Why Are Savings Rates so Low and Interest Rates so High in Brazil? The Role of Institutions *

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Abstract

In this paper we examine the joint effect of compulsory savings and unfunded social security on the macroeconomic equilibrium in an economy populated by heterogeneous and overlapping generations individuals. Our model economy is informed by the Brazilian experience, where a social security structure with high and progressive replacement ratios combined with a forced savings system with lower than market returns remuneration. Those institutional features coexist with extremely high real interest rates and low voluntary saving rates. We examine the links between those facts by simulating such model calibrated with Brazilian data and performing counterfactual exercises. Reducing pensions replacement ratios to levels comparable to US leads to substantial increase in the savings rates and modest reduction in real interest rates. Similar results are obtained when forced savings interest rates are increased from the current below market levels. *Keywords: social security, pay as you go, individual retirement accounts, unemployment and longevity risks J.E.L. codes:* E21, H55.

1 Introduction

The growth and welfare effects of the pay-as-you-go (PAYG) and capitalization social security systems are extensively discussed in the literature, as well as the transition from the former to the latter. Among others, see the influent works of **?** and **?**.[1] Due to population aging that turns the current replacement ratios of PAYG unsustainable, its combination with individual capitalization accounts is one more time considered as a device to generate the additional savings necessary to support retirees' standard of living for longer (**?**).

Brazil features both a pay-as-you-go social security system with high and progressive replacement rates, and forced savings in capitalization accounts with lower than market reumuneration rates. At the same time, real interest rates are among the highest in the world, and voluntary savings are very low. In this paper we examine the joint effect of compulsory savings and unfunded social security on the macroeconomic equilibrium in an model economy with heterogeneous and overlapping generations individuals.

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[1]See **?** and **?** for analyses of the Brazilian case.

To further describe the social security in our model economy, individuals have their wage taxed at a fixed rate and retire at a certain age (65 years old in Brazil), when they start to receive pensions at replacement ratios that are inversely related to their permanent incomes.[2] Additionally, workers mandatorily get paid an extra 8% of their gross labor income in individual savings accounts, called Service Time Guarantee Fund (FGTS), that accumulates until retirement at rates below market. The FGTS savings cannot be freely withdrawn before the retiring age and thus constitute another mechanism to countervail the low propensity to save of myopic agents. [3]

Because individuals have different abilities, incomes and mortality risks, and are subject to borrowing constraints, aggregate and distributive equilibrium effects of those institutions are non-trivial. The progressive replacement ratios have distributive effects. However, compulsory savings earn below market returns and subsidize physical capital investments. Eventual deficits in the social security are financed by additional taxes.[4]

We study the effects of those institutional features by simulating such a model and performing counter-factual exercises. We examine the joint and separate role of those two social security devices in the equilibrium economy. First, we compare the mixed system (called "Benchmark") with one where the FGTS forced saving mechanism is abolished. Since forced savings help to finance subsidized loans in this economy, its extinction lowers the aggregate physical capital, output and wages, simultaneously generating a fiscal deficit that has to be compensated by a tax increase. Voluntary savings rates rise substantially, but not enough to compensate for the elimination of forced savings, resulting in lower total savings. With lower output and wages, all agents get worse off. The wealth inequality also increases.

In our second exercise we bring down the very high Brazilian pension replacement ratios to the U.S. levels. As expected, this reform lowers taxes, increases both voluntary and total savings rates, and substantially reduces interest rates. With greater equilibrium physical capital than in the Benchmark, the economy produces higher output and wages that makes agents better off.

Finally increasing the FGTS remuneration (another reform discussed in the Brazilian Congress) makes it a more efficient wealth accumulation instrument, resulting in higher forced and total savings and lower market interest rates. Again, with higher physical capital, output and wages, wealth inequality goes down. Given that workers' FGTS balances accounts are proportional to their wages, their welfare variation depends on the fiscal regime. While high ability workers always improve, low ability ones get much better in the proportional wage tax regime and much worse in the lump-sum financing regime.

Our counterfactual exercises point out that both a decrease in replacement ratios and an increase in the forced savings remuneration are reforms that could lead to substantial increases in the total saving rates and reductions in the market interest rates, with positive consequences for growth and wealth inequality.

The plan of the paper is as follows. Section 2 briefly reviews the related literature. Section 3 presents our model economy. The following section explains how we calibrate the model parameters in order to represent relevant features of the Brazilian economy. Section 5 displays

[2]This unfunded pension system compensates under-savings and insure families from unemployment and death risks. However, we do not model the unemployment insurance feature.

[3]In reality the fund can be withdrawal if the worker is fired or if she buys her first house, provided that its price is lower than a stipulated value.

[4]The fast aging process of the Brazilian population is placing the fiscal situation under increasing financial stress. Here we abstract from the demographic transition and thus from fiscal sustainability issues. With an "stable-equivalent" age distribution, we assume that the budget is balanced every period through wage or lump-sum taxes.

the results of our counter-factual equilibria. The last section concludes this version with some preliminary remarks and directions for future research.

2 Related Literature and the Brazilian Social Security

The macroeconomic effects of the pay-as-you-go (PAYG) and capitalization social security systems are extensively discussed in the literature. Effects on labor supply, savings behavior, taxes and interest rates, as well as their efficiency and welfare consequences were initially studied in a fully rational decision making framework, as in **?** and **?**, usually concluding that social security serves as a redistribution device across generations. [5]

Incorporating the fundamental aspects that social security was introduced to prevent myopic individuals from inadequate savings for retirement, and as a substitute to the missing market for annuities, the literature in the early 2000s was extended to allow for time-inconsistent individuals. Studies like **?**, **?**, and others show that time-inconsistent agents value social security as a commitment device to save.

Another limitation of the 1990s literature was to consider only steady state effects of social security. It is necessary to compute the transition path between steady states and separate intergenerational redistribution from efficiency effects. An exact calculation must compensate income effects of generations in a separate simulation, as in **?** or **?**.

Increasing concerns with population aging bring to the foreground the issues of suitability and sustainability of the PAYG system in an era of higher dependency ratio (i.e., the ratio of the number of people aged 0-14 and 65-over divided by the number of people aged 15-64). For example, **?** show that in an aging society, taxes would have to almost double (reaching more than 50% of the GDP) to afford for the current benefit-wage ratio of the PAYG retirement system, hindering investments and per-capita growth.

Capitalization accounts are again considered as a device to generate the additional savings necessary to fund more years in retirement. For example, **?** and **?** study the introduction of individual retirement accounts to supplement the existing PAYG system in the U.S. and Germany respectively.

Given the evidence on the importance of illiquid assets for life cycle savings modelling (see **?**), housing seems an important aspect of the problem (see **?** and **?**).

In this context, Brazil is particularly interesting, given its costly mixed PAYG-capitalization social security system that promisses high average replacement ratio, though not necessarily progressive or proportional. To finance current pensioners in a PAYG scheme, the Brazilian formal sector worker is taxed 8% to 11% of his (her) wage, with an additional 20% contribution made by the employer. Additionally, the employer mandatorily deposits an extra 8% of the worker wage in an individual account, called the Service Time Guarantee Fund (FGTS). The worker cannot withdraw from this account unless she is fired, become disable or retire. Thus, FGTS is a particular kind of illiquid asset and constitutes an endowment for retirement, like an individual retirement account (IRA). Although the FGTS accumulates at below market interest rates, **?** show that it significantly supplement wealth to finance constant consumption habits during retirement.

