EXPLOSIVE, RECURRENT AND INTRINSIC BUBBLES IN EXCHANGE RATES FOR BRICS

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Abstract

We test for the presence of three types of rational bubbles in the BRICS exchange rates against the US dollar. For the fundamental value of the exchange rate, we use two structural specifications: the exchange rate defined by the pure PPP rule, and by a PPP rule adjusted for the interest rate differential between the two countries. For the bubble dynamics, we consider explosive bubbles, periodically collapsing bubbles and intrinsic bubbles. We find evidence consistent with of the presence of at least one of these bubbles for each country in the group, except for South Africa, confirming the results of other periodically recurring bubble tests for this dataset.

Keywords: Exchange rates. Periodically collapsing bubbles. Explosive bubbles. Intrinsic bubbles. BRICS.

JEL Classification: F31, C32.

Resumo

O objetivo do presente trabalho consiste em testar a presença de três tipos de bolhas racionais na taxa de câmbio dos BRICS contra o dólar americano: bolhas explosivas, bolhas que estouram periodicamente e bolhas intrínsecas. Foram utilizados dois modelos para determinação do valor fundamental: a taxa de câmbio definida pela paridade de poder de compra (PPP), e pela PPP ajustada pelo diferencial de juros entre os dois países. Foram encontradas evidências da presença de pelo menos um tipo de bolha racional em cada país do grupo, exceto África do Sul.

Palavras-chave: Taxa de câmbio. Bolhas que estouram periodicamente. Bolhas explosivas. Bolhas intrínsecas. BRICS.

Classificação JEL: F31, C32.

Área ANPEC: 4 - Macroeconomia, Economia Monetária e Finanças.

1. INTRODUCTION

Rational bubbles are present in asset prices when they deviate from their fundamental values, which is determined by the fundamental economic factors driving their long term behavior, which is derived from a partial or general equilibrium model. It is important to detect their presence and understand their behavior because they may distort agents' decisions, as shown by Brunnermeier and Oehmke (2013), or lead to economic fluctuations, recessions, and financial crisis when the bubble bursts, as exemplified by Kindlerberg and Aliber (2005).

The model for the fundamental value must be based on the principles of profit and/or utility maximization, inter-temporal budget balance, the risk-return tradeoff characterized by the CAPM (capital asset pricing model) in one of its versions, and the inexistence of riskless arbitrage opportunities. The exogenous variables which appear in the fundamental price model are called the fundamentals.

The theory of such phenomena started with Blanchard (1979), that showed rational decision making in a stochastic dynamic environment can produce rational bubbles that display an explosive behavior. Blanchard and Watson (1982) generalize that model and consider a specification that allows for an exogenous probability of the bubble collapsing. Several tests for explosive bubbles were proposed by them, and several other have been proposed since then and applied to a number of different asset markets, as surveyed by Brunnermeier (2008). Flood and Garber (1980) perform some the first tests for the presence of deterministic bubbles in prices, using the framework of the Cagan (1956) model of the German hyperinflation, and conclude that there is no evidence of the presence of bubbles. Diba and Grossman (1984) propose the use of stationarity tests to reject the presence of explosive bubbles. Diba and Grossman (1988) apply such tests to stock prices using a model that considers allows for the presence of unobservable variables among the fundamentals. Froot e Obstfeld (1991) argue for the use of intrinsic bubbles, i.e. bubbles where all of their variability is derived from exogenous economic fundamentals and not from extraneous factors nor expectations of long run arbitrage, and apply it to modeling stock market prices. They claim it is a more plausible empirical account of deviations from present-value pricing than that of the earlier types of rational bubbles.

Evans (1991) developed a model where bubbles that collapse periodically which avoids the theoretical restrictions of Blanchard (1979) whereby rational bubbles cannot be negative and, once they emerge they must burst and eventually disappear. Whenever the bubble exceeds a certain value, it enters into a regime that shifts its behavior between two trajectories: it can either grow faster, or it can collapse and revert to the mean. He shows that the tests developed by Diba and Grossman (1984) are unable to detect the presence of this class of rational bubbles. Van Norden and Schaller (1993) propose an alternative to Evans' specification, where the probability of the regime switch is a function of the size of the bubble, rather than being fixed, and where the regime is unobservable. They apply their model to a sample of stock exchange returns, and are unable to reject the hypothesis that bubbles are present. Bohl (2003) tests for the presence of Evans bubbles in a sample of the US stock market index using the momentum threshold autoregressive (MTAR) model proposed by Enders and Siklos (2001), and is unable to reject their presence.

Periodically collapsing bubbles have been used to model exchange rates series. Van Norden (1996) specifies a particular probabilistic model for the regime change and for the size of the bubble in each regime and tests for the presence of speculative bubbles in the exchange rates of Canada, Japan and Germany against the US dollar from 1977 to 1991. He uses the long term exchange rate from a modified PPP model as the fundamental and use an exogenous value to anchor its level. Maldonado, Tourinho and Valli (2012) extend that model by using a non-linear specification for the size of the bubble in the survival regime and allowing the endogenous determination of the fundamental value level. One advantage of the extended model is that it allows the rationality in the formation of expectations in the foreign exchange market to be tested.

The BRICS countries group (Brazil, Russia, India, China and South Africa) is often singled out for special consideration in the set of emerging market economies because of the size and importance of the countries that compose it, and because they are at a roughly similar stage of economic development. Also, economic crisis in the countries of the group display some correlation, which can be assessed by examining episodes in stock and foreign exchange markets where these asset prices display bubble-like behavior. This can be done by estimating regime-switching bubbles or by using semi-parametric testing procedures. An example of the first approach Maldonado, Tourinho and Abreu (2016), that use the Maldonado, Tourinho and Valli (2012) methodology to model to the exchange rates to the BRICS against the US dollar, and find that the model passes most of the standard specification tests for each country, yielding evidence of the presence of rational bubbles in all of them. They also explore the existence of regularities spanning all the countries in the group regarding the timing of the inception and collapse of bubbles by testing for cointegration between the relative (proportional) bubbles in the exchange rates, and estimate a vector error correction model (VEC) that captures the interaction between them. They find evidence of cointegration, and argue this means that there is transmission of shocks between the foreign exchange markets for the currency of these countries.

This paper tests the presence of other types of bubbles in the exchange rates of BRICS countries: Blanchard's (1979) explosive bubbles, Froot e Obstfeld (1991) intrinsic bubbles, and Evans (1991) bubbles periodically collapsing bubbles. The paper proceeds as follows. Section 2 describes the theoretical specification of the three types of rational bubbles considered here, section 3 presents the data, and section 4 discusses the estimation and tests of the models. We present the conclusions in section 5.

