Endogenizing non-price competitiveness in a model with capital accumulation and BoPC on growth

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Abstract

At a theoretical level this article revisits Thirlwall's rule (or law) proposing a new channel through which it is possible to endogenize non-price competitiveness in the BoPC framework. We develop a model that formalizes the inverted-U relationship hypothesis that non-price competitiveness rises as countries move from a primary productive structure to light manufactures and then decreases as richer countries get locked into antiquated industrial structures. We name it *the stratification mechanism.* Finally we incorporate the supply side of the economy into the structure of the model in order to avoid the so called *inconsistency problem.*

Resumo

O artigo revisita a lei de Thirlwall propondo um novo mecanismo para endogeneinização da competitividade não-preço dentro do referencial teórico BoPC. Nós desenvolvemos um modelo que formaliza a hipótese do U-invertido em que a competitividade não-preço aumenta a medida em que um país migra de estruturas primárias para industriais e depois se reduz a medida em que a estrutura produtiva se atrofia. Nós o chamamos o mecanismo de estratificação. Finalmente incorporamos o lado da oferta da economia no modelo de modo a evitar o chamado problema de inconsistência.

Keywords: Growth, Balance-of-Payments Constraint, Thirlwall´s law.

Palavras chave: Crescimento, Restrição do Balanço de Pagamentos, Lei de Thirlwall.

Área 6: Crescimento, Desenvolvimento Económico e Instituciones

JEL: O10, O11, O40.

1 Introduction

One of the main aims of growth theory has been to explain why growth rates differ across countries and overtime. In the traditional neoclassical framework and also in modern New Growth Theory, growth is said to be supply constraint in the sense that it is limited either by the accumulation of production factors and/or by technology. In both cases the figure of the production function in neoclassical lines is at the center of the stage.

In the other hand, Post Keynesian models have emphasized over the years the importance of demand constraints on growth. One of the most successful empirical regularities among them is Thirlwall's rule (or law). According to it, since countries cannot systematically finance permanent balance-of-payments imbalances, there is an adjustment in aggregate demand that constrain its expansion and consequently output's growth. Therefore, in the long run, growth is Balance-of-Payments Constrained (BoPC).

Despite its simplicity and powerfulness in explaining the great income divergence between advanced and developing/poor economies, there are at least two main issues still to be solved. The first of them consists in the fact that foreign trade income elasticities - which represent the non-price competitiveness of a country (or region) - are taken as exogenous. The second one is the *inconsistency* (or over determination) *problem* that may arise if the supply side of the economy is not properly incorporate.

There have been a recent number of efforts trying to formally endogenize non-price competitiveness in a BoPC framework (e.g. Palley, 2003; Setterfield, 2011; Ribeiro et al, 2016; Oreiro, 2016). In some cases there were also explicit concerns about the consistency of the BoPC growth rate and the capital accumulation problem. However we are still far from a consensus. It is well understood that distributive and technological questions matter and should be taken into account. But the subject is still open.

This paper aims to contribute to both issues developing a model that formalizes the inverted-U relationship hypothesis that non-price competitiveness rises as countries move from a primary productive structure to light manufactures and then decreases as richer countries get locked into antiquated industrial structures. The relationship above was first proposed by Thirllwall himself even though never in a formal way. We name it the *stratification mechanism*. It is a way through which divergent forces on growth are diluted over time, opening space for the appearance of falling-behind behavior or convergence clubs.

Finally we incorporate the supply side of the economy in a dynamic fashion in order to study the interaction between capital accumulation and the external constraint avoiding the so called *inconsistency problem*. This is an innovation with respect the contributions of Palley (2003) and Oreiro (2016) who explicitly address the over determination problem in static models.

The paper's next section is dedicated to revisit the basic Thirlwall's rule (or law) proposing a particular way to derivate it. Section 3 brings our model of the *stratification mechanism* and its simulation exercise. In section 4 we extend the model incorporating the supply side of the economy and the capital accumulation process. Once again we run some numerical simulations. The last section brings some final considerations.

2 Growth and the BoPC

The idea of a BoPC on growth has been a crucial component of much demand-led growth theory since at least Prebisch (1959) and Thirlwall (1979). However it is the role of the demand side in defining the nature of the constraint that distinguishes the BoPC approach from other growth models. Thirlwall begins proposing that exports and imports are functions of the exchange rate and the level of income. He also adopts as simplifying hypothesis that equilibrium in the balance-of-payments is given by equilibrium in foreign trade.

The hypothesis that trade is balanced over the long run is crucial for the model and is quite plausible. In this section we show that in fact Thirlwall's law (or rule) can be obtained from the so called *fundamental identity*. We suggest that this may help us to enlighten some strange results showing that countries like the United States are BoPC (e.g. Bagnai, 2010; Gouvea and Lima, 2013).