The Brazilian state paternalism also extends to firms' investments, and a significant portion of the FGTS (and other government managed "workerks' support" funds) is used to finance capital investments at below market interest rates, as studied in **?**.

[5]**?** and **?** are examples of similar analyses for Brazil.

In this paper, we incorporate the Brazilian social security system and investments subsidies into a general equilibrium model with time-consistent consumers that are borrowing constrained, have different abilities, income and mortality risks. We study the aggregate and distributive equilibrium effects of those institutions, their consequences to growth and welfare.

3 The Environment

At each point in time, the economy is inhabited by multiple cohorts of individuals of different ages. Each cohort is comprised of a continuum of measure one of individuals who live for a finite, albeit random, number of periods.

3.1 Demography

Each period, t, a new generation is born. For an individual born in date t, uncertainty regarding the time of death is captured by the fact that he or she faces a probability ψ_{t+1} of surviving to the age $t+1$ conditional on being alive at age t. Hence, an individual born in t is alive in $t+J$ with probability $\prod_{k=1}^{J} \psi_{t+k}$. We also assume that there is $T > 0$ such that $\psi_{T+1} = 0$.

For most of our analysis we will focus on the steady-state allocations. Since it greatly simplifies the presentation we shall drop all time indices from aggregate variables and use t to represent age.

We may map the survival probability into the time invariant age profile of the population denoted $\{\mu_t\}_{t=0}^{T}$. Letting g_n denote the population growth rate, the fraction of agents t years old in the population is found using the following law of motion

$$\mu_t = \frac{\psi_t}{1+g_n}\mu_{t-1},$$

with $\mu_t \geq 0$, and $\sum_{t=0}^{T} \mu_t = 1$.

3.2 Households

Preferences Individuals derive utility from consumption, c, and leisure, $1-l$. We use a quasi-hyperbolic preference in line with **?**. In this set-up, per period utility is discounted by $1, \varrho\beta, \varrho\beta^2, \varrho\beta^3, \ldots$. Given that ϱ is used only to discount utility from the current period and the next and β is used to discount future utility every period, they are called short-term and long-term discount factors, respectively.

The preference over lifetime consumption and leisure of an individual of age t is defined as follows:

$$U_t = u_t(c_t, 1-l_t) + \varrho\mathbb{E}_t\left[\sum_{j=t+1}^{T}\beta^j\left(\prod_{k=t+1}^{j}\psi_k\right)u_j(c_j, 1-l_j)\right], \tag{1}$$

where $\mathbb{E}_t$ is the expectation operator conditional on the information available to the individual at t.

We allow preferences over consumption-leisure bundles to vary with age by indexing the flow utility by t.[6] More specifically, flow utility will be of the form

$$u_t(c_t, 1-l_t) = \frac{(c_t^{1-\rho_t}(1-l_t)^{\rho_t})^{1-\gamma} - 1}{1-\gamma}, \quad (2)$$

for $\gamma > 0$ and $\gamma \neq 1$, where $\rho_t \in (0,1)\ \forall t$ allows preferences over consumption-leisure to vary with age.

Budget constraints Every period, individuals choose labor supply, consumption and asset accumulation to maximize their objective, (**??**), subject to a budget constraint which we shall explain momentarily.

An individual of age t who works for l hours supplies to the market a total of $l_t s_t e^{u+z_t}$ efficiency units which are paid at a wage rate w. The variable $u \sim \mathcal{N}(0, \sigma_u^2)$ is a permanent component of an individual's skills. It is realized at birth and retained throughout one's life. On the other hand, z evolves stochastically according to an AR(1) process, $z_t = \varphi_z z_{t-1} + \varepsilon_t$, with innovations $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Whereas u aims at capturing the heterogeneity at birth, everyone's most relevant lottery, z is the main source of uncertainty that affects one's choices. The parameter φ_z accommodates the empirically observed persistence of productivity shocks. s_t is the age-efficiency profile, which is deterministic and intend to capture the age component of the life-cycle earnings.

Each person has a unit time endowment which can be used in market related activities, or directly consumed in the form of leisure, $1-l$. Labor productivity shocks are independent across agents. As a consequence, there is no uncertainty regarding the aggregate labor endowment even though there is uncertainty at the individual level.

All workers in this economy pay taxes $(\tau_w, T_{ss}(.), \tau_{fgts})$. Ideally, the revenue from τ_w finances overall government expenditures not related to the social security system. The $T_{ss}(.) = \tau_{ss} min., y_{max}$ is the used to finance the benefit payments to the retirees, where τ_{ss} is the contribution rate and $y_{max} = 2.3 y_m$ with y_m for the average wage. In addition, employers make a compulsory contribution, τ_{fgts}, to their employees FGTS accounts. Households can also be taxed lump-sum, $\mathcal{T}$. Thus, the worker after-tax labor income is given by:

$$y_t = (1-\tau_w)(1-\tau_{fgts}) w_t l_t s_t e^{u+z_t} - \tau_{ss} min\{(1-\tau_{fgts}) w_t l_t s_t e^{u+z_t}, y_{max}\} - \mathcal{T} \quad (3)$$

At the age of $T_r = 65$, individuals retire and start collecting social security benefits, which we denoted by $b(u)$. In order to capture the correlation between pensions and life-cycle earnings, we allow the benefits to depend on the permanent shock. In particular, we assume that:

$$y_t = b(u) = \theta(u) y_m(u) - \mathcal{T} \quad (4)$$

where $\theta(u) = \theta_0 + \theta_1 u$ is the retirement replacement ratio and $y_m(u)$ is the average life-cycle earnings conditioned on u. The system is progressive, with $\theta_1 < 0$.

Individuals can trade an ordinary risk-free asset which holdings we denote by $a_{ord,t}$. Ordinary asset holdings are subject to an exogenous lower bound. More precisely, we assume that agents are not allowed to contract debt at any age, so that the amount of ordinary assets carried over from age t to $t+1$ is such that $a_{ord,t+1} \geq 0$. Because no agent can hold a negative position in ordinary assets at any time, we assume without loss that assets take the form of capital, as in **?**.

[6]The reason for allowing ρ to depend on age is to better match the actual behavior of hours worked over the life-cycle. This is important in our case given the nature of the exercises we study.

At each age, individuals total wealth, $a_t = (a_{ord,t} + a_{fgts,t})$, is the sum of the ordinary assets, $a_{ord,t+1}$, and the illiquid FGTS account balance, $a_{fgts,t}$. The budget constraints for workers aged $t = 1, ..., T_r - 1$ are:

$$\begin{aligned} c_t + a_{ord,t+1} = (1+r)a_{ord,t} + (1-\tau_w)(1-\tau_{fgts})w_t l_t s_t e^{u+z_t} \\ - \tau_{ss} min\{(1-\tau_{fgts})w_t l_t s_t e^{u+z_t}, y_{max}\} - \mathcal{T} + \epsilon \end{aligned} \tag{5}$$

and

$$a_{fgts,t+1} = (1+r_{fgts})a_{fgts,t} + \tau_{fgts} w_t l_t s_t e^{u+z_t} \tag{6}$$

where ϵ is a lump-sum involuntary bequests left by those who die before reaching age $T+1$, and the rate of return on the FGTS r_{fgts} is defined by the government. For simplicity, all assets of agents who died are collected and redistributed among all agents alive in the economy on a lump-sum basis:

$$\epsilon = \sum_{t=1}^{T} \mu_t \int_{\Omega} (1-\psi_{t+1})[(1+r)a_{ord,t} + (1+r_{fgts})a_{fgts,t}] d\lambda_t \tag{7}$$

