-Jr. and Gomes (2005) and Araújo, et al. (2008).

Other popular methods to extract the cyclical component of time series include the BN-decomposition (Beveridge and Nelson, 1981), based on unconstrained ARIMA models and estimated by Campbell and Mankiw (1987), and Morley, et al. (2003) for the U.S., and by Cribari-Neto, (1990; 1993) for the Brazilian economy; the band-pass filter (Baxter and King, 1999), used by Basu and Taylor (1999) and Ellery-Jr., et al. (2002); and the unobserved components model, UC, due to Clark (1987) and considered by Morley, et al. (2003) and Perron and Wada (2009). As far as we know, the UC model has not yet been applied to the Brazilian GDP decomposition. Nevertheless, Kannebley and Gremaud (2003) employ such type of methodology in an interesting study concerning the secular trend of the Brazilian terms of trade.

Although the trend-cycle decomposition has become computationally simpler due to all abovementioned methods, practitioners still face some key problems. For instance, one can show that the HP-filter outcome is extremely dependent on the smooth parameter (λ) and, most of the time, the rule of thumb for setting up its value, provided by Hodrick and Prescott (1997) and broadly employed, is not adequate (Perron and Wada, 2009).

Another notorious problem is that distinct decomposition procedures may lead to rather different trend–cycle components and stylized facts about economic activity. Canova (1998), for example, studies this question for the U.S. economy, showing that some evidence is not robust to changes in the filter. Besides, the dissimilarities between the cyclic component of the BN and UC methods are examined by Morley et al. (2003). The BN-cycle tends to be quite noisy and leaves more of the fluctuation for the trend component, while the UC models lead to larger and more persistent cyclic oscillations. Therefore, the choice of a specific method for filtering the data is by no means inconsequential, and probably affects the main results and brings policy implications.

The Brazilian GDP trend and cycle decomposition may be also facing these problems. In this respect, some recent results contradict the common view that, since the Real Plan implementation in July 1994, the Brazilian economy has become more stable. Araújo, et al. (2008) and Ellery-Jr. and Gomes (2005), for example, after using a HP-filter (λ=100), found that the Brazilian GDP volatility did not decrease in the post-war period. Additionally, Cribari-Neto (1990, 1993), while studying the annual Brazilian GNP and GDP data, respectively, from 1900 to 1990, and using the BN-filter, argued that the cyclic component of the Brazilian economic activity is small; that is, most of its oscillations are driven by "real-long-term" shocks to the trend. This conclusion does not match with Cunha and Ferreira (2004), who found that the welfare losses due to output fluctuations are significant in Brazil and reached up to 10%, a value higher than those estimated for the U.S. economy (see, e.g., Barlevy, 2004). Consequently, there is still room for improvements in the understanding of the Brazilian business cycle, which is a deep concern of the present work.

Two main contributions are provided. First, our estimation uses a new quarterly time series measured by Bonelli and Rodrigues (2012) for the period between 1947 and 1979, and by the Brazilian National Accounts System from then on (*Instituto Brasileiro de Geografia e Estatística*, IBGE, 2012). The first set of observations was calculated so as to be readily comparable to the second one, which avoids approximation errors. However, it should be clear that, when analysing a quarterly time series, we are able to cover oscillations shorter than one year and to compare our business cycle dates to those established by the Brazilian Business Cycle Dating Committee, CODACE, for the years between 1980 and 2009[1]. Thus, as in the U.S. case, the Brazilian economy also has a natural benchmark for the trend-cycle decomposition evaluation when studying higher frequency data.

Second and more importantly, based on the theories of model and forecast combination, and by exploring the flexibility of the HP-filter, we provide a new scheme to compute average trend and cycle components in which the problems of filter selection are reduced. The basic idea is that, by varying the filter's smoothness parameter properly, one can potentially reproduce the results of almost, if not all, the other methods (from the BN-filter oscillating trend, to the smooth deterministic trends). In this sense, we calculate a large variety of HP-filters with different values for λ, and then we take an average from the decomposition outcomes. The combination of different trend and cycle series is appealing since it produces components that are more adaptable in the presence of structural breaks, and it can be understood as a way to make the filtering procedure more robust against misspecification biases and measurement errors, when compared to individual methods[2]. Hence, the evidence reported here tends to be more reliable than that which is grounded on "once and for all" decompositions.

The remainder of this paper is organized as follows. Section 2 presents and discusses the trend-cycle decomposition procedure. Section 3 presents some preliminary results, unit root tests and comparisons between our method and the CODACE business cycle dates. Section 4 brings the filtered Brazilian GDP's facts and information, using a quarterly data set ranging from 1947 to 2012, while subsection 4.1 implements a number of robustness and structural change tests. Finally, section 5 shows the main conclusions and implications of the research work.

2. The Hodrick-Prescott filter and the average cycle

This section presents the trend-cycle decomposition method applied in the paper, covering briefly the mathematics of the HP-filter, then discussing its main caveats, and finally showing how we can overcome them.

The Hodrick-Prescott filter is a method developed for extracting a smoothed version, τ_t, from some given original series, say y_t. The τ_t component is considered as being the long-term trend, while the residuals, y_t - τ_t, contain the cyclic components. A parameter, λ, controls the smoothness of the trend series. As it tends to infinity, τ_t approaches the linear trend case (Hodrick and Prescott, 1997).

The value to be chosen for λ is still an open question in the economic literature, and it can have profound practical consequences. A wrong λ, for instance, may impute the greater part of the y_t's variation to the trend, leaving the cyclic component economically irrelevant. Nevertheless, most of the researchers, for the sake of simplicity, just follow the recommendations of Hodrick and Prescott (1997), setting it as 1600 for quarterly data[3] (e.g. Kydland and Prescott, 1990; Backus and Kehoe, 1992; Ellery-Jr., et al., 2002; Ellery-Jr. and Gomes, 2005; Araújo, et al., 2008; and Michaelides, et al., 2013; amongst several others).

On the other hand, Perron and Wada (2009) show that a λ=800,000 is good choice for detrending the U.S. quarterly real GDP during the period between 1947:01 and 1998:02. While Pedersen (2001), making an effort to find the best value for λ based on the theory of optimal filtering, suggests a value around 1,000 and 1,050 for this parameter on quarterly data.

Possibly, the only consensus economists have reached regarding λ is that its value must represent the underlying structure of some data generating process (DGP). However, since a DGP may vary across countries and across the time, it turns out to be extremely difficult to elect a particular smoothness parameter as the true one. Indeed, a good procedure shall consider a set of

[1] CODACE's committee provides the Brazilian business cycle chronology between 1980 and 2009. Its researches will expand the dating process for the 1947-2012 years using the same data set employed in this work.

[2] This is a usual result in the model and forecast combination fields (Pesaran and Timmermann, 2005; Timmermann, 2006).

[3] They found this value by squaring the ratio of the cyclical component's variance, set as 5% per quarter, and the variance of second differenced term, set as 1/8% per quarter.

conceivable values and find out a way to use this information in order to highlight features of the data set. This is exactly how we proceed in the present paper.

The method proposed here comprises two steps. First, it decomposes the original series, in our case the Brazilian quarterly GDP, into trend and cycle components by using the HP-filter with a variety of smoothness parameters. Second, it takes the mean over the outcome of these filtering processes, which can provides series with remarkable features, as discussed later. It is a simple method, nevertheless, strongly based on results obtained by the theory of model combination. Now, let us turn to details of each step, beginning with the second one.