Consider an open economy without government. Using standard notation, aggregate expenditure is defined as:

$$Y_t = C_t + I_t + X_t - E_t M_t \tag{1}$$

where Y is aggregate demand, C is consumption, I stands for investment, X are exports, M imports, and E is the exchange rate.

Rearranging we have:

$$(S_t - I_t) - (X_t - E_t M_t) = 0 \tag{2}$$

where $S_t = Y_t - C_t$ is simply private domestic savings. From the equation above the sum of domestic and foreign net lending must equal zero. This expression sometimes is called the *fundamental identity* and must always hold.

Now, define domestic net borrowing as the opposite of net lending, $D_t = -(S_t - I_t)$ so that $-D_t = X_t - E_t M_t$. It is straightforward the relation between domestic borrowing and current account imbalances. Dividing both sides by Y, we obtain:

$$\frac{-D_t}{Y_t} = \frac{X_t - E_t M_t}{Y_t} \tag{3}$$

A look at individual country data reported in the UNCTAD *Handbook of Statistics* for the period 1980-2014 reveals that current account imbalances as proportion of GDP are limited to the $\pm 5\%$ range for most countries. This in turn indicates that, for the long run, the hypothesis of equilibrium in foreign trade is actually a proxy for the fundamental identity.

We proceed separating countries in two groups. The first one corresponds to those economies that in the long run do not take $\frac{D_t}{Y_t}$ as given. This implies they have fully convertible currencies. The second group is formed by countries that in the long run take $\frac{D_t}{Y_t}$ as given, and may or may not have fully convertible currencies. We say that for this second group growth is BoPC.

The proposition above may help to understand why countries like the United States appear to be BoPC in empirical exercises. It has strong implications in terms of interpretation of the model and also of the data. As pointed out by Ramzi (2015) among others, empirical tests on Thirlwall's law are actually testing the hypothesis that foreign trade is balanced. What we are saying is that *if* for a certain country $\frac{D_t}{Y_t}$ is exogenously given - reflecting the long run capacity of a country (and a limited willingness of the rest of the world) to finance it - *then* trade must be balanced and as showed by Thirlwall growth *must* be BoPC. In this case balanced trade and BoPC growth directly follow from the *fundamental identity*. On the other hand, *if* a country does not take $\frac{D_t}{Y_t}$ as given, *then* still we may find that trade is balanced but growth *cannot* be considered BoPC because the external constraint becomes endogenous.

In order to complete the model we define as does Thirlwall a function for exports and imports:

$$X_t = E_t^{-\beta} Z_t^{\varphi} \tag{4}$$

$$M_t = E_t^{\delta} Y_t^{\pi} \tag{5}$$

where β and $\delta < 0$ are price elasticities, φ and $\pi > 0$ are income elasticities, and Z corresponds to income of the rest of the world.

Suppose an economy that takes $\frac{D_t}{Y_t}$ as given, being determined by historical reasons, and is fairly stable in time. Taking logarithms and time derivates of equation (3) to (5) we have:

$$\theta \left(\frac{\dot{X}_t}{X_t} - \frac{\dot{Y}_t}{Y_t} \right) = \frac{\dot{E}_t}{E_t} + \frac{\dot{M}_t}{M_t} \tag{6}$$

$$\frac{\dot{X}_t}{X_t} = -\beta \frac{\dot{E}_t}{E_t} + \varphi \frac{\dot{Z}_t}{Z_t} \tag{7}$$

$$\frac{\dot{M}_t}{M_t} = \delta \frac{\dot{E}_t}{E_t} + \pi \frac{\dot{Y}_t}{Y_t} \tag{8}$$

where $\theta = \frac{X_t}{X_t + D_t}$. Solving the system and assuming that in the long run terms of trade are constant we obtain:

$$\frac{\dot{Y}_t}{Y_t} = \rho \frac{\dot{Z}_t}{Z_t} \tag{9}$$

with $\rho = \frac{\theta \varphi}{\pi + \theta}$. For instance, this result is very similar to the aggregate BoPC growth models that include capital flows like Thirlwall and Hussain (1982) or Moreno-Brid (2003).

Income elasticities are dependent upon the level of specialization of the country's (or region) productive structure. A high level of specialization is associated with a high marginal propensity to import which in turn implies in a high income elasticity of imports. It is also associated with low elasticity of exports, since the economy will have few different types of goods to export in face of increasing world demand. The expression above is a simple but powerful instrument to explain the great divergence in income between advanced and developing/poor economies[1].

The foreign trade income elasticities ratio, ρ, represent the non-price competitiveness of a country (or region) and in a sense is comparable to "Solow's residual" because even though occupy a central position in the final expression, originally is taken as exogenous. After its empirical success in both aggregate and disaggregate versions (e.g. Bagnai, 2010; Gouvea and Lima, 2010, 2013; Romero and McCombie, 2016), a next and natural step consists in endogenize then.