Individuals are allowed to withdraw their FGTS account balances only upon retirement at the age T_r. Thus, the budget constraints for new retirees are:

$$c_{T_r} + a_{ord,T_r+1} = (1+r)a_{ord,T_r} + (1+r_{fgts})a_{fgts,T_r} + b(u) - \mathcal{T} + \epsilon \tag{8}$$

and

$$a_{fgts,T_r+1} = 0 \tag{9}$$

with the budget constraint for retirees aged $t = T_r + 1, ..., T$ simplifying to:

$$c_t + a_{ord,t+1} = (1+r)a_{ord,t} + b(u) + \epsilon - \mathcal{T} \tag{10}$$

Recursive Formulation of Households' Problem

Let $\omega = (a_{ord,t}, a_{fgts,t}, u, z_t)$ denote the individual state-space. The choice problem of workers, aged $t = 1, ..., T_r - 1$, can be recursively represented as follows:

$$V_{w,t}(\omega) = \max_{l, a'_{ord} \geq 0} : \left[U_t(c, 1-l) + \varrho\beta\psi_{t+1} E_{z'} \bar{V}_{w,t+1}(\omega') \right], \tag{11}$$

subject to the budget (**??**) and (**??**).

Let $a'_{ord} = d_{a,t}(\omega)$ and $l = d_{l,t}(\omega)$ denote the solution of the problem above. Thus, the value function on the right-hand side is updated as follows:

$$\bar{V}_{w,t}(\omega) = U_t(c, 1-l) + \beta\psi_{t+1} E_{z'} \bar{V}_{w,t+1}(\omega') \tag{12}$$

where

$$\begin{aligned} c + d_{a,t}(\omega) = (1+r)a_{ord} + (1-\tau_w)(1-\tau_{fgts})w d_{l,t}(\omega) s e^{u+z} \\ - \tau_{ss} min\{(1-\tau_{fgts})w d_{l,t}(\omega) s e^{u+z}, y_{max}\} - \mathcal{T} + \epsilon \end{aligned} \tag{13}$$

Notice that, due to the time-inconsistency problem associated with the quasi-hyperbolic discounting preference, the recursive formulation is such that agents choose next period ordinary assets and hours worked applying the discount factor $\varrho\beta$, but the actual value function is evaluated with the discount factor β.

The optimization problem of retirees, aged $t = T_r, ..., T$, can similarly be written as follows:

$$V_{r,t}(\omega) = \max_{a'_{ord} \geq 0} : \left[U_t(c, 1) + \varrho\beta\psi_{t+1}\bar{V}_{r,t+1}(\omega')\right], \tag{14}$$

subject to the budget constraints (**??**) and (**??**), or (**??**), where $\bar{V}_{r,t}$ is updated according to:

$$\bar{V}_{r,t}(a_{ord}, u) = U_t(c, 1) + \beta\psi_{t+1}\bar{V}_{r,t+1}(a'_{ord}, u) \tag{15}$$

It should be stressed that we have imposed non-negativity constraints on asset holdings. We have thus taken an extreme (albeit plausible) position with regards to capital markets. Relaxing a little the assumption by allowing some exogenous limit is likely to have little effect on our conclusions, at the cost of introducing a whole new set of issues that would have to be dealt with to maintain the internal consistency of the model.

Also important is the fact that we have only used individual state variables in ω. It is apparent that prices do enter the value function. Indeed, in solving the model we will need to find the equilibrium prices by explicitly taking into account how they enter the policy functions associated with (**??**) and (**??**).

3.3 The Government

In our economy, there is one government budget constraint that encompasses both the tax system and the pension system. It is given by:

$$\begin{aligned} G + G_{ss} + r_{fgts}K_{fgts} = \sum_{t=1}^{T}\mu_t \int_{\Omega} ([\tau_{fgts} + \tau_w(1 - \tau_{fgts})] w_t l_t s_t e^{u+z_t} + \\ \tau_{ss} min\{(1 - \tau_{fgts}) w_t l_t s_t e^{u+z_t}, y_{max}\}) d\lambda_t + \bar{r}K_{fgts} + \mathcal{T} \end{aligned} \tag{16}$$

where G is an exogenous stream of regular government spending, $G_{ss} = \sum_{t=T_r}^{T}\mu_t \int_{\Omega} b(u) d\lambda_t$ is the social security expenditure on pensions and K_{fgts} is the total wealth deposited in the FGTS. The above equation implies that the government consolidates its regular government spending together with the mixed PAYG-capitalization social security system. The pension benefits are financed through the combination of wage-proportional social security and regular labor income taxes, respectively τ_{ss} and τ_w, an eventual interest rate spread $(\bar{r} - r_{fgts})$ on the total FGTS managed, and a lump-sum tax $\mathcal{T}$. The amount of benefits received by each retired agent was presented in the last section.

In the simulations below, we allow two forms of fiscal budget adjustment: (i) τ_w adjusts to ensure that government budget constraint is satisfied; or (ii) the lump-sum tax $\mathcal{T}$ adjusts to ensure that government budget constraint is satisfied. We assume τ_{ss} is exogenous. As explained before, the government manages a compulsory savings program represented by the Service Time Guarantee Fund (FGTS). Under this program, employers deposit 8% of the payroll in an account in the name of the employee. The interest rate on these deposits, r_{fgts}, is determined by the government and thus is a policy parameter in our model.

3.4 Production

Except for some government subsidy to capital, the production side is standard. The technology for producing the consumption good is summarized by a Cobb-Douglas production function with constant returns to scale,

$$Y = BK^{\alpha}N^{1-\alpha},$$

where K is aggregate capital, N is aggregate efficient units of labor, and B is a scale parameter.

Every period, the representative firm solves the static optimization problem

$$\max_{K,N} \left\{ BK^{\alpha}N^{1-\alpha} - \delta K - wN - r(1-\phi)K - \bar{r}\phi K \right\},$$

where r is the market rental rate of physical capital, $\bar{r}$ is the government subsidised interest rate, w is the wage rate and ϕ is the share of the aggregate capital that is subsided. Given the government has K_{fgts} to lend, $\phi K = K_{fgts}$. Note that we assume that the rental rate of capital is net of depreciation costs which are born directly by the firm.

The first order conditions for the firm's profit maximization problem are,

$$w = (1-\alpha)BK^{\alpha}N^{-\alpha}, \tag{17}$$

and

$$r = \frac{\alpha BK^{\alpha-1}N^{-\alpha} - \delta - \bar{r}\phi}{1-\phi} \tag{18}$$

3.5 Recursive competitive equilibrium

In all that follows we describe the recursive equilibrium in a steady state. This greatly simplifies the presentation. Moreover it dispenses the distinction between age and time thus significantly reducing the notational burden.

At each point in time, agents differ from one another with respect to age t and to state vector $\omega = (a_{ord,t}, a_{fgts,t}, u, z_t) \in \Omega$. Agents of age t, identified by their individual states ω, are distributed according to a probability measure λ_t defined on Ω, as follows.

Let $(\Omega, \mathcal{F}(\Omega), \lambda_t)$ be a space of probability, where $\mathcal{F}(\Omega)$ is the Borel σ-algebra on Ω. For each $\eta \in \mathcal{F}(\Omega)$, $\lambda_t(\eta)$ denotes the fraction of agents aged t that are in η.

Given the age t distribution, λ_t, $Q_t(\omega, \eta)$ induces the age $t+1$ distribution λ_{t+1} as follows.