The true trend and cycle components of an economic time series are unknown. Practitioners try to model them, employing a specific method and choosing some parameters of control, such as λ in the HP-filter case. Now, suppose that we are interested in examine the variable y_c, say, the GDP cyclic component of any country, and that two options, y_{c1} and y_{c2}, are available, namely, the outcome of two different HP-filters provided by distinct analysts. Let the first guess be based on some N_1-vector of information x, i.e., $y_{c1} = g_1(x)$ while the second is based on some N_2-vector of information z, i.e., $y_{c2} = g_2(z)$[4]. If $\{x, z\}$, the full information set, were observable, it would be natural to construct a model based on all variables contained in x and z, i.e., $y_{c3} = g_3(x,z)$. On the other hand, if only the values of y_{c1} and y_{c2} are observed by the modeller (while the underlying variables are not), then the theories of model and forecast combination states that the better strategy is to combine these guesses, using a model of the type $y_{c,m} = g_c(y_{c1},y_{c2}; w)$, where w refers to the combination weights (cf., Clemen, 1987; Timmermann, 2006; and Moral-Benito, 2015).

In this paper, we assign equal weights for w, since this assumption has been providing good results even when comparing to others more elegant combinations of weights (Clemen, 1989, p.559; Moral-Benito, 2015). By their turn, Palm and Zellner (1992) find that adopting a simple average method is interesting because in many situations it will achieve a substantial reduction in the variance and bias of the forecasts. In addition, Timmermann (2006), analysed a quarterly data set comprising up to 43 time series for the G7 economies between 1959 and 1999, and found that the out-of-sample mean square forecast error (MSFE) for the arithmetic mean forecast is about 10% lower than the MSFE obtained by the best single model.

Another positive effect may arise from the combination of filter outcomes, namely, the higher degree of flexibility when facing structural breaks. Structural breaks are an important question when decomposing a time series and their incidence is almost certain during long time spans. Breaks can affect the stationarity of the series, introduce spurious correlations among its points, and make the trend-cycle decomposition troublesome, thus affecting analyses that do not account for this question.

Typically, it is difficult to timely detect structural breaks, but it is plausible that, on average, combinations of filter outcomes with different degrees of adaptability to breaks will outperform decompositions emerging from individual models. Some decomposition procedures have a more oscillating trend that will be only temporally affected by the break, while others have a smoother trend that will slowly adjust. As long as more data points are available after the break occurrence, slow-adapting models will perform better than fast-adapting ones, since the parameters of the former are more precisely estimated. On the other hand, if the data window from the most recent break is short, fast-adapting models can be expected to produce the best trend-cycle representations (see Pesaran and Timmermann, 2005). This intuition is easily expanded to the case where parameter and model uncertainty are present. As long as the DGP is a mutable process, the best decomposition model for a given economy will possibly change over time, in a sense that combining different filter outcomes can make the decomposition components more robust against such misspecification errors.

In short, the combination of filters is a way around the uncertainties arising from a complex data generating process. As stated by Winkler (1989, p.606): "(…) *in many situations there is no such thing as a 'true' model for forecasting purposes. The world around us is continually changing,*

[4] Here, these two analysts use their information when choosing the smoothness parameter.

with new uncertainties replacing old ones." This insight implicitly assumes that one could not identify the underlying process, but that different filtering procedures are able to capture various aspects of the information contained in the time series, and produce more consistent business cycle facts by using an averaging scheme.

As said before, the λ parameter of the HP-filter controls the smoothness of the trend. In this sense, we can benefit from its flexibility in order to mimic the trend and cycle components of other filtering methods. An illustration may explain this statement. First, we simulate a time series model as an autoregressive process taking the form $y_t = 0.2+0.99y_{t-1}+\varepsilon_t$, where ε_t is a Gaussian white noise. Then, we decompose the latter time series by utilizing two extreme case procedures: the noisy BN-ARIMA(2,1,0) filter, depicted in part "a" of Figure 1, below; and, the smooth third order polynomial trend, illustrated in part "c" of the same graph. Next, we use the HP-filter trying to replicate these two previous results. Part "b" shows the trend of a HP(λ=1)-filter, and part "d" a HP(λ=960,000)-filter. As we can see, the HP-filter is a good approximation for both methods, and by varying λ properly, one can reproduce an even larger range of filtering outcomes.

Moreover, we save time when making use of this approach since it avoids subsequent, and necessary, adjustments on the time series when averaging components from distinct trend-cycle decomposition methods (see, e.g., Lamo, et al., 2013, who employ this alternative procedure).

An extra detail should be noted with respect to the algorithm that averages the HP-filter components. In order to extract the best results from such method, the degree of overlap information between the series must be low. That is, the more new features are presented by the different filtering outcomes, the more useful and general are the combined series (see the intuition in Winkler, 1989). Nevertheless, the differences between two specific HP-filter outcomes decrease as the smoothness parameter increases. A series implied by a λ=10,000, for example, is quite similar to another obtained by setting a λ=11,000. Therefore, if the algorithm's step increases together with λ, the method will provide averaged trend and cycle components that will hold richer information about business cycle.

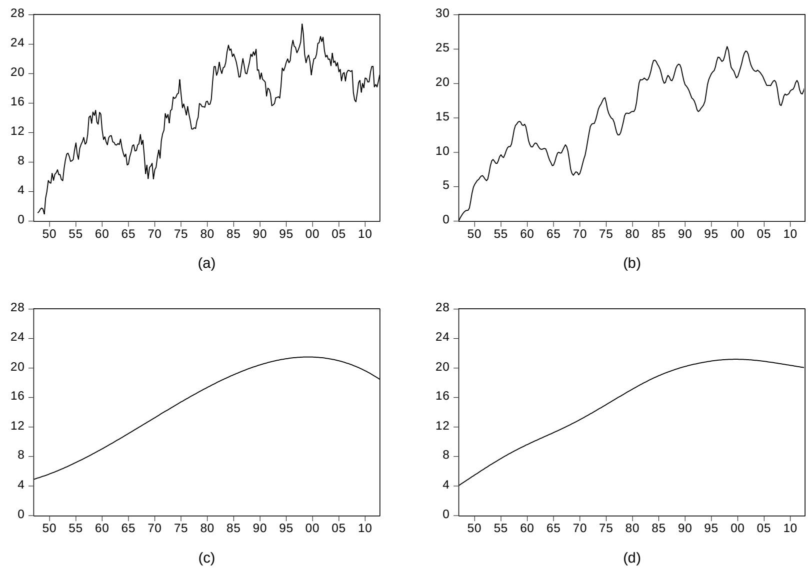

Fig.1: Trend component of the autoregressive process $y_t = 0.2+0.99y_{t-1}+\varepsilon_t$ according to the following models: (a) BN-ARIMA(2,1,0) filter; (b) HP(λ=1)-filter; (c) deterministic cubic trend; and, (d) HP(λ=960,000)-filter.

In this regards, we divide the HP-filter into five groups related to the degree of the trend smoothness, as is shown below, in Table 1. In each group, the algorithm's step was chosen as a simple function of the λ originally proposed by Hodrick and Prescott (1997).

