The Effects of Fiscal Policy on Income Inequality in a model with Heterogeneous Agents

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Resumo
A desigualdade de renda é um dos principais obstáculos ao desenvolvimento econômico. Compreender a dinâmica desse fenômeno social é central para entender o crescimento econômico também. Assim com base em modelos que se relaciona a política fiscal, a desigualdade e crescimento que propõe-se o seguinte problema: Dada a distribuição inicial do capital, qual é o efeito de um conjunto de tributação fixa, sobre a desigualdade? A fim de responder a esta pergunta constrói-se um modelo seguindo a tradição de estudos nesta área. O modelo proposto é uma versão de um equilíbrio competitivo do modelo neoclássico básico de crescimento, que incorpora desigualdade e agentes heterogêneos: pobres e ricos, permitindo compreender este problema de uma forma dinâmica. Usamos o problema de Ramsey para determinar as sequências ótimas para os três tipos de impostos, capital, trabalho e consumo em uma economia não estocástica. A solução analítica encontrada sugerem que, a taxação ótima no estado estacionário sobre o capital deve ser sempre zero, independentemente do favoritismo do governo em relação a um agente em particular. Além disso, o governo deve financiar suas transferências por diferentes combinações de impostos sobre o consumo e de trabalho.

Palavras-chave: Desigualdade, Política Fiscal, Problema de Ramsey, Crescimento Econômico, Distribuição de Renda.

Abstract
The income inequality is one of the main obstacles to economic development. Understanding the dynamics of this social phenomenon is central to understand economic growth also. Thus based on models that relates fiscal policy, inequality and growth we proposed the following problem: Given the initial distribution of capital, what is the effect of a set of flat-rate taxation, on inequality? In order to answer this question we built a model following the tradition of studies in this area. The model we propose is a version of a competitive equilibrium of the basic neoclassical growth model, which incorporate inequality and heterogeneous agents: poor and rich, allow us to understanding this problem in a dynamic way. We use Ramsey problem to determine the optimal sequences for the three types of tax, capital, labor and consumption in a nonstochastic economy. The analytical solution found suggest that in steady state optimal tax on capital should always be zero regardless of the government’s favoritism towards particular agents. Also the government should financed his transfers using different combinations of taxes on consumption and labor.

KeyWords: Inequality, Fiscal Policy, Ramsey Problem, Economic Growth, Income Distribution.

JEL codes: D63, E62, O40

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1 Introduction

Since the 1990s, Brazil has experienced great changes in its economic policy, these transformations brought economic stability and the possibility of real income gains. But these changes was not enough to change the process of income concentration and increased inequality. Since the beginning of the 2000s, Brazil’s economic policy has turned its attention to social programs, particularly income transfers. So understand dynamics of inequality has an important role in Brazilian economic policy.

As pointed out by Shin (2012), income inequality\(^1\) refers to disparities in the distribution of income, that is, the gap between the rich and poor in a country. Many studies in economic theory has turned its attention to the relationship between inequality and economic growth.

It is importante to point out that inequality can be harmful to the economy’s growth rate. This result has brought social equity to center of policy debate in Brazil. Through appropriate use of taxes and transfers, government can correct socially undesirable distributive outcomes arising from market forces. The evidence shows that most developed economies are more effective at this redistributive function, than developing economies (GONI; LOPEZ; SERVEN, 2011).

Many studies in economic theory has turned its attention to the relationship between inequality and economic growth. One of the most known theory on the international literature it is the study of Kuznets (1955), this author assumes that in the early process of capital accumulation the income and wealth distribution becomes unequal, but after wealth become enough accumulated, wealth and income distribution equalize and inequality decreases. The hypothesis proposed by Kuznets indicates that there is a relationship between the level of inequality and economic growth, although this relationship is not linear\(^2\). Many studies have indicated that initially, in the short run inequality standard increases with economic growth and, in the long run decreases from a turning point. This effect is attributed by the authors to two factors, wage growth and the decrease in the return of investment.

However there are other theories used to explain the relationship between inequality and economic growth, but they are different and there is no consensus of how this relationship occurs. Among these theories, those based on existing transition between the agricultural and industrial sectors, imperfections in the financing system and progress technological stands out.

The several theories presented while different, show there is a strong evidence that economic growth and inequality are related, it is also possible that different economic policies influence the relationship between this two variables. Nevertheless the way how this relationship occurs still is controversy, as point by Muinelo-Gallo and Roca-Sagales (2011) the issue of the sign and magnitude of theses effects is an open question. Garcia-Penalosa and Turnovsky (2007) argues that earlier evidence suggested a negative trade-off between growth and inequality, therefore recent studies have tended to support a positive relationship. Shin (2012) shows that empirical evidences in studies of East Asian and South American countries presents a negative relationship between income inequality and economic growth. However in developed countries, such as United States and France this relationship is positive.

More recently, some authors have shown two ways of how inequality and growth are not conflicting related. Chen and Turnovsky (2010) indicate that growth rate and inequality will be depend of the set of forces to which both are reacting. Barro (2000) shows that the positive or negative relationship depends on the development stage of the economy. This author also shows the effect of

\(^1\) The term inequality extends over various economic and social aspects, such as income distribution, access to education and health services, however in this work inequality indicates that income distribution is not equal, so whenever we refer inequality we are saying that income distribution is not equal.

\(^2\) This non-linearity is described as a curve in form of \(U\) inverted.
income inequality on economic growth is negative in poor countries, but is presented in a positive way in rich countries.