The function $Q_t(\omega, \eta)$ determines the probability of an agent at age t and state ω to transit to the set η at age $t+1$. In turn, $Q_t(\omega, \eta)$ depends on the policy functions in (**??**) and on the exogenous stochastic process for z.

A recursive competitive equilibrium for the economy is as follows.

Definition 1. Given the policy parameters $\{G, b(u), r_{fgts}, K_{fgts}, \tau_w, \tau_{ss}, \bar{r}, \mathcal{T}\}$, a **recursive competitive equilibrium** for the economy is a collection of value functions $\{V_{w,t}(\omega), V_{r,t}(\omega)\}$, policy functions for ordinary asset holdings $d_{a,t}(\omega)$ and labor supply $d_{l,t}(\omega)$ (implying consumption $c_t(\omega)$ and FGTS assets $a_{fgts,t+1}(\omega)$), prices $\{w, r\}$, age dependent but time-invariant measures of agents $\lambda_t(\omega)$, and accidental bequest ϵ such that:

(i) $d_{a,t}(\omega), d_{l,t}(\omega)$ solve the dynamic problems in (**??**) and (**??**) subject to the given constraints;

(ii) individual and aggregate behaviors are consistent, that is:

$$K = \sum_{t=1}^{T} \mu_t \int_{\Omega} [d_{a,t}(\omega) + a_{fgts,t+1}(\omega)] d\lambda_t = \sum_{t=1}^{T} \mu_t \int_{\Omega} d_{a,t}(\omega) d\lambda_t + K_{fgts}$$

$$N = \sum_{t=1}^{T} \mu_t \int_{\Omega} d_{l,t}(\omega) e^{u+z_t} d\lambda_t$$

$$C = \sum_{t=1}^{T} \mu_t \int_{\Omega} c_t(\omega) d\lambda_t$$

(iii) $\{w, r\}$ are such that they satisfy the optimum conditions (**??**) and (**??**);

(iv) given the decision rules, $\lambda_t(\omega)$ follows the law of motion:

$$\lambda_{t+1}(\eta) = \int_{\Omega} Q_t(\omega, \eta) d\lambda_t \;\; \forall \eta \subset \mathcal{F}(\Omega);$$

(v) the distribution of accidental bequests is:

$$\epsilon = \sum_{t=1}^{T} \mu_t \int_{\Omega} (1 - \psi_{t+1})[(1+r)d_{a,t-1}(\omega) + (1 + r_{fgts})a_{fgts,t}(\omega)] d\lambda_t$$

(vi) the share of subsidized capital is given

$$\phi = \frac{K_{fgts}}{K}$$

(vii) τ_w, τ_{ss} and $\mathcal{T}$ are such that the government's budget constraint,

$$G + G_{ss} + (r_{fgts} - \bar{r})K_{fgts} = \sum_{t=1}^{T} \mu_t \int_{\Omega} ([\tau_{fgts} + \tau_w(1 - \tau_{fgts})] w_t d_{l,t}(\omega) s_t e^{u+z_t} +$$

$$\tau_{ss} min\{(1 - \tau_{fgts})] w_t d_{l,t}(\omega) s_t e^{u+z_t}, y_{max}\}) d\lambda_t + \mathcal{T}$$

is satisfied every period.

(viii) the goods market clears:

$$Y = C + (1 + g_n)K - (1 - \delta)K + G$$

4 Calibration

To carry out our quantitative analysis, we need first to find values for all the parameters of the model. We accomplish this by calibrating the model to the Brazilian economy.

The population age profile $\{\mu_t\}_{t=1}^{T}$ depends on the population growth rate, g_n, the survival probabilities, ψ_t, and the maximum age, T, that an agent can live. Agents enter the economy at age 20 and may live for 71 years, $T = 71$, so that the maximum age is 90 years old.

Parameter	Value	Source/Target
β	0.995	$K/Y = 3.00$
γ	4.00	Micro evidence
ρ	0.62	life cycle profile of mean hours
σ_u^2	0.15	Gini at age 25 (0.33)
φ_z	0.95	Kaplan (2012)
σ_ϵ^2	0.022	Gini at age 60 (0.54)
δ	0.05	see text
α	0.36	NIPA
B	0.91	$w = 1$
ϱ	0.087	progressivity
τ_k	0.15	Paes and Bugarin (2006)
τ_c	0.20	Paes and Bugarin (2006)
τ_{ss}	0.11	social security law
θ_{cr}	0.74	Afonso (2016)
θ_{ar}	0.84	Afonso (2016)
y_{max}	$2.23y_m$	
y_{min}	$0.29y_m$	

Data on survival probability by age are from Brazilian Institute of Geography and Statistics (IBGE). Given the survival probabilities, the population growth rate is chosen so that the age distribution in the model replicates the dependency ratio observed in the data. By setting $g_n = 0.012$, the model generates a dependency ratio of 13%, which is close to the dependency ratio observed in the data for 2008.

[plain] Calibration

To calibrate the preference parameters we proceed as follows. First, we set $\varrho = 1$ and choose the discount factor β in such a way that the equilibrium of our benchmark economy implies a capital-output ratio around of 2.5, which is the value observed in the data. Then we fix the parameter γ to 4.0, from micro evidence, and choose the share of leisure in the utility function, ρ_t, to match average ours for different age groups. In particular, we assume that $\rho_t = \rho_0 + \rho_1 t$. To calibrate ρ_0, we use the average working hours for ages $12 - 40$ and for ρ_1 the average between $41 - 60$. The first group works on average 37.86% while the second 40.37% of their time endowment. For the last 5 years we specify a new profile $\rho_t = \rho_{60} + \rho_2 t$. We calibrate ρ_2 to match the average time worked during those last five years equal to 35.16%.[7]

To explore the implications of hyperbolic discounting, we also study economies in which $\varrho < 1$. In particular, we consider the cases where $\varrho = 0.85$ and $\varrho = 0.60$. For each value of ϱ, we only adjust β to generate the same capital to output ratio of 2.5.

The parameters that characterized the stochastic component of individuals productivity are $(\sigma_u^2, \varphi_z, \sigma_\epsilon^2)$. Several authors have estimated similar stochastic process for labor productivity using US data. Controlling for the presence of measurement errors and/or effects of some observable characteristics such as education and age, the literature provides a range of $[0.89, 0.97]$ for φ_z and of $[0.10, 0.25]$ for σ_ϵ^2. In Brazil, due to the lack of a household panel data survey, such as the Panel Study of Income Dynamics in the U.S., we cannot estimate φ properly. Thus, we set $\varphi_z = 0.95$ based on the evidence for the U.S. economy and then calibrate σ_ϵ^2 to match the income Gini at age 60, which was 0.54 in 2008 according to the PNAD. This procedure provides a value for σ_ϵ^2 equal to 0.022. Finally, σ_u^2 is chosen in such a way that the labor income Gini at age 20 in the model matches its counterpart in the data, which is nearly 0.32. We obtain a value of 0.15 for σ_u^2. [8] We discretize the two shocks in order to solve the model, using 5 states to represent the permanent shock and nine states for the persistent shock. For expositional convenience, we refer to the two extremes of the grid for the permanent shock as low and high ability/type.

[7] The data for hours worked are from 2008 Brazilian National Household Survey - PNAD (Pesquisa Nacional por Amostra de Domicilios).

[8] For the sake of comparison, using the PSID data, **?** finds $\varphi_z = 0.94$, $\sigma_\epsilon^2 = 0.016$ and $\sigma_u^2 = 0.056$.

