MODELING THE LOCATION CHOICE: EVIDENCES FROM AN EVOLUTIONARY GAME BASED ON REGIONAL INPUT-OUTPUT ANALYSIS[1]

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Resumo: A intenção deste artigo é desenvolver um novo modelo de jogo evolutivo baseado na análise regional de Insumo-Produto, que pode oferecer uma explicação razoável sobre o padrão de decisões de localização das empresas que competem no setor automotivo dos países do Mercosul. Para atingir esse objetivo, dividimos o Mercosul em duas regiões estratégicas: o Brasil e o Resto do Mercosul. As matrizes de *payoff* deste jogo foram construídas com base em informações recebidas das matrizes Insumo-Produto regionais obtidas no banco de dados do GTAP 9 e *International Trade Index*. Com o objetivo de capturar os efeitos de variáveis exógenas que podem afetar as decisões de localização estratégica, os *payoffs* incorporam alguns fatores (potencial de mercado, interdependência produtiva local, incentivo fiscal governamental e estabilidade macroeconômica), cujos pesos variam com cada cenário que construímos. Os resultados obtidos a partir do jogo dinâmico sugerem que, quando as empresas dão pesos homogêneos aos fatores, o equilíbrio ocorre com a disputa do mercado no local em que há o incentivo fiscal governamental. Uma vez que os fatores são atribuídos com pesos diferentes, o equilíbrio a longo prazo do jogo muda, o que fornece evidências de que disputar o mercado onde existe uma isenção de imposto nem sempre é uma decisão de localização ideal. Isso pode contradizer o senso comum sobre essas questões, além de contribuir com a literatura sobre a decisão de localização das firmas.
Palavras-Chave: Decisão Locacional, Interação Estratégica, Análise Insumo-Produto.

Abstract: The intention of this paper is to develop a new model of an evolutionary game based on regional input-output analysis, which can offer a reasonable explanation about the location decisions pattern of firms that compete in the automotive sector of Mercosur countries. To achieve this goal we divide Mercosur into two strategic regions: Brazil and Rest of Mercosur. The payoffs of this game were build based on information received from the regional input-output matrix obtained in the GTAP 9 database and from the International Trade Index. With the purpose of capturing the effects of exogenous variables that can affect strategic location decisions, the payoffs incorporate some factors (market potential; local productive interdependence; government tax incentive; and macroeconomic stability), whose weights vary with each scenario we built. The results obtained from the dynamic game suggest that when firms give homogeneous weights to the factors, the equilibrium occurs with both disputing the market in which there was the governmental tax incentive. Once the factors are assigned with different weights, the long-term equilibrium of the game changes, which provides evidence that disputing the market where there is a tax exemption is not always an optimal location decision. This may contradict the common sense about these issues, as well as bring some contribution to the literature on firms location decision.
Keywords: Location Decision, Strategic Interaction, Input-Output Analysis.

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1 Introduction

The discussion about firms' strategic location decision is not a new topic in many fields of Economic Literature. The Industrial Organization (IO) approach for example, is mostly concerned with the theoretical understanding of the competition nature in markets when firms strategically decide where to locate. By its turn, Regional Economics focuses on the understanding of development as consequence of an incentive that is given to an economic activity of the industrial sector in a specific region. Another important field of research, International Economics, contributes to the theoretical discussion related to the Location Theory (in general) and to the location decision of multinational firms (in particular)[6].

In the case of the interface between the research done by trade economists and urban/regional economists, we can mention Mayer *et al.* (2010), who show that the first tradition tries to understand why firms invest abroad while the second usually uses a pattern of inter-regional and inter-city choices to estimate agglomeration economies[7].

Despite the vast literature that has been established about location decision in these three fields of study, to the best of our knowledge, there are no studies about spatial competition lying on the interface of evolutionary game theory (EGT) and regional science, since the EGT models have been applied mainly in IO researches. Although it may still be in its infancy, this theme should attract more interest from regional analysis, because the competitive locational problem emerges as a prototype of many economic situations involving dynamically interacting decisions in which firms can learn with their own choices over time[8].

While the traditional theory of games requires that players have a very high level of rationality, the EGT[9] model has been used to successfully explain a number of aspects of agents' behavior. More specifically, EGT may accomplish better success in describing and predicting the choices of locational decisions, since it is better equipped to handle the weaker rationality assumptions.

Considering the existent literature and the lack of contributions of EGT to spatial theories as mentioned before, in this work we develop a new model that considers the projection made from the Regional Economic analysis for dynamically guiding firms to the Evolutionary Stable Strategy (ESS), i.e., to the optimal strategy location decision in the long term. The idea behind an ESS is to ensure that a so-called mutant strategy will not be able to dominate a competitive environment that embraces the incumbent strategy. Furthermore, as explained in Friedman (1991), the EGT provides a refinement of the dynamic approach applied in the traditional game theory, allowing an inference about which Nash Equilibrium (NE) corresponds to an ESS.

According to Samuelson (2002), the EGT model is a convenient tool especially in anti-coordination games, which is a model of conflict that fits well in our case of study. To illustrate and offer more intuition about the reason why the anti-coordination game could be applied in strategic locational decision, let us consider a scenario in which there are two representative competing multinational firms. Each of those intends to build a new plant and choose simultaneously between two different countries with similar potential markets. If they choose the same country, they will split the market share. If they choose different countries, they will avoid the competition for the market share and their payoffs will probably be higher. Consequently, in this scenario, the best response is to adopt the opposite strategy of their opponent. On this point, the model proposed here allows us to infer which country should be selected by each representative firm to build their new plant.

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[6] To illustrate the interface of these fields, we can mention D'Aspremont; Gabszewicz and Thisse (1979), who considered a slightly modified version of Hotelling's model, in which exists a tendency for both sellers to maximize their differentiation. This constitutes a counterexample to the conclusions originally presented by Hotelling (1929). By its turn, Gabszewicz and Thisse (1992) provided the framework for a spatial competition model and the location of firms.

[7] Mayer *et al*. (2010)is an example of Regional Science studies and, in the field of Trade Economics, one example is the work of Chen and Moore (2010).

[8] In Chan (2001) and Fischer and Nijkamp (2014) there is a useful compendium of spatial analysis techniques which points out the commonalities among models used to locate facilities one at a time and to forecast the economic development pattern in an entire region. In this regard, it unifies the models applied in spatial science, which is defined by the author as the analytical techniques that explicitly recognize the spatial elements and examine the determinants of location decision.

[9] A small sampling of topics that have been analyzed from the evolutionary perspective includes altruism (BOYD *et al.*, 2003) and behavior in public goods game (CLEMENS; RIECHMANN, 2006; HUBERMAN; GLANCE, 1995).

Therefore, we present a study of case about automotive plant location patterns in Mercosur countries[10]. There is a vast literature[11] that provides a description of the location decision patterns of the North American and European auto industry over the last several decades. On the other hand, we can say that this matter has not been widely debated yet in South America, where the market we analyzed in this paper is located.

The objective of this paper is to offer a reasonable explanation about location decision patterns of firms that compete in the automotive sector under a new methodology. In the evolutionary game presented here, we built the payoffs based on information received from the input-output methods using the GTAP database and from the Global Enabling Trade Report[12]. With the purpose of capturing the effects of exogenous variables that can affect strategic location decisions, the payoffs incorporate weighted factors, whose weights vary with each scenario we built.