2. THREE TYPES OF SPECULATIVE BUBBLES

As discussed above, the bubble is the difference between the observed exchange rate and its fundamental value. A test for the presence of bubbles in the time series for the price of a given asset is in fact a joint test of the presence of the specified bubble and of a particular model for the fundamental (long run) value of the asset. Here, since we are interested in exchange rates, we adopt the simplest and most universal model for the long term interest rate, namely, Purchasing Power Parity (PPP). This approach is also adopted in many other studies, and finds its empirical support in an extensive literature. In particular, we consider here two of the three models proposed by Maldonado, Tourinho and Valli (2012), and also used in Maldonado, Tourinho and Abreu (2016). They rationalize the use of pure PPP by requiring the absence of riskless arbitrage opportunities for firms operating in the foreign exchange market. They further extend that model to derive an extended PPP rule which takes into consideration the interest rate differentials between the home and foreign country. This is done by extending their model to take into consideration that the firm, in setting up the arbitrage portfolio, may have the opportunity to borrow funds. These two rules can be formalized as follows.[1]

The fundamental value of the pure PPP model, denoted $FUND_t^{(1)}$, is given by:

$$FUND_t^{(1)} = k_1 \left(\frac{IPD_t}{IPF_t}\right) = k_1 I_t^{(1)} \tag{1}$$

where IPD_t and IPF_t are, respectively, the domestic and foreign price indices, and k_1 is a normalization constant. In the van Norden model it is determined by the ratio of the exchange rate and the relative price index at a reference date when he considers a bubble is not

[1] For more details, see the Appendix in Maldonado, Tourinho and Valli (2012).

present. In general, it should be endogenously estimated, if the specified bubble model is able to identify that parameter. The second model is similar to the first one but adds a correction for the interest rate differential between the two countries:

$$FUND_t^{(2)} = k_2 \left(\frac{IPD_{t+1}}{IPF_{t+1}}\right)\left(\frac{1 + IRF_t}{1 + IRD_t}\right) = k_2 I_{\mathrm{t}}^{(2)} \quad (2)$$

where IRF_t e IRD_t are the foreign and domestic interest rates and the other variables are as before.

These fundamental values satisfy the following non-arbitrage condition:

$$FUND_t^{(i)} - EXR_t = aE_t\left[FUND_{t+1}^{(i)} - EXR_{t+1}\right]$$

where $a \in (0,1)$ is a discount factor. Successive forward substitutions yields:

$$EXR_t = FUND_t^{(i)} + B_t^{(i)}$$

where $B_t^{(i)} = \lim_{\mathrm{T}\to+\infty} a^T E_t\left[EXR_{t+T} - FUND_{t+1}^{(i)}\right]$. Under regularity conditions for the stochastic processes involved in the above expression, it is not difficult to prove that the exchange rate bubble

$$B_t^{(i)} = EXR_t - FUND_t^{(i)}, \quad (3)$$

satisfies the classical explosive bubble equation:

$$E\left(B_{t+1}^{(i)} | \Omega_t\right) = a^{-1} B_t^{(i)}, \quad (4)$$

where Ω_t is the information set at t. Thus, in expected value, the rational bubble will grow indefinitely over time.

2.1. Explosive Bubbles

The main difficulty in testing for the presence of rational bubbles in series represented by equation (3) is to decompose the observed price between the bubble and the fundamentals, since the latter is unobservable. The approach developed by Diba and Grossman (1988) address this situation and test null hypothesis of presence of explosive bubbles in the price series (in their case, the stock market index), even when some fundamentals are not observable. They observe that, over time, prices are expected to move further away from the asset's fundamental value (equation (4)) and propose the use of unit root tests and cointegration to test for the presence of explosive bubbles.

If the price and the fundamental value have a long-term equilibrium relationship, the presence of explosive bubbles is ruled out. When this does is not the case, it is not possible to reject the hypothesis that bubbles are present in the series. However, it is not possible to state that they are present, since the absence of a long-term relation can be due to other factors (for example, it can be due to the non-stationarity of the fundamental).

2.2. Periodically Collapsing Bubbles

Several models of periodically collapsing bubbles are available in the literature, as indicated earlier. For exchange rates, one of the most successful is the van Norden (1996) model which is based in a dynamics characterized by the occurrence of non-observable events (collapse and survival of the bubble). An extended version ot that model has been applied to the exchange rates of BRICS countries in Maldonado, Tourinho and Abreu (2016).

To better characterize the nature of these differences, and maybe understand better their implications, we estimate an alternative bubble model which has a sharper dynamic behavior, and more clearly reflects the change from a mostly stationary phenomenon to an essentially explosive one. This is the case of the Evans (1991) model, which can be formalized as:

$$\begin{cases} B_{t+1} = (1+r)B_t u_{t+1} & \text{if } B_t \leq \alpha \quad (5) \\ B_{t+1} = [\delta + \pi^{-1}(1+r)\theta_{t+1}(B_t - (1+r)^{-1}\delta)]u_{t+1} & \text{if } B_t > \alpha \quad (6) \end{cases}$$

where δ is the constant discount factor, $0 < (1+r)^{-1} < 1$, and α is a positive parameter, such that $0 < \delta < (1+r)\alpha$, u_{t+1} is an independent and identically distributed positive random variable with $E_t[u_{t+1}] = 1$, θ_{t+1} is an independent and identically Bernoulli random variable which is equal to 1 with probability π, and 0 with probability $1-\pi$, where $0 < \pi \leq 1$. Whenever B_t is smaller than α, the bubble will grow at the average rate $1+r$, and the collapse probability is null. However, after the size of the bubble exceeds α, the bubble starts growing faster, at the average rate $(1+r)\pi^{-1}$ and can collapse with a probability $(1-\pi)$. When it does collapse, B_t does not become null, but rather assumes a small value δ, and the process restarts.

The threshold autoregressive model (TAR) and the momentum threshold autoregressive model (MTAR) developed by Enders and Siklos (2001) can be used to test for the presence of this type of bubble, as indicated by Bohl (2003) for the case of stock prices. Applying the same methodology to our exchange series, we estimate the cointegration relation between the spot exchange rate and the fundamental value divided by the scaling factor:

$$EXR_t = \hat{\beta}_0 + \hat{\beta}_1\left(FUND_t^{(i)}/k_i\right) + \mu_t = \hat{\beta}_0 + \hat{\beta}_1 I_t^{(i)} + \mu_t; \quad (7)$$

where $I_t^{(i)}$ is the index associated with the $FUND_t^{(i)}$ according to equations (1) and (2). The dynamics of this type of bubbles suggest there is asymmetry of the residuals of the cointegration regression which can be captured by cumulative deviations μ_t above the threshold, followed by an abrupt fall to the threshold. This can be detected by estimating the following error correction model:

$$\Delta\mu_t = \Gamma_t\rho_1\mu_{t-1} + (1-\Gamma_t)\rho_2\mu_{t-1} + \sum_{i=1}^{p-1}\gamma_i\,\Delta\mu_{t-1} + \epsilon_t; \quad (8)$$