The values of the technological parameters (α, δ) are also in Table **??**. We chose a value for the capital, α, of 0.36 based on **?**. The depreciation rate, in turn, is obtained by $\delta = \frac{I/Y}{K/Y} - g$. We set the investment-product ratio I/Y equal to 0.19 and the capital-product ratio K/Y equal to 2.5. The economic growth rate, g, is constant and consistent with the average growth rate of GDP over the second half of the last century. Based on data from Penn-World Table, we set g equal to 2.2%, which yields a depreciation rate of nearly 5%.

The values for the actual age-efficiency profile are constructed similarly to **?** and **?**. We use annual earnings and annual hours worked for the age groups 15-24, 25-34,..., 75-84 from PNAD. First, we construct hourly wages by dividing annual earnings by annual hours for each age group. Afterwards, we use a second order polynomial to interpolate the points to obtain the age-efficiency profile by exact age.

Finally, we specify the others parameters related to government activity. First, we set government consumption, G, to 20% of the output of the economy under the baseline calibration. The interest rate on FGTS assets is 3% plus the reference interest rate (TR), which was nearly 1%. Thus, considering an inflation rate of 7%, we have that the real return on FGTS deposits is -3%, which is the value we use for r_{fgts} in the benchmark economy.

As for Social Security, contributions are of the form $T_{ss}(y) = \tau_{ss} \min\{y, y_{\max}\}$. The tax rate for social security is 11% for the employee and 20% for the employer, so we set $\tau_{ss} = 0.31$. To calibrate the contribution ceiling, we use the fact that in the data, $y_{\max} = 2.30 y_m$, where y_m is the average labor income. We assume that the retirement replacement ratio is given by $\theta(u) = \theta_0 + \theta_1 u$. We choose θ_0 and θ_1 in such a way that the replacement ratio of the lowest ability individual is 1.00 and the average replacement ratio is 0.70, as is observed in the data.

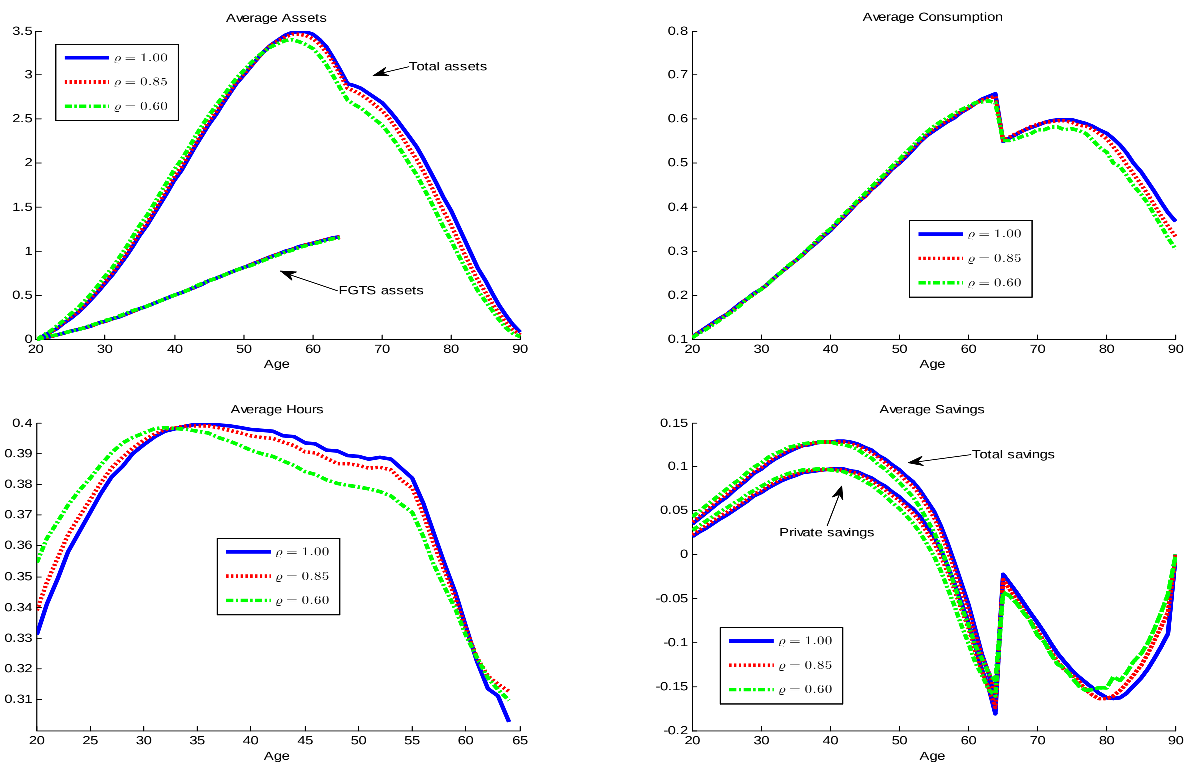

Figure 1: **Average life-cycle profiles for the benchmark economy: Exponential vs Hyperbolic discounting**

Figure **??** compares the average life-cycle profiles induced by the benchmark policies for both the exponential and the hyperbolic discounting models. The top right graph displays the assets profile for each of model, while the top left graphs displays the consumption along age. The graph in the bottom left displays the corresponding hours worked, whereas the bottom right displays the total savings. In Figure **??**, we see that hyperbolic accumulate less assets, have less wealth to spend when retired, and end up consuming less in the elderly age.

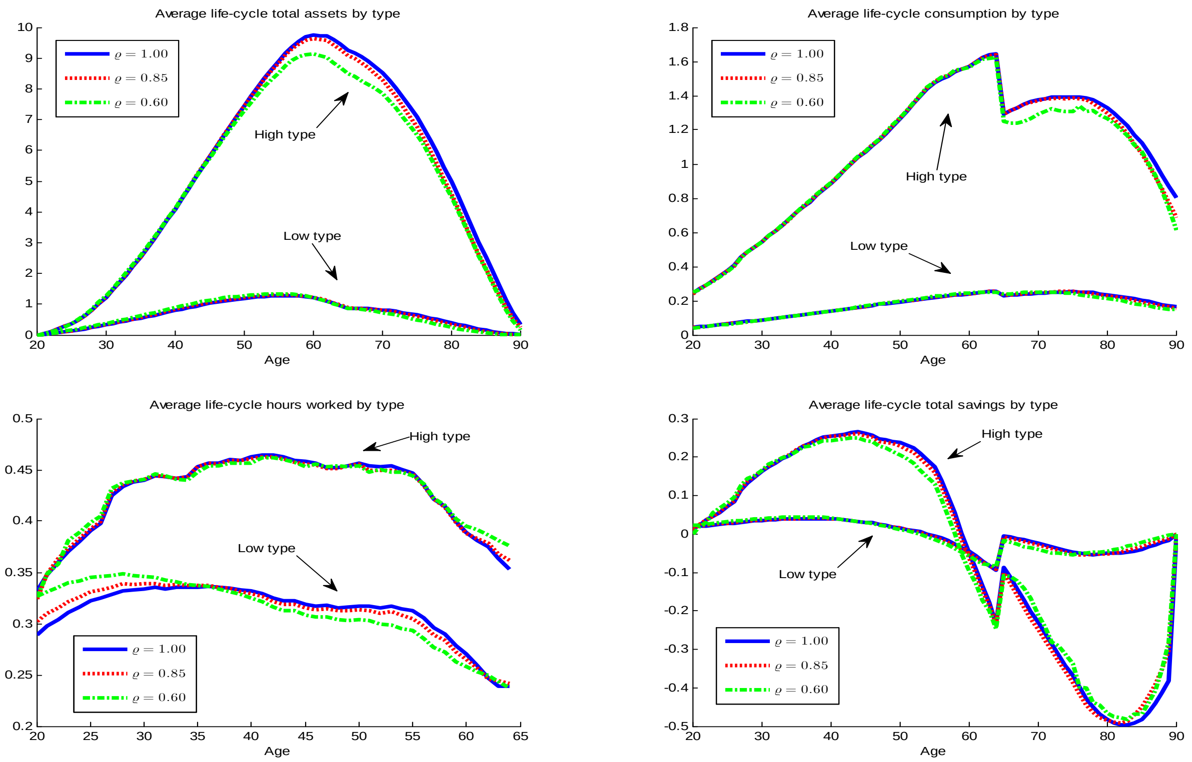

Figure 2: **Average profiles for the benchmark economy by type: Exponential vs Hyperbolic discounting**