Since there are multiple possible outcomes that vary with each weight we consider, our results show that the scenario and its inherent uncertainty may affect the strategic decisions when the outputs cannot be predicted. In this sense, we assume in our analysis representative multinational firms that have to decide between Brazil and rest of Mercosur for building a new plant. To make that decision, the following factors are considered: (i) potential market; (ii) local productive interdependence; (iii) government tax incentive and (iv) macroeconomic stability.

In order to reach our purpose and develop the discussion presented in this introduction, the remainder of this paper is organized as follows. Section 2 describes the algorithm used to design the evolutionary game and the structure of the payoff matrix. Section 3 introduces the regional input-output analysis framework and the empirical model applied to the automotive sector in Mercosur countries. Section 4 brings the results of the empirical analysis and Section 5 presents our conclusions.

2 Methodology

In this section, we present some basic background on Evolutionary Game Theory and an Evolutionary Stable Strategy that will be used to reach the objective specified before. The mathematical idea behind the dynamic replicator, which is a model of evolution and prestige-biased learning in games[13] is analyzed. It is composed by a system of ordinary differential equations (ODE). The analytical solution of the ODE system will govern the dynamic of the evolutionary game and will make it possible to do inference about the ESS. In addition, the Malthusian dynamics of the replicator is discussed. The software Matlab was used for plotting the phase diagrams[14], responsible for mapping the dynamic equilibria of the proposed games.

2.1 Evolutionary Game Theory Model – EGT Model

As written by Friedman (1991), the evolutionary game is effectively dynamic since it is based on a mechanism that allows one to understand how the strategies followed by the players can change as the game evolves. In this case, an important element is added to those considered by the classical game theory. In addition to players, strategies and payoffs, it is now also considered a dynamic rule that can change payoffs and, therefore, the way players interact with each other over time.

In evolutionary games, according to Maynard Smith and Price (1975), a convergence to the dominant long-run equilibrium is expected. In this equilibrium, achieved after a period of dynamic interaction, players must have adopted an evolutionary stable strategy (ESS) that is a strategy where players have no incentive to abandon, unless some external force disturbs the underlying conditions of the game.

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[10] Currently, Mercosur (Common Market of the South) promotes the integration between Argentina, Brazil, Paraguay, Uruguay and Venezuela, considered as States Parties, through the free movement of goods, services and factors, the establishment of a Common External Tariff (CET), the adoption of a common commercial policy, the coordination of macroeconomic and sectorial policies and the harmonization of legislation.

[11] See Lagendijk (1997), Klier and McMillen (2015, 2008), Domanski and Lung (2009) and Bentley *et al.* (2013).

[12] The Enabling Trade Index is made up of four sub-indexes: (1) market access; (2) border administration; (3) transport and communications infrastructure; (4) business environment.

[13] Developed by Taylor and Jonker (1978).

[14] The phase diagrams of the scenarios presented in this paper are in the appendix.

Then, if classical game theory can be defined as the science that studies strategic behavior, with the theory of evolutionary games it takes a step forward, since we now have the science that studies the robustness of strategic behavior. In an evolutionary game it is assumed bounded rationality, a large population, n, of players ($n \rightarrow \infty$) and an implicit recognition that agents learn. Every period, a player is randomly matched with another player and they play a two-player game. Each agent is assigned a strategy and they cannot choose their strategies. In other words, they are "programmed" to play a strategy in the initial period ($t = 0$) and it may not maximize their utility function. However, the systematic interaction with other agents will lead them to modify or update their behavior over time by choosing a given strategy. Thus, we can say that players imitate others' strategies.

In this way, Friedman (1991) presents the replicator dynamics as an efficient analytical tool, capable of inferring the pattern of evolutionary behavior of a population of players. Considering a utility function given by $u(s_i, \sigma)$ as the fitness, i.e., the number of descendants of an individual of type i adopting strategy s_i, against the average population fitness,σ, and let t be time instant:
$N(t)$: Population size;
$N_i(t)$: Number of individuals playing the strategy s_i;
$p_i(t)$: Proportion of the individuals playing s_i, i.e., $p_i(t) = \frac{N_i(t)}{N(t)}$;
d: Population mortality rate (constant);
$u(s_i, \sigma)$: The expected number of descendants (fitness) of an individual adopting s_i when it competes in a population whose average fitness is $\sigma(t)$.

The reduced form of the replicator dynamic is given by:

$$\dot{p}_i(t) = \frac{d}{dt}\frac{N_i(t)}{N(t)} = \frac{\dot{N}_i(t)N(t) - N_i(t)\dot{N}(t)}{[N(t)]^2} = p_i(t)[u(s_i, \sigma) - u(\sigma, \sigma)] \tag{1}$$

According to Binmore (1992), the results obtained by equation (1) say that the more successful a strategy that competes in a population in a state $\sigma(t)$, the greater its payoff[15] and consequently the greater its relative growth. Thus, the replicator dynamics provides the rate of growth of the proportion of players that adopt a certain strategy available in the game over time. To find the asymptotically stable points, that is, points of equilibrium, starting from a system of nonlinear differential equations, we must (i) find the stationary points by doing $\dot{p}_i = 0$ and (ii) verify its stability. In the next subsection, we will begin a more detailed discussion about the stability of the system when we have two populations of players competing against each other, which is the situation that depicts the competitive context we are modelling in this paper.

2.2 Two-Dimensional Games

In this class of games, we have two populations competing against each other, i.e., players from one population do not compete against their peers. They will only compete with players from the rival population. In this sense, imagine a situation where each population has the same two available strategies. What we present in equation (2) is a system of ODEs where each population has its replicator dynamic. So that:

$$\begin{aligned} \dot{p} &= p[u(s_i, \sigma) - u(\sigma, \sigma)] \\ \dot{q} &= q[u(s_i, \sigma) - u(\sigma, \sigma)] \end{aligned} \tag{2}$$

To check the stability of the points that are candidates for an ESS, i.e., an asymptotically stable steady state, we must use the Jacobian matrix (Ω). Calculating the eigenvalues:

$$\Omega(p, q) = \begin{bmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial q} \\ \frac{\partial \dot{q}}{\partial p} & \frac{\partial \dot{q}}{\partial q} \end{bmatrix}; \text{ doing the determinant } det(\Omega - \lambda i) = \begin{vmatrix} \frac{\partial \dot{p}}{\partial p} - \lambda & \frac{\partial \dot{p}}{\partial q} \\ \frac{\partial \dot{q}}{\partial p} & \frac{\partial \dot{q}}{\partial q} - \lambda \end{vmatrix} = 0$$

[15] As a synonym of fitness or aptitude, we interpret payoff as rate of reproduction.

We finally have that $\lambda_{1,2} = \frac{tr\Omega \pm \sqrt{tr\Omega^2 - 4det\Omega}}{2}$. In order for the stationary point to be asymptotically stable, the eigenvalues $\lambda_{1,2}$ of the matrix (Ω) evaluated at points that hold the condition $\dot{p}_i = 0$ and $\dot{q}_i = 0$ must have negative real parts.

As written in Friedman (1991), in a two-dimensional game where each population has the same two strategies available, S^1 corresponds to the state of the first population and is given by $S^1 = \{p, 1-p\} \therefore p \in [0,1]$. The state of the second population is given by $S^2 = \{q, 1-q\} \therefore q \in [0,1]$. The matrices A and B, shown below, represent the payoffs of population 1 (row player) and population 2 (column player), respectively, $A = \begin{bmatrix} 0 & a_1 \\ a_2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b_1 \\ b_2 & 0 \end{bmatrix}$.

