The Sraffian Supermultiplier as an Alternative Closure to Heterodox Growth Theory*

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RESUMO

O trabalho mostra que o modelo do supermultiplicador sraffiano prove um fechamento alternativo para a análise das relações entre crescimento, distribuição, utilização da capacidade e demanda efetiva em modelos heterodoxos de crescimento. O novo fechamento vem da variabilidade da razão entre as propensões média a marginal a poupar decorrente da hipótese da existência de gastos autônomos que não criam capacidade para o setor privado (que crescem a uma taxa exógena). Isto permite que a propensão marginal a investir determine a taxa de poupança da economia sem necessidade de mudanças na distribuição ou na propensão marginal a poupar. Junto com a hipótese adicional de que as mudanças na propensão marginal a investor são induzidas pela pressão competitiva que leva ao ajustamento gradual da capacidade a demanda, este ajustamento através the mudanças endógenas na razão entre as propensões média e marginal a poupar (a fração) nos prove um fechamento que permite a reconciliação de crescimento liderado pela demanda, distribuição exógena e uma tendência a utilização normal da capacidade produtiva entre steady states. Uma análise comparativa tenta mostrar que este fechamento é uma alternativa mais satisfatória aos fechamentos tanto dos modelos de crescimento da tradição de Cambridge quanto dos modelos kaleckianos.

ABSTRACT

The paper shows that the Sraffian Supermultiplier model provides an alternative closure for the analysis of the relationships between growth, distribution, capacity utilization and effective demand in heterodox growth models. The new closure comes from the variability of the average to the marginal aggregate propensities to save entailed by the assumption of the existence of (independently growing) autonomous expenditures that do not generate capacity for the private sector. This allows the marginal propensity to invest to determine the savings ratio without the need of changes in distribution or in the marginal propensity to save. Provided that it is also assumed that changes in the marginal propensity to invest are induced by the competitive need to gradually adjust capacity to demand, this adjustment through endogenous changes in the ratio of the average to the marginal propensity to save (the fraction) gives us a closure that allows us to reconcile demand led growth, exogenous distribution and a tendency towards normal capacity utilization, even across steady states. A comparative analysis will try to show that this is a more satisfactory alternative to the closures provided by both the Cambridge and the kaleckian growth models.

Key words: Effective Demand; Economic Growth; Income Distribution.

JELCode: E11; E12; O41

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Introduction

The paper has two purposes. The first is to show how the Sraffian Supermultiplier model provides an alternative closure for the analysis of the relationships between growth, distribution, capacity utilization and effective demand in heterodox growth models. This new closure comes from the variability of the average to the marginal aggregate propensities to save entailed by the assumption of the existence of (independently growing) autonomous expenditures that do not generate capacity for the private sector. This allows the marginal propensity to invest to determine the savings ratio without the need of changes in distribution or in the marginal propensity to save. Provided that it is also assumed that changes in the marginal propensity to invest are induced by the competitive need to gradually adjust capacity to demand, this adjustment through endogenous changes in the ratio of the average to the marginal propensity to save (the fraction) gives us a closure that allows us to reconcile demand led growth, exogenous distribution and a tendency towards normal capacity utilization, even across steady states.
The second purpose of the paper is to compare and constrast this new closure with the closures provided by both the Cambridge and the kaleckian growth models which operate , respectively, through the endogeneity of distribution or in the actual degree of capacity utilization. It will be argued that the closure based on the Sraffian Supermultiplier model and the fraction is a more satisfactory alternative to the closures provided by both the Cambridge and the kaleckian growth models.
The rest of the paper is organized as follows. Section 1 first presents the Sraffian Supermultiplier model (in the version used in Freitas & Serrano (2015)) and how its closure operates, as capacity adjusts to demand and then shows the results of permanent effects of changes in the rate of growth of autonomous consumption and in income distribution (or in the marginal propensity to save) in the model. Section 2 compares and constrasts the different closures and results of the Sraffian supermultiplier model with the Cambridge and the kaleckian growth models. Section 3 contains brief final remarks.

1. The Sraffian Supermultiplier Growth Model

1.1. The supermultiplier and the long period position

We shall present the Sraffian supermultiplier growth model in its simplest possible form in order to facilitate the identification of the most relevant properties of the model and the comparison with alternative growth models. Hence, we assume a closed capitalist economy without a government sector. The only method of production in use requires a fixed combination of a homogeneous labor input with homogeneous fixed capital to produce a single homogeneous output. Natural resources are supposed to be abundant; we assume constant returns to scale and no technological progress. We also assume that growth is not constrained by labor scarcity. Moreover, all variables are measured in real terms and output, income, profits, investment and savings are all presented in gross terms. The formal analysis will use continuous time for mathematical convenience.

In this very simple analytical context, the level of capacity output of the economy depends on level of the capital stock and on the technical capital/output ratio according to the following expression:

$$Y_{Kt} = \left(\frac{1}{v}\right) K_t \qquad (1)$$

where Y_{Kt} is the level of the capacity output of the economy, K_t is the level of capital stock installed in the economy and v is the technical capital/output ratio. Since v is given, then the rate of growth of capacity output is equal to the rate of capital accumulation

$$g_{Kt} = \left(\frac{I_t/Y_t}{v}\right)u_t - \delta \tag{2}$$

where g_{Kt} is rate of capital accumulation, $u_t = Y_t/Y_{Kt}$ is the actual degree of capacity utilization defined as the ratio of the current level of aggregate output (Y_t) to the current level of capacity output**;** I_t/Y_t is the investment share in aggregate output defined as the ratio of gross aggregate investment (I_t) to the level of gross aggregate output; and δ is exogenously given depreciation rate of the capital stock.[1] According to equation (2) the rate of capital accumulation depends on the actual degree of capacity utilization and on the investment share.[2] Given the capital/output ratio, the change in the actual degree of capacity utilization through time is then described by the difference between the rate of growth of output and the rate of capital accumulation following the differential equation below:

$$\dot{u} = u_t(g_t - g_{Kt}) \tag{3}$$

where g_t is the rate of growth of aggregate output.

Aggregate income in the model is distributed as wages and gross profits. We shall assume that besides the single technique in use, income distribution (either the normal real wage or the normal rate of profits) is also given exogenously along classical (Sraffian) lines. We accordingly assume that there is free competition and that output (but not capacity) adjusts fairly quickly to effective demand such that that market prices are equal to normal prices that yield a uniform rate of profits on capital using the dominant technique when the actual degree of capacity utilization u_t is equal to the normal or planned degree μ.[3] Note that we can make the assumption of normal prices at this stage of our analysis even when dealing with situations in which the actual degree of capacity utilization can be quite different from the normal or planned degree; for under classical competition, individual firms do not have the power to sustain higher-than-normal market prices when the actual degree of capacity utilization of a particular firm is below (or very much above) the normal level and their actual unit costs are higher than normal.[4] Indeed, at higher than normal prices other firms already in the market may be operating at the planned degree of utilization and can easily increase their market shares by undercutting the firms that have raised prices above the normal price. Moreover, these higher prices may also attract new entrants to the market which would also be able to operate their appropriately sized new capacities at the planned degree of utilization and reap higher than normal profits by undercutting incumbent firms that have raised prices to pass along their higher than normal actual average costs to market prices. Thus, both actual competition of existing firms as well as potential competition of new firms would ensure that effective demand will be met at the normal price even if the actual degree of capacity utilization is quite different from the normal or planned degree.[5]

Since we are assuming that output adapts quickly to demand, when normal prices prevail aggregate demand determines the level of output. Effective demand is composed of real aggregate consumption and gross real aggregate investment. We suppose that aggregate consumption has an induced component and an autonomous one. The former is related to the purchasing power introduced to the economy by the production decisions of capitalist firms when they pay wages. Given consumption habits (i.e., a given marginal propensity to consume out of wages, c_w) and the wage share of output (ω), we assume that there is a positive relationship between induced consumption and aggregate output

[1] The equation of capital accumulation is derived from the equation $I_t = \dot{K} + \delta K_t$ that defines the level of aggregate gross fixed investment. Dividing both sides of the equation by K_t we obtain $I_t/K_t = g_{Kt} + \delta$. Solving this last equation for the rate of capital accumulation gives us $g_{Kt} = (I_t/K_t) - \delta = (I_t/Y_t)(Y_t/Y_{Kt})(Y_{Kt}/K_t) - \delta = ((I_t/Y_t)/v)u_t - \delta$.