Table 1: Number of HP-filters utilized for building the average trend and cycle series

Group	Values of λ	Algorithm step(1)	Number of filters
1	**1**; 17; 33; 49; 65; 81; 97; 113; 129; 145	$1{,}6x10$	10
2	160; 320; 480; 640; 800; **960**; **1,120**; 1,280; 1,440	$1{,}6x10^2$	9
3	**1,600;** 3,200; 4,800; 6,400; 8,000; 9,600; 11,200; 12,800; **14,400**	$1{,}6x10^3$	9
4	16,000; 32,000; 48,000; 64,000; 80,000; 96,000; 112,000; 128000; 144000	$1{,}6x10^4$	9
5	160,000; 320,000; 480,000; 640,000; **800,000**; 960,000	$1{,}6x10^5$	6
Total			43

Notes: (1) Algorithm step stands for the interval between two sequential λ's.

In Table 1 we highlight some interesting values for λ, fetched by the calculations. First we emphasize λ=1, number that leads to an extremely noisy trend component; next we have λ=960 and 1,120, which are closely related to the values suggested by Pedersen (2001); finally we utilize a variety of other λ, including λ=1,600 and 14,440 as recommended by Hodrick and Prescott (1997) for quarterly and monthly data, respectively, and λ=800,000, considered by Perron and Wada (2009) for the quarterly U.S. real GDP data. In all, we implement 43 different types of HP-decompositions that, on average, shall bear a good resemblance with the true Brazilian GDP's trend and cycle components[5].

3. Preliminary results: unit roots and decomposition evaluation

Perron (1989), in a seminal paper, brought light on the effects of structural breaks while analysing unit roots in economic time series. One major problem with Perron's (1989) methodology, however, is that he considers the breakdate as exogenous, i.e., known beforehand. Thus, one could choose the breakpoint dates based on its prior observation of the time series, generating problems such data-mining and the like. Zivot and Andrews (1992), instead, propose a test that circumvents this problem, by estimating, rather than fixing, the breakpoint date. In this paper, we use their statistics, as well as the Kwiatkowski, et al. (KPSS, 1992) tests. Table 2 shows the test results.

Table 2: Unit root tests, Brazilian quarterly GDP, 1947:01 – 2012:04

Panel (a): Zivot and Andrews (1992) tests on level GDP					
Break Hypothesis	*Lags SIC (Max.: 8)*	*Critical values*			*Calculated t-statistic*
		1%	*5%*	*10%*	
Intercept	5	-5.34	-4.93	-4.58	-3.38
Trend	5	-4.80	-4.42	-4.11	-3.95
Both	5	-5.57	-5.08	-4.82	-4.78
Panel (b): KPSS tests on level GDP					
Test assumption	*Bandwidth*	*Critical values*			*Calculated t-statistic*
		1%	*5%*	*10%*	
Intercept	12	0.74	0.46	0.35	2.05
Trend and intercept	12	0.21	0.15	0.12	0.51

As we can see above, Panel (*a*) in Table 2 shows calculated t-statistics less than the 10% critical value for each one of the assumptions, that is, failing to reject the null of a unit root with a structural break. By its turn, Panel (*b*) displays KPSS tests rejecting the null of stationarity in both

[5] When estimating λ from a model of local linear trend plus AR(2) cycle (Harvey and Trimbur, 2003), as the inverse of signal-to-noise ratio, we found a value of 168. Thus, our algorithm also covers such estimated parameter.

assumptions. Consequently, all tests are confirming the existence of a unit root in the log-levels of the Brazilian GDP data, even when one takes account to the possibility of one structural change.

As the log-level of the Brazilian quarterly GDP is non-stationary around a stochastic trend, we can turn our attentions to the decomposition components, since the HP-filter is able to remove up to four unit roots of a given series (see, e.g., Baxter and King, 1999; and Pedersen, 2001).

Figure 2, below, brings four series. First, part "a" shows the log-GDP time series with CODACE recession dates illustrated in the shaded areas. Part "b" depicts our first average time series, namely, our results for the Brazilian GDP long-term trend (solid line) and its two standard error bands (dashed lines). Part "c" brings our mean cyclic series (solid line) and its two standard error bands (dashed lines); and, part "d" illustrates the mean cyclic series with CODACE recession dates in the shaded areas.

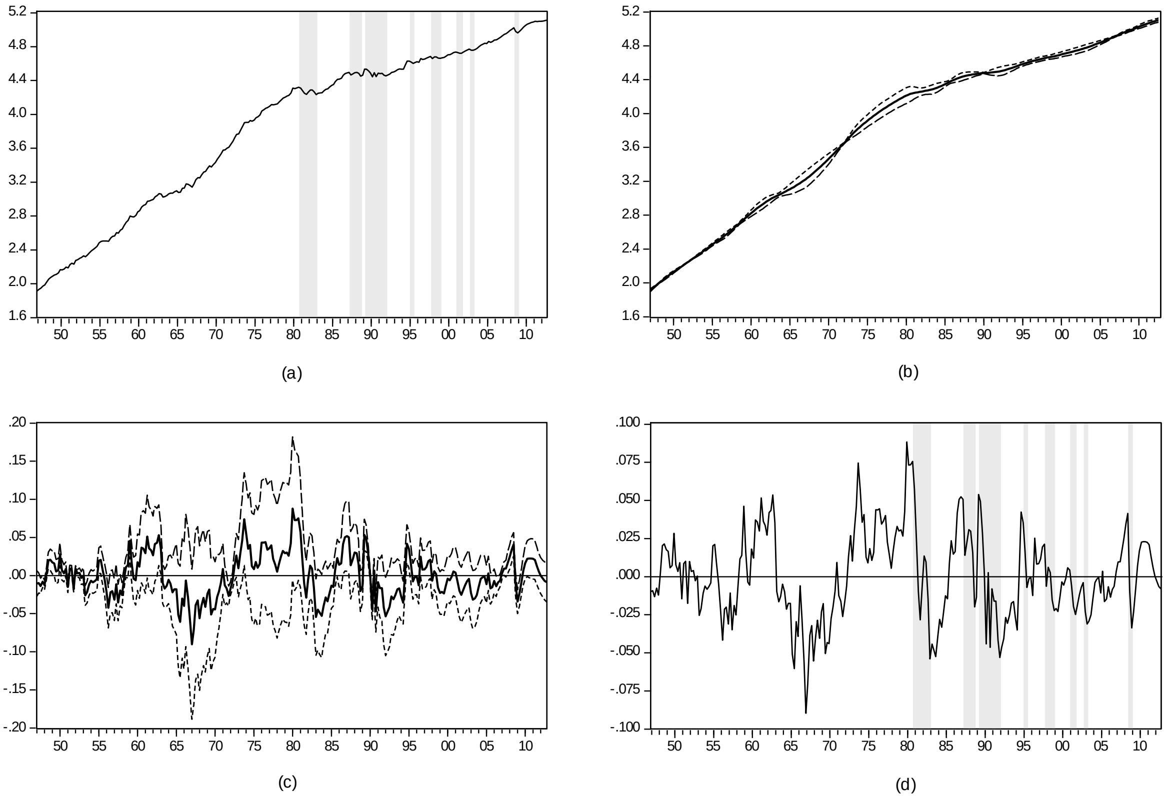

Fig.2: Brazilian time series. (a) Log of Brazilian quarterly real GDP; (b) average trend series (solid line) and two standard error bands (dashed lines); (c) average cycle series (solid line) and two standard errors bands (dashed lines); and, (d) average cycle series. Shaded areas are the CODACE recession dates.

By visually inspecting Figure 2, part "b", one can clearly see a very smooth trend with a flatter slope after the 1980s. In fact, this reduction was remarkable: Brazilian long-term trend is now about 50% times less steep than it used to be before the mentioned year. According to Perron (1989), this type of smooth stochastic trend is well-described as a unit root process with strong mean-reversion and fat-tailed distribution for the error sequences. As a result, most of shocks have a small, if any, long-term effect; only a few numbers of events can modify the trend permanently, as in the case of the highly unstable period of 1980s, in which Brazil underwent a political regime switching (military to democratic), and several years of hyperinflation.