According to the values of a_1, a_2, b_1 and b_2 there are, in total, nine possible cases[16]. Here, only two of them will be analyzed, as shown in Table 1, which are appropriate under the circumstances presented in session 3 of this article.

Table 1- Cases analyzed in the two-dimensional game

Case	**Payoff Structure**
I	$a_1, a_2\ e\ b_1, b_2 > 0$
II	$a_1 > 0, a_2 < 0\ e\ b_1 > 0, b_2 < 0$

Source: Elaborated by the authors.

Case I matches with the anti-coordination game and case II matches with the Prisioner's Dilemma. Both of them represent a social dilemma that have attracted significant interest in social and behavioral science, and the evolutionary game theory approaches provide useful complementary insights into decision-making in social dilemmas.

Figure 1 shows the matrix representation of a two-dimensional game. Note that both populations have the same strategies e^1 and e^2. To find the replicator dynamics for each population, we must rewrite equation (2) as $\dot{p} = p(u(e^1, S^2) - u(S^1, S^2))$. In words, the proportion (p) of players in population 1 that will adopt strategy e^1 depends on how good their performances are against the state of population 2, S^2. If e^1 is a better response to S^2 than e^2, then the proportion of players in population 1 adopting e^1 will raise in the long term. Rewriting $u(e^1, S^2)$ and $u(S^1, S^2)$ as follows, and after some trivial algebra we find the replicator equation.

$$u(e^1, S^2) = e^1 * \text{A} * S^2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & a_1 \\ a_2 & 0 \end{bmatrix} \begin{bmatrix} q \\ 1-q \end{bmatrix} = a_1(1-q)$$

$$u(S^1, S^2) = S^1 * \text{A} * S^2 = \begin{bmatrix} p & 1-p \end{bmatrix} \begin{bmatrix} 0 & a_1 \\ a_2 & 0 \end{bmatrix} \begin{bmatrix} q \\ 1-q \end{bmatrix} = a_2(1-p)q + a_1 p(1-q)$$

		Population 2	
		$e^1\ (q)$	$e^2\ (1-q)$
Population 1	$e^1\ (p)$	$(0\ ,0)$	a_1, b_2
	$e^2 (1-p)$	a_2, b_1	$(0\ ,0)$

Figure 1 - Two-dimensional game matrix representation

Source: Elaborated by the authors.

For population 1, the replicator is given by $\dot{p} = p(1-p)[a_1(1-q) - a_2 q]$ and by symmetry, for population 2, the replicator is $\dot{q} = q(1-q)[b_1(1-p) - b_2 p]$. The system formed by $\dot{p}$ and $\dot{q}$ is linearized by the Jacobian matrix (Ω) and then, as explained above, the neighborhood of the candidate points to asymptotic equilibrium is studied through the trace and the determinant of Ω.

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[16] For the complete analysis and discussion of all nine cases, see Friedman (1991).

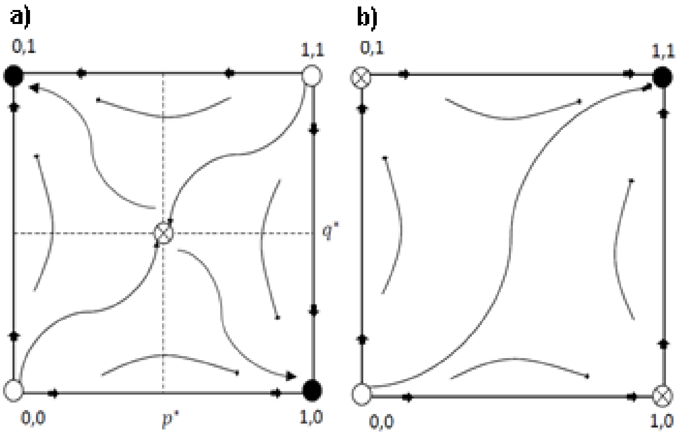

Figure 2 - Phase diagrams for case I (a) and case II (b)
Source: Elaborated by the authors.

Figure 2 presents the phase diagram for cases I and II. We can see that case I matches with the anti-coordination game, which is in figure 2a. Typically, it has two asymptotically stable points (attractor points): (0,1), (1,0), meaning that the best action for a player is to adopt the opposite strategy of your opponent. The points (0,0) and (1,1) are unstable and there is a possibility for a solution to start arbitrarily close to that stationary point and eventually leave its neighborhood over time. The critical point (p^*, q^*) is a saddle point and is unstable. On figure 2b, we can see that for case II the only asymptotically stable point is (1,1). The points (1,0), (0,1) are saddle points and (0,0) is unstable.

3 The model: an evolutionary game based on regional input-output analysis

We start this session by bringing the model designed to deal with the situations described in the introduction and in session 2. Let us suppose two representative firms, called F_A and F_B, which are the players of the game and produce homogeneous goods. They are competing for larger market shares in the automotive sector. Both firms are evaluating a region where they will install their new production plant. We assume that given the costs involved in re-location, once the investment decision about the regional location is made it is irreversible. We will limit firms' strategies between two possible regions: Brazil (BRA) and rest of Mercosur (RMSUR). The final decision will be based on a set of factors and their respective weights. In the next sub session, we present four categories of locational factors taken into account in our model[17].

3.1 Factors Considered for Strategic Locational Decision

As the main objective is to offer a reasonable explanation about the location decision patterns of competing firms, in the EGT model presented here, we built the payoffs based on information received from the regional input-output matrix and from the Global Enabling Trade Report. It incorporates weighted factors, whose weights vary with each scenario we built in order to capture the uncertainty inherent to a dynamically competitive environment. To take that decision, the following factors are considered.

i. Market Potential

The firms consider the influence of market conditions, since resources must be allocated to regions with better economic returns (CHRISTALLER, 1966; LOSCH, 1954). Thus, a measure of the absorption capacity of a particular good or service in a region is necessary. We observe the demand present in R_1 and R_2 in order to design a potential market to be serviced post-installation. The metric used follows the

[17] Naturally, the factors considered will depend on the activity under consideration and the time horizon of the investment. In this way, adjustments in the presented model are recommended.

gravitational model of Isard (1960) in which each of the regions s, located around city r, contribute to the composition of their market potential (ρ_r). It is assumed that the attribute Z of the k regions (*e.g. income*) is weighted by the inverse distance ($1/d_{rs}$) to r in order to obtain $\rho_r = \sum_{s=1}^{k} Z_s/d_{sr}$.

Thus, the market potential was calculated based on the Gravity Model, which took into account for each Mercosur member country: (a) GDP per capita in the PPP - US $ concept and (b) minimum road distance between economic capitals[18]. The values were normalized based on the highest observed value. The database used is from the year of 2010.

ii. Local productive interdependence

The firm will be located in an environment of greater productive integration, characterized by the ease in the acquisition of inputs - *backward effects* (HIRSCHMAN, 1958; WEBER, 1909). This information can be easily captured through the Input-Output matrix (IO). The Production Multiplier[19] will be considered as a proxy for productive interdependence since it is defined as the total value production of the good j, taking into account all sectors of the economy that are necessary to satisfy the increase of final demand of the productive sector in question. The equation $\pi_{j,r} = \sum_{i=1}^{N} \alpha_{ij}$ represents the Production Multiplier of good j in the region $r(\pi_{j,r})$ given by the sum of the rows of the Leontief matrix (direct and indirect production requirement), which is represented by the term a_{ij}. The greater the level of interdependence of this sector in relation to the others, the greater the systemic impact in the economy.