where the indicator variable Γ_t is defined (for the TAR model) as follows:

$$\Gamma_t = 1 \;\; if \;\; \mu_{t-1} \geq \tau, \quad (9)$$

$$\Gamma_t = 0 \;\; if \;\; \mu_{t-1} < \tau, \quad (10)$$

where τ represents the threshold value of the cumulative deviations. The MTAR model is similar to TAR, but the indicator variable is as follows:

$$\Gamma_t = 1 \;\; if \;\; \Delta\mu_{t-1} \geq \tau, \quad (11)$$

$$\Gamma_t = 0 \;\; if \;\; \Delta\mu_{t-1} < \tau, \quad (12)$$

According to Enders and Siklos (2001) in many economic applications the value $\tau = 0$ can be adopted. Alternatively, τ can be chosen using the methodology in Chan (1993), which is based on the time series of estimated residuals, as follows. The 15% larger and smaller valued residuals are discarded, the TAR and MTAR models are estimated for each of the remaining values, and the threshold which yields the smaller residual sum of squares is chosen. The models for which the threshold is estimated in this manner are denominated MTAR-consistent (MTARC) and TAR-consistent (TARC).

Two hypothesis tests are sequentially performed. First, no-cointegration can be tested either by testing $H_0\colon \rho_1 = 0$ and $\rho_2 = 0$ with the t-statistic, or by testing $H_0\colon \rho_1 = \rho_2 = 0$, with the ϕ-statistic, using the critical values provided by Enders and Siklos (2001). If the no-cointegration hypothesis is rejected, a second test is performed where the null is the presence of symmetry: $H_0\colon \rho_1 = \rho_2$. If the estimate of the coefficient $\hat{\rho}_1$ is significantly different from zero, has a negative sign, and its absolute value is larger than the estimate of $\hat{\rho}_2$, the hypothesis of a symmetrical adjustment is rejected. If both hypothesis (no cointegration and presence of symmetry) are rejected, the inexistence of periodically collapsing bubbles is rejected, and their presence is accepted.

2.3. Intrinsic Bubbles

The bubble examples discussed above do not contemplate an interaction between the bubble and the fundamentals, but there may exist situations where there is reason to believe such an interaction exists. Froot and Obstfeld (1991) specify a type of rational bubbles, the intrinsic bubble, where this possibility if modeled in and extreme form by assuming the bubble is a deterministic function of the fundamentals, and does not depend on extrinsic factors. For a given (constant) level of fundamentals, the bubble remains constant over time but, when fundamentals change, an overreaction of asset price may occur. For example, in their application of this formulation to model the US stock market index, the bubble depends only on the dividends stream. Following this approach, we assume the fundamental value $\left(f_t^{(i)} = \ln FUND_t^{(i)}\right)$ follows a random walk with drift:

$$f_{t+1}^{(i)} = \mu + f_t^{(i)} + \xi_{t+1}, \tag{13}$$

where $\xi_{t+1} \sim N(0, \sigma^2)$. The intrinsic bubble for the exchange rate is then defined by:

$$B_t = c \cdot \left(FUND_t^{(i)}\right)^{\lambda} \tag{14}$$

which must satisfy the fundamental bubble equation (4):

$$E_t\left[\left(FUND_{t+1}^{(i)}\right)^{\lambda}\right] = a^{-1}\left(FUND_t^{(i)}\right)^{\lambda} \tag{15}$$

For this equantion to be satisfied, the parameter λ must satisfy certaind conditions. To see this, recall that equation (13) implies the following:

$$FUND_{t+1}^{(i)} = FUND_t^{(i)} \exp\{\mu + \xi_{t+1}\}, \text{ and}$$

$$\left(FUND_{t+1}^{(i)}\right)^{\lambda} = \left(FUND_t^{(i)}\right)^{\lambda} \exp\{\lambda\mu + \lambda\xi_{t+1}\}$$

Taking the expected value of both sides of this expression above and recalling that the right-hand side is a log-normal random variable, yields:

$$E_t\left[\left(FUND_{t+1}^{(i)}\right)^{\lambda}\right] = \left(FUND_t^{(i)}\right)^{\lambda} \exp\left\{\lambda\mu + \frac{\lambda^2\sigma^2}{2}\right\}.$$

Substituting for the expectation in equation (15), it is clear that λ must satisfy the following quadratic equation $(\sigma^2/2)\lambda^2 + \mu\,\lambda + \ln a = 0$, which always has a real solution, since $a < 1$. Therefore, the intrinsic bubble is defined by (14) for that particular value for λ. Using the definition of the bubble (equation (3)), yields the following equation for the spot exchange rate:

$$EXR_t = FUND_t^{(i)} + c \cdot \left(FUND_t^{(i)}\right)^{\lambda} \tag{16}$$

Recalling that the fundamental value of the exchange rate has been defined here by equations (1) and (2), which can be written more concisely as $FUND_t^{(i)} = k_i I_t^{(i)}$, where $I_t^{(i)}$ is either the index of relative prices or the index of relative prices adjusted for the interest rate differential, equation (16) it can be written as:

$$\frac{EXR_t}{I_t^{(i)}} = k^{(i)} + c_i \cdot \left(I_t^{(i)}\right)^{\lambda-1} \tag{17}$$

where $c_i = c\left(k^{(i)}\right)^{\lambda}$ is a parameter that characterizes the existence of the intrinsic bubble: it is present only if it is found to be significantly different from zero in the estimation of (17). It is important to note that it also yields an estimate of $k^{(i)}$, which determines the level of the fundamental value.

3. THE DATABASE

To test the model, data is required for the spot (SPOT) and for the fundamental exchange rates (FUND1 and FUND2) of each of the five BRICS countries against the US dollar. As indicated earlier, we will test two competing models for the fundamental rate for each country. Model I, specified by equation (1), assumes that it satisfies the purchasing power parity (PPP) relation while Model II, specified by equation (2), modifies that relation to include the interest rates differential between in the country and the US. We use monthly data, and the time sample varies between countries, as indicated in Table 1, due to data availability.

Table 1: Time sample for each BRICS countries

Country	Model I	Model II
Brazil	March/1999 – August/2016	March/1999 – July/2015
Russia	July/2005 – August/2016	July/2005 – September/2015
India	March/1999 – August/2016	March/1999 – July /2015
China	July/2005 – August/2016	July/2005 – September /2015
South Africa	March/1999 – August/2016	March/1999 – July /2015

For China the sample starts at the date when the exchange rate ceased to be pegged to the US dollar and began fluctuating with respect to a basket of currencies of trade partners. For Russia the sample starting date was chosen to exclude the period when the exchange rate band precluded the market determination of the exchange rate.

