MEASURING THE NEUTRAL REAL INTEREST RATE IN BRAZIL: A JOIN ESTIMATION WITH POTENTIAL OUTPUT, NAIRU AND NAICU

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Abstract

This paper applies a multivariate Kalman Filter to estimate the Neutral Real Interest Rate (NRIR) in Brazil, from 2002Q1 to 2016Q4. Our state space representation combines macroeconomic models with the Hodrick-Prescott (HP) filter approach. This framework differs from previous literature in two aspects: (1) departing from a production function with two factors (capital and labor), we introduce more structure in the potential output state equation; (2) this procedure results in a state space representation that allow us to make a join estimation of four latent variables: NRIR, potential output, NAIRU (non-accelerating inflation rate of unemployment) and NAICU (non-accelerating inflation rate of capacity utilization).

Keywords: Neutral Real Interest Rate; Monetary Policy; State-Space Models.

JEL codes: E1; E4; E5.

Resumo

Esse artigo aplica um filtro de Kalman multivariado para estimar a Taxa de Juros Real Neutra (NRIR) no Brasil, entre o 1T2002 e o 4T2016. A representação em espaço de estado construída combina modelos macroeconômicos com o Filtro Hodrick-Prescott (HP), difenciando-se da literatura prévia em dois aspectos: (1) tomando uma função de produção com dois fatores, introduziu-se maior estrutura na equação de estado do produto potencial; (2) o procedimento resulta em uma representação que permite fazer uma estimativa conjunta de quatro variáveis latentes: NRIR, produto potencial, NAIRU (*non-accelerating inflation rate of unemployment*) e NAICU (*non-accelerating inflation rate of capacity utilization*).

Palavras-chave: Taxa de Juros Neutra; Política Monetária; Modelo Espaço de Estados.

Classificação JEL: E1, E4, E5.

Área Anpec: Área 4 - Macroeconomia, Economia Monetária e Finanças.

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1 Introduction

The neutral real interest rate (NRIR)[1] was first introduced by Wicksell (1936), whose work basically defines this variable as the one associated with long run equilibrium in the economy. The concern about NRIR in monetary policy grows after Taylor (1993) implicitly showed that there is an interest rate associated with steady state in terms of inflation and income. In turn, Woodford (2003) enhanced the role of the NRIR in a New Keynesian environment as the real interest rate that would prevail in the absence of price and wage rigidity in short to medium term.

However, as Blinder (1999) emphasized, once this variable is hard to measure and its estimates are usually accompanied by a great deal of uncertainty, the NRIR should be thought more as a way of thinking monetary policy than as the basis for a mechanical rule. Orphanides and Williams (2002) makes us aware of this by showing that the adoption of policy rules optimized under the false presumption that the uncertainties regarding the NRIR are small proves particularly costly in terms of stabilizing inflation and unemployment.

Anyway, this controversy over the NRIR is not apparently enough to diminish its role as a guidance for central bankers. The variable is often identified in public communications of monetary authorities around the world, both in developed and emerging economies. As an example for the former, Yellen (2017) underline the importance of the NRIR for assessing the monetary policy stance in US economy. As an example for the latter, Central Banks in India (Reserve Bank of India (2017)) and Brazil (Central Bank of Brazil (2017)) have mentioned the NRIR dynamics in the minutes of their monetary policy committee meetings as a reference for its decisions. In this sense, this variable still occupies an outstanding position in monetary policy, which justifies the efforts in measuring its level.

Usually, the literature sheds light on how the deviations of the short term interest rate from its estimated long-run value is related to inflation and business cycle variables. Woodford (2003) enhanced this framework, where the NRIR arises from as interest rule and varies continuously in response to real economy shocks. In this sense, if the short term interest rate is set under (above) its neutral value, we would expect to observe prices rising (declining) and key economic activity variables skipping from its steady-state values: the output should be above (under) the potential output, the unemployment rate should be under (above) the NAIRU (non-accelerating inflation rate of unemployment) and the installed capacity utilization should be above (under) the NAICU (non-accelerating inflation rate of capacity utilization). These relationships highlight an important issue: any deviation of the mentioned observed variables from its long run values could be seen as a clue that the real interest rate (RIR) is not at its neutral level.

However, once the long run variables cannot be directly observed, how to measure its values is a challenge for economists. Regarding the NRIR, Giammarioli and Valla (2004) present an overview about the methods usually applied to estimate its level. Among these methods, we focus on the one that applies Kalman filter in order to obtain a proxy for the latent variable using information provided by activity and prices dynamics.

Laubach and Williams (2003) is one of the key studies in this group. Originally

[1] As enhanced by Bernhardsen and Gerdrup (2007), the concepts "neutral real interest rate", "natural real interest rate" and "normal real interest rate" are used interchangeably in the literature. We use the expression Neutral Real Interest Rate (NRIR) in this article.

developed for the U.S. economy, in their framework the NRIR is related with the growth rate of output trend and an unobserved variable introduced in order to measure effects linked to other of its components (as the intertemporal consumer's preference). This arrangement arises from traditional economic relationships often observed in mainstream macroeconomic theory: an IS curve (Aggregate Demand), a Phillips curve (Aggregate Supply) and an interest rate rule.