The acquisition of the labor factor is also observed by the company (WEBER, 1909). We will assume that it is non-mobile and its contracting will take place in the destination region of the firm. It is also necessary to differentiate these workers into skilled and unskilled workers, given their productivity difference. The chosen region will be the one with the lowest labor cost per unit of production, either qualified ($\psi 1$) or not ($\psi 2$), calculated from the respective expressions $\psi 1_{j,r} = L1_{j,r}/Y_{j,r}$ and $\psi 2_{j,r} = L2_{j,r}/Y_{j,r}$ where $L1_{j,r}$, $L2_{j,r}$ and $Y_{j,r}$ are, respectively, the total remuneration received by the skilled and unskilled workers in addition to the total produced from the asset j in region r. The Input-Output matrices as well as the sectorial labor force were obtained by the GTAP 9 database[20].

iii. Government tax incentive

It is perceived that firms tend to seek the public agency ex ante the decision of a new investment. In this way, they might decide to locate where they receive greater incentives. In our model, this advantage must be expressed in terms of the market share obtained in Brazil and in the rest of Mercosur countries. Since taxes are a burden to the firms, when they are exempted we typically observe lower prices, which lead to an increase in the demand. We will name this percentage gain as $\mu_{j,r}$, and if the firms decide to go to the same region, the share will be equally divided, i.e., $\mu_{j,r}/2$.

In our model, a negative shock of 100% was simulated on the variable to (tax on the product Vehicles and Parts in the regions BRA and RMSUR) - symbolizing the tax exemption - granted to the sector

[18] Cities adopted as centroids: São Paulo (Brazil), Buenos Aires (Argentina), Asunción (Paraguay), Montivideo (Uruguay) and Caracas (Venezuela). For rest of Mercosur it was calculated by the mean of Argentina, Paraguay, Uruguay, and Venezuela.

[19] For more details see Miller and Blair (2009). It was also tested the Extraction Method, whose results were similar to the Production Multiplier.

[20] The national IO tables provided by GTAP 9. The values are in millions of (2004, 2007 and 2011) current U.S. dollars and has as reference more current 2011. First, the data were aggregated into 11 sectors (Grains and Crops, Livestock and Meat Products, Mining and Extraction, Processed Food, Textiles and Clothing, Motor Vehicles and Parts, Light Manufacturing, Heavy Manufacturing, Utilities and Construction, Transport and Communication and Other Services) and two regions Brazil (BRA) and Rest of Mercosur (RMSUR) using the program GTAPAgg. This same base presents the sectoral expenditure in 5 factors of production (land, unskilled labor, skilled labor, capital and natural resources). For this, the GTAPAgg was used again. For more details see: https://www.gtap.agecon.purdue.edu/databases/v9/default.asp..

of vehicles and parts. Subsequently, the variation of qds (domestic sales of Vehicles and Parts for each region) was observed[21]. For this, we used the Computable General Equilibrium Model[22], GTAP.

iv. Macroeconomic stability

Any investment in fixed capital must be done from a long-term perspective. In this decision-making process, one must take into account both the microeconomic variables, directly linked to the firm's performance, and the macroeconomic and institutional variables, which show the specific factors of the country receiving the investment. The firm, therefore, may decide to locate where there is less macroeconomic uncertainty. We will call this variable ε_r . Of course, the observation of this stability is imprecise, due to the existence of imperfect information.

The macroeconomic indicator will be composed by two contributions: (i) The Trading Economics Credit Rating[23], which assigns a score of 100 to countries whose investment grade is called "high grade" and 0 to countries that are in financial default. In this work, the indicator was normalized to a scale ranging from 0 to 1; (ii) Enabling Trade Index[24] from the years of 2012 to 2014, which measures the favoring of trade in goods and services from borders to final destination. For the composition of the index, it is taken into account the market access, the administration of bilateral negotiations, the transport and communications infrastructure and the business environment. Originally, the index is provided on a scale from 1 to 7. Here, to simplify the analysis, the values were normalized in the interval between 0 and 1.

3.2 The Payoff Matrix

After presenting all the factors incorporated in the composition of the game payoffs, the matrix that represents the strategic interaction between the players and their respective parameters are arranged as shown in figure 3.

F_A \ F_B	BRA	$RMSUR$
BRA	$(\beta_{A,1}, \beta_{B,1})$	$(\beta'_{A,1}, \beta_{B,2})$
$RMSUR$	$(\beta_{A,2}, \beta'_{B,1})$	$(\beta_{A,2}, \beta_{B,2})$

Figure 3 - The Payoff Matrix

Source: Elaborated by the authors.

Where[25]:

$$\beta_{A,1} = W_\rho(\rho_1/2) + W_\pi[0.5(\pi_{A,1}/2) + 0.5(0.5\psi1_{A,1} + 0.5\psi2_{A,1})] + W_\mu(\mu_{A,1}/2) + W_\varepsilon\varepsilon_1$$
$$\beta_{B,1} = W_\rho(\rho_1/2) + W_\pi[0.5(\pi_{B,1}/2) + 0.5(0.5\psi1_{B,1} + 0.5\psi2_{B,1})] + W_\mu(\mu_{B,1}/2) + W_\varepsilon\varepsilon_1$$
$$\beta'_{A,1} = W_\rho\rho_1 + W_\pi[0.5\pi_{A,1} + 0.5(0.5\psi1_{A,1} + 0.5\psi2_{A,1})] + W_\mu\mu_{A,1} + W_\varepsilon\varepsilon_1$$
$$\beta_{B,2} = W_\rho\rho_2 + W_\pi[0.5\pi_{B,2} + 0.5(0.5\psi1_{B,2} + 0.5\psi2_{B,2})] + W_\mu\mu_{B,2} + W_\varepsilon\varepsilon_2$$
$$\beta_{A,2} = W_\rho\rho_2 + W_\pi[0.5\pi_{A,2} + 0.5(0.5\psi1_{A,2} + 0.5\psi2_{A,2})] + W_\mu\mu_{A,2} + W_\varepsilon\varepsilon_2$$
$$\beta'_{B,1} = W_\rho\rho_1 + W_\pi[0.5\pi_{B,1} + 0.5(0.5\psi1_{B,1} + 0.5\psi2_{B,1})] + W_\mu\mu_{B,1} + W_\varepsilon\varepsilon_1$$
$$\beta_{A,2} = W_\rho(\rho_2/2) + W_\pi[0.5(\pi_{A,2}/2) + 0.5(0{,}5\psi1_{A,2} + 0.5\psi2_{A,2})] + W_\mu(\mu_{A,2}/2) + W_\varepsilon\varepsilon_2$$
$$\beta_{B,2} = W_\rho(\rho_2/2) + W_\pi[0.5(\pi_{B,2}/2) + 0.5(0{,}5\psi1_{B,2} + 0.5\psi2_{B,2})] + W_\mu(\mu_{B,2}/2) + W_\varepsilon\varepsilon_2$$

[21] The objective of the simulation is to verify the percentage effect of the total tax exemption in Vehicle and Parts sector (represented by the variable to = -100 in the BRA and RMSUR regions separately) on the domestic sale of Vehicles and Parts (qds "MotorVcles") implemented through RunGTAP software.