[2] The logic underlying this equation appears to have first been outlined by Garegnani (1962). In his SVIMEZ report, Garegnani argued that what raises the rate of growth of capacity output is a rise in the investment share in *capacity* output rather than in *actual* output. He furthermore noted that these two ratios can differ substantially from one another if the actual degree of capacity utilization can diverge from the normal or planned degree.

[3] Following Ciccone (1986, 1987), we interpret the normal or planned degree of capacity utilization as determined, among other things, by the historically 'normal' ratio of average to peak demand. This latter ratio, because it is presumably based on the observation of the actual cyclical pattern of the market over a very long period of time, is assumed to be unaffected by current oscillations of demand.

[4] If the actual degree of utilization is below the normal degree, fixed cost per unit of output will be higher than normal. If the degree of utilization is just above the normal degree unit costs will keep falling (giving rise, at normal prices, to extra profits) until at capacity utilization rates substantially above normal, the cost will begin to rise due to the extra expenses involved in operating capacity way above its cost minimizing range (Ciccone 1987).

[5] Normal price is thus a kind of entry-preventing 'limit price' in the language of the old industrial organization literature. Some Sraffians make the same argument in terms of a presumption of a uniformity of expected rates of profit on new investment (Garegnani 1992; Ciccone 1986, 2011).

regulated by the marginal propensity to consume out of income $c = c_w \omega$ (with $0 < c < 1$, since $0 < c_w \leq 1$ and $0 < \omega < 1$). In order to simplify the analysis of the model, we suppose that all wages are expended in consumption (i.e. $c_w = 1$ and, hence, $c = \omega$). On the other hand, the autonomous component Z_t is that part of aggregate consumption financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions.

We further assume that firms undertake all investment expenditures in the economy (i.e., we abstract from residential investment) and that the level of aggregate investment is an induced expenditure according to the following expression

$$I_t = hY_t \tag{4}$$

where h (with $0 \leq h < 1$) is the marginal propensity to invest of firms, which we *provisionally* assume to be determined exogenously.

Since we are thinking of a demand-led system, we assume that the marginal propensity to spend $c + h$ is strictly lower than one (if it was equal to one, Say's Law would obtain).[6] We must also assume that there is a positive level of autonomous consumption, otherwise no positive level of output could be profitably sold.

The demand-determined level of output in a long-period position is:

$$Y_t = \left(\frac{1}{s - h}\right) Z_t \tag{5}$$

where $s = 1 - \omega$ is the given aggregate marginal propensity to save and the term within the parenthesis is the supermultiplier that captures the effects on the level of output associated with both induced consumption and investment.[7]

Given the existence of autonomous consumption expenditures, we have that the marginal propensity to save does not determine the actual saving ratio (the average propensity to save) in this simple framework. The saving ratio is instead determined by and is equal to the marginal propensity to invest:

$$\frac{S_t}{Y_t} = s - \frac{Z_t}{Y_t} = sf = \frac{I_t}{Y_t} = h \tag{6}$$

where $f_t = I_t/(I_t + Z_t)$ is what Serrano (1995b) called 'the fraction', the ratio between the average and the marginal propensities to save $f_t = (S_t/Y_t)/s$.[8] With positive levels of autonomous consumption (i.e., $Z_t > 0$), it follows that $f_t < 1$ and $S_t/Y_t < s$. Therefore, the given marginal propensity to save defines only the upper limit to the value of the saving ratio of the economy corresponding to a given marginal propensity to save. In this case, the saving ratio depends not only on the marginal propensity to save but also on the proportion between autonomous consumption and investment. Thus, an increase (decrease) in the levels of aggregate investment in relation to autonomous consumption leads to an increase (decrease) in the saving ratio. As a result, the existence of a positive level of autonomous consumption is sufficient to make the saving ratio an endogenous variable.

In this context, for a given level of income distribution (and consumption habits) and, hence, a given marginal propensity to save, the given marginal propensity to invest h (equal to the investment share of output) uniquely determines the saving ratio of the economy. Hence, an exogenous increase

[6] Neo-Kaleckians call this assumption 'Keynesian stability' (see, for example, Lavoie, 2014; Allain 2014). But in fact it is much more than that. It is actually what we mean when we say that output is demand-determined. For if the marginal propensity to spend were equal to one and there no autonomous demand we would have Say's Law and if the marginal propensity to spend was lower than one but with no autonomous demand the economy would collapse and output would fall to zero (see Serrano 1995a; López and Assous 2010, chapter 2). Neo-Kaleckians like to distinguish such 'Keynesian stability' from 'Harrodian instability' due to induced investment accelerator effects. This distinction is much less useful than it may appear to be, for as we shall see the latter type stability also depends crucially on the aggregate marginal propensity to spend being lower than one when the economy is in the vicinity of the equilibrium position.

[7] On the marginal propensity to consume, see Serrano (1995a); on the (not fully adjusted) long-period supermultiplier with a given marginal propensity to invest, see Cesaratto, Serrano and Stirati (2003). Note that equation (5) can determine the level of output only if $u_t \leq 1$. So for all $t \geq 0$ we must have $Y_{Kt} \geq Y_t = [1/(s - h)]Z_t$ and, therefore, $Z_t \leq [(s - h)/v]K_t$.. That is, the level of output determined by effective demand is restricted by the level of capacity output for given values of s, h and v.

[8] According to (6), if there were no autonomous consumption (i.e., $Z_t = 0$) then $f_t = 1$ and $S_t/Y_t = s$. That is, the marginal propensity to save determines the savings ratio. Note also that, in this extreme case the equilibrium between aggregate demand and aggregate output with a given income distribution also implies the endogenous determination of the investment share of output by the marginal propensity to save.

(decrease) in the investment share of output in relation to the marginal propensity to save would raise (reduce) the fraction f, and, therefore, would cause a decrease (increase) in the ratio of autonomous consumption to output Z_t/Y_t and an increase (decrease) in the saving ratio.

Now let us suppose that autonomous consumption grows at an exogenously determined rate $g_Z > 0$. Since the marginal propensities to consume and to invest are given exogenously, the supermultiplier is also exogenous and constant. Therefore, aggregate output, induced consumption and investment grow at the same rate as autonomous consumption. The capital stock also tends to grow at this same rate since the growth rate of gross investment governs its trend rate of growth.[9] Thus, starting from any given investment share of output h, if initially the growth of demand is equal to the growth of the capital stock, the actual degree of utilization will remain constant over time. On the other hand, if initially the rate of growth of demand is higher (lower) than the rate of growth of the capital stock then the growth of the capital stock will increase (decrease) towards the rate of growth of demand (since a given h implies that gross investment is growing at the same rate as demand). In the context of our model with a given marginal propensity to invest, this implies that capacity output tends to grow at the same rate as autonomous consumption. Moreover, the actual degree of capacity utilization will tend to a constant value as can be calculated from equation (3) above and its trend level can be determined using equation (2) as follows:[10]

$$u^* = \frac{v(g_Z + \delta)}{h} \qquad (7)$$

According to equation (7) above, given the marginal propensity to invest h, a higher (lower) rate of growth of autonomous consumption leads to a permanently higher (lower) actual degree of capacity utilization. Moreover, for any exogenously given marginal propensity to invest, there is no reason why the actual degree of capacity utilization should tend to its normal or planned level. Therefore, in this simplified long-period supermultiplier model, the growth of productive capacity follows the growth of (autonomous consumption) demand, but the level of capacity may be quite different from the level of aggregate demand.

1.2. The fully adjusted positions and the supermultiplier

Let us now move on to the analysis of the slower, but always present, competitive tendency of capacity to adjust to demand. From what we saw above (equation (7)), we know that given the growth rate of autonomous consumption, a higher (lower) propensity to invest will lead to a permanently lower (higher) actual degree of capacity utilization. Therefore, changes of the marginal propensity to invest would appear in principle to allow the adjustment of the actual degree of capacity utilization to its normal or planned level.