Averages may, of course, hide a rich diversity in life-cycle patterns. We split the individuals in our economy in five different ability groups. We group the agents in the high extreme of the grid of the distribution of innate ability, u, and label them the high ability group. The agents on the lowest extreme of the grid are labeled low ability. In Figure **??**, we plot the same variables considered in Figure **??** for those to groups to get a sense of the role of heterogeneity in our model.

Life-cycle patterns are qualitatively similar for all groups and all models. However, the humps in Figure **??** are more pronounced for the high ability individuals in all different specifications. That means high ability agents do work more hours, save more during the working age, and can consume more during retirement than low ability ones. Those differences in life cycle patterns between high and low ability households reflect the difference in social security replacement rates, which is much higher for low ability workers.

5 Results

We present our analyses for two types of consumer preferences: exponential and hyperbolic discounting. For each preference and alternative scenarios, we describe the respective steady state under two fiscal policies: (A) τ_w is adjusted to balance government budget; or (B) the government balances its budget through a lump-sum tax.

5.1 Time-consistent households

Table ?? presents the steady state values of the benchmark scenario in column 1 and some counter factual exercises in the following columns: (i) to eliminate the FGTS in column 2; (ii) to reduce the replacement ratio of social security pensions in column 3; (iii) to simultaneously eliminate the FGTS and reduce the replacement ratio in column 4; (iv) to increase the remuneration of the FGTS in column 5; and (v) to simultaneously reduce the replacement ratio and increase the remuneration of the FGTS in column 6.

Table 1: **Results** The table displays the values of the relevant variables – GDP (Y), capital-output ratio (K/Y), average hours worked (Avg. hours), wages (w), real interest rates (r), wage tax (τ_w), subsidized capital share (ϕ), average voluntary and total savings rates, wealth Gini coefficient, and consumption equivalent variation (C.E.V.)

2.A. τ_w is adjusted to balance government budget						
Variable	Benchmark	$FGTS = 0$	$\theta = \theta_{US}$	$FGTS = 0$ & $\theta = \theta_{US}$	$r_{fgts} = 4$	$\theta = \theta_{US}$ & $r_{fgts} = 4$
Y	0.740	0.692	0.781	0.739	0.793	0.818
K	2.059	1.723	2.269	1.953	2.592	2.700
Avg. hours	0.378	0.377	0.387	0.387	0.366	0.374
K/Y	2.782	2.491	2.905	2.643	3.265	3.303
w	1.016	0.954	1.041	0.987	1.112	1.119
r	8.00%	7.95%	7.23%	7.12%	7.47%	7.13%
τ_w	5.65%	10.39%	0.63%	4.84%	14.16%	9.79%
ϕ	0.196	0.000	0.185	0.000	0.396	0.386
Voluntary savings	8.58%	10.80%	9.67%	12.49%	3.45%	4.17%
Total savings	11.36%	10.80%	12.91%	12.49%	14.58%	15.21%
Wealth Gini	0.577	0.606	0.565	0.586	0.532	0.533
CEV	-	-2.67%	3.53%	0.92%	3.33%	6.77%
CEV low type	-	-3.39%	4.19%	0.68%	7.29%	10.67%
CEV high type	-	-1.53%	3.02%	1.59%	0.76%	4.27%
2.B. Lump-sum financing						
Variable	Benchmark	$FGTS = 0$	$\theta = \theta_{US}$	$FGTS = 0$ & $\theta = \theta_{US}$	$r_{fgts} = 4$	$\theta = \theta_{US}$ & $r_{fgts} = 4$
Y	0.740	0.716	0.752	0.734	0.850	
K	2.059	1.723	2.181	1.942	2.790	
Avg. hours	0.378	0.397	0.365	0.383	0.407	
K/Y	2.782	2.492	2.901	2.646	3.281	
w	1.016	0.955	1.040	0.987	1.114	
r	8.00%	7.95%	7.24%	7.119%	7.46%	
τ_w	5.65%	5.65%	5.65%	5.65%	5.65%	
ϕ	0.196	0.000	0.185	0.000	0.398	
Voluntary savings	8.58%	10.92%	9.52%	12.47%	3.59%	
Total savings	11.36%	10.92%	12.77%	12.47%	14.77%	
Wealth Gini	0.577	0.611	0.563	0.586	0.530	
CEV	-	-5.28%	6.71%	1.48%	-1.43%	
CEV low type	-	-11.05%	13.35%	2.31%	-8.94%	
CEV high type	-	-0.97%	2.18%	1.17%	2.08%	
Policy Parameters						
τ_{fgts}	8.0%	0.0%	8.0%	0.0%	8.0%	8.0%
Average θ	0.70	0.70	0.40	0.40	0.70	0.40
r_{fgts}	-3.0%	-3.0%	-3.0%	-3.0%	4.0%	4.0%

In column 2, without the compulsory FGTS, the total (private) savings of the economy coincide with the voluntary savings. Voluntary savings rates increase substantially, but not enough to compensate the elimination of forced savings, resulting in lower total savings. Market interest rates only marginally decrease and the equilibrium physical capital is smaller than in the benchmark economy. Because forced savings financed subsidized loans to capital, FGTS extinction causes the aggregate capital level to go down. To pay a higher average return on capital, the marginal product of capital has to be higher, which implies lower levels of K/Y, less productive labor and lower wages. With fixed government and social security expenditures, the fiscal authority has to raise taxes to balance its budget. With lower output and (after-tax) wages, all agents get worse off. Because most of the low ability individuals income comes from their wages and wages are now lower, the Gini inequality also increases. When τ_w is adjusted in panel 2.A, high ability agents end up paying more of the fiscal adjustment and low ability agents do not lose as much as they would have if the fiscal adjustment choice were lump-sum financing in panel 2.B. Given taxes in panel 2.B are not dependent on hours worked, workers choose to increase the hours worked under this regime.