[22] It is a general multi-regional and multi-sectorial equilibrium model. The global database represents the world economy for the year 2011 (version 9). GEMPACK software was used to implement the model. The standard closure and Gragg Solution Method 2-4-6 were applied.

[23] More information available on: www.tradingeconomics.com/country-list/rating.

[24] More information available on: https://www.weforum.org/reports/global-enabling-trade-report-2012/.

[25] Note that $\beta_{A,1} = \beta_{B,1}$, $\beta'_{A,1} = \beta'_{B,1}$, $\beta_{A,2} = \beta_{B,2}$ e $\beta_{A,2} = \beta_{B,2}$.

W_ρ, W_π , W_μ e W_ε represent the weights attributed to each of the factors. The sum of all weights equals one unit. It should be emphasized that the weights depend on the nature of the activity studied. This structure is very flexible, since we can define in different ways the regions and sectors to be analyzed. When changing any of these components, adjustments must be made[26].

3.3 The Empirical Model

In this section, an empirical exercise will be presented using the approach described above, in order to validate it as an adequate study instrument for the locational decision. In this way, suppose that two firms (F_A, F_B) belonging to the automotive sector are deciding to locate in some Mercosur country, aiming to meet the potential demand of the market. To do so, our analysis will make the spatial division between Brazil (BRA) and rest of Mercosur (RMSUR). When locating in any of these regions the firms will incur a unique transaction fee among the participants of the block (member countries). Thus, the locational decision will be based on internal attributes of each region.

As explained before, in order to analyze the strategic decision on the location of firms, scenarios were created according to the attribution of weights to the four factors that make up the final payoff to be inserted in the matrix of the game. This framework is in accordance with the mechanical system of Varignon, as presented by Weber (1909). It is known that these are dependent on the specific production characteristics of each firm. In this way, we consider five distinct scenarios, as presented in Table 2.

Table 2 - Evaluated Scenarios

Factors	**Scenario**				
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
a) Market Potential	0.25	0.50	0.17	0.17	0.17
b) Location Interdependence*	0.25	0.17	0.5	0.17	0.17
b.1) Production Multiplier	0.50	0.50	0.50	0.50	0.50
b.2) Cost of labor factor**	0.50	0,50	0.50	0.50	0.50
b.2.1) Qualified labor	0.50	0.50	0.50	0.50	0.50
b.2.2) Unqualified labor	0.50	0.50	0.50	0.50	0.50
c) Government tax incentive	0.25	0.17	0.17	0.50	0.17
d) Macroeconomic Stability	0.25	0.17	0.17	0.17	0.50
Sum (weights)	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**

Notes:* (b) is formed by (b.1) + (b.2); ** (b.2) is formed by (b.2.1) + (b.2.2).
Source: Elaborated by the authors.

As seen in Table 2, in scenario 1, firm A and firm B give equal importance to observed factors. In scenario 2, the firms emphasize the demand side, observing the potential market. Scenario 3 is where they observe possible gains through productive interdependence. In scenario 4, there is a strong influence of state action through fiscal policy, and finally in scenario 5, firms observe the results of macroeconomic stability presented by region r, which would be closer to the decision strategy of multinational firms. In the next section, we will present the results of the evolutionary game obtained from the empirical model.

4 Results

This section brings the results of the empirical exercise. We analyze and discuss scenarios one through five in sequence.

Scenario 1 – Homogeneous weights: Note in the left side of Figure 4, that if we assign homogeneous weights to all factors and if the Brazilian government grants tax exemption, the Nash Equilibrium of the game coincides with the Evolutionary Equilibrium. In other words, the coordinate (1,1) emerges as the only

[26] Equal weights are given to the acquisition of inputs and labor power, as well as the use of skilled and unskilled labor.

asymptotically stable point. Therefore, there is a dominant strategy for both representative firms, which is to locate in Brazil. Thus, both firms will compete for the Brazilian market share and none of them has incentives to install their plants in RMSUR. On the other hand, from the right side of Figure 4, it is possible to conclude that if the tax exemption is given in RMSUR, the evolutionary stable strategy is the point (0,0). Thus, the best response for both firms is to decide to locate in RMSUR.

Tax exemption in BRA

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(\mathbf{0.3}, \mathbf{0.3})$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(-0.76, -0.76)$

Tax exemption in RMSUR

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(-0.77, -0.77)$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(\mathbf{0.025}, \mathbf{0.025})$

Figure 4 - Payoff Matrix for Scenario 1

Source: Elaborated by the authors.

Figure 9, available in the appendix, shows the phase diagram, where we can see the dynamics of the game over time when firms are facing scenario1 with the tax exemption given in Brazil. Figure 10, also in the appendix, corresponds to the dynamics for tax exemption in RMSUR. Therefore, giving homogeneous weights to the observed factors, the evolutionary stable strategy is to decide to locate where the government tax exemption is granted.

Scenario 2 – Emphasis on the potential market: In case the firms attribute greater weight to the region's sales potential and if the tax exemption is given in Brazil, what we observe from the left side of Figure 5 is that the asymptotic equilibrium of the game corresponds to the coordinate (1,1). Thus, both representative firms will decide to build their new plant in Brazil. In Figure 11 of the appendix, the phase diagram is presented and we can see the dynamic of the game under the situation mentioned above.

Tax exemption in BRA

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(\mathbf{0.33}, \mathbf{0.33})$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(-0.82, -0.82)$

Tax exemption in RMSUR

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(-0.39, -0.33)$	$(\mathbf{0}, \mathbf{0})$
	$RMSUR$	$(\mathbf{0}, \mathbf{0})$	$(-0.82, -0.82)$

Figure 5 - Payoff Matrix for Scenario 2

Source: Elaborated by the authors.

However, if the tax exemption is granted by the rest of Mercosur countries, what is configured is an anti-coordination game. Under these circumstances, typically, when analyzing the results based on the Nash Equilibrium approach, we verify the existence of two N.E. in pure strategies, given by the points (1,0) and (0,1). There also exists an N.E. in mixed strategy (p^*, q^*). From the refinement provided by evolutionary game theory, it is possible to eliminate one of the N.E., once we see in Figure 12 (appendix) that the point (p^*, q^*) corresponds to a saddle point. Thus, it is not asymptotically stable.

Therefore, we managed to eliminate the Nash equilibrium in mixed strategy of the game, which allows us to infer that the best response for a representative firm is to adopt the (pure) strategy opposite to that chosen by its opponent. In practice, what we observe is an equilibrium where one type of representative firm located in BRA and another in RMSUR. This result suggests that, from the perspective of potential demand, it is not always an optimal decision to locate and compete in the market where there is a tax exemption.

Scenario 3 – Emphasis on the local productive interdependence: In this scenario, where greater weight is attributed to the gains obtained through productive interdependence, the same asymptotic equilibriums of the previous scenario were found. Thus, given a tax exemption in Brazil, both representative firms will choose it as the destination region. Once the government tax exemption is given in RMSUR, a game of anti-coordination is set up again and, therefore, the best response of a firm will be to choose the region opposite to that chosen by its opponent. Thus, the ESSs' of the game are given by the points (1,0) and (0,1) (p^*, q^*) is said to be a saddle point.

As seen in the previous scenario (Figure 6), this result suggests that, from the perspective of local productive interdependence, it is not always an optimal decision to locate and compete in the market where there is a tax exemption.