Thus, it is more likely that in countries like Brazil inequality impacts negatively on economic growth. So inequality is a barrier to economic growth and also to development in Brazil. The inequality can reduces the potential economic growth in long-run and welfare too, Muinelo-Gallo and Roca-Sagales (2011) argues that distribution of wealth can affect the economic growth rate in the long-run by modifying the size and the composition of aggregate demand.

A central concern of this discussion is the role that government policies may play in reducing economic inequalities, and determining the effects on economic growth rate. Important issue raised in this context are what are the most efficient ways to reduce inequality? and also what are the main instruments capable of reducing inequality?

The economic theory has suggested that fiscal policy can affect inequality in the long run. This is because fiscal policy can impact aggregate demand, distribution of wealth and income, in addition to the production capacity of the economy. Thus, the choice of a distributive fiscal policy strategy has become of crucial importance in achieving economic growth and reduce inequality at the same time (MUINELO-GALLO; ROCA-SAGALAS, 2013).

One of the ways in which tax policy impacts inequality is presented by Rebelo (1991). He shows that redistribution of income by using income tax on has negative effect on growth because reduces the gap of income and wealth and consequently reduces growth. Income redistribution financed through taxes on income also reduces the incentive to accumulate wealth.

Using an endogenous growth model with elastic labor supply, Garcia-Penalosa and Turnovsky (2007) investigate how different ways of financing an investment, impact the income distribution, they show that growth is driven by policies that increase the return of capital. The authors also argue that capital is more unequally distributed than labor, hence, higher returns to capital represent greater income inequality.

There are others authors that argue inequality decreases as consequence of the tax policy chosen by median voter as show by Muinelo-Gallo and Roca-Sagales (2011, p. 10) “greater fiscal redistribution through distortionary taxes, while it may reduce investment incentives, also decreases social conflict and contributes to greater stability that encourages productive activities and capital accumulation”. Therefore more unequal democratic societies demand redistribution financed by distortionary taxes.

The reduction of inequality and, therefore, a policy of income redistribution more equitable may be achieved through the fiscal policy. A progressive tax policy, for example, can modify the inequality in the long run. What is the optimal fiscal policy, which with less distortions on growth decreases income inequality? the answer to that question is part of the optimal tax theory. This theory, the well known Ramsey’s problem is the study of what amount of tax causes minimum distortion and reduces inefficiency.

The literature of optimal taxation focus on the effects of fiscal policy on economic growth, there is also several studies that focus on the impact of other macroeconomic variables such as income inequality. An example of this theory is the seminal study of Chamley (1986), which shows that the optimal tax on capital in the long run should be zero. Though there are other studies as conducted by Aiyagari (1995) that show in a model with incomplete insurance markets and borrowing constraints, optimal capital tax should be positive, even in the long run.

The effects of fiscal policy considering endogenous saving rates is used by Judd (1985) to show that fiscal policy has transitory effect on output, and has no permanent effect on economic growth. In other study Oh (2013) showed that Real Business Cycle models with only shocks to total factor
productivity can not explain the inequality. Furthermore, inequality is moderately countercyclical in
United States of America with data on an annual basis.

Fochezatto and Bagolin (2006) indicate that a progressive tax policy have small effects on
income distribution. On the other hand, a policy of income redistribution, made by government
transfers thought income taxation, have a significant impact on inequality.

There are few authors that estimate the impact of fiscal policy on inequality, this studies focus
primarily in econometrics estimates for data panel, but the results are mostly not robust, presenting
evidence that the size of the impact and its significance depends on the set of control variables used
and the initial conditions of the economy.

As stated high inequality can be harmful to the economy’s growth rate. This result has brought
social equity to center of policy debate in Brazil. Through appropriate use of taxes and transfers,
government can correct socially undesirable distributive outcomes arising from market forces. The
evidence shows that most developed economies are more effective at this redistributive function, than
developing developing economies (GONI; LOPEZ; SERVEN, 2011). Through the necessary fiscal
reforms, its redistributive function effectively. The redistributive impact of country’s fiscal system
is shaped by three factors: (1) The feasible volume of transfers. (2) Incidence of taxation and (3)
incidence of transfers.

Despite its importance, little has been researched about the impact of fiscal policy on income
inequality. Thus investigate the relationship between fiscal policy and income inequality can show
important mechanisms through which the latter can be reduced. It also highlights the impact of these
variables on economic growth, because more equitable initial allocations lead to a higher growth rate.

There is a lack in the national literature of analyzes the impact of fiscal policy on income
inequality, especially studies that analyze the issue dynamically. To understand the linkages between
fiscal policy, inequality and economic growth, it is necessary to adopt a structural, consistent dynamic
general equilibrium approach.

Thus based on models that relates fiscal policy, inequality and growth we propose the fol-
lowing problem: Given the initial distribution of capital, what is the effect of a set of flat-rate taxation,
on inequality? Besides to answer this question, this work aims to understand how the government ob-
tains resources from the private sector through taxation and redirects them to current spending and
transfers.

Therefore, this work intends to contribute to literature, presenting a model that captures the
effect of a set of fiscal policies on inequality and consequently economic growth. This type of analy-
sis because they capture the key interactions between agents and the economic system. The model
introduced here aim to describe the behavior for the economy as a whole by analyzing the interaction
of many microeconomic decisions.