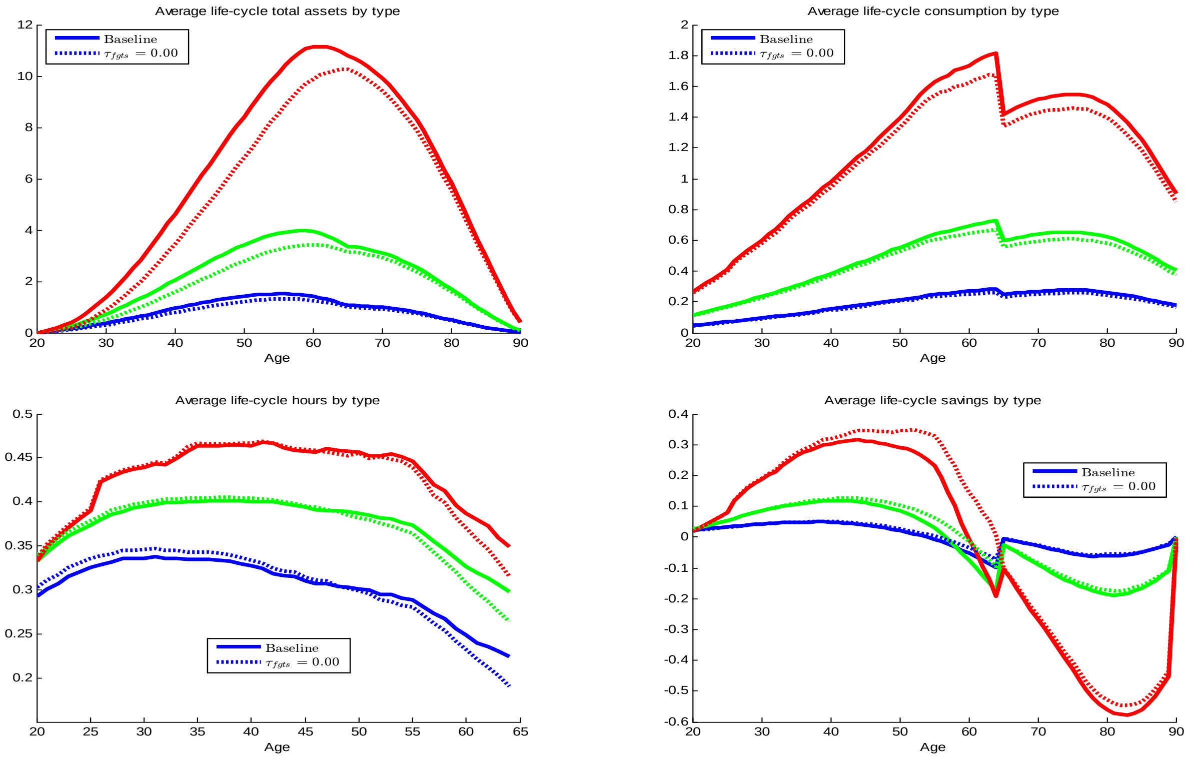

Figure 3: **Benchmark versus Eliminate the FGTS**

In column 3 of Table **??**, we see that a lower replacement ratio (from the Brazilian $\theta_{Br} = 0.70$ to the U.S. $\theta_{US} = 0.40$) lowers the taxes needed to finance pensions and considerably increases the need of voluntary savings for retirement. By accumulating more wealth, the ratio of capital to output increases and the interest rate decreases. With greater equilibrium physical capital than in the benchmark, the economy produces higher output. Labor is now more productive, wages are higher, and wealth inequality slightly lower than in the benchmark. Because in the endogenous τ_w regime (in panel 2.A), agents are now less taxed for working, they choose to work more. Agents are better off in general, and the low ability types get even better in the lump-sum tax regime (panel 2.B), where they enjoy a higher (than proportional to income) tax reduction.

Column 4 of Table **??** shows the joint consequences of no compulsory FGTS savings and a lower replacement ratios. The effects of the latter seem to dominate in terms of improved welfare, although output and before-tax wages are sligthly lower. These measures have the

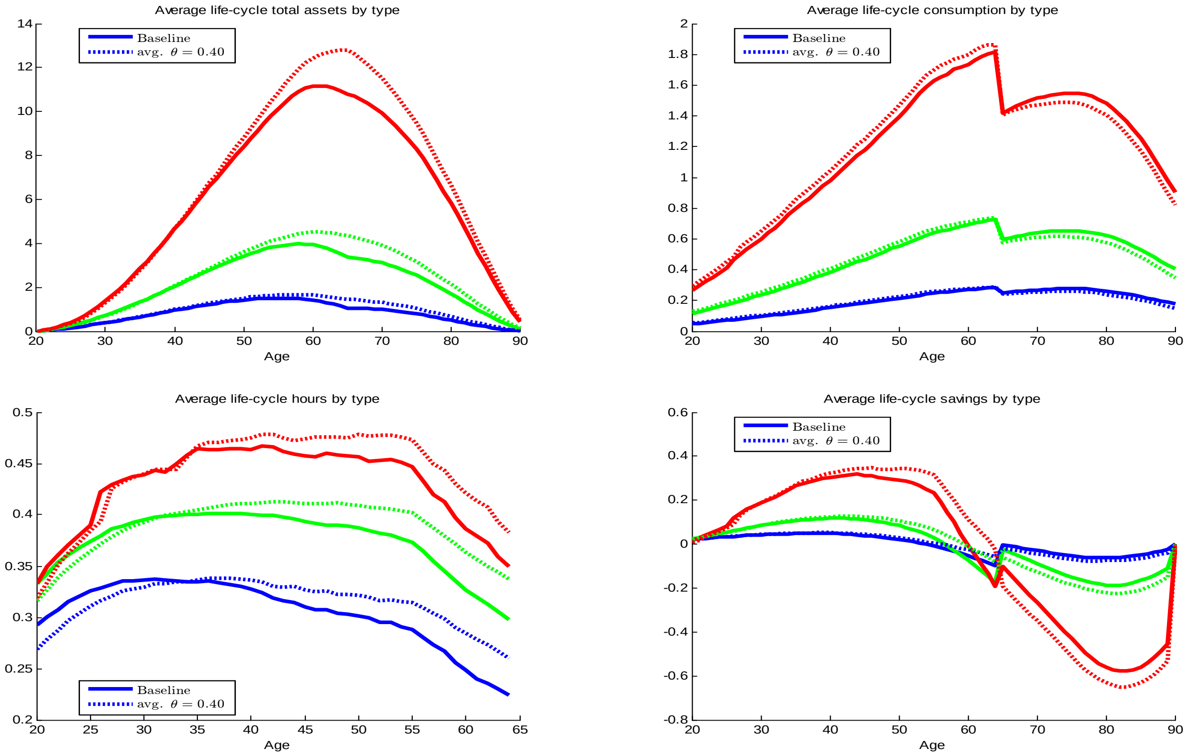

Figure 4: **Benchmark versus Reduce Average** θ **to** 40%

net effect of a tax reduction, that more than compensate the lower wages.