Tax exemption in BRA

		F_B: BRA	F_B: $RMSUR$
F_A	BRA	$(\mathbf{0.11}, \mathbf{0.11})$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(-0.75, -0.75)$

Tax exemption in RMSUR

		F_B: BRA	F_B: $RMSUR$
F_A	BRA	$(-0.60, -0.60)$	$(\mathbf{0}, \mathbf{0})$
	$RMSUR$	$(\mathbf{0}, \mathbf{0})$	$(-0.22, -0.22)$

Figure 6 - Payoff Matrix for Scenario 3

Source: Elaborated by the authors.

Scenario 4 – Emphasis on the government exemption tax: What is observed in the game payoff matrix for scenario 4 is the configuration of an equilibrium where the region selected by both representative firms is exactly the one in which there is the fiscal incentives.

This can be visualized in the phase diagram presented on Figures 15 and 16 (appendix). For the tax exemption given in Brazil, the asymptotic equilibrium of the game is given by the coordinate (1,1) and, for the fiscal incentive given in RMSUR, what is observed by the diagram is that the equilibrium is obtained in the coordinate (0,0).

Tax exemption in BRA

		F_B: BRA	F_B: $RMSUR$
F_A	BRA	$(\mathbf{0.53}, \mathbf{0.53})$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(-0.95, -0.95)$

Tax exemption in RMSUR

		F_B: BRA	F_B: $RMSUR$
F_A	BRA	$(-1.62, -1.62)$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(\mathbf{0.63}, \mathbf{0.63})$

Figure 7 - Payoff Matrix for Scenario 4

Source: Elaborated by the authors.

Scenario 5 – Emphasis on the macroeconomic stability: When firms attribute greater weight to macroeconomic stability, for a given tax exemption in Brazil we observe that both representative firms will be located there. On the other hand, given a shock in RMSUR, a game of anti-coordination is characterized.

Once again, the interior point (p^*, q^*) does not set up an ESS and the mixed strategy can be eliminated from the set of solutions. The evolutionary stable strategies are given by the coordinates (1,0) and (0,1). Thus, as shown before, the best strategic decision for a firm is to play the strategy opposite to that chosen by its opponent.

Tax exemption in BRA

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(\mathbf{0.23}, \mathbf{0.23})$	$(0, 0)$
	$RMSUR$	$(0, 0)$	$(-0.54, -0.54)$

Tax exemption in RMSUR

		F_B	
		BRA	$RMSUR$
F_A	BRA	$(-0.49, -0.49)$	$(\mathbf{0}, \mathbf{0})$
	$RMSUR$	$(\mathbf{0}, \mathbf{0})$	$(-0.01, -0.01)$

Figure 8 - Payoff Matrix for Scenario 5

Source: Elaborated by the authors.

5 Conclusion

In this paper we proposed an evolutionary game theory model to explain the decision-making process about the location pattern of firms. In this context, we analyzed the automotive sector in Mercosur Countries, where under a government tax exemption, the representative multinational firms had to decide between Brazil and rest of Mercosur countries for building a new plant. We also designed scenarios where firms assigned weights to the factors (market potential; local productive interdependence; government tax incentive; and macroeconomic stability) that influence the location decision.

The results obtained from the dynamic game suggest that when firms give homogeneous weights to the factors, the equilibrium occurs with both competing in the market in which there was the governmental tax incentive. Once the factors are assigned with different weights, the long-term equilibrium of the game changes, which provides evidences that competing the market where there is a tax exemption is not always an optimal location decision. This may contradict the common sense about these issues.

The largest contribution of the Evolutionary approach can be seen in Scenarios 2, 3 and 5, specifically when the tax incentive is given in RMSUR. What is observed is the formation of an anti-coordination game. By eliminating the Nash equilibrium in mixed strategies (p^*, q^*) of the set of best responses, only the pure strategies BRA and RMSUR are supported as ESS. In this way, firms that compete in this market can make a more controlled and less risky decision on which region to settle. Put another way, firms can infer about which game, in fact, is played in order to achieve the long-term dynamic equilibrium and correctly decide in which region they must install their new plant.

In terms of investment policy, the outcomes essentially expose the importance of these policies when inserted in a favorable economic environment. In other words, the results suggest that, besides the tax exemption, the firms attribute a significant relevance to market potential, local productive interdependence and macroeconomic fundamentals. This may justify the results observed in Scenarios 2, 3 and 5, in which, despite tax incentives offered by the other Mercosur countries, there is still an ESS characterized by firms locating in Brazil.

One limitation of this work was the fact that it did not use the information derived from the regional input-output matrix in a disaggregated form for each of the Mercosur countries. By doing so, it would be possible to say which country would be chosen for the installation of the automotive plant due to a government tax incentive. For future research, a modeling to be developed can consider stochastic elements in the EGT model proposed in this paper by using an Agent Based Simulation approach. Another possibility is to advance in the framework presented here in terms of the factors considered in the decision-making process, as well as the elaborated scenarios, available regions and expand the study to other sectors of the economy.

References

BENTLEY, G., BAILEY, D., MACNEILL, S. (2013). The changing geography of the European auto industry. *Handbook of Industry Studies and Economic Geography*. Cheltenham: Edward Elgar Publishing..

BINMORE, K. (1992). *Fun and games, a text on game theory*. Lexington: DC Heath and Company.

BOYD, R. *et al.* (2003). The evolution of altruistic punishment. *Proceedings of the National Academy of Sciences of the United States of America*, v. 100, n. 6, p. 3531–3535.

CHAN, Y.(2001). *Location theory and decision analysis*. 2º ed ed. Cincinnati: Springer.
CHEN, M., MOORE, M. (2010). Location decision of heterogeneous multinational firms. *Journal of International Economics*, v. 80, n. 2, p. 188–199.
CHRISTALLER, W. (1966). *Central places in southern Germany*. New Jersey: Prentice-Hall.
CLEMENS, C., RIECHMANN, T. (2006). Evolutionary Dynamics in Public Good Games. *Computational Economics*, v. 28, n. 4, p. 399–420.
D'ASPREMONT, C., GABSZEWICZ, J. J., THISSE, J.F.(1979). On Hotelling's stability in competition. *Econometrica*, v. 47, n. 5, p. 1145–1150.
DOMANSKI, B., LUNG, Y. (2009).The changing face of the European periphery in the automotive industry. *European Urban and Regional Studies*, v. 16, n. 1, p. 5–10.
FISCHER, M. M., NIJKAMP, P. (2014). *Handbook of regional science*. Berlin: Springer.
FRIEDMAN, D. (1991). Evolutionary Games in Economics. *Econometrica*, v. 59, n. 3, p. 637.
GABSZEWICZ, J. J., THISSE, J.F. (1992). *Handbook of game theory with economic applications*, v. 1, p. 281–304.
HIRSCHMAN, A. O. (1958). *The strategy of economic development*. New Haven: Yale University Press, v. 44.
HOTELLING, H. (1929). Stability in competition. *The Economic Journal*, v. 39, n. 153, p. 41–57.
HUBERMAN, B. A., GLANCE, N. S. (1995). The dynamics of collective action. *Computational Economics*, v. 8, n. 1, p. 27–46.
KLIER, T., MCMILLEN, D. (2015). Plant Location Patterns in the European Automobile Supplier Industry. *Growth and Change*, v. 46, n. 4, p. 558–573.
KLIER, T., MCMILLEN, D. P. (2008). Evolving agglomeration in the U.S. auto supplier industry. *Journal of Regional Science*, v. 48, n. 1, p. 245–267.
LAGENDIJK, A. (1997). Towards an Integrated Automotive Industry in Europe A'Merging Filiere'Perspective. *European Urban and Regional Studies*, v. 4, n. 1, p. 5–18.
LOSCH, A. (1954). *Economics of location*. . New Haven: Yale University Press.
MAYER, T., MEJEAN, I., NEFUSSI, B. (2010). The location of domestic and foreign production affiliates by French multinational firms. *Journal of Urban Economics*, v. 68, n. 2, p. 115–128.
MAYNARD SMITH, J., PRICE, G. R. (1975). The logic of animal conflict. *Nature*, v. 254, n. 5499, p. 463–463.
MILLER, R. E., BLAIR, P. D. (2009). *Input - Output Analysis: Foundations and Extensions*. New Jersey: Prentice-Hall.
SAMUELSON, L. (2002). Evolution and Game Theory. *Journal of Economic Perspectives*, v. 16, n. 2, p. 47–66.
TAYLOR, P. D., JONKER, L. B. (1978). Evolutionarily stable strategies and game dynamics. *Mathematical Biosciences*, v. 40, n. 1–2, p. 145–156.
WEBER, A. (1909). *Theory of the Location of Industries*. Chicago: The University of Chicago Press.