Following Laubach and Williams (2003), other studies presented similar strategies to estimate the natural interest rate in other developed economies. For the Euro Area, Mésonnier and Renne (2004) introduced a new equation for the NRIR, comprised by a constant parameter and a total productivity factor component. Bernhardsen and Gerdrup (2007) estimate the NRIR in Norway by means of the Kalman filter, using a model that comprises four equations: the first split the RIR in the NRIR and a ciclycal i.i.d component; the second decomposes the NRIR in a constant that is set as the productivity growth and a disturbance that explains the deviations of the neutral from the long-term real interest rate (LRIR); the third defines the last disturbance as an AR(1) process; the fourth is an IS curve. In this simpler framework, there is a distinction between the NRIR and the LRIR. While the latter is determined by factors like potential output and long-term savings, the former is additionally influenced by various disturbances linked to the supply and demand side of the economy in medium term.[2] [3] Holston et al. (2016) estimates a version of the Laubach and Williams (2003) model using data from the United States and three other advanced economies (Canada, the Euro Area, and the United Kingdom). They highlight four key results: (i) all four economies presented evidence of time-variation in the NRIR; (ii) all four economies registered large declines in trend GDP growth and natural rates of interest over the past 25 years; (iii) although estimation is done on a economy-by-economy basis, there is substantial comovement in the estimates of the natural rates of interest and trend GDP growth across economies; and (iv) the estimates of the NRIR are highly imprecise, reinforcing a key finding of the original Laubach and Williams (2003) paper.

In similar approaches, but applied for Brazilian economy, Barcellos Neto and Portugal (2009), Magud and Tsounta (2012), Araújo and Silva (2014), Gottlieb (2013) and Machado and Portugal (2014) present versions of the Laubach and Williams (2003) model to estimate the NRIR. In turn, Ribeiro and Teles (2013) applies both Laubach and Williams (2003) and Mésonnier and Renne (2004) to estimate the NRIR in Brazil.

A common point in these state space models is that all of them make a join estimation of the NRIR and the potential output, imposing for the latter a structure that depends on unobservable components that follows a random walk or have an autoregressive pattern.

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[2] As Bernhardsen and Gerdrup (2007) enhances, in the neoclassical theory, the Ramsey–Cass–Koopmans model express that the real interest rate in the long term is related to economic fundamentals, been directly proportional to potential output (given by the sum of productivity and population growth) and household rate of time preference (concept associate with the long-term savings rate). A more detailed discussion about this model can be found in Romer (2001).

[3] Another important remark from Bernhardsen and Gerdrup (2007) is that in small open economies, due to free movements of capital hypothesis, the NRIR is dependent on global interest rate and the exchange rate. This last issue can be related with Galí and Monacelli (2005) finding that both the natural levels of output and interest rates in the small open economy are generally a function of both domestic and foreign disturbances.

Other part of the literature, although uses alternative methods to the ones mentioned before, highlights issues that are relevant for the determination of the NRIR in Brazil. Bacha et al. (2009), Bacha (2010) and Goldfajn and Bicalho (2011) present evidences that some structural factors as the dollarization, the country risk rating and the public debt are relevant to explain the RIR level in Brazil. More recently, Barbosa et al. (2016) claims that some studies that uses the Taylor Rule as a reference to obtain the NRIR in Brazil do not consider the reaction function properties' in a small open economy, where variables linked to overseas can affect the domestic fundamentals, which leads to a specification error. In this sense, making reference to the Covered Interest Rate Parity (CIRP), the authors calculate a measure of the NRIR applying a Hodrick-Prescott (HP) Filter over a sum of variables related to the international real rate of interest, the premium due to country risk and the premium due to the exchange rate risk.

Considering these aspects, the aim of this study is to present an expanded version of the Mésonnier and Renne (2004) model, built over Brazilian data. Our model differs from the mentioned literature in two aspects. First, using as reference Areosa (2008), we introduce more structure in the potential output state equation. Areosa (2008) proposes a methodology that combines the production function approach and the HP filtering in a multivariate Kalman Filter, which simultaneously produces estimates for the potential output and its unobservable components. Second, this procedure results in a state space representation that allow us to make a join estimation of four latent economic variables: NRIR, potential output, NAIRU and NAICU.

Applying our model to evaluate the monetary policy stance in our sample period, we observed that in the first half of our data, tighter monetary policy prevailed. The last period of policy easing began in the aftermath of 2008 financial crisis and sustained till the first half of 2014. Since then, Brazil Central Bank has been adopting a tighter monetary stance. Also, we highlight increasing idle productive capacity linked to both capital and labor factors.

The rest of the paper is structured as follows. Section 2 provides the theoretical background, presenting the models proposed by Laubach and Williams (2003), Mésonnier and Renne (2004) and Areosa (2008). In section 3, we introduce our state space model. Section 4 focuses on data description. Section 5 depicts the results (i) comparing the latent variables estimates from our model with an original HP filter framework and (ii) using our state space model to evaluate the monetary policy stance in Brazil during our sample period (2002Q1-2016Q4). Section 6 concludes.