Palley (2003) was the first one to do it in a model with capital accumulation, proposing that the income elasticity of imports is a positive function of the level of capacity utilization. He explicitly addressed both the endogeneity of non-price competitiveness and the *inconsistence problem*. A different route is followed by Setterfield (2011). Taking as departure point the Kaldorian cumulative causation model, he endogenized foreign trade elasticities as a function of labour productivity growth rate.

In order to take into account technological issues, Cimoli and Porcile (2014) proposed to endogenize non-price competitiveness as a function of an index of technological complexity. A similar route is followed by Ribeiro et al (2016) also including the distributive dimension into the equation. Recently, Oreiro (2016) recalled Palley's solution and proposed an extension putting the exchange rate at the center of stage.

As this briefly revision shows, there have been a recent number of efforts trying to endogenize non-price competitiveness in a BoPC framework. In some cases there were also explicit concerns about the consistency of the BoPC growth rate and the capital accumulation problem. However we consider that we are still far from a consensus.

The BoPC growth literature has taken two different routes in terms of mathematical modeling. In one hand authors like Araújo and Lima (2007) and Nishi (2016) have developed multi-sectoral versions of the rule. In this approach, non-price competitiveness is in a sense already endogenized since aggregate foreign trade elasticities can change due to changes in the productive structure of the economy. Based in a multi-sectoral Pasinetian approach the model is inherently consistent and does not suffer with the over determination problem.

On the other hand a second group of scholars have incorporated the other elements of the BoP to the aggregate model (e.g. Barbosa-Filho, 2001; Moreno-Brid, 2003, Alleyne and Francis, 2008). In this paper we focused on the aggregate version of Thirlwall's rule. This allows us to present in a clearer and pedagogical way the mechanisms we intend to show.

[1] For a further discussion of the implications behind Thirlwall's law and the BoPC growth literature see Thirlwall (2011).

3 Non-price competitiveness and the *stratification mechanism*

3.1 The model

After the empirical consolidation of Thirlwall's rule (or law) during the eighties and the beginning of the nineties, the behavior of the foreign trade income elasticities came to the discussion. In the late nineties a particular hypothesis was proposed by Thirlwall among others. Thirlwall (1997) and Setterfield (1997) argued that the elasticity of exports growths as the country moves from the production of primary products to manufactures and decreases when the economy gets locked in antiquate industrial structures.

In the same line, McCombie and Roberts (2002) considered that while low growth rates generate pressures to an increase in the elasticities ratio, high growth rates would encourage the lock-in of the productive structure. They assume that actually is the ratio of foreign trade income elasticities that is endogenous. Even though discussed, this "inverted-U" relation was never properly formalized in a mathematical model[2].

We propose the following set of equation that summarizes two basic forces acting on the system:

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z, \tag{10}$$

$$\frac{\dot{\rho}_t}{\rho_t} = -\varepsilon \left(\frac{\dot{Y}_t}{Y_t} - \alpha \right) + \beta. \tag{11}$$

Equation (10) corresponds to Thirlwall's rule (or law), where z stands for the growth of the rest of the world and is taken for simplification as constant. The only difference between our formulation and the original one is the subscript t that follows the foreign trade elasticities ratio. The interpretation of the expression is the same.

Equation (11) is the representation of the hypothesis discussed above and that we name as the *stratification mechanism.* Low growth rates - bellow α - generate pressures to an increase in non-price competitiveness. In the other hand, high growth rates – above α - would encourage the lock-in of the productive structure. All this process is intermediate by the stratification coefficient (or elasticity), , that captures the impact of growth on the non-price competitiveness. Finally, β aggregates all other forces not represented.

Since it is a crucial proposition we shall briefly discuss it here. It may sound at first implausible that structural change is induced by an economy with low growth. However, we argue that growth bellow certain rates can be expected to give rise to a pressure for reform; while high growth rates give incentives to "keep things like they are". This represents the struggle between reform vs *status quo* that we consider is part of growth and structural change processes. Bellow α

[2] McCombie and Roberts (2002) do provide for expositional purposes the sketch of a model in a broader discussion of the role of balance of payments in economics growth.

reform prevails while above α *status quo* prevails. As result of the prevalence of *status quo* we shall expect a stratification of the productive structure.

While ε is a technical structural parameter, α can be understood as a social construction. That is, a threshold that society implicitly chose of what is a "satisfactory" growth rate. A lower α is associated with countries that do not value GDP growth that much. On the other hand it is reasonable to suppose that a higher ε is associated with countries with complex and diversified productive structures because the economy takes more time to get locked in old structures.