Appendix - Phase Diagrams

Scenario 1 - Homogenous weights

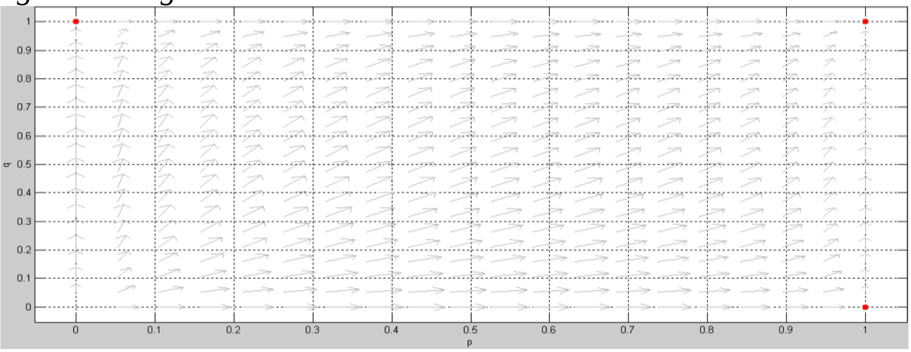

Figure 9 - Phase diagram for tax exemption in BRA

Source: Elaborated by the authors, using Matlab.

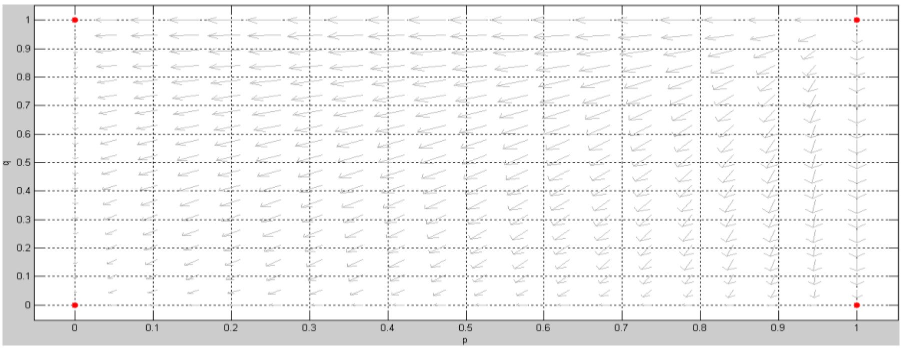

Figure 10 - Phase diagram for tax exemption in RMSUR

Source: Elaborated by the authors, using Matlab.

Scenario 2 - Emphasis on the potential market

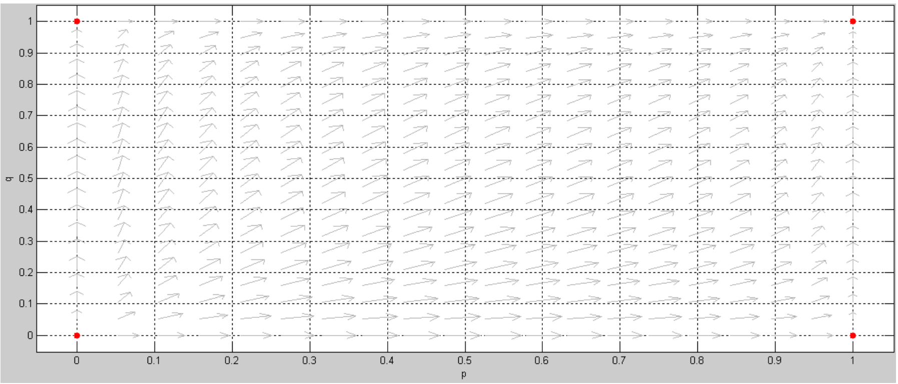

Figure 11 - Phase diagram for tax exemption in BRA

Source: Elaborated by the authors, using Matlab.

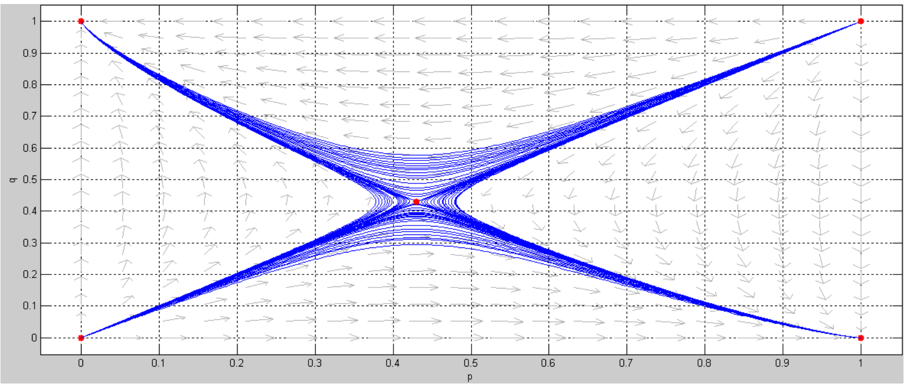

Figure 12 - Phase diagram for tax exemption in RMSUR

Source: Elaborated by the authors, using Matlab.

Scenario 3 - Emphasis on the local productive interdependence

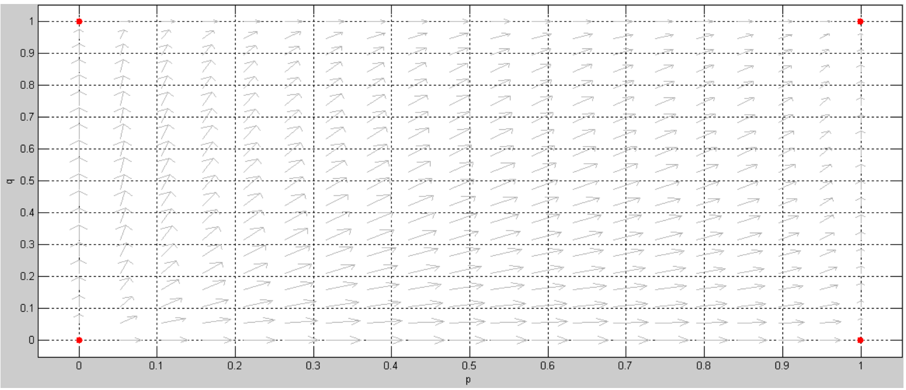

Figure 13 - Phase diagram for tax exemption in BRA

Source: Elaborated by the authors, using Matlab.