Parts "b" and "c" also show that the trend and cycle components are less accurately calculated during the periods of 1965-1970, and 1974-1980. These periods may be associated with many political and economic instabilities, such as the beginning of the Military Regime; a high and increasing inflation (in 1964, Brazilian General Price Index - Internal Availability -reached a peak of 92% per year, for the whole 1947-1979 period); the extended slowdown in the GDP growth rate that Brazilian economy has passed through during 1963-1967, due to a large package of restrictive fiscal and monetary policies, aiming to control the soaring prices; an even longer period of high growth, over the 1968-1973 period, also referred as the Economic Miracle, in which the real GDP grew at a rate of 11% per year (Abreu, 1989); and, the break started around 1980.

Such sources of uncertainties would have affected the usual trend-cycle decomposition methods whenever it is assumed a particular specification for the filtering procedure. However, as we argued before, the average series tends to be more robust to them, since different decomposition outcomes can adapt faster to the vast complexity of the real world economic time series.

In Figure 2, part "d", we focus on the average cyclic series as well as on the CODACE recession dates (the Brazilian committee provides us the business cycle chronology for the 1980-2009 period), in which one can notice a high degree of matching between both series. Still, in order to access an exact measure of their correspondence, we need to transform our cyclic series in a binary indicator of the business cycle phases. For such, first, we find the turning point dates (peaks and troughs), and, second, we create our own dummy series, defining expansions as the period from a previous trough to the most recent peak, and recessions as the period from a previous peak to the most recent trough.

In order to find the turning points, we employ a simple rule created by Wecker (1979), and also employed by Pagan (1997), among others, where a peak is equal 1 when $\{\Delta y_{c,t} > 0; \Delta y_{c,t+1} < 0; \Delta y_{c,t+2} < 0\}$, where $y_{c,t}$ refers to the average cyclic component. Conversely, a trough series is equal to 1 when $\{\Delta y_{c,t} < 0; \Delta y_{c,t+1} > 0; \Delta y_{c,t+2} > 0\}$. We also require that peaks and troughs alternate, so if two or more peaks (troughs) are subsequent we select the one with a higher (smaller) value for y_c. Thus, the algorithm replicates a common view among media members and politicians that expansions (recessions) involve at least two quarters of positive (negative) growth.

Table 3 presents comparisons between our cyclic series and CODACE's business cycle dates. This table also refers to the binary indicators for expansions and recessions, S_t^E and S_t^R, respectively, derived from our decomposition after applying Wecker's (1979) rule. In a total of 120 quarters, our dummy indicator has a correspondence rate of 86% ((79+24)/120) during expansions, and 88% ((44+61)/120) in recessions.

Table 3: Decomposition-CODACE comparisons, 1980:01 – 2009:04

Expansions				*Recessions*			
	CODACE				*CODACE*		
S_t^E	1	0	Total	S_t^R	1	0	Total
1	79	8	87	1	44	11	55
0	9	24	33	0	4	61	65
Total	88	32	120	Total	48	72	120
Correspondence	86%			Correspondence	88%		

Moreover, while the errors in the course of expansions are quite similar, i.e., 8 cases in which our cycle series indicates a false upswing, and 9 cases in which it misses a real one; during recessions the average cyclic series misses only 4 in a total of 48 CODACE dates, i.e., a matching of 92%. Finally, it shall be noted that both series, S_t^R and CODACE, are perfectly coordinated during the 1989:02-1992:01, 2002:01-2003:02 and 2008:03-2009:01 recessions.

In order to compare the performance of our method, we show below, in Table 4, correspondence rates of other four different methods, using the same Wecker's (1979) rule. The alternative decomposition methods considered are: a) the classical HP(λ=1600)-filter; b) an ARIMA(1,1,1)-BN filter; c) the Baxter and King (1999) band pass filter, assuming that business cycles lasts from 1.5 to 8 years; and, d) the unobservable components decomposition, considering that the cyclic component behaves as an AR(2), process (that is, the UC(2) model).

Table 4: Comparative performance of decomposition methods

Filtering method	Coincidence with CODACE in: Expansions (%)	Recessions (%)
Mean cycle	88	86
HP(λ=1600)	82	85
ARIMA(1,1,1)-BN	31	20
Baxter-King	75	76
UC(2)	73	73

As can be seen in Table 4, our averaged decomposition outperforms all the other methods, especially when it comes to expansions. Including, we have gotten qualitative gains even when comparing with the classical HP-filter, the second best option. It is important to note that if one had means to check methods' performance between 1965 and 1980, a period with large filtering instability, as Figure 2 shows, is possible to think that our method could have obtained even better results.

4. Features of Brazilian cyclic component from 1947 to 2012

In order to examine features of the Brazilian business cycle, we divide the sample into four distinct subsamples, namely, the years before the Military Regime, 1947-1963; the Military Regime itself, 1964-1984; the Democratic period before the Real Plan, 1985-1993; and the Real Plan period itself, 1994-2012. It is a natural way to divide a sample in Brazil, since it takes into account the several changes in the political field. However, we relax this assumption later.

In Table 5, which brings patterns of the business cycle phases, the average duration, a measure of the phase length, is calculated using the formula $\hat{D}^i = 1/(1-\hat{\alpha}^i - \hat{\beta}^i)$, where $i = E, R$, meaning expansions and recessions, respectively. In this case, parameters are obtained by the OLS regression $S_t^i = \alpha^i + \beta^i S_{t-1}^i$, where $i = E, R$, and S_t^E and S_t^R represent the same binary variables analysed in Section 3. By its turn, the average range during the business cycle phases, $\hat{A}^i$, is estimated as the slope of the linear regressions between ΔGDP_t and S_t^i, and indicates the height of expansions or the depth of recessions, in a given period of time. Having the mean duration and range, total gains and losses of the business cycle phases follow directly by using the triangle approximation, C_{Ti}=0.5($\hat{D}^i * \hat{A}^i$), where C_{Ti} refers to the cumulative movements inside a cycle's phase (aforementioned calculations follow Harding and Pagan, 2002).

Table 5: Phase patterns, Brazilian business cycle

Statistics/Period	Pre-1964	Military	1985-1993	Real Plan	Full sample
Panel (a): Expansions					
Duration (quarters)	8.40	9.00	7.67	8.00	8.36
Range (%)	1.14	2.27	1.84	1.36	1.62
Total gain (%)	4.79	10.23	7.04	5.45	6.75
Mean growth (%)	2.09	2.09	1.26	1.13	1.70
Panel (b): Recessions					
Duration (quarters)	6.00	5.00	6.33	5.50	5.59
Range (%)	-1.23	-1.76	-1.80	-1.17	-1.44
Total loss (%)	-3.69	-4.41	-5.72	-3.21	-4.05
Mean growth (%)	1.10	0.49	-0.21	0.10	0.46

Expansions in Brazil last for approximately eight quarters, or two years, and recessions have a mean duration of six quarters, or 1.5 years. The three longest expansions occurred from 1956:02 to 1961:02, related to the "*Plano de Metas*", a consistent and comprehensive plan of public investments; from 1967:01 to 1971:01 and 1971:04 to 1973:04, related to the Brazilian Economic Miracle period; and from 2003:02 to 2008:03, which was interrupted by the Great Recession.