Substituting equation (10) in (11) and rearranging, we obtain:

$$\dot{\rho_t} = \rho_t \left(\varepsilon\alpha + \beta - \varepsilon z \rho_t \right). \tag{12}$$

The expression above corresponds to a logistic one (see appendix 1). The story it tells is basically the inverted-U. In the beginning an increase in non-price competitiveness increases competitiveness itself up to a limit when there is a stratification of the economic structure. The speed of this process and the final level of the elasticities ratio depend on the parameters of the economy

When the system comes to apparent rest (or steady-state) . We have one non-trivial solution given by:

$$\rho^* = \frac{\varepsilon\alpha + \beta}{\varepsilon z} \tag{13}$$

with $\frac{\partial \rho^*}{\partial \varepsilon} < 0$.

As Thirlwall's rule (or law) itself, the result above is very simple but with a straight message. Higher stratification elasticity implies lower non-price competitiveness. We shall expect that economies with higher ε would lock-in themselves faster, conditional to a given threshold. This brings us to our second point. Higher threshold imply in higher foreign trade elasticities. Finally, there is a direct and positive relation between the autonomous growth rate and the equilibrium level of non-price competitiveness.

Therefore the BoPC growth rate is given by:

$$\frac{\dot{Y_t}}{Y_t} = \rho^* z = \frac{\varepsilon\alpha + \beta}{\varepsilon}. \tag{14}$$

Equation (14) shows that even though the original expression for Thirlwall's rule depends on the growth rate of the rest of the world, once the stratification mechanism is taken into account it does not anymore. This is an important (and not obvious) result that comes from the fact that equilibrium non-price competitiveness depends inversely on .

3.2 Numerical Simulations

We shall proceed to some numerical simulations in order to study the dynamic properties behind the model so far developed. The software employed was the E&F Chaos in its 2012 version.

First we represent a situation in which the world economy growths at a rate of $z = 3\%$, the stratification elasticity is $\varepsilon = 1$, threshold growth level $\alpha = 1.5\%$, and the autonomous component of non-price competitiveness growth is $\beta = 1.5\%$. We set it as a benchmark case. Two different initial conditions are established, namely, $\rho_0 = 1.5$ and $\rho_0 = 0.5$. The results are plotted in figure 1.

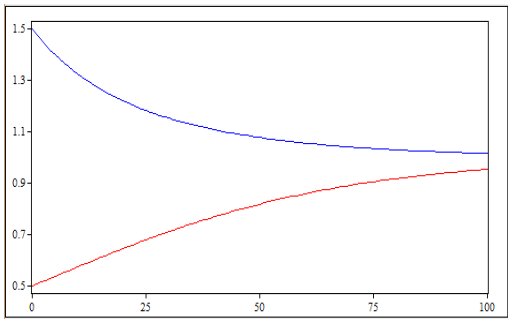


Figure 1: Benchmark economy

For both initial conditions the model shows slowly convergence. The blue line corresponds to an economy that use to growth faster than the rest of the world but is converging to the same growth rate. The red line represents the opposite situation. After one hundred periods both economies still have not reached the equilibrium.

A second case corresponds to an economy that in equilibrium is falling behind, that is, growths less than the rest of the world. We keep all the parameters and the initial conditions the same, except the stratification elasticity that now equals $\varepsilon = 3$. Consequently we have that $\rho^* \approx 0.67$. The results are plotted in figure 2.

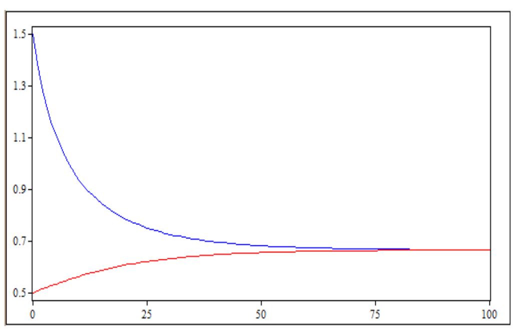


Figure 2: Falling behind economy

Once again we observe convergence to equilibrium. However in this case non-price competitiveness comes to a state of rests in 50 periods. Therefore an increase in the stratification elasticity not just reduces the elasticities in level but also increases the speed of converge. The blue line represents the path of an economy that use to growth faster than the rest of the world but after 10 periods starts to fall behind. The red line corresponds to an economy that is able to increase its growth rate but not enough to overcome the falling behind process.

Finally we consider the case of an economy that in equilibrium growths faster than the rest of the world. It could be a developing country in catching up or an already developed economy differentiating itself from the leading group. Once more we keep the same parameters except the stratification elasticity that now equals $\varepsilon = 0.6$. Non-price competitiveness at equilibrium approximates $\rho^* \approx 1.3$. The results are plotted in figure 3.