2 Theoretical Background

2.1 The Laubach and Williams (2003) model

The basic specification for the NRIR in Laubach and Williams (2003) is inspired on the neoclassical growth model. The Euler Equation that results from the household intertemporal CRRA (Constant Relative Risk Aversion) utility maximization yields a relationship that shows this latent variable being explained by shifts in preferences and the growth rate of output. Based on this theoretical link, the following state-equation gives the law of motion for the NRIR (r_t^*) :

$$r_t^* = \phi g_t + z_t, \tag{2.1}$$

where g_t denotes the trend growth rate of the potential output, ϕ is interpreted as the constant relative risk aversion [4] and z_t is a control for other determinants of the NRIR. This last component follows a random walk, which originates another state equation:

$$z_t = z_{t-1} + \epsilon_{z,t}. \tag{2.2}$$

As the remaining state equations, the log potential output is specified as a random walk with a stochastic drift given by the trend growth rate of the potential output, that itself follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t}. \tag{2.3}$$

$$g_t = g_{t-1} + \epsilon_{g,t}. \tag{2.4}$$

The next step is to define functions for demand and supply side of the economy. Following Holston et al. (2016), we will present the reduced-form equations of, respectively, an IS (equation 2.5) and a Phillips curve (equation 2.6) that arise from the open economy version of the New Keynesian model (Galí (2008)). These specifications follow closely the small semi-structural model presented by BCB (2015) and comprise the measurement equations:

$$h_t = \alpha_1 h_{t-1} + \alpha_2 h_{t-2} + \frac{\alpha_3}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \epsilon_{h,t}, \tag{2.5}$$

$$\pi_t = \beta_1 \pi_{t-1} + (1-\beta_1)\pi_{t-2,4} + \beta_3 h_{t-1} + \epsilon_{\pi,t} \tag{2.6}$$

In this setting, h_t is the output gap, given by the output deviation from its potential. r_t and r_t^* are the RIR and NRIR, respectively. π_t is the consumer price inflation. $\pi_{t-2,4}$ is the average consumer price inflation of its second to fourth lags. $\epsilon_{h,t}$ and $\epsilon_{\pi,t}$ are error terms ($i.i.d. \sim$ N(0,σ_i^2), for i=h,π). The long-run verticality of the Phillips curve is imposed in equation 2.6. Besides, these two equations are consistent with the hypothesis that price stability is reached when the RIR and the output are equal to their respective long term equilibrium (NRIR and potential output).

2.2 The Mésonnier and Renne (2004) model

Mésonnier and Renne (2004) follow a simpler structure than Laubach and Williams (2003). The main difference are the state equations, that hold the following specifications:

$$r_t^* = \mu_r + \phi g_t \tag{2.7}$$

[4] This parameter corresponds to the inverse of the intertemporal elasticity of substitution, that arises from the Ramsey model applying a standard CRRA utility function for the representative household

$$y_t^* = \mu_y + y_{t-1}^* + g_t + \epsilon_{y,t}. \tag{2.8}$$

$$g_t = \rho g_{t-1} + \epsilon_{g,t}. \tag{2.9}$$

$$y_t = y_t^* + h_t. \tag{2.10}$$

The variables have the same interpretation of the ones in last section, with few adaptations. Once more, the errors terms are *i.i.d.* and normally distributed. In this setting, the NRIR and the potential output dynamics have a common component linked to trend growth rate (subject to low frequency fluctuations), which follows an AR(1) process (equation 2.9). For instance, ϕ is interpreted as the constant relative risk aversion [5]. The parameters μ_r e μ_y stand for the rate of time preference of consumers [6] and the average potential output growth rate, respectively. The measurement equation remains similar to the ones presented in last section.

In this structure, unlike Laubach and Williams (2003), there is no need to extract two non-observable variables (g_t and z_t) from another variable that is also non-observable (r_t^*). Accordingly to the authors, this strategy makes the estimation easier and more reliable.

2.3 The Areosa (2008) model

Areosa (2008) uses Brazilian data to apply a methodology that combines the production function approach and the HP filtering in a multivariate Kalman Filter, which simultaneously produces estimates for the potential output, the NAIRU and the NAICU. In order to assess these non-observable variables, this framework departs from a Cobb-Douglas production function with constant returns to scale and defines the following equation:

$$y_t = y_t^* + \gamma(e_t - e_t^*) + (1-\gamma)(c_t - c_t^*) \tag{2.11}$$

Following the same terminology from previous sections, y_t and y_t^* are the log output and its potential. As we introduce for the first time, c_t is the log capacity utilization, c_t^* is the log NAICU, e_t is the log employment rate and e_t^* is the log NAIRE (non-accelerating inflation rate of employment). The NAIRU is obtained as 1-E_t^*, with the upper-case variable denoting the exponential of the corresponding lower-case variable. In this approach, the equation 2.11 is explored as one of the measurement equations. The output elasticities of labor and capital are represented by γ and (1-γ), respectively.