In order to analyze the adjustment of capacity to demand we now add to the model a rule for the changes of the marginal propensity to invest. We will use the simplest possible (and yet not unreasonable) rule. We will assume that the process of inter-capitalist competition will lead to a tendency for the growth rate of aggregate investment to be higher than the rate of growth of demand (and hence of output) whenever the actual degree of capacity utilization is above its normal or planned level and *vice-versa*. Competition would ensure that firms as a whole will be pressed to invest in order to ensure they can meet future peaks of demand when the degree of utilization is above the normal or planned degree and the margins of spare capacity are getting too low. Conversely, firms will not want to keep accumulating costly unneeded spare capacity when the actual degree of capacity utilization remains below the profitable normal or planned level.

We shall also assume that such endogenous changes in the marginal propensity to invest are gradual rather than drastic and therefore compatible with the view expressed in the principle of the

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[9] The fact that the rate of growth of the stock of capital follows the rate of growth of gross investment can be shown as follows. From the definitional equation $I_t = \dot{K} + \delta K_t$ we can obtain the differential equation $\dot{g}_K = (g_{Kt} + \delta)(g_{It} - g_{Kt})$ relating the rate of growth of the flow of gross investment to the rate of capital accumulation. From this differential equation it can be seen that, if initially the rate of capital accumulation is different from the rate of growth of investment, the rate of capital accumulation begins to move towards the rate at which investment grows. We can also see that the rate of capital accumulation only constant when it becomes equal to the given rate of growth of gross investment.

[10] From equation (2) we can write the actual degree of capacity utilization as $u_t = v(g_{Kt} + \delta)/h$. But since g_{Kt} converges to g_Z, the actual degree of utilization tends to the value described in equation(7).

adjustment of the capital stock associated with *flexible* accelerator models of induced investment. Thus, in our version of the flexible accelerator investment function the marginal propensity to invest changes as follows:[11]

$$\dot{h} = h_t\gamma(u_t - \mu) \tag{8}$$

where μ is the normal degree of capacity utilization discussed above (with $0 < \mu < 1$), and $\gamma > 0$ is a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of utilization u_t from the normal or planned level μ. From equations (4) and (8) we can see that the growth rate of investment g_{It} is then given by the following expression:

$$g_{It} = g_t + \gamma(u_t - \mu) \tag{9}$$

Given our investment function we can obtain the following equation for the growth rates of aggregate output/demand:[12]

$$g_t = g_Z + \frac{h_t\gamma(u_t - \mu)}{s - h_t} \tag{10}$$

Equation (10) shows that when the actual and normal degrees of capacity utilization are different, the rate of growth of output and demand is determined by the rate of expansion of autonomous consumption plus the rate of change of the supermultiplier given by the second term on the right-hand side of the equation.

Let us now substitute equations (10) and (2) into equation (3). From the combination of the resulting equation and equation (8) we obtain a system of two first order nonlinear differential equations in two variables, the share of investment h and the actual degree of capacity utilization u, which we present below:

$$\dot{h} = h_t\gamma(u_t - \mu) \tag{8}$$

$$\dot{u} = u_t\left(g_Z + \frac{h_t\gamma(u_t - \mu)}{s - h_t} - \left(\frac{h_t}{v}\right)u_t + \delta\right) \tag{11}$$

We will use the above system to develop the analysis of our fully adjusted version of the supermultiplier growth model.

1.2.1. The Fully Adjusted equilibrium

The equality between actual and normal degrees of capacity utilization characterizes the fully adjusted position of the economy according to the supermultiplier model. Setting $u_t = u^* = \mu$ in equation (8) we obtain $\dot{h} = 0$, and since μ is constant we also have $\dot{u} = 0$. Thus, if $u_t = u^* = \mu$ then $\dot{h} = \dot{u} = 0$ and we can verify that the fully adjusted position is an equilibrium of the supermultiplier model represented by the system comprising equations (8) and (11).

From equations (9) and (10) we can also verify that in the fully adjusted position of the supermultiplier model the growth rates of output, demand and investment are equal to the growth rate of autonomous consumption. Moreover, since $\dot{u} = 0$ and $\mu > 0$ then, from equation (3), the rate of capital accumulation is equal to the growth rate of output. Hence, in the fully adjusted position we have $g_K^* = g_I^* = g^* = g_Z$. That is, the growth rate of autonomous consumption determines the equilibrium growth rates of the capital stock, investment and output and demand. The model generates an equilibrium path where economic growth is consumption-led and normal utilization of productive capacity prevails.

[11] On the flexible accelerator investment function see Goodwin (1951), Chenery (1952) and Koyck (1954). We have elsewhere explored other specifications of a flexible accelerator induced investment function; see Cesaratto, Serrano and Stirati (2003), Serrano and Wilcox (2000), Serrano (2007), Serrano and Freitas (2007) and Freitas and Dweck (2010). The particular function used here was chosen as the simplest for the purpose at hand, i.e. the elucidation of the adjustment of capacity to demand; but the essential features of the stability conditions discussed below are basically the same for all these variants of the induced investment function.

[12] The equation is deduced as follows. Taking the time derivatives of the endogenous variables involved in expression $Y_t = Z_t + cY_t + h_tY_t$ and dividing both sides of the resulting equation by the level of aggregate output, we obtain $g_t = cg_t + h_tg_t + \dot{h} + (Z_t/Y_t)g_Z$. If $Z_t/Y_t = s - h_t > 0$, then we can solve the last equation for the rate of growth of aggregate output and demand, obtaining $g_t = g_Z + \dot{h}/(s - h_t)$. Finally, we can substitute the right-hand side of equation (8) in the second term on the right-hand side of the last equation, which gives us equation (10).

The last result shows that, according to the model, the growth of autonomous consumption drives the pace of capital accumulation and, therefore, the growth of productive capacity. Thus it is able to represent a sustainable process of demand-led growth. Such a result is compatible with the maintenance of a normal or planned degree of capacity utilization throughout the equilibrium path because the investment share of output (i.e., the marginal propensity to invest) assumes a required value $h*$ given by:

$$h^* = \frac{v}{\mu}(g_Z + \delta) \tag{12}$$

The required investment share is uniquely determined by the rate of growth of autonomous consumption, the technical capital/output ratio, the normal degree of capacity utilization and the rate of depreciation.

The maintenance of the equilibrium between aggregate output and demand in the fully adjusted position of the model requires the endogenous determination of the saving ratio (or average propensity to save). As we mentioned above, the endogeneity of the saving ratio in the model is a consequence of the hypothesis of the existence of a positive level of autonomous consumption. Actually, the latter makes it possible for the fraction $f_t = h_t/s$ to change its value according to the modifications of the investment share of output. As a result, $Z_t/Y_t = s(1 - f_t) = s - h_t$, the ratio of autonomous consumption to aggregate output can change, making the saving ratio an endogenous variable and allowing its adjustment to the investment share of output. In fact, along the equilibrium path of the model, once the investment share is determined we can obtain the equilibrium values of the fraction, of the aggregate autonomous consumption/output ratio and, accordingly, of the equilibrium value of the saving ratio as follows

$$f^* = \frac{h^*}{s} = \frac{\frac{v}{\mu}(g_Z + \delta)}{s} \tag{13}$$

$$\left(\frac{Z_t}{Y_t}\right)^* = s(1 - f^*) = s - h^* = s - \frac{v}{\mu}(g_Z + \delta) \tag{14}$$

and

$$\left(\frac{S_t}{Y_t}\right)^* = s - (Z_t/Y_t)^* = sf^* = h^* = \frac{v}{\mu}(g_Z + \delta) \tag{15}$$

We saw that the equilibrium investment share of output is positively related to the equilibrium rate of growth of output. Thus, given income distribution (and, therefore, the marginal propensity to save), according to equations (13), (14) and (15), a higher (lower) equilibrium rate of economic growth entails, on the one hand, higher (lower) equilibrium values of the fraction and of the saving ratio, and, on the other, a lower (higher) equilibrium value of the autonomous consumption/output ratio.