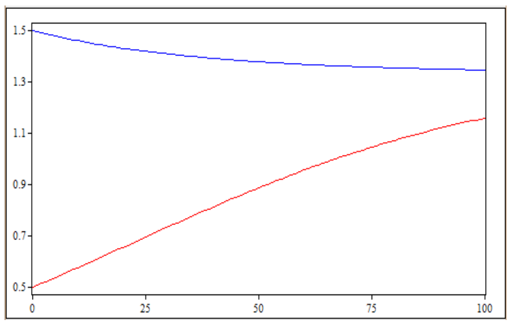


Figure 3: Catching up/differentiating economy

One more time the model shows convergence to its steady state values. However, since we are taking a lower stratification coefficient, convergence takes more time and goes to higher values of non-price competitiveness. The blue line corresponds to an economy that maintains its growth rate above the rest of the world. The red line represents an economy that was falling behind but after 50 periods is able to start catching up.

The model so far presented crucially depends on the stratification coefficient. For a given ε the dynamics are pretty clear. It is a way through which divergent forces on growth among countries are diluted over time, opening space for the appearance of falling-behind behavior or convergence clubs. However, even though it allows us to go one step further in the study of the dynamics behind non-price competitiveness in a BoPC growth framework, one might ask how it interacts with the rest of the productive structure. This issue is discussed in the next section.

4 Incorporating the supply side

According to the Keynesian hypothesis, investment is in the short and long run independent of savings that would be forthcoming from normal utilization of productive capacity. Traditionally the argument has been justified on the grounds that the level of capacity utilization is endogenous and its variations are modeled as the difference between investment and savings.

Recently it has been proposed that the Keynesian Hypothesis could be sustained through the endogeneity of the share in GDP of an autonomous component of aggregate demand. It implies that the effective demand principle relies on the existence of autonomous expenditures that do not generate capacity for the private sector. This proposition has gained some attention in the literature (e.g. Allain, 2015; Dutt, 2015; Pariboni, 2015; Lavoie, 2016) and is going to be used in this section to model the relation between the BoPC growth rate and capital accumulation.

Our exercise is in line with a recent contribution of Nah and Lavoie (2016) that extend this approach to a small open economy framework. However, our effort is different in the sense that we are interested exclusively in the long run dynamics of non-price competitiveness in economies that are BoPC.

It is necessary to incorporate the supply side of the economy in order to avoid the *inconsistence* (or over determination) *problem.* We begin stating the following production function:

$$Y_t = \min\left\{a_t K_t u_t ; q_t L_t\right\}, \tag{15}$$

where capital, K, and labour, L, are weighted by their respective productivity coefficients, a and q. Variable u stands for the level of capacity utilization and it is equal to the ratio of current output, Y, and potential output, Y^*. If inputs are used efficiently then the economy must be operating with a level of output that satisfies the following condition:

$$Y_t = a_t K_t u_t = q_t L_t. \tag{16}$$

Using the traditional definition of capital, empirical evidence seems to indicate that the capital productivity coefficient is fairly stable (e.g. Kaldor, 1957; Homburg, 2015; Barbosa-Filho, 2015). Taking logarithms and time derivatives:

$$\frac{\dot{u}_t}{u_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \tag{17}$$

$$\frac{\dot{q}_t}{q_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{L}_t}{L_t}. \tag{18}$$

From equation (17) we have that variations in the level of capacity utilization are given by the difference between the growth rate of output and capital accumulation. Equation (18) on the other hand states that the growth rate of labour productivity is obtain as a residual of the difference between the growth rate of output and labour. Both expressions directly come from the Leontief efficiency condition.

Turning back to equation (17) it follows that:

$$\frac{I_t}{K_t} = a\frac{I_t}{Y_t}u_t. \tag{19}$$

Defining $h_t = \frac{I_t}{Y_t}$ as the ratio between investment and output (or the propensity to invest), and disregarding capital depreciation so that $I_t = \dot{K}_t$ we have:

$$\frac{\dot{K}_t}{K_t} = a h_t u_t. \tag{20}$$

An increase in the investment-output ratio or in the level of capacity utilization directly increases capital accumulation. This comes again from the efficiency condition. In the limit, if there is no investment or no capacity utilization is being use, obviously there is no capital accumulation.

Recall the aggregate expenditure identity (equation 1). Define $C_t = cY_t$, where $0 < c < 1$ is the behavioral propensity to consume. As already defined, investment is given by $I_t = h_t Y_t$. Exports and imports are given by equations (4) ad (5). Assume constant terms of trade. Substituting those definitions in the identity, taking logarithms and time derivates we have:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \frac{\dot{h}_t}{1 - c - h_t + \theta_M \pi}, \tag{21}$$

where $\frac{\dot{A}_t}{A_t} = \theta_X \varphi z$ is the autonomous component growth rate, and θ_X and θ_M are the shares of exports and imports in aggregate demand, respectively. For simplicity we assume that the product $\theta_M \pi$ is constant. Moreover $\frac{1}{1-c-h_t+\theta_M \pi}$ is the Keynesian multiplier. In steady state $\dot{h}_t = 0$ and aggregate demand follows the weighted growth rate of exports, i.e. the autonomous component growth rate.