The hypotheses are: (i) there is a causal relationship between fiscal policy and income inequa-
ity regardless of the type of tax used by the government; (ii) the type of flat-rate tax: consumption,
capital and labor income, can simultaneously affect the growth and inequality rate of the economy in
different magnitudes; and (iii) consumer tax has a larger impact on inequality.

2 Theoretical Framework

2.1 Fiscal Policy and Inequality

The analysis of fiscal policy effects on the economy is usually done in order to study economic
growth. One of mains instruments of fiscal policy are changes in the level and composition of taxation.
These changes can affect crucial macroeconomic variables, such as: aggregate demand, savings and investment and the distribution of income.

The studies of economic growth theory, focus in factors that cause inequality. In this context inequality can be explained due to imperfections in the credit market, there are many studies that explain the distribution of income and consequently inequality by the limited participation of financial markets (GALOR; ZEIRA, 1993; MOTTA; TIRELLI, 2012; FOCHÉZATTO; BAGOLIN, 2006).

As point out by Mankiw (2000), households with low-wealth fail to smooth consumption over time and do not accumulate wealth, on the other hand households with high wealth smooth consumption not only from year to year, but also from generation to generation. This process is capable of generating income inequality, and shows the importance of fiscal policy that redistribute the income correcting this problem.

In recent literature of optimal taxation, the debate has turned to the role of fiscal policy in distributions issues. Many authors have questioned the adequacy of the representative agent model to analyze these questions. It is considered necessary the use of models with heterogeneous agents. Introduce another type of agent in a economic growth model, in special the case of our model with poor and rich agents, does not alter the assumptions usually made in this type of analysis. This type of modeling brings a gain in terms of analytical results since differentiate between rich and poor agents, allowing a better understanding of fiscal policy effects on inequality.

However, before we build a model that incorporates these hypotheses is necessary to build a standard economic growth model according to the literature, to show the changes which we propose on this model and its implications to the mains conclusions. In the standard models of optimal growth theory, where the economy has only one sector, a representative household solves a version of the savings problem with constant returns to scale production function. This function usually is: $F(K, L)$, where $L$ is labor input and $K$ represents the physical capital. When the return on capital is tax-free, the gross rate of return is equal to the marginal product of capital, less depreciation.

According to Sargent and Ljungvist (2000) we have: $R_t+1 = F_{k,t+1} + (1 - \delta)$, where $R_t+1$ is gross rate of return, $F_{k}(k, t + 1)$ is the marginal product of capital and $\delta$ is the depreciation rate. We use a general utility function $U(C, l)$, where $l$ is the household’s leisure and $C$ represent the consumption level.

$$\max_{C_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t, l_t)$$ (1)

The utility function is concave and twice differentiable. Using this assumptions Sargent and Ljungvist (2000) build a simple growth model with no taxes. The model proposed by these authors it is a maximization of social planer problem (equation 1). This maximization is subject to a resources constraint as shown in equation (2).

$$c_t + k_{t+1} = f(k_t)$$ (2)

With these equations we can make the Lagrangian and solve the maximization problem. After solve the maximization problem and find the First Order Necessary Conditions - FONC, we can find the Euler condition (Equation 3) that is:

$$U_{C_t} = \beta U_{C_{t+1}} \left[ F_{k_{t+1}} + (1 - \delta) \right]$$ (3)
Where \( U_{C_t} \) is the marginal utility consumption at period \( t \) and is the marginal utility of consumption at \( t + 1 \). Because of constant returns to scale property, \( F_k(K,N) = f'(k) \), where \( N \) is the labor demand on this economy, \( k = K/N \) and \( F(K,N) = Nf(K/N) \). In steady state, with no population growth where \( k \) is constant in time, the equation 3 must satisfy:

\[
\rho + \delta = f'(k) \tag{4}
\]

Where \( \beta^{-1} = (1 + \rho) \). To solve this equation is necessary to find the value of \( k \) know as augmented golden rule that show the level of capital-labor ratio of steady state. Equation (4) represents how technology, \( f \), \( \delta \) and time preference, \( \beta \), are determinants of level of capital when both factors are not subject to taxation in steady state. An important conclusion of the model is that the cross-distribution of wealth and consumption replicates itself over time, so that each individual share always occupies the same position, so there is no mobility. The initial distribution of wealth perpetuates the initial inequality remains over time.

Then it is necessary a model where there is a mechanism for changes in inequality, and this can occur through fiscal policy. We can introduce fiscal polices in this model with a tax of flat-rate marginal rate \( \tau_{k,t+1} \) . Though if income from capital is taxed at the flat rate marginal rate, then the Euler equation (3) becomes:

\[
U_{C_t} = \beta U_{C_{t+1}} \left[ F_{k_{t+1}}(1 - \tau_{k,t+1}) + (1 - \delta) \right] \tag{5}
\]

If the flat rate tax on capital is constant, so in steady state, we can find a result similar of case before, it must satisfy this condition:

\[
\rho + \delta = (1 - \tau_k)f'(k) \tag{6}
\]

We now can conclude that taxing capital diminishes the steady state capital labor ratio. The equations (4) and (6) are the most important of studied in the dynamic theory of optimal taxation and show that optimal is do not tax capital on long run. Based in this model, Chamley (1986) and Judd (1985) proposes a model with flat rate taxes on labor and capital income at different rates and force the government to finance itself with a stream of flat-rate taxes on capital and labor. This modeling of optimal taxation is known as Ramsey (1927) problem and studied the behavior limits of optimal taxation.