On the other hand, the three longest recessions occurred from 1950:01 to 1953:02, related with unbalanced public accounts and a cambial crisis started around 1952; and the periods of 1981:01-1983:04, and 1989:02-1992:01, where political and economic instabilities led to hyperinflation, and negative growth (Brazilian GDP reduced -4.25 in 1981, -2.93 in 1983, -0.06% in 1988, -4.35% in 1990, and -0.47% in 1992, according data from the Brazilian System of National Accounts, IBGE)[6]. After that, 2009 was the only year in which Brazilian economy presented a negative growth, with -0.33% of variation in its GDP.

By closely inspecting Table 5, one can note that, while the business cycle phases change across the subsamples, the duration of a full cycle is quite constant, around 3.5 years, i.e., 14 quarters. Another interesting fact is that the duration of expansions are longer than that of recessions, even during the 1980s. Additionally, total gains are always higher than total losses, depicting a manifested asymmetry between the business cycle phases in the country. Besides, after the Real Plan, Brazilian economy has had milder expansions and recessions (see total gain and loss, in Table 5), which shows some evidence of increased stability in the country since the mid-1990s.

Persistence[7], by its turn, has been quite stable in Brazil, usually oscillating around the 0.7-0.8 bands, as can be seen in Table 6. However, during some recessions, such as those occurred in 1989:02-1992:01, 1997:04-1999:01 and 2008:03-2009:01, the cyclic series correlation has a tendency to reduce, probably due to a higher degree of uncertainty (as depicted by Figure 3, part "c", below).

Table 6: Volatility and Persistence of the Brazilian Business Cycle

Statistic/Period	Pre-1964	Military	1985-1993	Real Plan	Full sample
Stand. Deviation (%)	2.22	3.85	3.23	1.79	2.88
Persistence	0.74	0.89	0.73	0.71	0.82
N. Obs.	68	84	36	76	264

Taking a closer look into the stability question, Table 6 shows that Brazilian volatility, measured as the standard deviation of the cyclic series, has surely decreased after 1994. In fact, the quarterly volatility after this year is less than half of that observed during the 1964-1984 period, and about 40% lower than its full sample value. In order to make international comparisons, we can reproduce statistics calculated by Aguiar and Gopinath (2007) for a set of developed countries, who found standard deviations in the order of 1.39% for Australia, 1.64% for Canada, 2.18% for Finland, 1.52% for Sweden, and 1.34% overall, between 1980 and 2003. Thus, our results indicate that Brazilian volatility has been converging to a number comparable to those estimated in some developed countries.

Figure 3, below, provides additional information about Brazilian business cycles. In that figure, we estimate moving standard deviation, variance, and persistence of our cyclic series using a 14-quarter window, which is the same average duration of a full cycle, besides the GDP 14-quarter mean growth-rate. These time series are depicted in parts "a", "b", "c", and "d", respectively.

As one can see, parts "a" and "b" of Figure 3 confirm that Brazilian instability has decreased since mid-1990s. Moreover, during periods of recessions, standard deviation and variance tend to increase, but after the mentioned period, peaks of volatility are consistently lower than the previous one (the only exception is the last international crises peak). In this sense, all the evidence reported so far seems to support the incidence of a delayed great moderation in Brazil, in lines with that found by several authors for the U.S. economy (see, e.g., McConnell and Perez-Quiros, 2000; and Stock and Watson, 2002).

[6] Data is available on http://www.ipeadata.gov.br/.
[7] Persistence is defined as the first autoregressive coefficient of the cyclic component as in Pivetta and Reis (2007).

According to Stock and Watson (2002), there are generally three main explanations for this phenomenon. The first one is related with structural changes that might have affected the economy; for example, the shift in output from goods to services (Zarnowitz and Moore, 1986), improvements in the inventory management made possible by the advancements in information-technology (McConnell and Perez-Quiros, 2000), and financial innovations that facilitates intertemporal smoothing of consumption and investment (Blanchard and Simon, 2001). The second reason would be improved monetary policy (e.g., Cogley and Sargent, 2005). And the third category is good luck, that is, a number of exogenous shocks that reduced U.S. volatility (Leeper and Zha 2003; Sims and Zha 2006). However, a consistent investigation of this question, applied to the Brazilian case, is beyond the objectives of the present paper. Here we focus on the mapping of the business cycles facts, providing information that can be utilized in future research.

Additional information is found in Figure 3, part “d”. In the full sample, Brazilian economy has had 14 quarters of negative growth. The historical growth-rate peak was found in 1973:04, which equals to 3.09% per quarter, while the lowest value occurred in 1983:03, equalling to –0.42% per quarter. Before the 1980s, the 14-quarter growth rate of the GDP was swinging around 2% per quarter, a value closer to that believed as being the Brazilian natural growth-rate up to this year, which equals to 7% per year, as pointed out in many sections of Abreu’s book (1989, p.222, for instance). Since then, the Brazilian’s GDP long-term growth-rate seems to have converged to half of this value, about 1% per quarter.

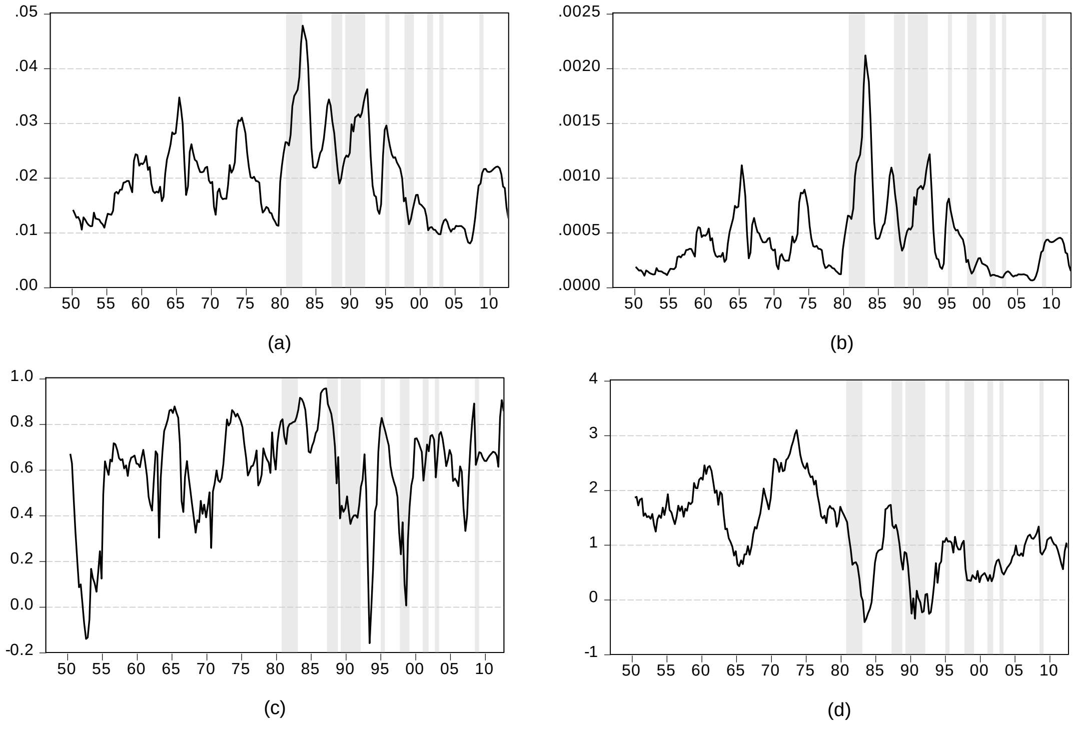

Fig.3: Brazilian moving statistics: (a) standard deviation; (b) variance; (c) persistence; and, (d) GDP growth rates. Note: moving window of 14 quarters, one full cycle period. Shaded areas represent CODACE recession dates.