Finally, by introducing the required level of the investment share h^* in equation (5) we obtain the fully adjusted (final equilibrium) level of output:

$$\frac{\mu K}{v} = Y_t^* = Y_t = \left(\frac{1}{s - \frac{v}{\mu}(g_Z + \delta)}\right) Z_t \tag{16}$$

Equation (16) shows that at each moment t the level of autonomous consumption and the fully adjusted (final equilibrium) level of the supermultiplier (the term in parenthesis) determine the levels of output of the fully adjusted positions (final equilibria) towards which the economy is slowly gravitating.[13] Thus, contrasting with the situation represented by the model in the long-period position, in the fully adjusted positions not only aggregate output but also the levels of capacity output and the capital stock adjust to the levels of aggregate demand.

[13] In terms of the old long-period method the fully adjusted position is thus a classical 'secular' rather than a long-period position (Ciccone 1986) just as a Solow steady state is a secular not a long-period neoclassical equilibrium path. In the terms of the recent neo-Kaleckian literature the fully adjusted equilibrium is a type of 'final equilibrium' and the long-period position would be a 'medium- run equilibrium' (see Lavoie 2014).

1.2.2. The tendency towards the Fully Adjusted Position

Let us now look at the stability of the fully adjusted position to see if the long-period positions of the economy tend to move towards it. There are two conditions for this process of adjustment to occur. First, the investment share of output must be susceptible to change via the influence of the competitive process over capitalist investment decisions. This is guaranteed by the existence of an autonomous consumption component of aggregate demand, which allows the proper adjustment of the saving ratio. In fact, from equations (9) and (10) we can obtain the following relations:

$$g_{It} \gtreqless g_t \gtreqless g_Z \text{ as } u_t \gtreqless \mu$$

According to them, if the actual degree of capacity utilization is above (below) the normal level the marginal propensity to invest increases (decreases). At the same time, the saving ratio ($S_t/Y_t = sf_t$) also increases (decreases) because the growth rate of investment is higher (lower) than the growth rate of autonomous consumption and, therefore, the fraction $f_t = I_t/(I_t + Z_t)$ increases (decreases) while the marginal propensity to save is constant. These changes in the investment share of output in response to deviations from the normal degree of capacity utilization are necessary for the adjustment of capacity to demand and the corresponding tendency towards a fully adjusted position of the model. However, they are not sufficient to assure such a result since the intensity of the adjustment must also be considered. The latter point leads us to the condition discussed below.

The second condition is that the marginal propensity to invest changes *gradually* as a response to deviations of capacity utilization from its planned degree. The latter is required to assure that the value of the marginal propensity to spend remains lower than one throughout the process of convergence to the fully adjusted position. The reason for that is quite simple. Given that empirically it is plausible that normal technical capital/output ratios for fixed capital tend to be greater than one, an immediate and full reaction of induced investment that tried to adjust capacity fully and immediately to any current deviation from normal utilization (what is known as a 'rigid accelerator') would certainly lead to a marginal propensity to invest greater than one. Since the marginal propensity to consume is positive, the overall marginal propensity to spend in the vicinity of the fully adjusted position would be much greater than one, implying that any positive level of autonomous expenditure would endogenously generate an infinite demand stimulus, and any fall in output would induce the collapse of the economy. Such drastic adjustment however is highly unrealistic, both because of the durability of fixed capital (which means that producers want normal utilization only on average over the life of equipment and not at every moment) and also because producers know that demand fluctuates a good deal and therefore do not interpret every fluctuation in demand as indicative of a lasting change in the trend of demand.[14] The precise meaning of this second sufficient condition is that the aggregate marginal propensity to spend in the vicinity of the fully adjusted position must be lower than one (see Freitas & Serrano (2015) for the proof). Therefore we assume that:

$$c + \frac{v}{\mu}(g_Z + \delta) + \gamma v < 1 \tag{17}$$

We can interpret (17) as an expanded marginal propensity to spend that besides the final equilibrium propensity to spend ($c + \frac{v}{\mu}(g_Z + \delta)$) includes also a term (i.e., γv) related to the behavior of induced investment outside the fully adjusted position. The extra adjustment term captures the fact that the investment function of the model is grounded in the capital stock adjustment principle (a type of flexible accelerator investment function) and that outside the fully adjusted trend path there must be room not only for the induced gross investment necessary for the economy to grow at its final equilibrium rate g_Z, but also for the extra induced investment responsible for adjusting capacity to demand. Thus, *ceteris paribus*, for a sufficiently low value of the reaction parameter γ the system described above is stable.

[14] This is the pattern of behavior underpinning the flexible accelerator model of investment.

1.2.3 Effects of changes in the rate of growth of autonomous consumption and in income distribution

We can now analyze the impact on the fully adjusted (final equilibrium) positions of changes in the growth rate of autonomous consumption (i.e., g_Z) and in income distribution (i.e. the wage share ω), assuming that the fully adjusted equilibrium is stable both before and after the change.

As we saw above, according to the supermultiplier growth model, the rate of growth of autonomous consumption determines the trend rate of growth of output/demand (i.e., g^*). So an increase (a decrease) in the rate of growth of autonomous consumption causes an increase (a decrease) in the trend rate of growth of output/demand. Therefore changes in g_Z have a *growth effect* on g^*. We also have seen that g_Z determines the value of the rate capital accumulation in equilibrium (i.e., g_K^*). Hence a permanent rise (fall) in g_Z causes a permanent increase (decrease) in g_K^*. Since, according to the model, there is a tendency for the economy to converge towards normal capacity utilization, this *growth effect* of g_Z on g_K^* occurs through the effect of g_Z on the equilibrium level of the investment share of output (i.e., h^*). In fact, from Equation (12) we can see that the there is a positive relationship between g_Z and h^*, such that a permanent increase (fall) in g_Z has a positive (negative) *level effect* on h^*. Moreover, from Equations (13) to (15) we can observe that a rise (fall) in g_Z has, on the one hand, a positive (negative) *level effect* on the equilibrium value of the fraction (i.e., f^*) and on the equilibrium level of the saving ratio (i.e. $(S_t/Y_t)^* = sf^*$) and, on the other, a negative (positive) *level effect* on the equilibrium level of the autonomous consumption to output ratio (i.e., $(Z_t/Y_t)^* = s(1-f^*)$). Therefore, the supermultiplier growth model implies that there is a positive causal relationship running from the trend rate of growth of output/demand to the the investment share of output and the saving ratio. As argued above such a relationship is essential to the obtainment of the distinctive results of the Supermultiplier growth model, according to which an economy can be demand-led, while it mantains a tendency of the actual degree of capacity utilization to converge towards its normal level with a given income distribution.

Now let us look at the effects of changes in income distribution. First, our analysis of the steady state of the model has shown that the equilibrium rates of growth of output/demand and of the capital stock are independent from the value of the wage share. Hence, in the supermultiplier model there is no relationship between income distribution and the trend growth rate of output and demand. Next, since, as we just saw, a change in income distribution does not have a permanent growth effect, then equation (12) shows that a variation in income distribution does not have a permanent effect on the equilibrium value of the investment share of output.

Nevertheless, although a change in income distribution does not have a permanent growth effect, such a change does have a *level effect* over the equilibrium values of all non-stationary variables of the simplified supermultiplier growth model here presented. This is so because a change in income distribution affects, through its influence over the marginal propensity to save, the equilibrium value of the supermultiplier and, hence, the equilibrium value of aggregate output. So a change in the wage share has wage led output level effect. Indeed, as can be easily verified, an increase (decrease) in the wage share reduces (raises) the marginal propensity to save, which yields an increase (a decrease) in the equilibrium value of the supermultiplier and, accordingly, an increase (a decrease) in the equilibrium level of output.[15]

On the other hand, we know that, according to the supermultplier growth model, the investment share of output determines the saving ratio. Thus, there is no effect of a modification in income distribution on the saving ratio also. Note, however, that a change in income distribution does affect the marginal propensity to save. In fact, since the latter variable is equal to the profit share (i.e. $s = 1 - \omega$), an increase (decrease) in the wage share would reduce (raise) the marginal propensity to save (i.e. $ds = -d\omega$). But making use of equations (13) and (15) we can verify how this latter result is reconciled with the invariability of the saving ratio in relation to an income distribution change. Thus, given the

[15] Note that, in this sense, the Supermultiplier growth model allows the extension of the validity of the paradox of thrift concerning the level of output to the context of analysis of the economic growth process.

value of the investment share of output, an increase (a decrease) in the marginal propensity to save leads to a fall (an increase) in the level of output that causes a decrease (an increase) in the fraction. This explains why the saving ratio is unaffected by change in income distribution (since $h^* = sf^*$).