The remaining state space model follow closely Boone (2000). Departing from a HP filter structure for the state equations based on Harvey (1985), a multivariate filter similar to Laxton and Tetlow (1992) is implemented, adding a Phillips Curve in the set of measurement equations. Considering the most complete specification presented by Areosa (2008), the resulting system is given by three transition equations and four measurement equations, as follows:

$$e_t^* = e_t^*(-1) + \epsilon_{e*,t} \tag{2.12}$$

[5] This parameter corresponds to the inverse of the intertemporal elasticity of substitution, that arises from the Ramsey model applying a standard CRRA utility function for the representative household

[6] Also, as arises from the Ramsey model applying a standard CRRA utility function for the representative household

$$c_t^* = c_t^*(-1) + \epsilon_{c*,t} \tag{2.13}$$

$$y_t^* = 2y_t^*(-1) + y_t^*(-2) + \epsilon_{y^*,t} \tag{2.14}$$

$$e_t = e_t^* + \epsilon_{e,t} \tag{2.15}$$

$$c_t = c_t^* + \epsilon_{c,t} \tag{2.16}$$

$$y_t = y_t^* + 0.6(e_t - e_t^*) + 0.4(c_t - c_t^*) \tag{2.17}$$

$$\pi_t = \theta_1 \pi_{t+1} + (1 - \theta_1)\pi_{t-1} + \theta_2(y_t - y_t^*) + \epsilon_{\pi,t} \tag{2.18}$$

The state equations above comprise the lowest order r-filter (r=1) to NAIRE (2.12) and NAICU (2.13). In turn, an HP filter structure is set for potential output (2.14).[7] Considering the measurement equations, the specification above introduces additional signals through the employment rate (3.9) and capacity utilization (2.16) dynamics. The long-run verticality is imposed in the Phillips curve (2.18), that includes the output gap $(y_t - y_t^*)$ as another control. Finally, the measurement equation given by equation 2.11 is included (2.17), where the parameter γ is set to 0.6.

3 A State Space Macroeconomic Model for the NRIR: a join estimation with potential output, NAIRU and NAICU

In this section, we present an expanded version of the Mésonnier and Renne (2004) model, built over Brazilian data. Using as reference Areosa (2008), we introduce more structure in the potential output state equation. This procedure results in a state space representation that allow us to make a join estimation of four latent economic variables: NRIR, potential output, NAIRU and NAICU. First, we depart from the same state equations in Areosa (2008):

$$e_t^* = e_t^*(-1) + \epsilon_{e*,t} \tag{3.1}$$

$$c_t^* = c_t^*(-1) + \epsilon_{c*,t} \tag{3.2}$$

$$y_t^* = 2y_t^*(-1) + y_t^*(-2) + \epsilon_{y^*,t} \tag{3.3}$$

Considering the measurement equations, we modify the Phillips curve specification. Our dependent variable is the freely-set consumer inflation. Also, in the set of controls, we add a variable related to external prices.[8] Then, we have the following set of equations:

$$e_t = e_t^* + \epsilon_{e,t} \tag{3.4}$$

$$c_t = c_t^* + \epsilon_{c,t} \tag{3.5}$$

[7] As Areosa (2008) mentions, the r-filters is a two-parameters family of filters in which the HP filter is considered as the second order member (r=2). The r=1 filter converges to a constant, been more indicated for stationary variables. As the order increases, the r-filter converges to higher order polynomial time trends. See Araújo et al. (2003) for details.

[8] This Phillips curve specification follows closely BCB (2015).

$$y_t = y_t^* + \gamma(e_t - e_t^*) + (1-\gamma)(c_t - c_t^*) \tag{3.6}$$

$$\pi_t^F = \beta_1\pi_{t-1}^* + \beta_2 E_t(\pi_{t+1}) + (1-\beta_1-\beta_2)\pi_{t-1} + \beta_3 h_{t-1} + \epsilon_{\pi^F,t} \tag{3.7}$$

In this setting, h_t is the output gap, given by the output deviation from its potential. π_t^F is the freely-set consumer inflation. π_t^* is the imported goods inflation. $E_t(\pi_{t+1})$ is the inflation expectation. π_t is the head consumer inflation. r_t is the RIR. $\epsilon_{\pi,t}$ is an error term (*i.i.d.* $\sim$ N(0,σ_i^2), for i=π. The long-run verticality of the Phillips curve is imposed in equation 3.7. Besides, these two equations are consistent with the hypothesis that price stability is reached when the RIR and the output are equal to their respective long term equilibrium (NRIR and potential output).

As we can observe in equation 3.7, these changes demand an adjustment in the restriction that stem from the long term verticality. The next step is to make the link between the NRIR and the potential output. Using as reference Mésonnier and Renne (2004), this relationship is denoted by the following state equation:

$$r_t^* = \mu_r + \phi(y_t^* - y_{t-4}^*) \tag{3.8}$$

Equation 3.8 is a modified version of equation 2.7 in Mésonnier and Renne (2004), where we replace the trend growth rate (g_t) by the potential output growth (year over year). The remaining equations are signals from an r-filter for the RIR and an IS curve:

$$r_t = r_t^* + \epsilon_{r,t} \tag{3.9}$$

$$h_t = \alpha_1 h_{t-1} + \alpha_2(r_{t-2} - r_{t-2}^*) + \alpha_3 h_{t-1}^* + \alpha_4 gov_{t-2} + \epsilon_{h,t}. \tag{3.10}$$

h_t^* is the world output gap. gov_t is the growth rate of the government expenditure in real terms. r_t is the RIR. $\epsilon_{h,t}$ is an error term (*i.i.d.* $\sim$ N(0,σ_i^2), for i=h. The remaining variables were defined before.