The spot exchange rates were obtained from the database of the International Monetary Fund (IMF). To calculate the index for the fundamental rate of Model I, for each country we used the Producer Price Index (PPI) as the domestic price index and the United States PPI as an approximate measure of the international price index. The sources of PPI from China and Russia were National Bureau of Statistics of China e Federal State Statistics Service - Russian Federation, respectively. To complete the information required for Model II, we require indicator of the relevant international interest rate and of the domestic interest rate. For the former we used the sum of the interest rates paid on the T-Bill with four weeks term, and the Emerging Markets Bond Index (EMBI+)[2] rate of each country. For India, we used the JACI (JP Morgan Asia Credit Index) rate as an approximation for EMBI+, which started being compiled only in 2012. For the latter, we used the interbank market interest rate of each country for the one month term bond with highest liquidity. The data for these interest rates are from Reuters, via Datastream.

4. ESTIMATION AND TESTS

Tests were performed to assess the presence of each of the three types of bubbles described in section 3, in exchange rate series of the BRICS countries currency against the US dollar.

4.1. Explosive bubbles

To test for the presence of explosive bubbles, first the ADF (Augmented Dickey-Fuller), PP (Phillips-Perron) and KPSS (Kwiatkowski, Phillips, Schmidt and Shin) unit root tests were performed to check for the presence of a unit root. Then the Johansen and Engle-Granger cointegration tests between the spot and fundamental rates were used to identify the existence of a long-term equilibrium relationship between these series.

[2] Calculated by J.P. Morgan.

For all countries, the ADF e PP tests did not reject the presence of unit root for the SPOT, FUND1 and FUND2 (level) series, and rejected it for their first difference. The KPSS test rejected the null hypothesis that the (level) series are stationary, but did not reject the hypothesis that its first difference stationary. Therefore, all the exchange rate series, for all the countries, were classified as I(1),[3] and it was possible to test for cointegration of the spot and fundamental rates.

The Engle-Granger cointegration test, using the critical values from MacKinnon (2010), show (Table 2) that the presence of a unit root in the residuals of the cointegration relation cannot be ruled out for any country, at a significance level of 5%, and that the cointegration hypothesis can be rejected. Therefore, this test is unable to reject the hypothesis that bubbles are present in the exchange rate series of all BRICS countries. The results of the Johansen cointegration test (Table 3 and 4) confirm these results and indicate the rejection of the cointegration hypothesis for all countries, except for South Africa. Therefore, it is not possible to rule out the presence of explosive bubbles in the exchange rates of Brazil, Russia, India and China. For South Africa, although the Engle-Granger test points to no cointegration, Johansen's test results indicate that the series cointegrate and the presence of explosive bubbles is discarded for that country.

Table 2: The Engle-Granger cointegration test between the SPOT and fundamental (FUND1 and FUND2) exchange rate series

Country	FUND1	FUND2	5% Critical values
Brazil	-1.60	-1.17	-3.37
Russia	-1.93	-2.41	-3.38
India	-2.53	-1.86	-3.37
China	-3.29	-1.83	-3.38
South Africa	-1.90	-2.06	-3.37

Table 3: The Johansen cointegration test between the SPOT and the Model I fundamental (FUND1) exchange rate series.

Country	Number of cointegration vectors	Trace test		Maximum eigenvalue test	
		statistic	5% Critical value	statistic	5% Critical value
Brazil	Zero	25.31	25.87	18.79	19.39
	At maximum 1	6.52	12.52	6.52	12.52
Russia	Zero	17.44	20.26	10.69	15.89
	At maximum 1	6.75	9.16	6.75	9.16
India	Zero	17.94	20.26	12.72	15.89
	At maximum 1	5.22	9.16	5.22	9.16
China	Zero	25.49	25.87	13.36	19.39
	At maximum 1	12.12	12.52	12.12	12.52
South Africa	Zero	18.20	12.32	15.56	11.22
	At maximum 1	2.63	4.13	2.63	4.13

[3] The tables containing the results of the tests are presented in Appendix A.

Table 4: The Johansen cointegration test between the SPOT and the Model II fundamental (FUND2) exchange rate series.

Country	Number of cointegration vectors	Trace test		Maximum eigenvalue test	
		statistic	5% Critical value	statistic	5% Critical value
Brazil	Zero	20.59	25.87	15.88	19.39
	At maximum 1	4.71	12.52	4.71	12.52
Russia	Zero	18.64	25.87	11.80	19.39
	At maximum 1	6.83	12.52	6.83	12.52
India	Zero	12.29	20.26	7.80	15.89
	At maximum 1	4.48	9.16	4.48	9.16
China	Zero	18.69	20.26	11.56	15.89
	At maximum 1	7.13	9.16	7.13	9.16
South Africa	Zero	14.19	12.32	11.58	11.22
	At maximum 1	2.61	4.13	2.61	4.13

4.2. Evans' Periodically Collapsing Bubbles

We tested the exchange rates of the BRICS for the presence of bubbles consistent with the Evans (1991) specification estimating TAR, MTAR, TARC and MTARC models of equation (8). Here we report only the results for MTAR and MTARC tests, using the critical values in Enders and Siklos (2001), because Bohl (2003) shows they are more powerful than the TAR and TARC tests. Recall also that the difference between the MTAR and MTARC tests is that in the former $\tau = 0$, while in the latter it is calculated from the series of estimated residuals. As will be seen below, both tests arrive at the same conclusion for all countries, except India. Table 5 shows the results of the estimation for each country.

Table 5 - Tests for the occurrence of periodically collapsing Evans (1991) type bubbles in the exchange rate of BRICS against the US dollar

		BRAZIL		RUSSIA		INDIA		CHINA		SOUTH AFRICA	
		FUND1	FUND2	FUND1	FUND2	FUND1	FUND2	FUND1	FUND2	FUND1	FUND2
MTAR	$\hat{\rho}_1$	0.075	0.069	0.026	0.079	-0.081	-0.069	-0.098	-0.058	-0.018	-0.010
	$\hat{\rho}_2$	-0.084	-0.063	-0.174	-0.199	-0.071	-0.054	-0.131	-0.154	-0.078	-0.064
	T	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	φ coint.	11.518	6.568	8.051	11.071	3.666	2.879	5.198	5.082	3.301	2.361
	φ crit (5%)	6.38	6.38	6.510	6.510	6.38	6.38	6.510	6.510	6.38	6.38
	φ symmetry	20.371	11.378	9.573	15.615	0.034	0.089	0.226	1.933	1.545	1.313
	φ crit (5%)	4.08	4.08	4.08	4.080	4.08	4.08	4.08	4.08	4.08	4.08
		FUND1	FUND2	FUND1	FUND2	FUND1	FUND2	FUND1	FUND2	FUND1	FUND2
MTARC	$\hat{\rho}_1$	0.129	0.167	0.359	0.373	-0.255	-0.224	-0.114	-0.102	0.007	0.004
	$\hat{\rho}_2$	-0.072	-0.058	-0.143	-0.157	-0.037	-0.030	0.185	0.396	-0.061	-0.048
	T	0.050	0.070	3.100	3.100	1.200	1.200	-1.800	-1.800	0.600	0.600
	φ coint.	13.807	11.298	16.666	19.621	8.684	6.986	5.328	4.943	2.944	1.921
	φ crit (5%)	6.63	6.63	6.860	6.860	6.63	6.63	6.860	6.860	6.63	6.63
	φ symmetry	24.897	21.382	21.020	31.919	9.725	8.066	0.468	1.672	0.848	0.448
	φ crit (5%)	4.08	4.08	4.08	4.080	4.08	4.08	4.08	4.08	4.08	4.08