2. A Comparison with other heterodox closures[16]

In this section we shall compare the main results obtained in our discussion of the Sraffian supermultiplier growth model with alternative heterodox growth models that also deal with the relationship between economic growth, income distribution and effective demand. More specifically, we will compare the supermultiplier, Cambridge and neo-Kaleckian growth models trying to point out their similarities and, more importantly, their main differences. The comparison will be made within the same simplified analytical context used in the discussion of the supermultiplier model. Thus, the following assumptions will be maintained: we have a closed capitalist economy without government; aggregate income is distributed in the form of wages and profits; the method of production in use requires a fixed combination of homogeneous labor input with homogeneous fixed capital to produce a single output; there is constant returns to scale and no technological progress; and there exists a permanent labor surplus. Moreover, as occurred in the case of the supermultipier model, the alternative growth models will be presented in their simplest form in order to facilitate the comparisons and to draw our attention to the different theoretical closures provided by each model.

2.1. Cambridge growth models

Let us start with Cambridge growth models. Maintaining the hypothesis of permanent labor surplus, the version of the Cambridge growth model presented here[17] supposes that the level of aggregate output is determined by the level of capacity output. So, contrary to the supermultiplier growth model, the equilibrium level of aggregate output is not determined by effective demand, but by the supply constraint associated with full utilization of productive capacity. That is, we have

$$Y_t = Y_{Kt} = \frac{1}{v} K_t$$

and thus

$$u^* = 1$$

Also differently from the supermultiplier growth model, there is no autonomous consumption and, additionally to the consumption induced by the wage bill, there is also a component of aggregate consumption induced by total current profits. We retain the assumption that the propensity to consume out of wages c_w is equal to one (i.e., $c_w = 1$) and suppose that the propensity to consume out of profits c_π is a positive constant and has a value lower than one (i.e., $0 < c_\pi < 1$). Thus the consumption function is given by the following expression

$$C_t = \big(\omega_t + c_\pi(1 - \omega_t)\big)Y_t$$

where $\omega_t + c_\pi(1 - \omega_t)$ is the marginal propensity to consume, which is equal to the average propensity to consume since there is no autonomous component in the consumption function. Moreover, in contrast to the supermultiplier growth model, aggregate investment is an autonomous expenditure in this version of the Cambridge model, and we suppose, for the sake of simplicity, that investment expands at an

[16]This section is based on and confirms the main findings contained in the more general comparative analysis presented in Serrano (1995b, chapter 3). See also Serrano and Freitas (2007), for an early discussion of the main differences between the supermultiplier, Cambridge and neo-Kaleckian growth models containing similar results to the ones here obtained.

[17] See Robinson (1962). For similar formalizations of the Cambridge growth model see Dutt (1990 and 2011) and Lavoie (2014).

exogenously determined rate g_I>0. From these hypotheses we obtain the aggregate demand equation of the Cambridge model

$$D_t = \big(\omega_t + c_\pi(1-\omega_t)\big)Y_t + I_t$$

Since aggregate output is determined by capacity output, the equilibrium between aggregate demand and output requires that the former adjusts to the latter. In the Cambridge model such an adjustment involves a change in the marginal propensity to consume through the modification of income distribution (i.e., of the wage share). Thus, according to the model, a situation of excess aggregate demand (supply) raises (reduces) the general price level and, with a relatively rigid nominal wage, it causes a decrease (increase) in real wages. So, given labor productivity, the excess aggregate demand (supply) causes a reduction (an increase) in the wage share ω and, since $0 < c_\pi < c_w = 1$, it causes a decline (rise) in the marginal propensity to consume. Therefore, the adjustment implies a tendency for the establishment of an equilibrium between aggregate demand and supply at the level of output capacity (i.e. the level of potential output), which in equilibrium determines the level of aggregate demand.[18] At the same time, in equilibrium between aggregate demand and output capacity the model endogenously determines the income distribution between wages and profits. So, in the Cambridge model, the determination of a required level of income distribution allows the adjustment of aggregate demand to potential output, while in the supermultiplier growth model it is the appropriate change in the level of aggregate output that explains the adjustment of aggregate output to the level of aggregate demand.

Now, in equilibrium between aggregate output and aggregate demand, we have $Y_t = \big(\omega^* + c_\pi(1-\omega^*)\big)Y_t + I_t$ and, thus

$$S_t^* = s^* Y_t = s_\pi(1-\omega^*)Y_t = I_t$$

where $s_\pi = 1 - c_\pi$ is the marginal propensity to save out of total profits. The last equation shows that, according to the Cambridge model, aggregate investment determines aggregate (capacity) savings, although, as we saw above, the level of potential output determines the level of real aggregate demand. Furthermore, dividing both sides of the last equation by the level of aggregate output, we have an expression relating the saving ratio to the investment share of output as follows

$$\left(\frac{S_t}{Y_t}\right)^* = s^* = s_\pi(1-\omega^*) = \frac{I_t}{Y_t}$$

From the above equation we can see that in the Cambridge model the investment-output ratio determines the saving ratio $s^* = s_\pi(1-\omega^*)$, a result shared with the supermultiplier growth model. But, since in the Cambridge model there is no autonomous consumption component, the marginal and average propensities to save are equal to each other. Hence, the burden of the adjustment of the saving ratio to the investment share relies on required modifications in the *marginal* propensity to save and, therefore, on appropriate changes in income distribution.

Let us now discuss the determination of the equilibrium level of the investment share of output. From the assumption of full capacity utilization, the growth rate of output is given by the rate of capital accumulation (i.e., $g_t = g_{Kt}$). Thus, if we have initially $g_{Kt} < g_I$ ($g_{Kt} > g_I$), then we also have $g_t < g_I$ ($g_t > g_I$). It follows that the investment share of output would increase (decrease) and, according to

[18]Note however that the adjustment mechanism based on endogenous modifications of income distribution only guarantees the adjustment of aggregate demand to the level of potential output and not necessarily the adjustment of the degree of capacity utilization to its full capacity level. As Kaldor (1955-6) argued in his seminal discussion of such adjustment mechanism, the latter can be viewed as an alternative to the usual Keynesian adjustment based on variations of the level of aggregate output leading to the equilibrium between aggregate output and demand. In the present version of the Cambridge growth model, the maintenance of a full utilization of capacity resulting from the operation of the adjustment mechanism involving changes in income distributions is a consequence of the assumption that the binding supply constraint in the economy is the availability of capital. If the operative supply constraint were the full employment of the labor force, then the adjustment of aggregate demand to potential output based on endogenous changes in income distribution would not guarantee the full (or the normal) utilization of the available capital stock. In Kaldor's full employment growth models (c.f. Kaldor, 1957, 1958 and 1962) the investment function is responsible for the adjustment of the degree of capacity utilization. For an analysis of this role of the investment functions in Kaldor's full employment growth models see Freitas (2002, chapter 2; and 2009) and Palumbo (2009, pp. 343-345).

equation (2) and with the degree of capacity utilization constant, the rate of capital accumulation would increase (decrease). Eventually this type of adjustment leads to the convergence of the rate capital accumulation to the investment growth rate.[19] Therefore in the equilibrium path of the model we have

$$g^* = g_K^* = g_I$$

Substituting this last result in equation (2), solving for the investment share of output and recalling that $u^* = 1$, we have the equilibrium value of the investment share given by

$$\left(\frac{I_t}{Y_t}\right)^* = v(g_I + \delta)$$

Thus, according to the Cambridge growth model, a higher (lower) rate of growth of investment implies higher (lower) equilibrium growth rates of capacity output and output. Moreover, with a constant capacity utilization rate, this result is possible because the equilibrium level of the investment share is positively related to the growth rate of investment and, therefore to the equilibrium growth rates of output and capacity output. This last result is shared by the supermultiplier growth model although the process by which it is achieved is different from the process featured in the Cambridge growth model.