Sargent and Ljungvist (2000) pointed out the important role of Euler equation in determining the limiting tax rate on capital in a non-stochastic economy and therefore, find a solution to the Ramsey (1927) problem. In the Ramsey problem the Euler equation takes a form similar to equation (5), therefore household’s Euler equation becomes:

\[
W_{C_t} = \beta W_{C_{t+1}} \left[ F_{k_{t+1}} + (1 - \delta) \right] \tag{7}
\]

\[
W(C_t, l_t) = U_C(C_t, l_t) + \Phi \left[ U_{C_t} C_t + U_{l_t} (1 - l_t) \right] \tag{8}
\]
Where $\Phi$ is a Lagrange multiplier on the government’s inter-temporal budget constraint. In the Ramsey problem the taxes must be such that both (3) and (8) always hold. Thus if a steady state exists, where $C_t, l_t, k_t, \tau_{kt}$ all converge and government expenditure sequence converges (CHAMLEY, 1986; JUDD, 1985). It also must satisfy equation (4) and (6). The main conclusion of these conditions is that, in optimal $\tau_k = 0$, in other words, steady state properties of two versions of consumption Euler equation results that it is optimal not to tax capital asymptotically (SARGENT; LJUNGVIST, 2000).

This conclusion is robust if the government issuing debt or execute a balanced budget in the sense of Ricardian equivalence in every period. However if the tax system is incomplete, the optimal tax on capital can be different from zero.

On the other hand, Aiyagari (1995) shows that with incomplete insurance markets and borrowing constraints, the optimal capital tax rate is positive in the long run. Jones, Manuelli and Rossi (1997) extend this model to allow the accumulation of human capital, assuming constant returns and a linearly homogeneous function in human capital, they show that a zero tax also applies to income from labor. Chari Lawrence J. Christiano (1994) found that taxing labor causes little fluctuation while taxing private capital cause large fluctuations. There are also studies that show the effects of taxation on consumption (see Mendoza, Milesi-Ferretti and Asea (1997)).

An important aspect of optimal taxation problem introduced above is determining income taxes, which can be regressive, flat, or progressive. Furthermore, there is one type of tax that does not create a any distortion, the lump-sum tax. The lump-sum tax does not create distortions because these taxes do not depend on agents decisions.

Although taxing capital in a progressive way is often considered an appropriate instrument for redistribution, Chamley (1986) argues that doing so is not an efficient policy in the long run (SWARBRICK, 2012). The Ramsey problem focus in how government maximize households’ welfare subject to raising set revenues through distortionary taxation. We propose a model with three types of flat rate tax: consumption, capital and labor, and the impact of theses taxes on income inequality.

### 3 Model with Heterogeneous Agents

In order to study the impacts of fiscal policies on inequality we built a model following the tradition of studies in this area. The model we propose is a version of a competitive equilibrium of the basic neoclassical growth model, which incorporate inequality and heterogeneous agents, allow us to understanding this problem in a dynamic way.

In a economy described by a representative agent, the household receives income from labor and capital and chooses a path of consumption and capital investment to maximize their utility, the government taxes both labor and capital income and also is subject to intertemporal budget constraint. Without any market imperfection, it is best not to distort the choices of that consumer, so a lump-sum should be implemented. In the real economies, lump-sum taxes are rarely because this tax falls equally on the rich and poor, placing a greater relative burden on the most poor.

The model we built introduces a new factor inequality in an attempt to show how fiscal policy can alter the distribution of wealth of an economy. This model assumes heterogeneous agents, opposing the use of representative agent as in Sargent and Ljungvist (2000). Krusell and Smith (1998) showed that while aggregate consumption behavior is enough to analyze most macroeconomic variables, in order to study the distributional effects of tax policies is necessary to introduce with different types of agents in the model.

A model with heterogeneous agents should consider the empirical evidence as suggested by Mankiw (2000), he show that significant part of the population lives without virtual wealth, spending
the majority of their income and saving a small portion. Meanwhile there is a small portion of the population with high level of wealth, the wealthiest Individuals are able to smooth consumption over time and save a large portion of his wealth.

Thus a model with heterogeneous agents must have, two types of agents: poor and rich, the difference between them is the amount of accumulated wealth and savings rate (marginal propensity to save). In fact the difference in the marginal propensity to save is critical to building a model that considers inequality as endogenous. This one of the main mechanisms by which fiscal policy can affect income inequality. Kaldor (1955) argues that economic growth can affect inequality through the marginal propensity to save of the agents, this is because the marginal propensity to save is greater for rich than the poor. Fiscal policy therefore can change not only the level of wealth of each agent, but also their marginal propensity to save, affecting simultaneously, income inequality and economic growth.

Muinelo-Gallo and Roca-Sagalas (2013) indicates that a fiscal policy which can redistributing wealth from the rich, who has marginal productivity of investment relatively low. Due to decreasing returns on individual investments, to the poor who has marginal productivity of investment relatively high, but who can not invest more than their limited endowments, would enhance aggregate efficiency and growth and manly reduce income inequality.

This relationship between agents wealth and their relative allocation of time to leisure it is also important for a redistributive fiscal policy. Wealthier agents have a lower marginal utility of wealth, they therefore increase leisure, and reduce labor supply. Given their relative capital endowments, this translates to an endogenously determined distribution of income. As we will see exist various structural changes through which a redistributive fiscal policy affect inequality and the economy as a whole, including an increase in productivity and savings.