Now, we are dealing with an economy that is BoPC, which according to our first proposition means that it takes $\frac{D_t}{Y_t}$ as exogenously given in the long run. This means that in the long run $\frac{\dot{A}_t}{A_t} = \rho_t z$ because aggregate demand must follow the external constraint. Therefore the share of exports in aggregate demand becomes endogenous accommodating both growth rates. Using the definition of ρ_t we have:

$$\theta_X = \frac{\theta}{\pi + \theta}. \tag{22}$$

The result above holds for a BoPC economy because of the *fundamental identity*. It is also in line with the Kaldorian tradition which considerers that "from the point of view of any particular region, the 'autonomous component of demand' is the demand emanating from outside the region; [...] the rate of economic development of a region is fundamentally governed by the rate of its exports" (Kaldor, 1970, p.342). We can rewrite equation (21) as:

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z + \frac{\dot{h}_t}{1 - c - h_t + \theta_M \pi}. \tag{23}$$

Changes in aggregate demand have lagged effects on investment that may give rise in the long run to the Harrodian instability problem. Skott (2010) shows that the Harrodian instability condition can be satisfied by the following special case:

$$\frac{\dot{h}_t}{h_t} = \gamma \left(u_t - u_n\right) \tag{24}$$

where u_n is the planned, desired or normal level of capacity utilization and γ is an adjustment parameter.

Substituting equation (20), (23) and (24) in (17) we obtain:

$$\frac{\dot{u_t}}{u_t} = \rho_t z + \frac{\gamma\left(u_t - u_n\right) h_t}{1 - c - h_t + \theta_M \pi} - a h_t u_t. \tag{25}$$

The dynamic non-linear system is formed by equations (12), (24) and (25). When the economy comes to a state of apparent rest, $\dot{h_t} = \dot{u_t} = \dot{\rho}_t = 0$. In this situation, for h_t, u_t, and $\rho_t \neq 0$ we have that:

$$0 = \gamma\left(u_t - u_n\right), \qquad \text{(Harrodian curve)}$$

$$a h_t u_t = \rho_t z + \frac{\gamma\left(u_t - u_n\right) h_t}{1 - c - h_t + \theta_M \pi}, \qquad \text{(Capacity utilization curve)}$$

and

$$\beta = \varepsilon\left(\rho_t z - \alpha\right). \qquad \text{(Non-price competitiveness curve)}$$

From the Harrodian curve it is straightforward that in equilibrium the level of capacity utilization equals its normal rate. On the other hand the capacity utilization curve corresponds to the combination of endogenous variables that bring aggregate demand and capital accumulation growth rates to equilibrium. Finally the Non-price competitiveness curve brings to equality the effect of growth on competitiveness with its autonomous component.

Equilibrium values are defined and given by:

$$h^* = \frac{\rho^* z}{a u_n}, \tag{26}$$

$$u^* = u_n, \tag{27}$$

$$\rho^* = \frac{\varepsilon\alpha + \beta}{\varepsilon z}. \tag{28}$$

All equilibrium values are positive. An increase in non-price competitiveness has no permanent impact on the level of capacity utilization but implies in a higher investment-output ratio. It is an interesting result because while the level of capacity utilization accommodates differences between demand growth and capital accumulation through the investment-output ratio, is the investment-output ratio that accommodates in steady state variations in non-price competitiveness.

4.1 Stability properties

To investigate the local stability of the system we linearized it around the equilibrium values. So that:

$$\begin{bmatrix} \dot{h_t} \\ \dot{u_t} \\ \dot{\rho}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & J_{12} & 0 \\ J_{21} & J_{22} & J_{23} \\ 0 & 0 & J_{33} \end{bmatrix}}_{J} \begin{bmatrix} h_t - h^* \\ u_t - u^* \\ \rho_t - \rho^* \end{bmatrix}, \tag{29}$$

$$J_{12} = \gamma h^* > 0, \tag{29a}$$

$$J_{21} = -a\,(u^*)^2 < 0, \tag{29b}$$

$$J_{22} = \frac{h^*\left[\frac{\gamma}{a} - (1 - c - h^* + \theta_M \pi)\right]}{1 - c - h^* + \theta_M \pi} < 0, \tag{29c}$$

$$J_{23} = z u^* > 0, \tag{29d}$$

$$J_{33} = -\varepsilon z < 0. \tag{29e}$$

Outside equilibrium there must be room for the induced investment necessary to growth at its final equilibrium rate, and for the extra induced investment responsible for adjusting capacity to the trend of demand. This implies that the Keynesian multiplier and the Harrodian coefficient are low enough so that $\frac{1}{1-c-h_t+\theta_M\pi} < \frac{a}{\gamma}$. This condition guarantees that $J_{22} < 0$.