4.1. Robustness tests on the decreased volatility and structural change tests

In this subsection, we seek to investigate further questions regarding the reduction of the Brazilian instability, since this is an interesting feature, also found in other countries that had not yet been detected by previous papers. Specifically, we intend to examine if this finding is robust to changes in the volatility measure, and when it has really happened.

Table 7 brings a variety of proxies for the volatility in the country. The first three series (“*Δgdp* variance (*i*)”, i = 4, 8, 14) refer to the variance of the GDP log growth-rate considering 4, 8 and 14 quarters moving windows, respectively. Next, two classical measures of instability are presented, i.e., the absolute and the squared returns. Table 7 also shows the one step ahead forecasts of the conditional variance emerging from an ARIMA(7,1,0)-Garch(1,1) model for the log-GDP time series, and an ARIMA(5,1,0)-Garch(1,1) model for the mean HP-cycle. Both specifications were chosen according to the Akaike (1987) information criteria, where, for the sake of simplicity, we assume normally distributed disturbances. Finally, we present, within parenthesis, the classical Student's *t* and Welch's (1951) statistics for the null that the estimated volatility in a certain period is equal to that found after 1994. Welch (1951) test considers the possibility of different variances across subsamples[8].

Table 7: More volatility statistics and mean equality tests

Statistic/Sample	Pre 1964	Military	1985-1993	Real Plan
Δgdp variance (4)	2.07 (1.95**; 4.03**)	2.51 (3.26*; 10.56*)	4.81 (4.49*; 11.33*)	1.47
Δgdp variance (8)	2.40 (2.25**; 5.61**)	3.15 (4.84*; 23.45*)	5.35 (5.55*; 17.22*)	1.82
Δgdp variance (14)	2.61 (3.08*; 11.08*)	3.32 (6.11*; 37.80*)	5.76 (8.16*; 37.28*)	2.01
Absolute returns	1.97 (3.77*; 13.75*)	2.08 (4.25*; 18.70*)	1.96 (2.88*; 6.07*)	1.24
Squared returns	0.055 (3.26*; 10.19*)	0.063 (3.87*; 15.67*)	0.063 (2.75*; 4.68*)	0.026
Garch(1,1) GDP	0.030 (1.35; 1,87)	0.034 (3.76*; 14.14*)	0.046 (4.68*; 12.40*)	0.028
Garch(1,1) HP-cycle	0.025 (2.96*; 9.48*)	0.028 (4.68*; 22.33*)	0.038 (5.53*; 16.98*)	0.022
Number of obs.	68	84	36	76

Notes: *, **, *** denotes statistical significance at 1, 5 and 10% levels, respectively. Inside parentheses are *t* and Welch (1951) mean equality test statistics. All figures refer to the Brazilian quarterly data, 1947-2012, and are in percentages.

The first pattern seen in Table 7 is that volatility increased from 1947 to 1993 and then, after 1994, it fell to lower levels. On average, the instability of the 1994-2012 years is 28% lower than that observed during 1947-1963; 41% lower than that under the Military regime; and, 63% lower than the volatility of the 1985-1993 years. The major difference across periods was measured by the variance of the growth rate within one year moving window, an approximation for the short-run volatility (*Δgdp* variance, 4 quarters). According to this proxy, instability has decreased about 40, 70 and 230% in the Real Plan, comparatively to the pre-1964, Military and 1985-1993 periods, in that order. Conversely, the minor differences were estimated by the Garch(1,1)-GDP proxy, wherein the instability after 1994 has decreased 7, 21, and 64%, relatively to the periods in the same order as before. A second feature is that the period from 1985 to 1993, in which inflation levels were extremely high, was the most unstable in the Brazilian history, for nearly all volatility measures[9].

Throughout this paper, we have assumed a fairly logical division in the data set, beginning with pre-Military period, then the Military regime itself, passing through the 1980s and, finally, the Real Plan's years. It seems to be a natural assumption, given the recent political history of the country. Nonetheless, statistically, this type of a priori division of the sample makes the breakdate endogenous (correlated with the data), and tests are likely to falsely indicate a break, when none in fact exists (Hansen, 2001).

In order to circumvent this difficulty, we apply three different classes of methods, which are the Nyblom's *L* test of structural changes with unknown breakdate (Hansen, 1992), the Quandt

[8] We do not include the log-squared returns because of their several problems, such as negative, large, and/or undetermined values for volatility.

[9] Exception for the absolute returns.

(1960)-Andrews (1993) procedure, and the Hansen (2001) tests. The first one evaluates a structural change in all the parameters of a model, without assuming a specific date, but it does not provide an estimated date of change. The Quandt (1960)-Andrews (1993) and Hansen (2001) methodologies, however, do estimate a breakpoint date, besides providing complementary pieces of information.

When computing Nyblom's *L* test, we follow Hansen (1992) and McConnell and Perez-Quiros (2000), assuming that the Brazilian GDP log-growth rates behave according an AR(1) process with a drift. It is a simple yet powerful model, as showed by Hess and Iwata (1997). Proceeding in this way, we are able to test for breaks in the mean, in the autoregressive coefficient and in the variance of the time series (μ, ϕ, σ^2), which are associated, respectively, to structural changes in the trend, persistence and volatility of the GDP. Hansen (1992) provides two types of statistics, one for testing the stability of each parameter individually, and other for testing the stability of all parameters jointly. Both are presented in Table 8, below.

Table 8: Nyblom's *L* test for stability of Brazilian real GDP growth – 1947:01 to 2012:04

Specification: $\Delta gdp_t = \mu + \phi\ \Delta gdp_{t-1} + \varepsilon_t$			
Panel (a): 1947:01 to 2012:04			
Parameter	Estimate	L_c	CV (5%; 10%)
μ	0.0103 (0.00)	1.268	0.47; 0.35
ϕ	0.1525 (0.01)	0.274	0.47; 0.35
σ^2	0.0003	0.363	0.47; 0.35
Joint L_c	1.7345		1.01; 0.85
Panel (b): 1960:01 to 2012:04			
Parameter	Estimate	L_c	CV (5%; 10%)
μ	0.0087 (0.00)	0.7643	0.47; 0.35
ϕ	0.1837 (0.09)	0.3911	0.47; 0.35
σ^2	0.0004	0.5634	0.47; 0.35
Joint L_c	1.5347		1.01; 0.85
Panel (c): 1970:01 to 2012:04			
Parameter	Estimate	L_c	CV (5%; 10%)
μ	0.0078 (0.00)	0.6332	0.47; 0.35
ϕ	0.2073 (0.01)	0.5784	0.47; 0.35
σ^2	0.0003	0.6917	0.47; 0.35
Joint L_c	1.7345		1.01; 0.85
Panel (d): 1980:01 to 2012:04			
Parameter	Estimate	L_c	CV (5%; 10%)
μ	0.0056 (0.00)	0.1426	0.47; 0.35
ϕ	0.0764 (0.25)	0.1363	0.47; 0.35
σ^2	0.0003	0.9688	0.47; 0.35
Joint L_c	1.1734		1.01; 0.85
Panel (e): 1990:01 to 2012:04			
Parameter	Estimate	L_c	CV (5%; 10%)
μ	0.0073 (0.00)	0.1273	0.47; 0.35
ϕ	-0.0651 (0.75)	0.3805	0.47; 0.35
σ^2	0.0003	0.9422	0.47; 0.35
Joint L_c	1.2292		1.01; 0.85

Notes: P-values are within parenthesis. L_c is the statistic for a break point in each of the parameters listed in the first column. CV is the critical value for both 5 and 10% of significance, according Hansen (1992).