3.1 Households

To understand the inequality is constructed a model with two types of agents: poor and rich. The literature on fiscal policy and income inequality usually supposes that the poor (P) consume all their income each period and do not keep savings and consequently wealth. The rich (W) are owners the capital and are able to smooth consumption in each period, as can be seen in models of Mankiw (2000), Swarbrick (2012). The model proposed by Judd (1985) for example has two classes of agents, one class are workers that do not save and other class call capitalists who do not work, there is not any kind of redistribution in the limit of all this model. Therefore, in there models inequality is exogenous given.

Different from this previously studies conducted such as Mankiw (2000), Gali, Salido and Valles (2007) is supposed in this work that both agents can own physical capital and that there is a difference $\gamma$ in return of savings for rich and poor agent, therefore:

$$R^P_t = R^W_t - \gamma$$

Where $R^P_t$ is return for the poor agent and $R^W_t$ is the marginal product of capital. We also assume $\overline{K}$ be the minimum level of capital to become a rich agent. So poor agents can also save and accumulate wealth. This difference in the model allows social mobility, in the form in which poor agents can become rich and rich agents may become poor. This change enables analyzing inequality as an endogenous variable in the model, allowing the study of the effects of fiscal policy on inequality.

Since the model has two types of agents is necessary to define the proportion of the population that corresponds to the poor $\theta_t$ which must satisfy the following condition $\theta_t < 1$, so the percentage
of the population will be rich is \((1 - \theta_t)\). The initial condition of \(\theta\), constitutes an aggregate state variable at the beginning of economy, but becomes an endogenous variable in the following periods and is determined by fiscal policy in the previous period and the agents decisions.

3.2 Rich Agent

For this problem it is considered that households want to maximize their utility subject to a budget constraint:

\[
\max_{\theta_t} \sum_{t=0}^{\infty} \beta^t U(W, C_t, H_t)
\]  

Where \(C_t\) is the consumption of wealthier households and \(H_t\) is the hours dedicated to work for wealthier agent on time \(t\), discounted by a intertemporal rate \(0 < \beta < 1\). We will use \(1 - H_t\) to represent the hours spent in leisure. The households maximize utility subject to budget constraint:

\[
(1 + \tau C_t)C_t + I_t = (1 - \tau H_t)H_t + (1 - \tau K_t)K_t, \quad \text{if} \quad K_t \geq \bar{K}
\]  

Rich agents are defined, in this model, as those agents who have a level of capital higher or equal than the threshold level \(\bar{K}\). Rich agents have a higher return of capital, \(R_t\) and different scheme of labor and capital income taxation, \(\tau_{H,W}, \tau_{K,W}\).
From FONCs we can find the marginal utility of consumption and leisure. Isolating \( \lambda_t \) in the first two derivatives, equation (14) and (15) we can find the labor supply:

\[
\frac{U_H(C_t^W, 1 - H_t^W)}{U_C(C_t^W, 1 - H_t^W)} = \frac{(1 + \tau_t^{H,W})W_t}{(1 + \tau_t^{C,W})} \tag{18}
\]

The labor supply show how the rich agent trade labor and leisure over time, and also show the relation between two type of tax and wage are important to determining this ratio. Divining equations (14) by (17) and substitution in equation (14) we can find the Euler Equation:

\[
U_C(C_t^W, 1 - H_t^W) = \beta U_C(C_{t+1}^W, 1 - H_{t+1}^W) \frac{(1 + \tau_t^{C,W})}{(1 + \tau_{t+1}^{C,W})} \left[ (1 - \tau_t^{K,W})R_t + (1 - \delta) \right] \tag{19}
\]

Equation (19) is the well-known Euler Equation. It shows that the intertemporal marginal rate of substitution between consumption at period \( t \) and period \( t+1 \) equals to the relative price between both consumptions. Note that even if consumption tax is constant over time, taxation on capital income distorts the relative price of intertemporal consumption (CHAMLEY, 1986).

### 3.3 Poor Agent

In this model poor agent has a similar maximization problem as the rich agent, that is, maximizing a utility function:

\[
\max \sum_{t=0}^{\infty} \beta^t U_P(C_t^P, H_t^P) \tag{20}
\]

Where the variables are the same of case of rich agent and superscript \( P \) indicate the type (Poor) of agent. The poor agent receive income from work and savings on capital, and want to maximize their utility subject to a budget constraint:

\[
(1 + \tau_t^{C,P})C_t^P + I_t^P = (1 - \tau_t^{H,P})W_t^P H_t^P + (1 - \tau_t^{K,P})R_t^P K_t^P + T_t^P, \quad \text{if } K_t < \mathcal{K} \tag{21}
\]

Where \( T_t^P \) represent the government’s transference funded by the raise of tax on over the rich agent. There is three forms of tax that is different of the rich agent: \((1 + \tau_t^{C,P}), (1 - \tau_t^{H,P})\) and \((1 - \tau_t^{K,P})\) over consumption, labor and capital respectively. The other variables are same for case of rich agent. Note that both problems, rich and poor are different only because fiscal policy and interest rate received on savings \((\tau_t^{C,W}, \tau_t^{C,P}, \tau_t^{H,W}, \tau_t^{H,P}, \tau_t^{K,W}, \tau_t^{K,P} \text{ and } R_t^P = R_t^W - \gamma)\). Based on the equations of the poor agent problem we can find the FONCs, \( \forall t \):

\[
\frac{\partial L^P}{\partial C_t^P} = \beta^t U_C(C_t^P, 1 - H_t^P) - \lambda_t^P (1 + \tau_t^{C,P}) = 0 \tag{22}
\]