The characteristic equation of the Jacobian matrix is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{30}$$

where λ denotes a characteristic root. Each coefficient of equation (30) is given by:

$$\begin{aligned} a_1 &= -\operatorname{tr} J = -\left(J_{22} + J_{33}\right), \\ a_2 &= \begin{vmatrix} J_{22} & J_{23} \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{22}J_{33} - J_{12}J_{21}, \\ a_3 &= -\det J = J_{12}J_{21}J_{33}. \end{aligned}$$

The necessary and sufficient condition for the local stability is that all characteristic roots of the Jacobian matrix have negative real parts, which, from Routh–Hurwitz condition, is equivalent to the following inequalities:

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0 \text{ and } a_1a_2 - a_3 > 0.$$

Let us examine whether these inequalities hold. It is easy to check that J_{22} and J_{33} are negative, therefore $a_1 > 0$, and the first inequality is always satisfied. It is also straightforward that $J_{22}J_{33} > 0$ and $J_{12}J_{21} < 0$. Therefore $a_2 > 0$, and the second inequality is always satisfied. Moreover, $J_{12}J_{21}J_{33} > 0$ so that $a_3 > 0$, and the third inequality is also fulfilled. Finally we have to check if $a_1a_2 - a_3 > 0$. Through direct computation we have that:

$$a_1a_2 - a_3 = -\left(J_{22} + J_{33}\right)\left(J_{22}J_{33} - J_{12}J_{21}\right) + J_{22}J_{12}J_{21} > 0.$$

Therefore the system is stable around the equilibrium values.

For an exogenous growth rate of employment, labour productivity growth is directly obtained as a residual from equation (18) so that there is no *inconsistency problem*. Moreover if we state that labour productivity follows a Kaldor-Verdoorn rule - as does Palley (2003) and Oreiro (2016) - employment growth adjust to the difference between output and productivity growth rates, and once again there is no inconsistency.

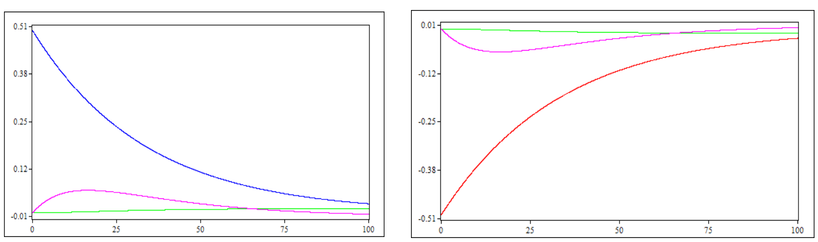
4.2 Numerical Simulations

In the third section we showed how the stratification mechanism operates in the BoPC framework for a given stratification coefficient while in section 4 we showed how it interacts with the supply side of the economy. We shall proceed to some numerical simulations in order to study the dynamic properties behind this broader structure.

First we represent a situation in which the world economy growths at a rate of $z = 3\%$, the stratification elasticity is $\varepsilon = 1$, threshold growth level $\alpha = 1.5\%$, and the autonomous component of non-price competitiveness growth is $\beta = 1.5\%$. Normal level of capacity utilization $u_n = 0.7\%$. Propensity to consume $c = 0.8$, and for simplicity the share of imports in output multiplied by the income elasticity of imports $\theta_M \pi = 20\%$. Finally the adjustment coefficient of investment $\gamma = 0.02$.

Our simulations are done in terms of deviations from the equilibrium values. As initial conditions we use the equilibrium values of h and u, and a deviation of 0.5 for ρ. From the combination of the parameters above we obtain $h^* = 0.215$, $u^* = 0.7$ and $\rho^* = 1$. We set it as a benchmark case. The results are plotted in figure 4:

Figure 4: Benchmark economy



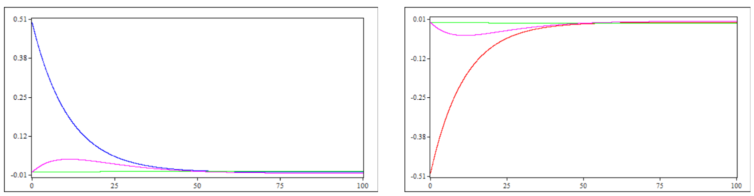
(4a) (4b)

For both initial conditions the model shows slowly convergence. Figure (4a) depicts an economy that use to growth faster than the rest of the world but is converging to the same growth rate. The blue line is non-price competitiveness while the green and purple lines are investment and capacity utilization, respectively. Because non-price competitiveness is above its equilibrium value, demand growths faster than capital accumulation and we observe an increase in the level of capacity utilization and of the investment ratio. However once non-price competitiveness converges to its equilibrium value, the other two variables also accommodate to their equilibrium values.

Figure (4b) represents the opposite situation, that is, an economy that used to growth less than the rest of the world but that is converging to the same growth rate. Because non-price competitiveness is bellow its equilibrium value, demand growths less than capital accumulation and we observe a decrease in the level of capacity utilization and of the investment ratio. However, once non-price competitiveness converges to equilibrium, the other two variables also accommodate to their equilibrium values.