For Brazil and Russia, both MTAR and MTARC reject the null hypothesis of no cointegration ($\mathrm{H}_0: \rho_1 = \rho_2 = 0$) and the hypothesis of symmetry ($\mathrm{H}_0: \rho_1 = \rho_2$) at the 5% significance level, but the hypothesis of absence of bubbles is not rejected because ρ_1 is positive. Therefore, there is no clear evidence of periodically recurring bubbles.

For India, MTAR and MTARC report different results. Since our time series is not long, the estimate of τ for MTARC may not be accurate, because its consistency relies on asymptotic results. Therefore, it will be more convenient the use of the MTAR test. Using this method, the hypothesis of no cointegration and symmetry cannot be rejected, so the hypothesis of no existence of periodically collapsing bubbles cannot be rejected for India and thus, it is accepted. For China and South Africa the hypothesis of no cointegration and symmetry cannot be rejected, so the hypothesis of inexistence of periodically collapsing bubbles cannot be rejected, and is accepted.

4.3. Intrinsic Bubbles

To test for the presence of intrinsic bubbles in the exchange rate series we fitted the non-linear equation (17) and obtained the maximum likelihood estimates of the parameters using the methodology in Davidson and MacKinnon (2003) to allow the errors to be AR(1) i.e. follow a first order auto-regressive process which was detected in a first stage estimation.

Namely,

$$\frac{EXR_t}{I_t^{(i)}} = k^{(i)} + c_i \cdot \left(I_t^{(i)}\right)^{\lambda-1} + u_t; \quad u_t = \rho u_{t-1} + \epsilon_t, \ \ \epsilon_t \sim NID(0, \sigma_\epsilon^2)$$

The analysis of the statistical significance of each parameter allowed us to test for the presence of this type of rational bubble. The results are shown in Table 6. We find significant positive values for c_{i} only for Brazil, for both models of the fundamental rate (FUND1 and FUND2), and for India, for the fundamental rate FUND2. The estimated parameter $\hat{\lambda} - 1$ was not significant for any of the countries. Therefore, there is evidence of the presence of intrinsic bubbles only for Brazil and India.

Table 6 - Estimated parameters for testing the presence of intrinsic bubbles in the exchange rate of BRICS countries against the US dollar

Country	Reference	$\boldsymbol{\rho}$	$\boldsymbol{\sigma_\epsilon^2}$	$\boldsymbol{k}$	$\boldsymbol{c_{\mathrm{i}}}$	$\boldsymbol{\lambda - 1}$
Brazil	FUND1	0.977	0.161	1.596	1.048	0.000
		(0.050)	(0.005)	(0.302)	(0.260)	(0.006)
	FUND2	0.981	0.147	1.590	1.160	0.001
		(0.038)	(0.003)	(0.211)	(0.415)	(0.012)
Russia	FUND1	0.979	1.808	24.373	0.000	11.681
		(0.225)	(0.360)	(5.930)	(0.001)	(4.070)
	FUND2	0.951	1.614	23.251	7.352	0.001
		(0.307)	(0.183)	(3.821)	(5.233)	(0.035)
India	FUND1	0.925	1.143	48.961	1.193	0.001
		(0.317)	(0.147)	(2.096)	(1.038)	(0.026)
	FUND2	0.941	1.018	44.165	7.043	0.003
		(0.262)	(0.107)	(2.480)	(2.303)	(0.018)
China	FUND1	0.908	0.091	7.010	0.010	0.005
		(0.023)	(0.002)	(0.037)	(0.017)	(0.003)
	FUND2	0.914	0.088	7.088	0.003	0.089
		(0.021)	(0.001)	(0.035)	(0.002)	(0.069)
South Africa	FUND1	0.947	0.449	7.414	1.476	0.000
		(0.139)	(0.011)	(0.895)	(0.963)	(0.041)
	FUND2	0.950	0.412	6.504	2.271	0.002
		(0.122)	(0.014)	(1.679)	(1.635)	(0.009)

The estimated value of $k^{(i)}$ and the relative price index of the relevant model for the fundamental rate allows us to recover the series for the fundamental exchange rate, which are displayed in Figures 1 to 5. We show the FUND2 in those graphics.

Figure 1 - Exchange rates for Brazil - 1999 to 2015
observed (SPOT) and fundamental rate (FUND) for the intrinsic bubble model