It is important to note, however, that the growth determining role of investment in the Cambridge model follows from its effect on productive capacity and *not* from its influence on aggregate demand. As we saw above, in the Cambridge growth model, the level of capacity output determines the level of aggregate demand. Thus, a higher (lower) growth rate of investment only causes a higher (lower) rate of output growth because it raises (reduces) the level of the investment share of output and, through this way, the pace of capital accumulation and the growth of capacity (or potential) output. Therefore, the Cambridge model displays, in fact, a supply (capacity) constrained pattern of economic growth, and hence, it is *not* a demand-led growth model.

Finally, we can substitute the expression for the equilibrium level of the investment share in the equation relating the investment share and the saving ratio. Doing so we can obtain the following result

$$(1 - \omega^*) = \frac{v(g_I + \delta)}{s_\pi} \qquad (18).$$

The last equation shows us the determinants of the required level of income distribution in the Cambridge model. In particular, we can see that, given v, δ and s_π, a higher (lower) growth rate yields a higher (lower) profit share of output. Thus, according to the Cambridge growth model and in contrast with the supermultiplier model, there exists a theoretically necessary relationship between income distribution and economic growth, the profit share of output (the wage share) is positively (negatively) related to the rate of economic growth. In the Cambridge model, such relationship is necessary for obtaining a growth path characterized by the equilibrium between aggregate demand and aggregate output with a constant degree of capacity utilization. Therefore, the closure provided by the Cambridge model requires the endogenous determination of an appropriate level income distribution, which must be compatible with various combinations of the values of the model's exogenous variables and parameters.[20]

[19] The adjustment process between the rate of capital accumulation and the expansion rate of investment can also be explained as follows. As we saw before, $\dot{g}_{Kt} = (g_{Kt} + \delta)(g_{It} - g_{Kt})$, so, since $g_{It} = g_I$, if initially $-\delta < g_{K0} \neq g_I$, then the capital accumulation rate would converge to the investment growth rate because from the last differential equation we can verify that $\dot{g}_{Kt} \gtreqless 0$ according to $g_{It} \gtreqless g_{Kt}$.

[20] Contrast equation (18) with equation (13) that represents the closure provided by the supermultiplier growth model.

2.2. Neo-Kaleckian growth models

We shall now compare the supermultiplier and the neo-Kaleckian growth models.[21] Like in the supermultiplier model, in the neo-Kaleckian growth model the level of aggregate demand determines the equilibrium level of aggregate output. Also, as occurs with the supermultiplier model, income distribution is exogenously determined and, therefore, cannot be part of the adjustment mechanism that allows the existence of the equilibrium path as in the Cambridge model. However, contrarily to the supermultiplier model, aggregate investment is autonomous and grows at an exogenously determined growth rate $g_I > 0$,[22] and also there is no autonomous component in aggregate consumption. In fact, in our representation of the basic neo-Kaleckian model we utilize the same specification for the consumption function used in the Cambridge model above. The only, but important, difference being that in the neo-Kaleckian model the wage share is exogenously determined and, accordingly, the marginal propensity to consume (equal to the average propensity to consume) is also an exogenous variable.

With these hypotheses, aggregate demand is given by the following expression

$$D_t = \big(\omega + c_\pi(1-\omega)\big)Y_t + I_t$$

and in equilibrium between aggregate demand and output we have

$$Y_t^* = \left(\frac{1}{s}\right) I_t = \left(\frac{1}{s_\pi(1-\omega)}\right) I_t \qquad (19).$$

According to the last equation aggregate investment is the main determinant of the equilibrium level of output. Further, given income distribution (i.e. the wage share), the value of the multiplier $1/s = 1/\big(s_\pi(1-\omega)\big)$is constant. Thus, in the neo-Kaleckian model, as can be verified from the last equation, the pace investment expansion determines the equilibrium output growth rate of the economy for a given level of income distribution. That is, we have

$$g^* = g_I$$

So like the supermultiplier growth model, the neo-Kaleckian model produces a demand-led growth pattern. But while in the supermultiplier model we have a consumption-led growth pattern, the neo-Kaleckian model generates an investment-led growth pattern.

From equation (19) we can also verify that

$$S_t^* = sY_t^* = s_\pi(1-\omega)Y_t^* = I_t$$

and

$$\frac{I_t}{Y_t^*} = \frac{S_t^*}{Y_t^*} = s = s_\pi(1-\omega)$$

Thus, according to the first of the two equations above, the level aggregate savings adjusts to the level of aggregate investment through the variation of the level of aggregate output, the only endogenous variable in the equation. Note however that, as we can verify from the second equation above, given s_π and ω, the saving ratio, equal to the marginal propensity to save s, is exogenously determined and thus it determines the investment share of output in the neo-Kaleckian model. This feature of the model contrasts with the

[21]These models descend from Kalecki (1971) and Steindl (1952 and 1979) original contributions. The modern neo-Kaleckian model was presented originally by Dutt (1984) and Rowthorn (1981). See also Dutt (1990) and Lavoie (2014) for a formalization and comparison of the neo Kaleckian model with alternative growth models. For a detailed survey of the literature, see Blecker (2002).

[22]Note that our presentation of the neo-Kaleckian model has some minor differences in relation to the usual presentation of these models. The main difference is that we specify the investment function in terms of the determinants of the investment growth rate, whilst the usual neo-Kaleckian specification is in terms of the determinants of the desired rate of capital accumulation. Observe that this difference doesn't affect the equilibrium values of the model's endogenous variables because, since by definition $g_{It} = g_{Kt} + \dot{g}_K/(g_{Kt} + \delta)$ and since, in equilibrium, we have $\dot{g}_K = 0$, then in equilibrium we obtain $g_I^* = g_K^*$. Moreover, as we shall see shortly, our specification of the investment function doesn't affect also the equilibrium stability condition. Therefore, we claim that nothing essential is altered by our specification of the investment function of the neo-Kaleckian model.

related result obtained from the Cambridge and supermultiplier growth models. Indeed, as we pointed out before, in these latter models the investment share of output determines the saving ratio. In the Cambridge model this result follows from changes in income distribution and in the marginal propensity to save, whilst in the supermultiplier growth model the same result follows from the existence of an autonomous component in aggregate consumption which makes the saving ratio endogenous even though income distribution and the marginal propensity to save are given exogenously. In contrast, the neo-Kaleckian model assumes that income distribution (and thus the marginal propensity to save) is exogenously determined and that there is no autonomous consumption component, which implies, in combination with the other assumptions of the model, the exogeneity of the saving ratio.