\[
\frac{\partial L^P}{\partial H_t^P} = -\beta^t U_H(C_t^P, 1 - H_t^P) + \lambda_t^P (1 + \tau_t^{H,P})W_t = 0 \tag{23}
\]

\[
\frac{\partial L^P}{\partial K_{t+1}^P} = -\lambda_t^P + \lambda_{t+1}^P \left[ (1 - \tau_{t+1}^{K,P})R_t^P + (1 - \delta) \right] = 0 \tag{24}
\]

\[
\frac{\partial L^P}{\partial \lambda_t^P} = -(1 + \tau_t^{C,P})C_t^P - I_t^P + (1 - \tau_t^{H,P})W_t^P H_t^P + (1 - \tau_t^{K,P})R_t^P K_t^P + T_t^P \tag{25}
\]
From FONCs we can find the marginal utility of consumption and leisure. Dividing equations (22) by (23) we can find the labor supply:

\[
\frac{U_H(C_t^P, 1 - H_t^P)}{U_C(C_t^P, 1 - H_t^P)} = \frac{(1 + \tau_t^{H,P})W_t}{(1 + \tau_t^{C,P})} \tag{26}
\]

Equation (26) shows how poor agent trades both goods: consumption and leisure at period \( t \). The marginal rate of substitution between consumption and leisure equals to the relative prices between consumption and leisure where the price of consumption good at period \( t \) is normalized to one and the wage rate, net of taxation, is the opportunity cost of leisure. Note that as long as taxation on consumption is different from taxation on labor income for poor agents, the taxation distorts the relative price of both goods. Divining equations (22) by (23) and substitution in equation (24) we can find the Euler Equation:

\[
U_C(C_t^P, 1 - H_t^P) = \beta U_C(C_t^{P+1}, 1 - H_t^{P+1}) \frac{(1 + \tau_{t+1}^{C,P})}{(1 + \tau_{t+1}^{C,P})} [(1 - \delta_{t+1} K_t^P) R_{t+1}^{P} + (1 - \delta)] \tag{27}
\]

Equations are (26) and (27) are the counterparts of equations (18) and (19) of rich agents. The Euler equation shows the relationship between intertemporal consumption and all type taxes, return of capital and depreciation.

### 3.4 Firm

The function of the firm can be assumed as Cobb-Douglas: \( Y_t = F(H_t, K_t) \), usually the marginal products are equal to factor prices, this condition is known as production function with zero profits. A representative firm rents capital and employs labor to produce a consumption good, this way we have the following equation:

\[
Y_t = AH_t^\alpha K_t^{1-\alpha} - W_t H_t - R_t K_t \tag{28}
\]

Where \( Y_t \) is the product, \( H_t \) are household labor provided at time \( t \) and \( K_t \) is capital stock, \( W_t \) and \( R_t \) are wages and capital income respectively. From the First Order Necessary Conditions - FONCs, \( \forall \ t \) we can have:

\[
W_t = \alpha A \left( \frac{H_t}{K_t} \right)^{\alpha - 1} \tag{29}
\]

\[
R_t = (\alpha - 1) A \left( \frac{H_t}{K_t} \right)^{\alpha} \tag{30}
\]

The equations (29) and (30) are the first order conditions for the firms problem with technical progress assumed to be constant. The first equation show the wage is equal to marginal product of labor, and the second equation show the rent tax is equal to marginal product of capital.
3.5 Government

The government’s goal is to maximize welfare of the economy subject to raising set revenues through distortionary taxation.

The government collects taxes on household income of labor and capital to exogenously fund transfers to the poor. The government also collects tax on consumption. The government is supposed to operate with a balanced budget constraint:

\[ G_t + T_t = \tau_H W_t H_t + \tau_K R_t K_t + \tau_C C_t \]  \hspace{1cm} (31)

Where \( T_t = \theta_t T^P_t \), is transference for the poor agent and \( G_t \) is spending of government given exogenously. The government tax labor, capital income and consumption of the both agents with: \( \tau_{H,W} t \), \( \tau_{H,P} t \), \( \tau_{K,W} t \), \( \tau_{K,P} t \), \( \tau_{C,W} t \), \( \tau_{C,P} t \), for the rich and poor agent respectively.

Therefore we have in the aggregate the sums of tax so that: \( \tau_H = \tau_{H,W} + \tau_{H,P} \), for labor, \( \tau_K = \tau_{K,W} + \tau_{K,P} \) for capital, and \( \tau_C = \tau_{C,W} + \tau_{C,P} \), for consumption.

3.6 Competitive Equilibrium

As the model incorporates two types of agents is necessary to define the behavior of the variables in the model. The variables \( C_t, K_t \) and \( H_t \) are divided between the two types of agents, so the aggregate should be added to take:

\[ C_t = \theta_t C^P_t + (1 - \theta_t) C^W_t \] \hspace{1cm} (32)
\[ H_t = \theta_t H^P_t + (1 - \theta_t) H^W_t \] \hspace{1cm} (33)
\[ K_t = \theta_t K^P_t + (1 - \theta_t) K^W_t \] \hspace{1cm} (34)
\[ \theta T_t = \theta T^P_t \] \hspace{1cm} (35)

Every period, the economy is bound by the resource constraint \( Y_t = C_t + I_t + G_t \). The competitive equilibrium government policy (tax) allocations and prices are set based on the following conditions:

- Families maximize their utility subject to the restrictions for both types of agents.
- Firms maximize profit using capital and labor.
- The government budget is balanced.