In Table 8, results are presented in five panels; from "*a*" to "*e*", in which Nyblom's tests are applied to different samples. First, Panel (*a*) shows the method for the whole data set. In this case, the estimation shows a clear break in the trend, with a calculated statistics of 1.27 that can be compared to the critical values of 0.47 (5%), or 0.35 (10%). Besides, we cannot find a significant break in the autoregressive parameter, and the variance presents a break only considering a critical value of 10%.

As presented in Table 8, if the sample is divided in decades, the volatility drop is clear, with the *L* statistic reaching a peak of 0.97 during the 1980s, and 0.94 after the 1990s. Besides, as long as we move in the sample, the structural change in the mean term, μ, disappears after 1980, which indicates that the break occurred around this year.

Results from this table for the autoregressive parameter are mixed, and it may not have suffered a major change during the period of the analysis of the sample, as shown in Table 8, Panel (a), Table 6 and Figure 3.

Hansen (1992) tests are informative and have a solid statistical basis, but they lack information regarding the timing of the breaks. In this sense, we apply two other tests, the first one due to Quandt (1960) and Andrews (1993, henceforth Quandt-Andrews), and the second one to Hansen (2001). Here we present the Quandt-Andrews test first, since Hansen (2001) utilizes some of its concepts.

The Quandt-Andrews method tests one or more unknown structural breakpoints in the sample for a specified equation. Quandt (1960) proposed a method that calculates Chow's statistics for every point of the sample between two dates, say, t_1 and t_2[10]. Through this search procedure, the breakdate can be found either using the maximum of the Chow's statistics (the original Quandt's test), or the exponential and average statistics, for which Andrews (1993) and Andrews and Ploberger (1994) calculated tables of critical values, while Hansen (1997) provided approximate asymptotic *p*-values. In this research work, we use the maximum and the exponential statistics, standard in the literature, denoted by MaxF and ExpF, respectively.

When applying Quandt-Andrews tests, we assume two specifications, one for breaks in the AR(1) model, $\Delta gdp_t = \mu + \phi \, \Delta gdp_{t-1} + \varepsilon_t$, which tests for breaks in μ and ϕ, and other for breaks in volatility, $\sigma^2_{t,proxy} = \kappa + \upsilon_t$, where the dependent variable is one of the proxies for the Brazilian instability listed below; κ is a parameter that refers to the average volatility; and, υ_t is an error term. The positive point about this approach it that it allows testing for breaks in a vast number of instability indicators.

The volatility proxies considered here are: moving variance cycle (*i*), with *i* = 4, 8, 14, denoting the variance of the mean cyclic series using moving windows of 4, 8, and 14 quarters; Δgdp variance (*i*), with *i* = 4, 8, 14, absolute returns, squared returns, and one step ahead forecasts of the conditional variance emerging from GARCH models are defined as before.

Finally, Hasen's (2001) test integrates both Hansen (1992) and Quandt-Andrews methodologies in a single framework. Assuming, again, an AR(1) process for the Brazilian GDP log-growth rates, Hansen (2001) procedure estimates and tests the time of a break occurrence for the full set of parameters (i.e., the OLS estimates for μ, ϕ and σ^2) by means of the maximum and exponential statistics. The results of Quandt-Andrews and Hansen (2001) tests are presented in Table 9, below.

Beginning with the break on the mean growth-rate of the process, μ, Table 9 shows that Quandt-Andrews method estimates it in the second quarter of 1980, while Hansen (2001) estimates a break in the first quarter of the same year. Besides, MaxF and ExpF Wald statistics for both methods are highly significant. In this sense, based on the findings of the trend-cycle decomposition, and on these results, one can be quite sure about the timing of the break on the trend: the first semester of 1980. The GDP's log-growth rate was estimated at 1.8% per quarter, before 1980, and 0.7% after that year, which is similar to the results found in Section 3. The evidence reported here agrees, therefore, with the Lost Decade view, which has imposed to the Brazilian economy a permanent slowdown in its rate of growth.

With respect to the autoregressive coefficient, Table 9 Panel (b) confirms our previous expectations, showing that at 10% of significance we cannot reject the null hypothesis of constancy

[10] Andrews' (1993) statistics diverge to infinity in probability if one uses them to the full sample. Thus, following his suggestions, we exclude from the analysis the first and the last 7.5% of the observations.

in the ϕ parameter. Specifically, MaxF statistic was calculated as 7.24, with a p-value of 14%, while ExpF was calculated as 1.37, with a p-value of 12%. By reviewing Tables 6 and 8, Panel (*a*), we are able to reach the same conclusion.

Now, we shall turn attention to the volatility question. In Table 9, Panel (a), almost all short-term volatility measures, namely, Moving var. cycle (4), *Δgdp* variance (4), Garch(1,1)-GDP, and Garch(1,1)-HP-cycle estimate one break around the years 1996 or 1997. This result is confirmed by the Hansen (2001) test in Panel (b), which estimates a break in 1996:04. All statistics are highly significant. The long-term instability measures, Moving var. cycle (8), Moving var. cycle (14), *Δgdp* variance (8), and *Δgdp* variance (14), by construction, postpone the timing of the break by few quarters.

Table 9: Breakdate tests

Panel (a): Quandt-Andrews methodology			
Specifications: i) $\Delta gdp_t = \mu + \phi\ \Delta gdp_{t-1} + \varepsilon_t$; and, ii) $\sigma^2_{t,proxy} = \kappa + \upsilon_t$ for variances			
Variable	Breakdate	MaxF (p-value)	ExpF (p-value)
Δgdp	1980:02	26.04 (0.00)	9.39 (0.00)
Moving var. cycle (4)	1996:01	13.42 (0.01)	3.98 (0.00)
Moving var. cycle (8)	1996:04	23.64 (0.00)	8.45 (0.00)
Moving var. cycle (14)	1997:04	40.98 (0.00)	17.09 (0.00)
Δgdp variance (4)	1997:03	20.81 (0.00)	7.37 (0.00)
Δgdp variance (8)	1998:03	30.83 (0.00)	12.15 (0.00)
Δgdp variance (14)	1999:01	49.60 (0.00)	21.71 (0.00)
Absolute returns	1991:02	25.25 (0.00)	8.98 (0.00)
Squared returns	1991:02	19.30 (0.00)	6.88 (0.00)
Garch(1,1) GDP	1997:03	18.53 (0.00)	6.71 (0.00)
Garch(1,1) HP-cycle	1997:03	28.40 (0.00)	11.49 (0.00)
Panel (b): Hansen (2001) methodology			
Specification: $\Delta gdp_t = \mu + \phi\ \Delta gdp_{t-1} + \varepsilon_t$			
Parameter	Breakdate	MaxF (p-value)	ExpF (p-value)
μ	1980:01	24.18 (0.00)	8.04 (0.00)
ϕ	1990:02	7.24 (0.14)	1.37 (0.12)
σ^2	1996:04	66.92 (0.00)	28.24 (0.00)

Note: In Panel (a), Hansen (1997) p-values.