Now, since the saving ratio is an exogenous variable and it determines the investment share of output, then the latter variable cannot be changed according to the requirements of the pace of economic growth. So, in contrast with the supermultiplier growth model, according to the neo-Kaleckian growth model, given income distribution, a change in the investment and output growth rates does not have any effect on the equilibrium value of the investment share of output. More importantly, from equation (2) we can verify that in the neo-Kaleckian model the rate of capital accumulation can only be reconciled with the output/demand growth rate if the degree of capacity utilization is properly adjusted. Indeed, since in the neo-Kaleckian model the saving ratio determines the investment share of output, then, according to equation (2), the rate of capital accumulation is given by

$$g_{Kt} = \left(\frac{s_\pi(1-\omega)}{v}\right)u_t - \delta$$

On the other hand, we saw that in the neo-Kaleckian model the growth rate of investment determines the equilibrium growth rate of output, so we have that $g^* = g_I$. Thus, using these results in equation (3), we obtain the following differential equation for the dynamic adjustment of the degree of capacity utilization

$$\dot{u} = u_t\left(g_I - \left(\frac{s_\pi(1-\omega)}{v}\right)u_t + \delta\right) \tag{20}$$

The last equation shows that if the investment growth rate is higher (lower) than the rate of capital accumulation, then the degree of capacity utilization increases (declines) and this raises (reduces) the pace of capital accumulation. As a result, the capital accumulation rate converges to the investment growth rate through changes in the degree of capacity utilization. Therefore, in the equilibrium path of the neo-Kaleckian model we have

$$g_K^* = g_I = \left(\frac{s_\pi(1-\omega)}{v}\right)u^* - \delta$$

Now, solving the last equation for the equilibrium degree of capacity utilization we obtain

$$u^* = \frac{v(g_I+\delta)}{s_\pi(1-\omega)} \tag{21}$$

Equation (21) shows the determinants of the required degree of capacity utilization u^* in the simplified version of the neo-Kaleckian model here presented. This latter rate is the one that reconciles the rate of capital accumulation with the pace of economic growth and, therefore, allows the existence of an equilibrium growth path in the model. Observe that, in its role as an adjusting variable, the equilibrium degree of capacity utilization has to able to assume any value between zero and one, however implausible it may be. Therefore the neo-Kaleckian model is not compatible with the related notions of planned spare capacity and normal (or desired) capacity utilization rate. Indeed, if we suppose the existence of a normal degree of capacity utilization, the closure provided by the model implies that it would be possible to have large and persistent deviations of the equilibrium degree of capacity utilization from its normal level and also that such divergence would not have any repercussion on capitalist investment decisions.[23] It is important to be remarked that the required long run endogeneity of the equilibrium degree of capacity utilization does not depend on the particular specification for the investment function adopted here, being in fact valid for all the usual specifications of the investment function in neo-Kaleckian models.[24] Actually, the necessity concerning the variability of the equilibrium degree of capacity utilization follows from the specification of the consumption function and *not* from the particular formulation of the investment function adopted in the model. As we argued above, it is the rigidity of the investment share of output implied by the exogeneity of the saving ratio that leads to the requirement of the long run variability of the equilibrium capacity utilization rate.

Further, note that admitting the possibility of an adjustment of the actual degree of capacity utilization to the normal one in the context of a neo-Kaleckian model only leads to an instability process of the Harrodian type.[25] Indeed, suppose, following Skott (2008), that in trying to adjust the actual degree of capacity utilization to its normal level, capitalist firms change the investment growth rate according to $\dot{g}_I = \eta(u_t - \mu)$, $\eta > 0$. Thus, in the equilibrium path of this particular model (i.e. with $\dot{g}_I = \dot{u} = 0$), we would have $u^* = \mu$ and $g_I^* = (s_\pi(1-\omega)/v)\mu - \delta$. Now, if initially we have $u_0 = \mu$ and $g_{I0} \gtreqless g_I^*$, then, according to equation (20), we would have $\dot{u} \gtreqless 0$, which implies that thereafter we would have $u_t \gtreqless \mu$ and, hence, $\dot{g}_I \gtreqless 0$ and $g_{It} \gtreqless g_I^*$. So the equilibrium rate of growth would be unstable. Thus, according to the neo-Kaleckian growth model, we would have a dilemma: either, on the one hand, we assume way the possibility of an adjustment of the actual degree of capacity utilization towards its normal level and admit the possibility of obtaining an equilibrium path with an implausibly high or low equilibrium degree of capacity utilization or, on the other, we allow an adjustment of the actual to the normal degree of capacity utilization and obtain an unstable growth trajectory as we just saw. Observe, however, that the dilemma exists only if we restrict ourselves to the set of assumptions of the neo-Kaleckian model. In fact, once we admit the existence of an autonomous component in aggregate consumption the saving ratio becomes an endogenous variable and the investment share of output can change allowing the adjustment of the actual degree of capacity utilization to its normal level, as we have in the supermultiplier growth model.[26]

Finally, we shall analyze the role of income distribution in the neo-Kaleckian growth model. Thus, in the very simple version of the model presented here there is no relationship between the pace of economic growth and the level of functional income distribution. Since the investment growth rate is supposed to be an exogenous variable in the model, the equilibrium rate of output growth does not affect and is not affected by the level of the wage share of output. Nevertheless, a change in the wage share has a level effect over the equilibrium value of output according to the simple neo-Kaleckian model under analysis. In fact, an increase (decrease) in the wage share, raises (reduces) the value of the multiplier $1/\big(s_\pi(1-\omega)\big)$ and, through it, such a change has a positive (negative) level effect on equilibrium output. These two latter results are shared with the supermultiplier growth model. On the other hand, in contrast with the latter model, from equation (21) we can see that, in the neo-Kaleckian model, an increase (decrease) in the wage share leads to an increase (a reduction) in the equilibrium degree of capacity utilization. A model that presents this type of result is classified in the neo-Kaleckian literature as a "stagnationist" or "wage led aggregate demand" model.[27]

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[23]Compare equation (21) with the theoretical closure associated with the supermultiplier growth model as represented by equation (13) above.

[24]For instance, the result under discussion is valid for the investment function given by equation (22) (below in the text). Indeed, the endogenous character of the equilibrium degree of capacity utilization and its implications are maintained as can be verified from equation (23) (below in the text) which is the equation that shows the determinants of the equilibrium degree of capacity utilization corresponding to the investment function represented by equation (22). The same point is valid for other investment functions that frequently appear in the neo-Keleckian growth literature and, in particular, it is valid in the case of the investment function given by $g_{It} = \alpha + \beta(u_t - \mu)$ with $\alpha,\beta > 0$ and μ exogenous, Note that in the latter investment function the normal degree of capacity utilization appears as an argument. Nonetheless, the endogenous character of the equilibrium degree of capacity utilization is also maintained in this case and the corresponding value of the equilibrium rate is given by $u^* = \big(v(\alpha + \delta - \beta\mu)\big)/(s_\pi(1-\omega) - \beta v)$.

[25]In this connection, see Hein, Lavoie and van Treeck (2012) for a survey on Harrodian instability and the tendency for the normal capacity utilization rate in neo-Kaleckian growth models.

[26] SeeAllain (2014) for a growth model that comes from the Kaleckian tradition pointing in the direction of a closure similar to the one provided by the supermultiplier growth model presented in this paper.

[27]See Blecker (2002) for discussion of the neo-Kaleckian models based on this type of classification.

We must say, however, that the independence between the pace of economic growth and income distribution in the simple version of the neo-Kaleckian model presented above is a direct consequence of the specific investment function adopted, which, as we saw, assumes the rate of investment growth to be completely exogenous. Indeed, if we consider the more usual formulations of the investment function in the neo-Kaleckian models, then we can obtain a causal relationship running from income distribution to the pace of economic expansion. So let us consider, for instance, a linear version of the investment function suggested by Marglin&Bhaduri (1990) and Bhaduri&Marglin (1990)

$$g_{It} = \alpha + \beta u_t + \rho(1-\omega) \tag{22}$$

where $\alpha > 0$ is an autonomous component of the investment function, $\beta > 0$ is a parameter measuring the sensibility of the growth rate of investment to the capacity utilization rate, and $\rho > 0$ is a parameter measuring the sensibility of the investment growth rate with respect to the profit share (i.e. $(1-\omega)$). The introduction of an induced component βu_t in the investment function turns the investment growth rate into an endogenous variable of the model thatpositively depends on the degree of capacity utilization. With such specification for the investment function, the equilibrium value of the degree of capacity utilization is given by

$$u^* = \frac{v\big(\alpha + \delta + \rho(1-\omega)\big)}{s_\pi(1-\omega) - \beta v} \tag{23}$$

From the equation above we can see that an equilibrium with a positive value for the degree of capacity utilization requires that $\beta < s_\Pi(1-\omega)/v$.[28] Also, it can be shown that, *ceteris paribus*, an increase (decrease) in the wage share raises (reduces) the equilibrium degree of capacity utilization.[29] Thus, the model still is classified as "stagnationist" or "wage led aggregate demand". Now, substituting this last result in the investment function, we obtain the equilibrium level of the investment growth rate as follows

$$g_I^* = \alpha + \beta\left(\frac{v\big(\alpha + \delta + \rho(1-\omega)\big)}{s_\pi(1-\omega) - \beta v}\right) + \rho(1-\omega)$$