The government solves the problem of Ramsey to find the allocations, prices and fiscal policy that maximizes social welfare.

The effects of policy affect prices \( \{ R, W \}_{t=0}^\infty \) allocations \( \{ H^W, H^P, C^W, C^P, K^W, K^P \}_{t=0}^\infty \).

The equilibrium conditions are used to solve the model, and find the solution of Ramsey’s problem.

Assuming there is a steady state condition in which \( C_t \) it is constant, \( C_t = C_{t+1} \) and \( \tau^C_t = \tau^C_{t+1} \) then we can modify the Euler equation of the rich and poor agents so that we have:

\[ (1 - \tau^K) F_K + (1 + \delta) = \frac{1}{\beta} \] \hspace{1cm} (36)
Where $F_K = r_t = (\alpha - 1)A\left(\frac{H_t}{K_t}\right)\alpha$, so we have that taxing capital diminishes the steady state for both agents. It is easy to prove that the same result can also be found for the poor agent. This result is the same as found by the majority of studies done in the literature, is consistent with theory. This famous equation shows how technology represent by $\delta$ and $F_K$ and time preference of time $\beta$ are important to determine of steady state rate level of capital. Another important point to note is that tax consumption and labor does not affect the steady state.

4 Ramsey Problem

The literature of optimal taxation has as objective to find a tax system that maximize a social welfare function of the economy subject to a set of constraints. In this section we formulate the problem of taxation dynamically, this approach is known in the literature as Ramsey problem, the commonly found solution is called Ramsey plan.

The government’s objective is to maximize households’ welfare subject to raising set revenues through distortionary taxation. By creating an ideal fiscal policy, the government takes into account the reactions of consumers and firms to the tax system. The problem is to determine the optimal sequences for the three types of tax, capital, labor and consumption in a nonstochastic economy, using a competitive equilibrium version of the basic neoclassical growth model. We assume that the government has a balanced budget.

Following the example of Chamley (1986) and Sargent and Ljungvist (2000), we can rewrite the budget constraint of the government in order to facilitate derivation of the central planner problem’s. It consists of using the FONC’s of the firm and equilibrium outcomes in factor markets. For this we take the equation (31) and first we add and subtract $W_tH_t, R_tK_t$ therefore have:

$$G_t + T_t = F(H_t^W, K_t^W, H_t^P, K_t^P) - (1 - \tau_H)W_t\left[\theta H_t^P + (1 - \theta)H_t^W\right]$$

$$- (1 - \tau_K)R_t\left[\theta K_t^P + (1 - \theta)K_t^W\right] - (1 - \tau_C)\left[\theta C_t^P + (1 - \theta)C_t^W\right]$$

(37)

Based on the equation (37) and the equations from the FONC’s for agents poor and rich, the firm and Euler conditions than is possible build the problem of Ramsey using estimation strategy similar to Sargent and Ljungvist (2000). But unlike these authors, we have not included resource constraint this because if government and household budget constraints holding imply that the resource constraint holds. The Ramsey problem in Lagrangian form is shown below where the objective function is constrained by the household first order conditions, the household budget constraints and the government budget constraints.

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4If the government present debt in each period the results show in this study do not change.
\[
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ V_t(U_t^W, U_t^P) + \varphi_t \left[ F(H_tK_t) - (1 - \tau_H)W_tH_t - (1 - \tau_K)R_tK_t + (1 - \tau_C)C_t - G_t - T_t \right] \\
+ \lambda_t^P \left[ - (1 + \tau_t^{C,P})C_t^P - I_t^P + (1 - \tau_t^{H,W})W_tH_t^P + (1 - \tau_t^{K,P})R_tK_t^P + T_t^P \right] \\
+ \lambda_t^W \left[ - (1 + \tau_t^{C,W})C_t^W - I_t^W + (1 - \tau_t^{H,W})W_tH_t^W + (1 - \tau_t^{K,P})R_tK_t^W \right] \\
+ \phi_t^P \left[ U_C(1 + \tau_t^{H,P})W_t - U_H(1 + \tau_t^{C,P}) \right] \\
+ \phi_t^W \left[ U_C(1 + \tau_t^{H,W})W_t - U_H(1 + \tau_t^{C,W}) \right] \\
+ \Psi_t^P \left[ \beta U_{C_{t+1}} \frac{(1 + \tau_t^{C,P})}{(1 + \tau_{t+1}^{C,P})} \left[ (1 - \tau_{t+1}^{K,P})R_{t+1}^P + (1 - \delta) \right] - U_{C_t} \right] \\
+ \Psi_t^W \left[ \beta U_{C_{t+1}} \frac{(1 + \tau_t^{C,W})}{(1 + \tau_{t+1}^{C,W})} \left[ (1 - \tau_{t+1}^{K,W})R_{t+1}^W + (1 - \delta) \right] - U_{C_t} \right] \right\} 
\]

Where \( V_t \) is the sum of the utility functions of all agents. Based on the Lagrangian of the Ramsey’s problem, it is possible derive the Lagrangian regarding the following variables: \( C, H, K, \tau^C, \tau^H, \tau^K \) and \( T \) for both agents, also is derived the same variables for periods \( t = 0 \) and \( t > 1 \). All mathematical work and the main results are shown in the annexes. We now can concentrate in find the optimal tax on capital. Utilizing the derivative of the Lagrangian with respect \( K_{t+1}^W \) and \( K_{t+1}^P \), it is find the condition of optimal taxation on capital \( \tau^K \), which for the rich agent is:

\[
\beta \varphi^W (1 - \theta) \left[ F_K - (1 - \tau^K)R^W \right] - \lambda^W + \beta \lambda^W \left[ 1 + (1 - \tau^K)R^W + (1 - \delta) \right] = 0 \quad (38)
\]