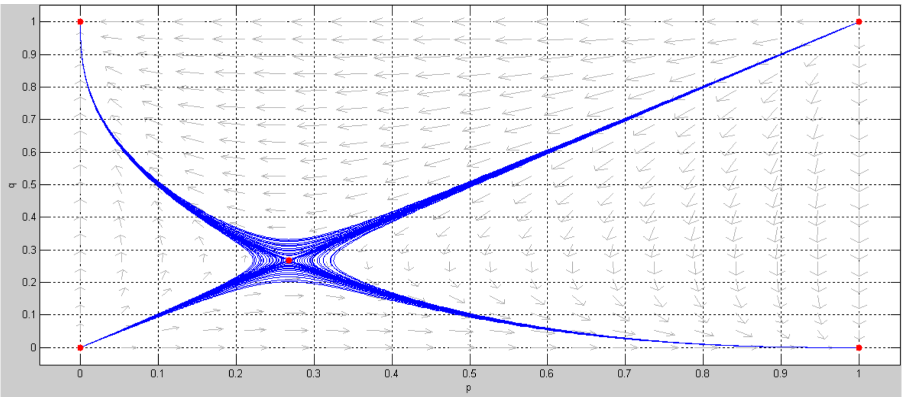

Figure 14 - Phase diagram for tax exemption in RMSUR

Source: Elaborated by the authors, using Matlab.

Scenario 4 - Emphasis on the government exemption tax

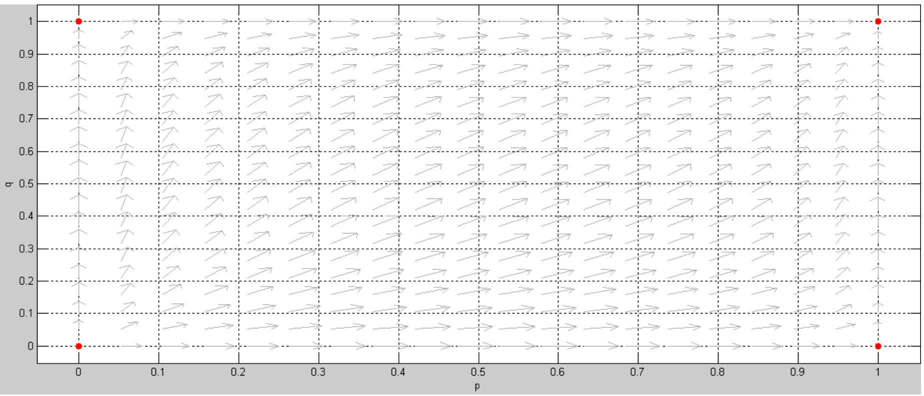

Figure 15 - Phase diagram for tax exemption in BRA

Source: Elaborated by the authors, using Matlab.

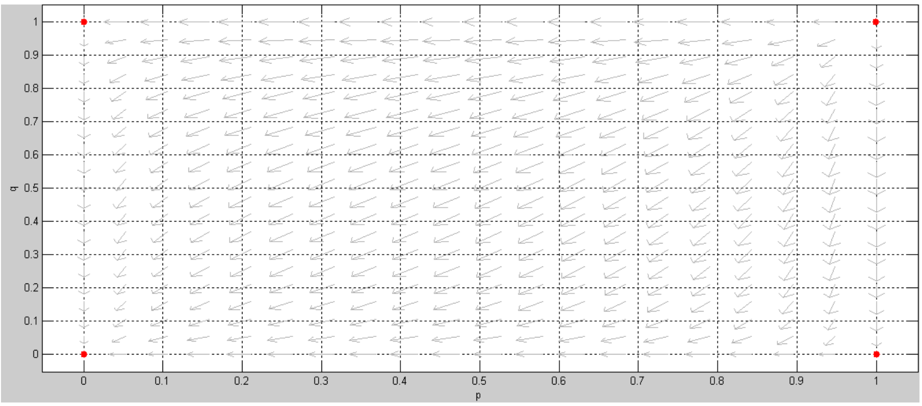

Figure 16 - Phase diagram for tax exemption in RMSUR

Source: Elaborated by the authors, using Matlab.

Scenario 5 - Emphasis on the macroeconomic stability

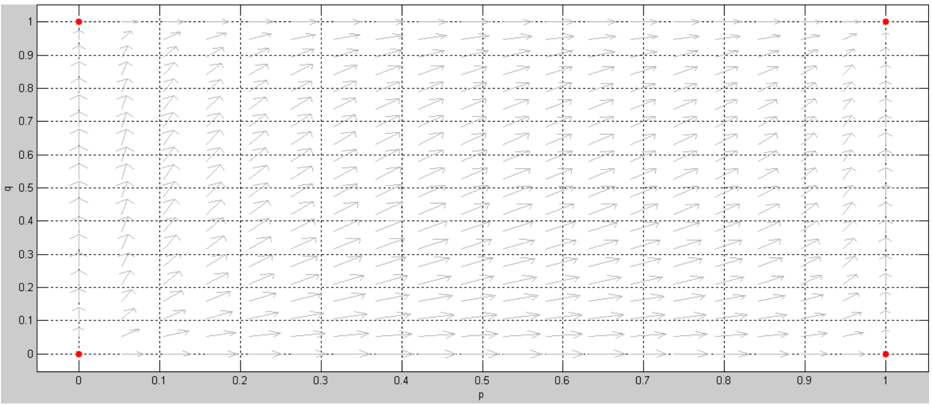

Figure 17 - Phase diagram for tax exemption in BRA

Source: Elaborated by the authors, using Matlab.

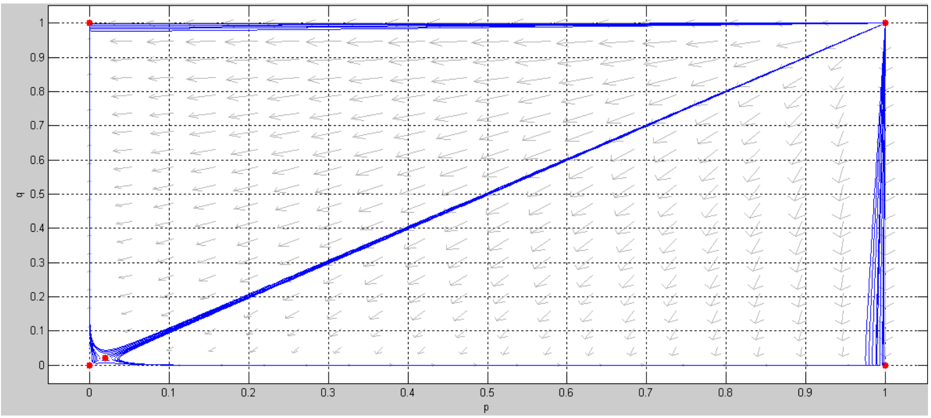

Figure 18 - Phase diagram for tax exemption in RMSUR

Source: Elaborated by the authors, using Matlab.