Also, similar to Areosa (2008), we introduce linear restrictions in the errors standard deviation of some transition and measurement equations (σ^ϵ) that shares the same state component, as follows: (1) $\sigma_{e_t}^\epsilon = \sqrt{\lambda_e}\sigma_{e_t^*}^\epsilon$; (2) $\sigma_{c_t}^\epsilon = \sqrt{\lambda_c}\sigma_{c_t^*}^\epsilon$; and (3) $\lambda_c = \lambda_r = 40$.[9]

4 Data description

Our data was collected for Brazilian economy from the first quarter of 2002 to fourth quarter of 2016, totalling 60 observations. On one hand, this sample interval excludes structural breaks as happened with the implementation of the real plan in 1994 and the change of the exchange rate regime in 1999. One another hand, the period comprises structural changes undergone by the economy after the Real plan and the last decade commodities' boom. As Ubiergo (2012) highlights, the adoption of an inflation-targeting regime and better economic fundamentals has helped Brazil sustain significantly lower real interest rates than in the past. In the aftermath of the 2008 financial crisis, prices pressures that stemmed from overstimulus policies triggered an monetary tightening cycle that lasted almost four years (2013Q1-2016Q3). More recently, the combination of a severe recession

[9]Setting $\lambda = 40$ on a filter with $r = 1$ is equivalent to $\lambda = 1600$ on the HP filter ($r = 2$). See Araújo et al. (2003) for details

and inflation deceleration opened room for Central Bank to begin an interest rate reduction cycle. All these events characterized the general economic background behind the policy rate fluctuations over the last years in Brazil, that can be observed in Figure 1.

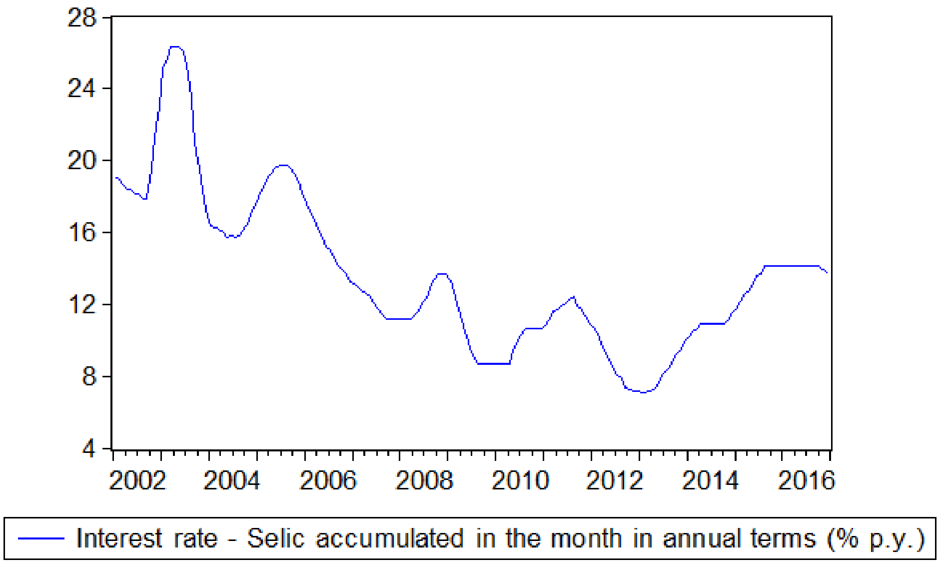

Figure 1: Policy rate cycle in Brazil 2002-2016

We use as nominal interest rate the quarterly average of Selic rate in annual terms. We also introduce the 12 months ahead inflation expectation from Focus, both to calculate our RIR and to add as a control in the Phillips Curve. The other domestic price measures applied in our framework was the head and the freely-set quarterly inflation (Broad National Consumer Price Index -IPCA). The CRB commodity price index growth in domestic currency is applied as the measure for external price inflation. The output measure is the Brazilian GDP, obtained in National Accounts. The unemployment rate was constructed exploring data from PNAD (Pesquisa Nacional por Amostra de Domicílios) and PME (Pesquisa Mensal de Emprego). Capacity utilization is provided by FGV (Fundação Getulio Vargas). The external output was calculated through the aggregation of the GDPs from Brazil's thirteen largest export destinations. The nominal exchange rate is quarterly average of the PTAX reference rate. The fiscal instance is captured by the recurring general government outlays in real terms, calculated as Oreng (2012). When necessary, the variables were seasonally adjusted using the X12 process.