It is worth mentioning that despite the result be displayed for the rich agent the same result is valid for the poor agent. As in Sargent and Ljungvist (2000), Swarbrick (2012), \( \lambda \) is interpreted as the marginal social value of goods and is strictly positive. Furthermore \( \varphi \) can be interpreted as the marginal value of reducing taxes and is nonnegative. Replacing the Euler equation of rich agent (26) in steady-state and remembering that steady-state: \( (1 - \tau_{K,W})R + (1 - \delta) = 1/\beta \) and also, \( F_K = R \), so we have from the expression (38):

\[
\varphi^W (1 - \theta) \left[ R^W - (1 - \tau^K)R^W \right] = 0 \quad (39)
\]

This implies that \( 1 - (1 - \tau^K) = 0 \) and thus we conclude that \( \tau^K = 0 \). Thus it can be concluded that the optimal capital tax is zero at steady state, a result that is equivalent to the poor and rich agents.

**Proposition 1.** In a economic growth model with heterogeneous agents and a nonstochastic economy, in steady state optimal taxing on physical capital should equal to zero for the both type of agents poor and rich.
Although the return ($R_t$) the capital of the rich agents is higher than the poor agents. This result is same as found by several authors most notoriously by: Judd (1985), Chamley (1986), Sargent and Ljungvist (2000) and is consistent with studies of optimal taxation\(^5\).

Note that the result of zero taxation on capital is independent of the government bias to the poor or the rich, represented by parameter $\gamma$ in our model, as previously seem the parameter $0 < \gamma < 1$ indicates the government’s favoritism for the rich or poor agents. So the transfers made by the government should be financed

Although there are studies showing different results of this, mainly because under certain assumptions taxes on capital would be optimize above zero as in Aiyagari (1995) that work with a incomplete markets model. Similar results can also be found in other studies such as Stiglitz (1987), Jones, Manuelli and Rossi (1997). This type of result is usually found when some kind of good is not tax. Therefore government spending, in particular the transfer to the poor should be financed by a combination of taxes.

However as it is possible to see this result is presented with the economy in the steady state, and does not explain how macroeconomics variables such as consumer, labor wage etc, react during the Real Business Cycle - RBC. Also optimal steady-state tax do not say much about how long it takes to reach the zero tax on capital and mainly how taxes and redistributive transfers are set during the transition period.

So it becomes necessary calibration a model with demand and supply shocks in order to know how differ the reaction of agents rich and poor to the mains macroeconomic variables. We therefore propose a model that simulates how a pro-poor government policy affects the optimal fiscal policy. When $\gamma$ is near to one, the government is highly favorable towards the poor. When $\gamma$ is close to zero, the government is highly favorable towards the wealthy.

A similar study was conducted by Giavazzi and McMahon (2012) which presents a model that contrary to the theory and other studies, low-income families tend to respond more to fluctuations cutting consumption and working more hours following a government spending shock, for example. This analysis suggests that the most wealthy families react more to fluctuations, although they can smoothen their spending over time.

Having focused on optimal tax policy, attention turns to the process of change in the composition of taxes on labor and capital. Domeij and Heathcote (2004) argue that combine different types of taxes implies significant distributional changes, this type of combination is used in the context of tax reform as shown abundance in the literature.

Swarbrick (2012) show that changing the government preferences did affect fiscal policy as shown. The government becoming more favorable towards the poor caused an increase in transfers to these agents. This was financed mainly by taxing labor but in part by increasing taxes on capital. Clearly, even in the case of a government very biased towards the rule-of-thumb agents, the optimal tax rate on capital should be close to zero.\(^6\)

### 5 Conclusions and Remarks

This paper solves a optimal fiscal policy in a model with agent heterogeneity and inequality. By introducing agents poor and rich, we show that the steady state optimal tax on capital in the long run should always be zero regardless of the governments favoritism towards particular agents.

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\(^{5}\)Sargent and Ljungvist (2000) point out that this model can be generalized to a finite of different classes of agents, $N$.

\(^{6}\)It is possible that, as the preference becomes too strong for the poor, there is political pressure to increase government transfers financed by taxes on capital. This is a non-optimal policy and causes the model to fail.
The fiscal policy has two main instruments through which affect the economy, mainly the following macroeconomic variables: aggregate demand and the level of economic activity, savings and investment and the distribution of income. First the government can increase or diminish they spending in various sectors. Second the government can changes in the level and composition of taxation. The analytical solution find in this paper suggest that in steady state the government should be financed the transfers by different combinations of taxes on consumption and labor.

So not all taxes should be zero at steady state. This result is usually explained by zero profit conditions, this condition comes from the assumption of linearity in the accumulation technologies. Note that the result of zero taxation on capital is independent of the government bias to the poor or the rich, represented by parameter $\gamma$ in our model.

Technological shocks impact the marginal product of both agents, and consequently their remunerations, which changes the income gap between the agents. Therefore it can be said that technological shocks impact the income inequality in the steady state, although it can not say for sure whether this relationship is positive or negative. For the better understanding of these relationships becomes necessary for calibration of this model.

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