Thus, since u^* is positively related to the level of the wage share, then the effect of a change in the wage share on the investment growth rate can be either positive or negative according to the value of the parameter ρ. As can be seen from the equation above, a higher value of the latter parameter reduces the positive (and indirect) effect of a change of the wage share exerted through the equilibrium degree of capacity utilization and increases the direct contribution of a change in the wage share through the third term on the RHS of the equation above. Hence, for a sufficiently *low* value of ρ the positive effect of a modification in the wage share on the investment growth rate through the capacity utilization rate dominates the direct negative effect related to the term $\rho(1-\omega)$. In this case, according to the neo-Kaleckian literature, the model would produce a wage led growth pattern. On the other hand, for a sufficiently *high* value of ρ we would have the opposite situation and the model would generate a profit led pattern of economic growth.[30] In both cases, a change in income distribution has a permanent growth

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[28] The inequality $\beta < s_\Pi(1-\omega)/v$ is also necessary for the stability of the model. To see this, note that, in the case of the present version of the model, the equation for the dynamic adjustment of the degree of capacity utilization (equation (20)) would be given by $\dot{u} = u_t(\alpha + \beta u_t + \rho(1-\omega)) - (s_\pi(1-\omega)/v)u_t + \delta) = u_t\big(\alpha + \rho(1-\omega) - \big((s_\pi(1-\omega)/v) - \beta\big)u_t + \delta\big)$. Observe that now a change in the capacity utilization rate affects both the investment growth rate and pace of capital accumulation in the same direction. Thus, if initially the investment growth rate were higher (lower) than the capital accumulation rate, then the capacity utilization rate would increase (decline). For its turn, this latter change would produce the adjustment between investment growth and capital accumulation rates only if the impact of a change in the capacity utilization rate over the capital accumulation rate is greater than the impact of such a change on the pace of investment growth. That is, only if $\partial g_{Kt}/\partial u_t = s_\pi(1-\omega)/v > \beta = \partial g_{It}/\partial u_t$, which is the inequality mentioned above.

[29] Thus taking the partial derivative of u^* with respect to ω we obtain that $\partial u^*/\partial\omega = (v[\rho\beta v + s_\pi(\alpha+\delta)])/(s_\pi(1-\omega) - \beta v)^2 > 0$, which justifies the "stagnationist" or "wage led aggregate demand" classification attributed to the model. Observe, however, that if a nonlinear specification of the investment function were adopted then it would be possible, according to neo-Kaleckian literature, to obtain a negative relationship between $u^*$and ω. In this case, following the suggestion of Marglin and Bhaduri (1990), the model would be classified as "exhilarationist" (or "profit led aggregate demand"). See Blecker (2002) for a detailed discussion of these topics.

[30] For sufficiently low or high value of ρ we mean a value of ρ respectively lower or higher than a critical value $\rho_c = \big(\beta v(\alpha+\beta)\big)/\big((s_\pi(1-\omega) - 2\beta v)(1-\omega)\big)$. Therefore, we would have a wage led (profit led) growth pattern as $\rho < \rho_c (\rho > \rho_c)$. That is we have $\partial g_I^*/\partial\omega \gtreqless 0$as $\rho \lesseqgtr \rho_c$.

effect. The existence of a relationship between economic growth and income distribution featured in the last version of the neo-Kaleckian model, also characterizes the Cambridge growth model as we saw. In the latter model there is an inverse relationship between the wage share and the rate of output growth, whereas the last specification of the neo-Kaleckian model admits either, a positive (in the wage led growth case) or a negative (in the profit led growth case) relationship between the two variables. In contrast with these results, the absence of any permanent relationship between income distribution and economic growth is an important feature of the supermultiplier growth model.

2.3. General comparison

Table 1 summarizes the principal results obtained from the above comparative analysis. The main conclusion that emerges from this analysis is that the supermultiplier growth model can be considered a true heterodox alternative to the Cambridge and neo-Kaleckian growth models in the analysis of the relationship between economic growth, income distribution and effective demand. In this sense, first of all, the supermultiplier growth model shows how is it possible for a heterodox growth model to obtain a tendency towards the normal utilization of productive capacity without relying on the endogenous determination of the level of income distribution as in Cambridge growth model. Thus, the supermultiplier model does not impose the existence of any necessary *a priori*relationship between income distribution and economic growth in the interpretation of economic reality and, therefore, leave open the space for the determination of income distribution from outside the model by political, historical and economic factors not directly and necessarily related to the process of economic expansion. Secondly, the model also shows that the existence of a stable process of demand-led growth does not require the endogenous determination of an equilibrium degree of capacity utilization as in the neo-Kaleckian growth model. Actually, the supermultiplier model shows that the existence of a demand-led pattern of economic growth is fully compatible with the tendency towards the normal utilization of productive capacity. Therefore, it shows that allowing the possible prevalence of arbitrarily high or low rates of capacity utilization and permanent deviations of the actual degree of capacity utilization from its normal level are not necessary requirements for the existence of a demand-led pattern of economic growth.

Table 1 here

3. Final Remarks

In this paper we have shown how the Sraffian supermultiplier model with its hypotheses of growing non capacity generating autonomous demand and induced investment provides us with a distinct closure for an heterodox growth model. Using this closure changes in the propensity to invest can determine the saving ratio through changes in the fraction, i.e., the ratio of the average to the marginal propensity to save. We have also shown how the gradual changes in the marginal propensity to invest that allow capacity to adjust to demand allow also for the reconciliation of demand led growth, exogenous distribution and a tendency to normal degree of capacity utilization even across steady states. This is in sharp contrast with the Cambridge model that has to resort to an implausible theory of distribution driven by accumulation in order to retain the idea of a tendency towards normal utilization. It constrast also with the kaleckian model that rightly maintains the idea of the lack of a necessary relation between distribution and accumulation but does so at the cost of the implausible property of the actual degree of utilization being in general always different from the normal degree, even across steady states. In our view (and also of Cesaratto(2015)) the Sraffian supermultiplier and its closure through an endogenous saving ratio offer a useful way of combining the classical surplus approach to distribution and the principle of effective demand in the same simple model that can be used as a tool to analyse the process of accumulation along the lines suggested by the pioneering work of Garegnani(1962[2015])).

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Table 1

	Cambridge Growth Model	**neo-Kaleckian Growth Model**	**Supermultiplier Growth Model**
Output capacity, aggregate output and aggregate demand	*Capacity (potential) output determines the levels of aggregate output and aggregate demand*	*Aggregate demand determines the level of aggregate output*	*Aggregate Demand determines the levels of aggregate output and capacity (potential) output*
Income distribution	*Endogenous*	*Exogenous*	*Exogenous*
Investment share of output and saving ratio	*The investment share of output determines the saving ratio*	*The exogenous saving ratio determines the investment share of output*	*The investment share of output determines the saving ratio*
Pattern of economic growth	*Supply (capacity) constrained growth*	*Demand (investment) led growth*	*Demand (consumption) led growth*
Degree of capacity utilization	*Tends to full capacity utilization rate*	*Tends to an endogenous equilibrium value*	*Tends to the normal degree of capacity utilization*
Investment share of output and the trend rate of economic growth	*There is a theoretically necessary and positive relationship between the two variables*	*No theoretically necessary relationship between the two variables*	*There is a theoretically necessary and positive relationship between the two variables*
Wage share of output (income distribution) and the rate of economic growth	*There is a theoretically necessary and negative relationship between the two variables*	*There is a positive relationship between the two variables in the "wage led growth" case and a negative one in the "profit led growth" case*	*No relationship between the two variables.* *There is a positive wage led output level effect*
Theoretical closure	*Endogenous determination of a required level of income distribution*	*Endogenous determination of a required equilibrium degree of capacity utilization*	*Endogenous determination of a required value of the fraction (i.e. the ratio between the average and marginal propensities to save)*