5 Results

5.1 Result and evaluation of the State-Space Macroeconomic Model

Applying the state space model described in section three on our data, we obtained paths for NRIR, potential output, NAIRU and NAICU in Brazil. Table 1 stands the estimated parameters out. Using as reference national accounts data from Brazil, the output elasticity of labor (γ) was set at 0.6.[10] Also, we assumed a logarithmic utility function for the representative household, setting the constant relative risk aversion (ϕ) equal to 1.[11]

The latent variables dynamics is presented in Figure 2. For comparison, we also produce the latent variables estimated by the HP filter. Table 2 and 3 shows the descriptive statistics for each long-term variable.

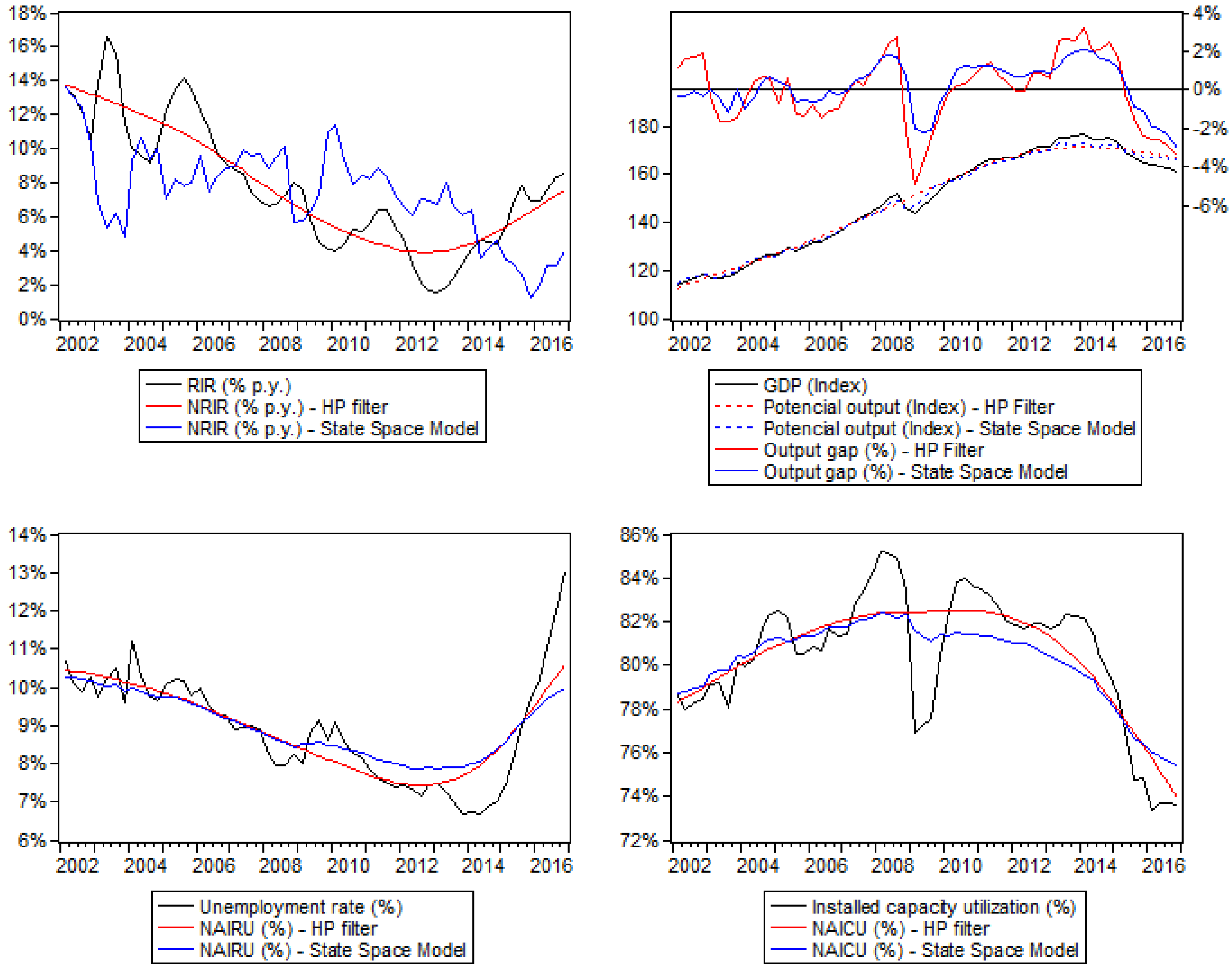

Figure 2: Non-observable variables - HP filter vs State Space Macroeconomic Model

[10]Analogously to one of the methods proposed by Gollin (2002), we calculated γ as the average for the period 2010-2014 of the following ratio: (employee compensation+operating surplus of private unincorporated enterprises)/(employee compensation+operating surplus of private unincorporated enterprises+gross operating surplus). The data is provided by Brazilian Institute of Geography and Statistics (IBGE (2016)). Considering Brazilian data, a similar approach was adopted by Bastos (2012).