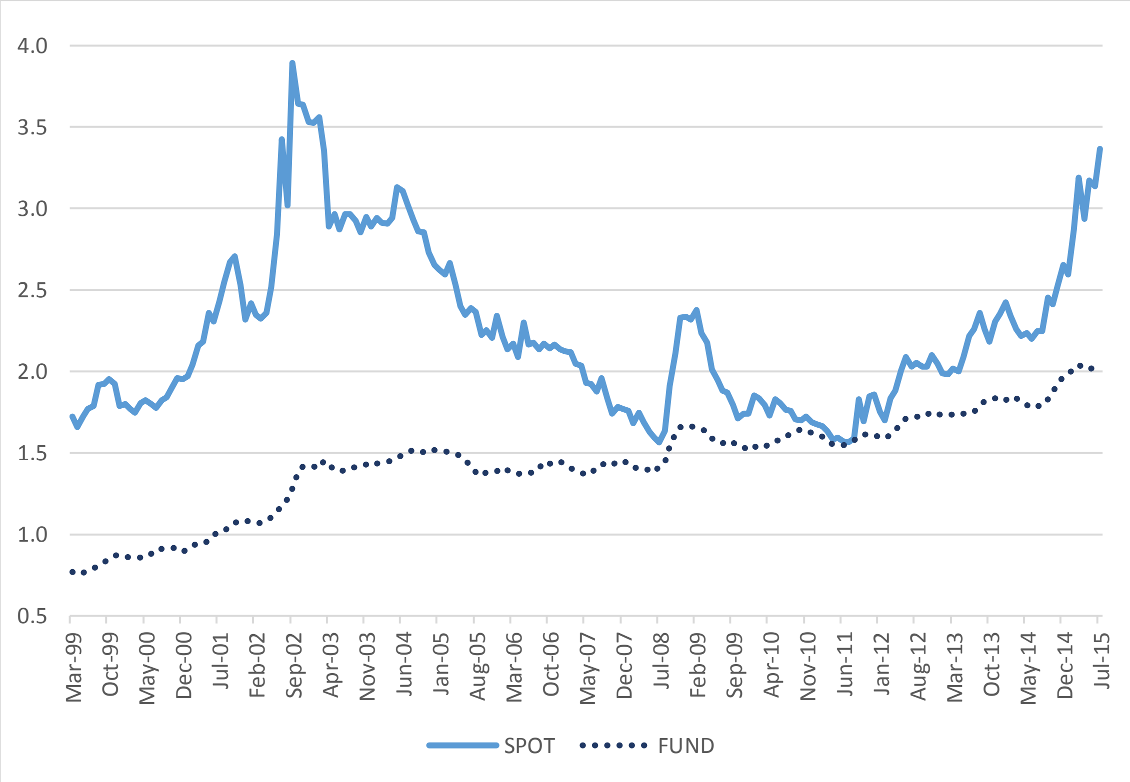


Figure 2 - Exchange rates for Russia - 2005 to 2015

observed (SPOT) and fundamental rate (FUND) for the intrinsic bubble model

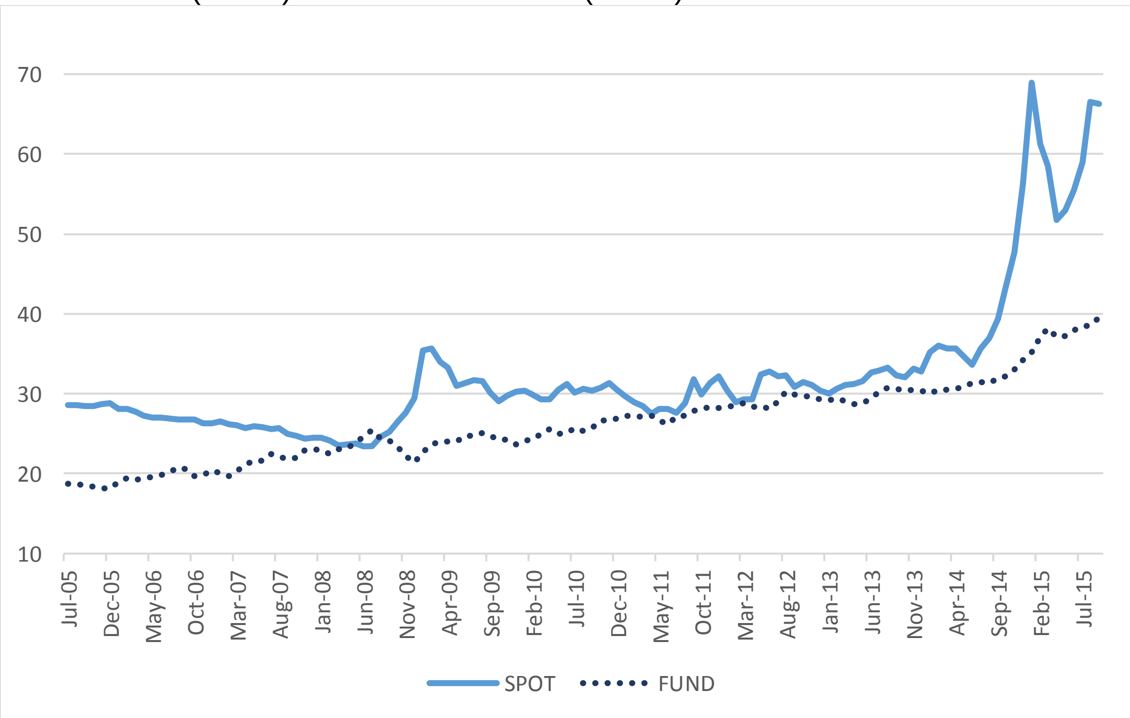


Figure 3 - Exchange rates for India - 1999 to 2015
observed (SPOT) and fundamental rate (FUND) for the intrinsic bubble model

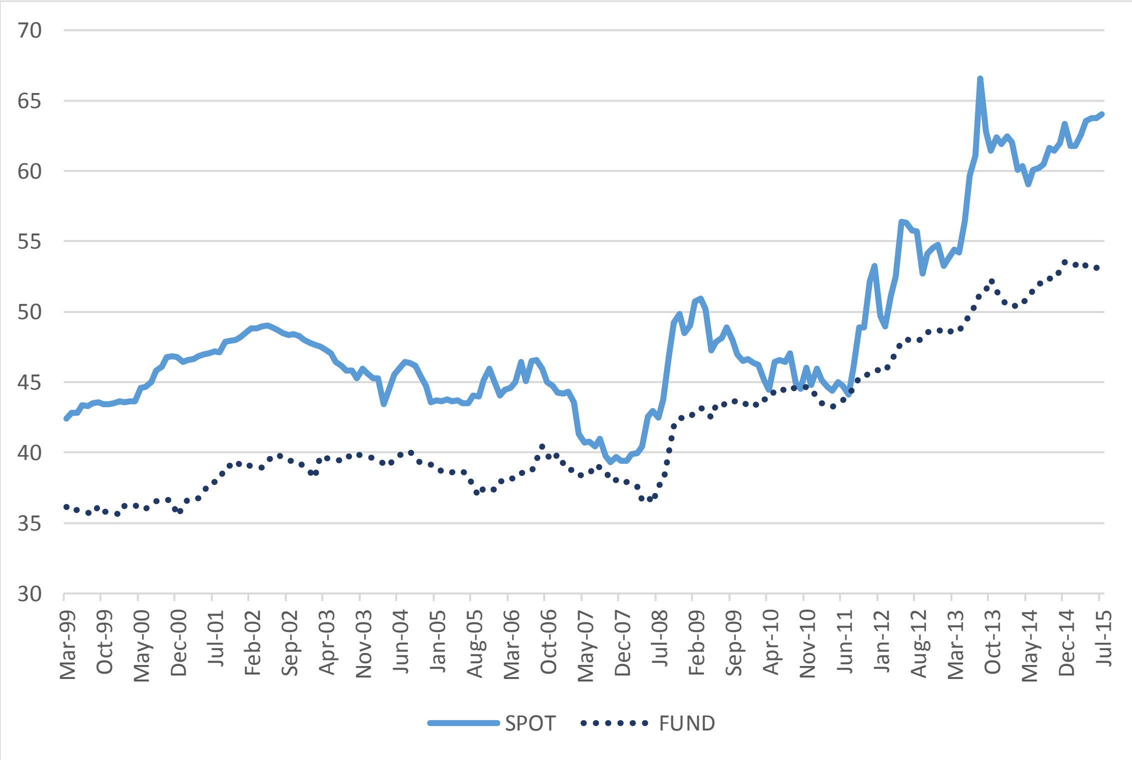


Figure 4 - Exchange rates for China - 2005 to 2015
observed (SPOT) and fundamental rate (FUND) for the intrinsic bubble model