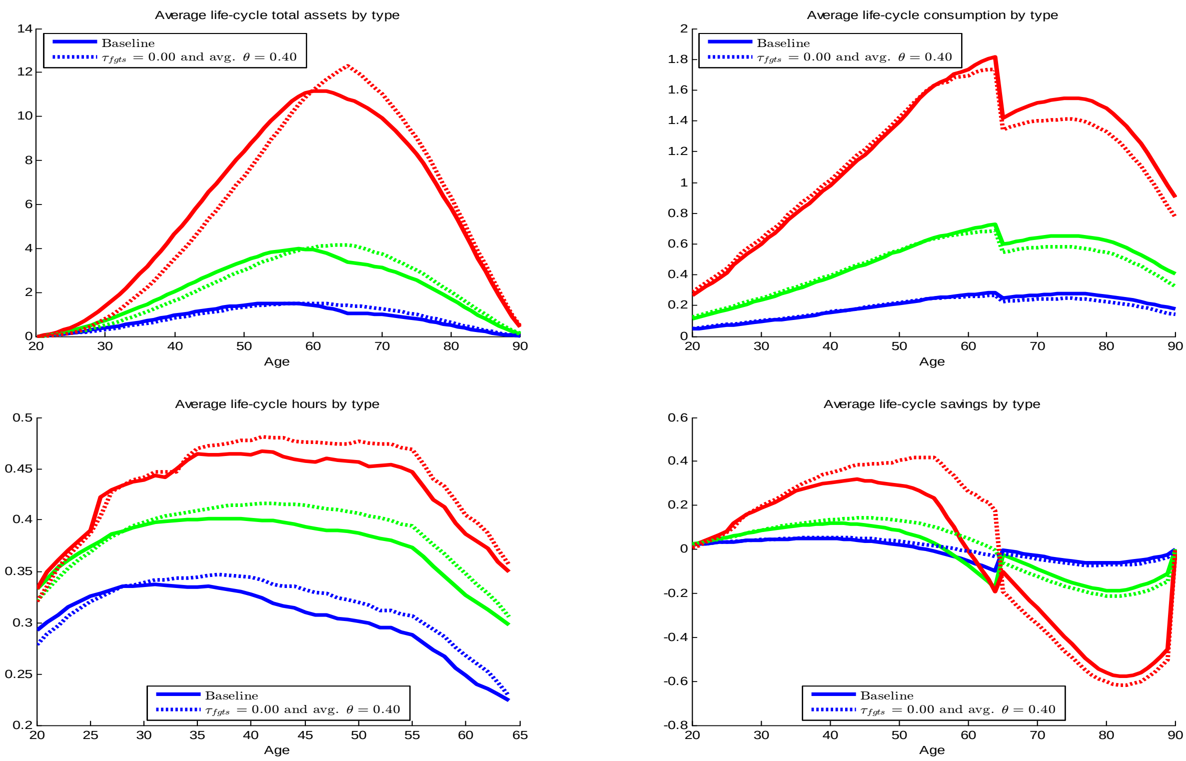

Figure 5: **Benchmark versus Eliminate FGTS and Reduce Average** θ

In column 5 of Table **??**, we analyze the effects of a higher remuneration to the FGTS (from -3% to 4.0% per year), which makes it a more efficient wealth accumulation instrument. The cumulative effect of higher interest on the FGTS is substantial and, although workers choose to save less voluntarily, total savings increase. The greater total savings lowers market interest rates and increases the physical capital stock. With much more productive labor, before-tax wages are higher and wealth inequality decreases. In this scenario, the government has to raise taxes to pay for the higher FGTS remuneration. The higher τ_w (in panel 2.A) distorts the labor market and workers choose to work less. They all get better off though, given workers' FGTS balances benefitting from higher remuneration are also proportional to their owners' wages. However, because the additonal lump-sum tax contribution (in panel 2.B) is not proportional to the workers' FGTS balance, low ability workers get much worse off, paying more on taxes than they earned from higher wages and FGTS remuneration.

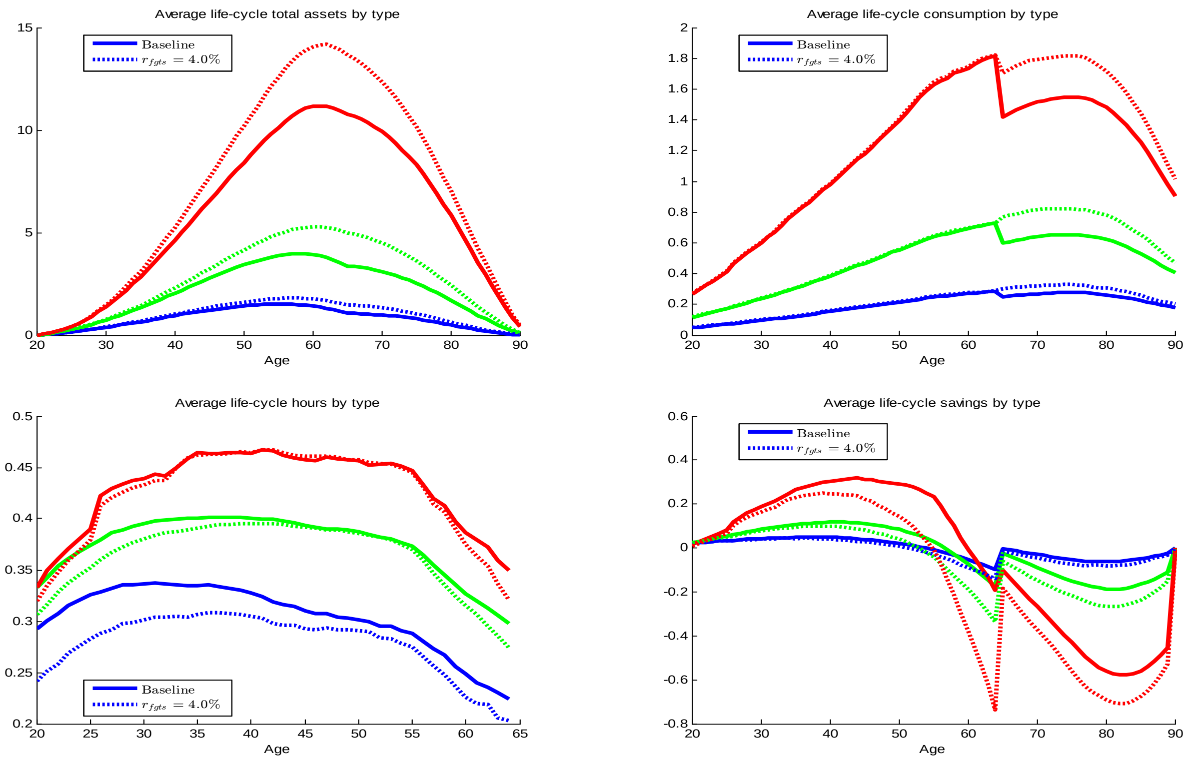

Figure 6: **Benchmark versus Change** r_{fgts} **to** 4%

In column 6 of Table **??**, we analyze the consequences of joint lower replacement ratios and a higher remuneration to the FGTS. Given these reforms increase total savings, their effects add to each other, and physical capital, output and before-tax wages are the highest among the considered counterfactuals. The wealth distribution improves almost as in column 5. The opposite preassures on the fiscal budget bring taxes to somewhere between their pure reforms, meaning tax increase relative to the benchmark. Recollecting that both the replacement ratio and the FGTS are proportional-to-wage benefits, if this joint reform is financed through higher τ_w, it proportionaly charges who benefitted more. The net effect on hours worked is small and the welfare improves to all agents. One can notice that real interest rates in the benchmark simulation is very high - 8%. This is even higher than the real rates found in Brazil. Thus, in some sense, our model is able to reproduce this particular feature. When we change parameters of the social security and compulsory savings, we are able to reduce this rate to 7.12%, still a very high rate. Thus, the Brazilian forced savings and high replacement retirement systems cannot explain why real interest rates are so high in Brazil.

5.2 Time-inconsistent households

To be completed

6 Conclusions

Our counterfactual exercises suggest that both a decrease in replacement ratios and an increase in the forced savings remuneration are sensible reforms that could lead to substantial increases in the total saving rates, physical capital and output. With more productive workers and higher before-tax wages, consumers are better off in the steady state if the necessary fiscal adjustments are made through wage income tax.

This research is still quite preliminary and we plan to extend the analysis to the case of myopic agents. So far, our results for exponential discounting agents seem to hold for time-inconsistency agents as well.

Additionally, we plan to model the agents budget constraint more accuratly, better reflecting the change in income and wealth from employment to unemployment.

Given the literature evidence on the importance of illiquid assets for life cycle savings modelling (see **?**), to better incorporate some important aspects of the FGTS seems another necessary next step. Mainly its interaction with housing and unemployment.