[11]Arguing that ϕ is not well identified in the data, Holston et al. (2016) chose to impose this coefficient equal to unity. The assumption behind this strategy is that we have a logarithmic utility function for the representative household. See Hamilton et al. (2016) and Laubach and Williams (2016) for further details.

Table 1: State Space Model estimated

Method: Maximum likelihood (Marquardt)
Sample: 2002Q1 2016Q4
Log likelihood: 948.6907

	Coefficient	Std. Error	z-Statistic	Prob.
$\sigma^{\epsilon}_{e*,t}$	-12.70508	0.163439	-77.73604	0.0000
$\sigma^{\epsilon}_{c*,t}$	-10.77062	0.356445	-30.21682	0.0000
γ	0.6	Calculated with national accounts data		
$\sigma^{\epsilon}_{y*,t}$	-8.568818	0.172210	-49.75804	0.0000
β_1	0.057728	0.011792	4.895419	0.0000
β_2	0.783757	0.224313	3.494033	0.0005
β_3	0.083293	0.043259	1.925445	0.0542
$\sigma^{\epsilon}_{\pi^L,t}$	-10.14113	0.207943	-48.76875	0.0000
μ_r	0.045141	0.006139	7.353434	0.0000
ϕ	1	Logarithmic household utility function		
$\sigma^{\epsilon}_{r_t}$	-10.03231	0.244445	-41.04121	0.0000
α_1	0.821494	0.114818	7.154749	0.0000
α_2	-0.071226	0.020311	-3.506775	0.0005
α_3	0.769099	0.159459	4.823193	0.0000
α_4	0.047232	0.023594	2.001890	0.0453
$\sigma^{\epsilon}_{h_t}$	-10.70132	0.307924	-34.75311	0.0000
	Final State	**Root MSE**	**z-Statistic**	**Prob.**
e^*_t	-0.104878	0.004401	-23.83085	0.0000
c^*_t	-0.282167	0.010668	-26.45075	0.0000
y^*_t	5.107972	0.014723	345.7206	0.0000

We can observe that for all variables, the State Space Model has standard deviation smaller than the original HP filter. Another point to emphasize is that the former is more robust to the trend estimates bias usually presented by the latter method at the end and beginning of samples.

Evaluating the NRIR dynamics, as we can observe in the Figure 2, the minimum NRIR value on the sample (1.3%) was registered in 2016Q1, reflecting the impact of the potential GDP contraction in that moment. Considering this variable and period, we also emphasize the difference between the NRIR estimated by the models (530 basis points), a result that exemplifies the sizeable error that we can incur in the measurement of this variable in periods marked by greater potential GDP variations. Analysing the remaining non observable variables, we highlight the growing idle productive capacity stemming from the recent recession, associated to both capital and labor factors.

Table 2: Non-observable variables descriptive statistics - State Space Model

Descriptive Statistic	NRIR (%)	Output gap (%)	NAIRU (%)	NAICU (%)
Mean	7.41	0.16	8.95	80.23
Median	7.58	0.33	8.82	81.02
Maximum	13.57	2.10	10.25	82.40
Minimum	1.18	-3.01	7.87	75.41
Std. Deviation	2.73	1.22	0.79	1.84

Table 3: Non-observable variables descriptive statistics - HP Filter

Descriptive Statistic	NRIR (%)	Output gap (%)	NAIRU (%)	NAICU (%)
Mean	7.71	-0.02	8.88	80.51
Median	6.80	0.28	8.83	81.30
Maximum	13.76	3.2	10.59	82.51
Minimum	3.90	-4.92	7.43	73.97
Std. Deviation	3.25	1.80	1.03	2.22

Assessing the remaining latent variables, the output gap measured by the State Space Model is smother than the one obtained with the HP filter. This effect is specially evident in the 2008 financial crisis. Considering the NAIRU and the NAICU, we highlight the differences in these variables levels estimated by both models between 2008 and 2013. This period coincides with a RIR holding sistematically under the NRIR in the state space model, resulting in a NAIRU over and NAICU under the respective HP filter measures.

5.2 Evaluating the Monetary Policy Stance in Brazil through the lens of a State-Space Macroeconomic Model

In this section we evaluate the stance of monetary policy in Brazil in our sample period. First, we compare the RIR gap (the difference between the RIR and the NRIR) with some price and activity variables explored in our Kalman Filter. Second, we identify the monetary policy easing and tightening cycles using the latent variables dynamics. Figure 3 allows a comparison of the RIR gap with: inflation, output gap, unemployment rate gap and capacity utilization gap.