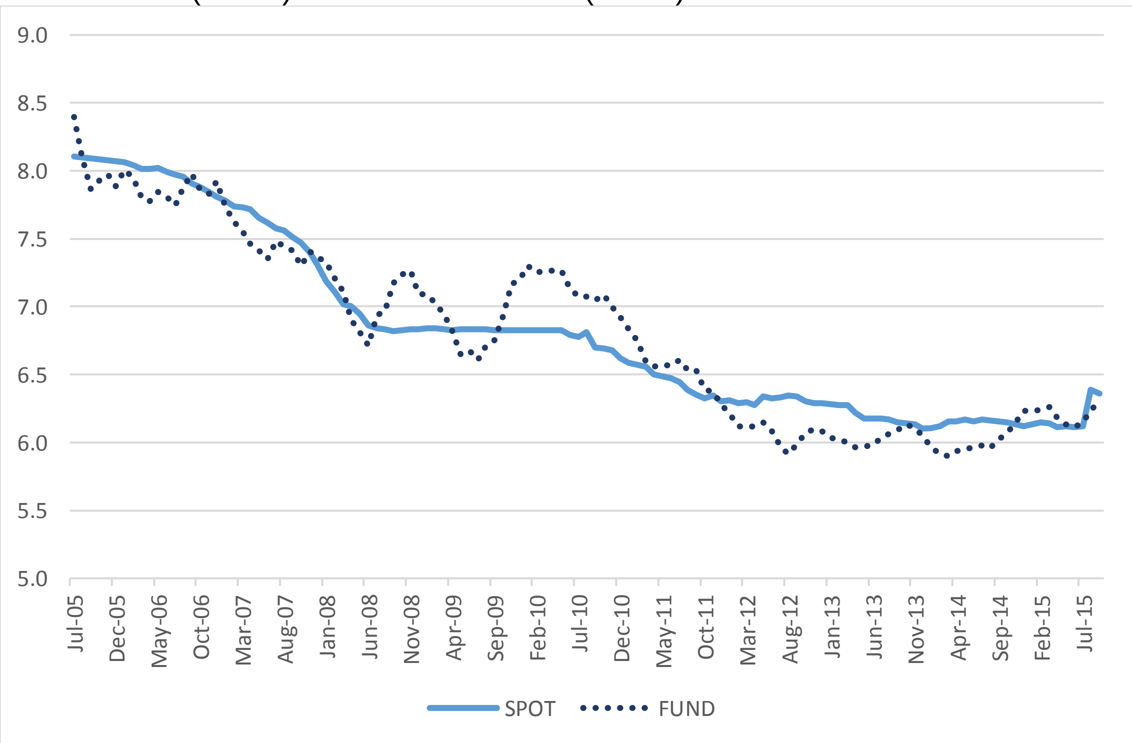
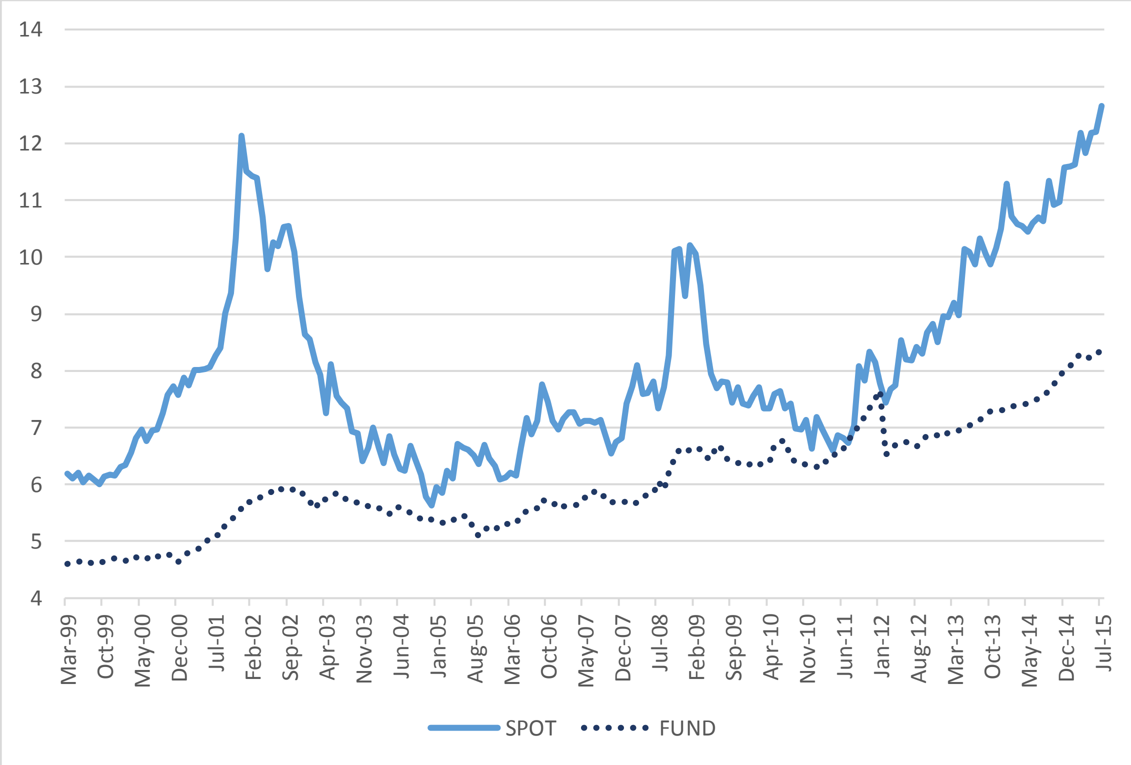


Figure 5 - Exchange rates for South Africa - 1999 to 2015
observed (SPOT) and fundamental rate (FUND) for the intrinsic bubble model



5. CONCLUSIONS

Rational bubbles are present in asset prices when persistent deviations between the observed price and the fundamental rate which satisfy the fundamental bubble dynamic equation. This paper tested for the existence of rational bubbles in the exchange rates of BRICS countries of three particular types: explosive bubbles, periodically collapsing bubbles of the type proposed by Evans (1991), and intrinsic bubbles proposed by Froot and Obstfeld (1991).

Two structural models were used to obtain produce the series for fundamental value of the exchange rate. One is based on the pure purchasing power parity (PPP) relation and the other is a modified version of that model that included a term to capture the effects of the interest differential between the BRICS country and the USA.