Two series disagree with this timing, estimating 1991:02 as the breakpoint; they are the squared returns, and the absolute returns. Nevertheless, these series seem to be an exception to the other findings, in a sense that we can be quite sure about a break around 1996 and 1997[11].

Next, we analyse the possibility of multiple structural changes in the Brazilian output volatility. When doing so, Bai and Perron (1998) procedure is employed. This method searches for breaks successively, aiming to reduce the regression's sum of squared residuals. Whenever a breakdate is identified, data is split and another break is assessed inside the smaller samples. The algorithm stops at a number of, say, *l* breaks, which minimizes some information measure (here, the Bayesian Information Criterion, BIC, is utilized). Trimming parameter is set at 15%, as usual. Results are shown in Table 10.

[11] With 1996/1997 as the breakdate, the estimated reduction in the volatility is 50% on average.

Table 10: Multiple breaks test in volatility

Volatility proxy	l^*	Breakdates	Volatility in sample:		
			1	2	3
Moving S.D. cycle (4)	2	1979:04; 1995:04	1.3	1.8	0.9
Moving var. cycle (4)	2	1979:04; 1995:04	1.5	3.2	1.0
Δgdp variance (4)	2	1987:03; 1997:02	2.3	5.3	1.0
Absolute returns	1	1991:01	2.3	1.3	-
Squared returns	1	1991:01	0.063	0.026	-
Garch(1,1) GDP	2	1987:03; 1997:01	0.032	0.049	0.026
Garch(1,1) HP-cycle	2	1988:03; 1997:04	0.027	0.040	0.019

Note: (*) *l* stands for the number of estimated breaks.

According to Table 10, most of the considered volatility proxies present two structural breaks (here we focus on short term proxies). The only exceptions are absolute and squared returns, in which, in both cases, only one breakpoint was detected (1991:01). The remaining variables indicate that Brazilian instability has an interesting inverted U-shape pattern, peaking in the period from 1980, or 1985, to the first half of the 1990's. Moreover, regarding dates of the last break, results in this table are consistent with the evidence previously presented, in which we estimate a regime of lower volatility beginning around 1996 or 1997.

Finally, Figure 4, below, displays the moving standard deviations of the HP-cyclic time series, when using only the 1990-2012 sample, part "a", and the 1994-2012 sample, part "b". In this figure, we compute the moving standard deviation, using, again, moving windows of four, eight, and 14-quarter-long, respectively.

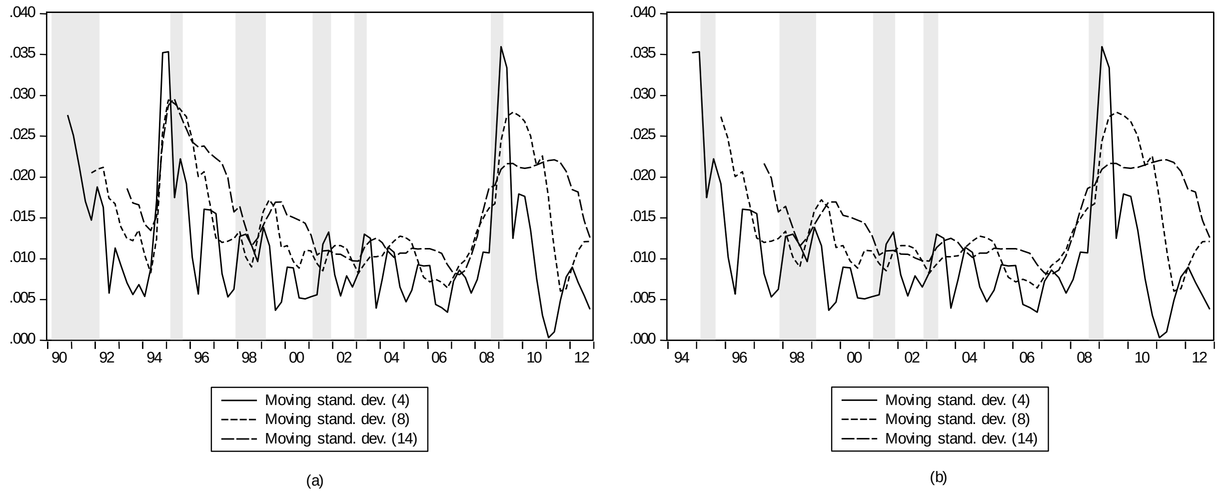

Fig.4: (a) Moving standard deviations after 1990:01; and, (b) moving standard deviations after 1994:01. Note: shaded areas refer to CODACE recession dates.

Our data show that, after the last break, Brazilian instability has become cyclic and has presented phases of higher volatility in recessions, which is the case of the first three quarters of 1995, when the economy was adjusting to the new monetary and fiscal scenario, and during the Great Recession; and phases of lower volatility. Besides, although the Great Recessions did not have the same impact on Brazilian instability as the crises of the 1980s, their effects were quite relevant in the post 1990s conjuncture.

5. Conclusions

The present work aimed to provide new facts about the Brazilian business cycle during the period of 1947 and 2012. Therefore, we have decomposed the quarterly and seasonally adjusted real GDP into its mean trend and cycle components, by estimating and averaging on a variety of HP-filter outcomes. In this sense, we provide many pieces of information which can be utilized in other models and studies.

Our main findings are the following: i) the model has almost matched the CODACE business cycle dates, with a correspondence of 88% and 86% during recessions and expansions, respectively; ii) the estimated trend component is noticeably flatter after the 1980s, depicting a major structural break that happened in that period, also known as the “lost decade”; iii) there is strong evidence that volatility decreased in the country (for example, we found significant structural breaks which occurred around 1996/97); iv) the persistence of the cyclic series tends to oscillate between the 0.7-0.8 bands; v) the business cycle phases have a different duration in the full sample, with expansions and recessions lasting for, respectively, 8 and 6 quarters on average, which implies a full cycle of 3.5 years; however, during the Military Regime, Brazilian economy presented mean expansions about 80% longer than recessions; and, vi) the mean growth rates of the phases are quite different, reaching a value of 1.7% per quarter (or 6.8% per year) during expansions, and 0.45% per quarter (or 1.8% per year) during slowdowns; however, between 1985 and 1993, the mean growth rate during recessions was around -0.2% per quarter, i.e., -0.8% per year.

The results described above are wide-ranging. For example, we found that the Brazilian business cycles are asymmetric, with expansions exhibiting longer duration and accumulated movements than recessions. These asymmetries across business cycle phases are also observed in the OECD countries, as documented in Chang and Hwang (2011), and Chauvet and Yu (2006). Moreover, we found that the Brazilian long-term trend is reasonably similar to that described by Perron and Wada (2009) for the U.S. economy, except that, for the latter, the major break occurred in 1973. Our expansion and recession growth rates are also parallel to those obtained by Chauvet (2002).

All these regularities, namely, phase asymmetries and a decreased volatility over the time, are also observed in the OECD countries, in a sense that Brazilian business cycle, although delayed (and contrary to the evidence reported by previous papers), does share central qualitative features with other decentralized market economies.

Specifically to the volatility reduction, it shall be clear that the present paper concluded that it happened *after* the Real Plan implementation. We are not stating, however, that the Real Plan *was* the only or the most important source of such phenomenon. As we discussed before, the strand of literature investigating on this issue usually finds that changes in the private sector, in the monetary policy conduction, and in the size of the shocks hitting the economies can, each one of them, be responsible for the structural break in the variance. Thus, we leave this topic for future research.

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