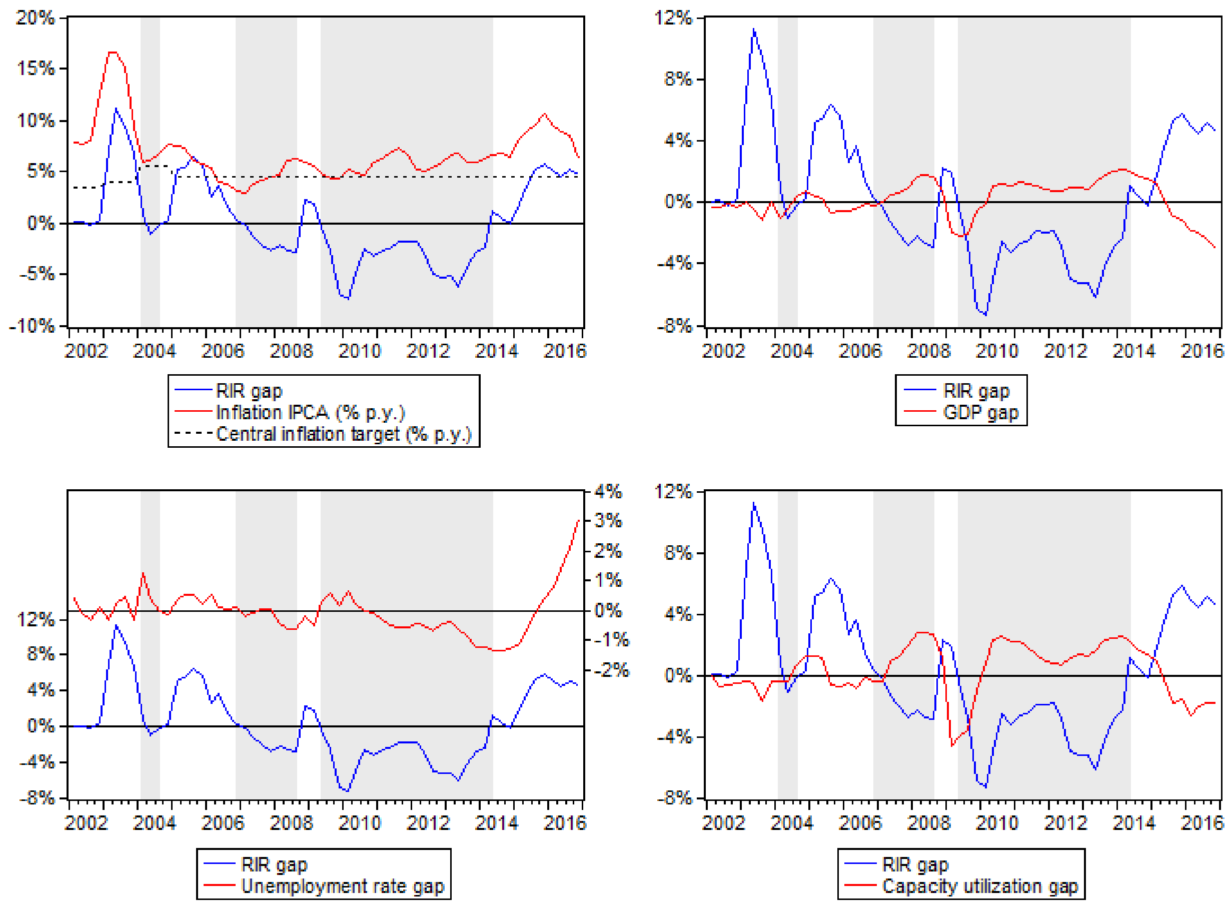

Figure 3: Evaluating the stance of monetary policy: RIR gap vs business cycle variables

As we can observe, the comovement among the variables occurs as expected. When the RIR is set under (above) the NRIR, we usually identify inflation above (under) the central target, the output above (under) its potential, the unemployment rate under (above) the NAIRU and the installed capacity utilization above (under) the NAICU.

In order to assess the monetary policy stance, we verify the RIR gap dynamics during our sample period. Periods of negative (positive) RIR gap are identified by the shaded (white) areas in the graphs, being equivalent to monetary easing (tightening) cycles. A more restrictive monetary policy prevails in the first half of our sample. The last (and longest) period of policy easing was 2009Q3-2014Q2. Since then, Brazil Central Bank has been adopting a tighter monetary stance.

6 Final Remarks

In this study, we employ a multivariate Kalman Filter to measure the Neutral Real Interest Rate (NRIR) in Brazil, from 2002Q1 to 2016Q4. Basically, we depart from Mésonnier and Renne (2004) and Areosa (2008) state space models in order to build a representation that allow us to make a join estimation of four latent economic variables: NRIR, potential output, NAIRU and NAICU. This method differs from previous literature in two ways. First, former studies usually make a joint estimation of two latent variables: NRIR and potential output. Second, instead of imposing for potential output structures that follow random walks or have autoregressive patterns, our state space model relies on the production function approach.

Comparing the latent variables estimated by our state space macroeconomic model with the ones that results from a HP filter, the former is more robust to the trend estimates bias usually presented by the latter method at the end and beginning of samples. Also, the difference between the NRIR estimated by the models at some moments exemplifies the sizeable error that we can incur in the measurement of this variable in periods marked by greater potential GDP variations. Analysing the remaining non observable variables, we highlight the growing idle productive capacity linked to both capital and labor factors.

Evaluating the monetary policy stance in our sample period with the state space macroeconomic model, we observed that in the first half of our data, tighter monetary policy prevailed. The last (and longest) period of policy easing began in the aftermath of 2008 financial crisis and sustained till the first half of 2014. Since then, Brazil Central Bank has been adopting a tighter monetary stance.

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