The results of the tests are summarized in Table 7; in the second line it is stated the null hypothesis that is being tested. In summary, the tests for explosive bubbles does not reject the existence of them for all countries except South Africa. The no rejection does not imply the acceptance of existence of explosive bubbles, since other factors may cause the absence of cointegration. There is no evidence of the presence of periodically collapsing bubbles of the Evans (1991) type for all the countries in the BRICS group. There is evidence of intrinsic bubbles only for Brazil, for both models of the fundamental rate (FUND1 and FUND2), and for India, for FUND 2. Looking at all tests for each country, we conclude that for Brazil and India there is evidence of the presence of explosive and intrinsic rational bubbles (they are either not rejected or present in at least one test), for Russia and China only explosive bubbles might exist and for South Africa there is no evidence of any of these types of exchange rate bubbles.

Table 7: Summary of the tests for the presence of three types of bubbles in the exchange rate of BRICS countries against the US dollar

Country	Type of bubble			
	Existence of Explosive	Absence of Periodically collapsing	Absence of Intrinsic	
	FUND1 & FUND2	FUND1 & FUND2	FUND1	FUND2
Brazil	Not rejected	Not rejected	Rejected	Rejected
Russia	Not rejected	Not rejected	Not rejected	Not rejected
India	Not rejected	Not rejected	Not rejected	Rejected
China	Not rejected	Not rejected	Not rejected	Not rejected
South Africa	Rejected	Not rejected	Not rejected	Not rejected

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APPENDIX A - RESULTS OF ADF, PP AND KPSS TESTS

Table A.1: Unit root test - SPOT series

SPOT		Period I*						Period II*					
		ADF		PP		KPSS		ADF		PP		KPSS	
		LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF
Brazil	Constant and trend	-1.28	-5.94	-1.76	-15.78	0.23	0.10	-0.94	-6.22	-1.47	-15.97	0.20	0.14
	Constant	-1.28	-5.92	-1.72	-15.81	0.57	0.11	-1.11	-6.21	-1.55	-15.99	0.60	0.17
	No constant of trend	0.44	-5.88	0.25	-15.81			0.57	-6.16	0.50	-15.97		
Russia	Constant and trend	-1.78	-7.23	-1.43	-8.94	0.26	0.06	0.16	-7.54	0.19	-8.28	0.21	0.13
	Constant	-0.74	-7.60	0.14	-8.36	0.92	0.28	1.73	-6.99	2.22	-8.06	0.79	0.41
	No constant of trend	0.75	-4.89	1.35	-8.46			1.81	-6.76	1.84	-8.05		
India	Constant and trend	-0.79	-5.22	-1.28	-13.61	0.39	0.06	-0.75	-5.06	-1.30	-12.86	0.34	0.07
	Constant	0.42	-5.01	-0.06	-13.53	1.16	0.21	0.23	-4.87	-0.28	-12.80	0.93	0.19
	No constant of trend	1.65	-4.72	1.49	-13.46			1.46	-3.00	1.30	-12.75		
China	Constant and trend	-0.35	-9.50	0.79	-9.81	0.30	0.09	-0.22	-9.50	0.01	-9.92	0.27	0.07
	Constant	-2.38	-8.33	-2.51	-9.15	1.19	0.44	-2.26	-8.68	-2.37	-9.44	1.20	0.41
	No constant of trend	-0.75	-2.04	-1.93	-8.87			-1.03	-7.84	-1.56	-8.88		
South Africa	Constant and trend	-1.13	-14.11	-1.22	-14.11	0.28	0.08	-1.29	-13.70	-1.47	-13.71	0.20	0.08
	Constant	-0.32	-14.08	-0.39	-14.07	0.81	0.19	-0.77	-13.69	-0.94	-13.71	0.52	0.15
	No constant of trend	1.17	-14.00	1.16	-14.01			0.96	-13.64	0.90	-13.65		

*Brazil, India e South Africa: 1999:03 – 2016:08. China e Russia: 2005:07 – 2016:08.
** Brazil, India e South Africa: 1999:03 – 2015:07. China e Russia: 2005:07 – 2015:09.
*** 5% Critical values of ADF and PP tests for each model are respectively -3,43, -2,88 e -1,94.
**** Critical values of the KPSS test for each model are respectively: 0,15 e 0,46. (5%)

Table A.2: Unit root test - Fundamental level series

SPOT		FUND1*						FUND2**					
		ADF		PP		KPSS		ADF		PP		KPSS	
		LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF	LEVEL	1st DIFF
Brazil	Constant and trend	-1.66	-6.28	-1.13	-8.76	0.18	0.148	-2.56	-7.98	-2.14	-8.03	0.20	0.10
	Constant	0.35	-6.24	0.29	-8.74	1.59	0.18	-1.19	-7.99	-0.99	-8.05	1.51	0.11
	No constant of trend	2.37[1]	-6.26	3.10[2]	-8.06			2.09[1]	-7.53	2.47[1]	-7.68		
Russia	Constant and trend	-1.93	-8.44	-1.37	-9.22	0.22	0.07	-1.51	-7.96	-1.06	-8.59	0.21	0.11
	Constant	1.21	-8.35	1.20	-9.09	1.35	0.25	1.63	-7.79	1.65	-8.49	1.29	0.27
	No constant of trend	3.94[2]	-8.68	3.45[2]	-8.64			1.50	-8.05	3.49[2]	-8.00		
India	Constant and trend	-1.19	-7.84	-1.00	-12.18	0.41	0.05	-1.65	-7.72	-1.43	-11.98	0.36	0.05
	Constant	1.06	-8.03	1.26	-12.09	1.59	0.38	0.20	-7.65	0.55	-11.91	1.47	0.23
	No constant of trend	2.64[2]	-7.59	2.93[2]	-11.74			1.98	-7.34	2.34[1]	-11.66		
China	Constant and trend	-0.40	-6.42	-1.54	-8.43	0.21	0.05	-1.49	-5.48	-2.40	-8.62	0.20	0.05
	Constant	-2.25	-5.90	-2.58	-8.10	1.22	0.40	-1.62	-6.13	-2.30	-8.56	1.22	0.24
	No constant of trend	-1.46	-5.75	-1.57	-8.07			-1.51	-5.82	-1.91	-8.49		
South Africa	Constant and trend	-0.84	-6.34	-0.96	-13.64	0.26	0.08	-1.72	-13.51	-1.83	-13.50	0.20	0.06
	Constant	1.05	-6.18	1.00	-13.53	1.61	0.27	0.21	-13.48	0.20	-13.48	1.56	0.14
	No constant of trend	2.40[1]	-5.64	2.83[2]	-13.18			2.29[1]	-13.18	2.28[1]	-13.19		

*Brazil, India e South Africa: 1999:03 – 2016:08. China e Russia: 2005:07 – 2016:08.
** Brazil, India e South Africa: 1999:03 – 2015:07. China e Russia: 2005:07 – 2015:09.
*** 5% Critical values of ADF and PP tests for each model are respectively -3,43, -2,88 e -1,94.
**** Critical values of the KPSS test for each model are respectively: 0,15 e 0,46. (5%)

[1]Does not reject the unit root hypothesis at the 1%.significance level. Critical level: 2.58.

[2] Reject the unit root hypothesis