Since we are interested in the impact of the *stratification mechanism* on the model we continue simulating an economy that in equilibrium is falling behind, that is, growths less than the rest of the world. We keep all the parameters and the initial conditions the same, except the stratification elasticity that now equals $\varepsilon = 3$. Consequently we have that $h^* = 0.14$, $u^* = 0.7$, and $\rho^* \approx 0.67$. The results are plotted in figure 5:

Figure 5: Falling behind economy

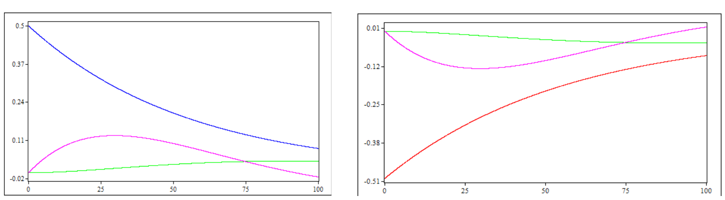


(5a) (5b)

Results are remarkable similar from the last simulation. Figure (5a) corresponds to an economy that use to growth faster than the rest of the world but after some periods starts to falling behind. Figure (5b) on the other hand represents an economy that starts a catching up process but interrupts it before it is complete. Because the stratification coefficient is higher we observe a faster convergence to equilibrium values.

Finally we consider the case of an economy that in equilibrium growths faster than the rest of the world. It could be a developing country in catching up or an already developed economy differentiating itself from the leading group. Once more we keep the same parameters except the stratification elasticity that now equals $\varepsilon = 0.6$. Equilibrium values are $h^* = 0.285$, $u^* = 0.7$ and $\rho^* \approx 1.3$. The results are plotted in figure 6:

Figure 6: Catching up/differentiating economy



(6a) (6b)

Once again the results are similar from previous simulations. For lower values of the stratification mechanism convergence is slower. However it is also possible to observe that the magnitude of the variation in capacity utilization and investment necessary to accommodate misalignments between demand and capital accumulation growth rates is higher. This suggests that the stratification mechanism influences not just the final values of investment and non-price competitiveness but also their path.

5 Final considerations

Even though there have been recent efforts in order to endogenize non-price competitiveness in the BoPC framework, we could say that we are still building a consensus. Most of the models strongly rely on *ad hoc* assumptions. It is well understood that distributive and technological questions matter and should be taken into account. But the subject is still open.

This paper aims to contribute to the discussion. We developed a model that formalizes the hypothesis that while growth rates generate pressures to an increase in the non-price competitiveness, high growth rates would encourage the lock-in of the productive structure. Such proposition was sustained on the grounds of the struggle between reform vs *status quo*. The inverted U relationship was first proposed by Thirllwall himself and discussed by authors like Setterfield and McCombie even though not in a formal way. We name it the *stratification mechanism*. Finally we incorporate the supply side of the economy into the structure of the model in order to explicitly avoid the over determination problem.

Our simulation exercises showed that under reasonable parameters the model behaves properly. In section 3 it is clear how the variation of the stratification elasticity impacts non-price competitiveness in different scenarios. Section 4 on the other hand shows how this mechanism depends on the productive structure of the economy.

We believe that our exercise was able to formalize the *stratification mechanism* under very general assumptions. Our results explore the non-linear dynamics of the model under the umbrella that the effective demand principle relies on the existence of autonomous expenditures that do not generate capacity for the private sector. We also provided a particular way to derivate and (maybe more appropriate) to interpret Thirlwall's rule (or law). Growth is as complex as fascinating and we hope that above everything this paper helps to enlighten a glance of it.

Appendix

The dynamics of non-price competitiveness are given by:

$$\frac{\dot{\rho}_t}{\rho_t} = \left(\varepsilon\alpha + \beta - \varepsilon z \rho_t\right).$$

This expression can be rewrite as:

$$\frac{\dot{\rho}_t}{\rho_t} = \left(\varepsilon\alpha + \beta\right)\left(1 - \frac{\varepsilon z}{\varepsilon\alpha + \beta}\rho_t\right). \tag{A1}$$

Define:

$$x_t = \frac{\varepsilon z}{\varepsilon\alpha + \beta}\rho_t, \tag{A2}$$

$$m = \varepsilon\alpha + \beta. \tag{A3}$$

Log-differentiating (A2) on time we have that:

$$\frac{\dot{x}_t}{x_t} = \frac{\dot{\rho}_t}{\rho_t}. \tag{A4}$$

Substituting (A2)-(A4) in (A1) and after some trivial algebraic manipulations we have:

$$\dot{x_t} = mx_t\left(1 - x_t\right). \tag{A5}$$

(A5) is the logistic function in its classical form.

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