July 2015

Precautionary and Long Run Risks for an Open Economy

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Abstract

This paper investigates open economy factors for precautionary and long run currency risk pricing. It is shown that precautionary savings in foreign currency arises because of the volatility differential between domestic and imported consumption growth. We present evidence that the variance of foreign consumption is much higher than the variance of domestic, durable and nondurable consumption growth for the domestic USA resident. These open economy factors are also shown to be strongly positively correlated with the USA business cycle (between 1967Q3 and 2012Q2). Estimation and simulation results imply - through implied parameters and implied risk - that the open economy model can explain the risk faced by a domestic USA resident with regards to Japan and the rest of the world.

Keywords: risk, exchange rate, consumption, imports.

JEL Classification code: F31. F36. E44

Resumo

Este artigo investiga a implicação para a teoria de precificação do risco cambial de longo-prazo e precaucionário em um modelo de agente representativo. A poupança precaucionária em moeda estrangeira surge nesse modelo por causa da diferença de volatilidade entre o crescimento do consumo doméstico e importado. Apresentam-se evidências de que a volatilidade do consumo de importados é muito maior do que a de bens domésticos, duráveis e não-duráveis. A taxa de crescimento de bens domésticos e importados também está correlacionada com os ciclos de negócios dos EUA (entre 1967T3 e 2012T2). Resultados de estimações e simulações mostram, através dos parâmetros e do risco implícito, que o modelo de economia aberta pode explicar o risco cambial de um norte-americano em relação à economia Japonesa e ao resto do mundo.

Palavras-chave: risco, câmbio, consumo, importações, juros.

Área ANPEC: 4; Macroeconomia, Economia Monetária e Finanças

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I acknowledge the invaluable support from the State of São Paulo research foundation FAPESP and from the University of São Paulo, USP. I would like to thank Prof. Fernando de Holanda Barbosa and Prof. Miguel León-Ledesma for comments on an earlier version of this paper and also seminar participants at the REAL (Regional Economics Applications Laboratory) at Illinois, Urbana-Champaign and at Queen's University Belfast. I would finally thank the University of Illinois at Urbana-Champaign and Queen's University Belfast for research facilities between 2012M3 until 2012M6 and 2013M2 to 2014M2, respectively. Data were collected from the repository Datastream while the author was visiting Queen's University Belfast and for which he is also very grateful. Usual disclaimers apply.

Introduction

Standard Stochastic Discount Factors (SDFs), such as the growth of nondurable and durable consumption, offer an incomplete motive for pricing a bond which pays monetary units in foreign currency. This paper explores the traditional foreign and domestic consumption growth factors, which follow from international trade theory, for precautionary and long run risk asset pricing[2].

There are interesting economic interpretations for domestic and imported consumption growth factors, as they are able to provide microeconomic foundations for buying a foreign bond: smoothing out the consumption of imported goods. These factors are theoretically superior when compared to the conventional durable/nondurable model, because precautionary savings in foreign currency arises as a response to an increase in the variability of future consumption. They are able to rationalize why a representative agent holds external wealth and how volatility in consumption and uncertainty regarding exchange rates can impact on currency risk. The present paper focus on risk derived from precautionary savings, i.e., risk due to changes in consumption growth variance. For this purpose, it analyzes currency risk assuming different functional forms for the instantaneous utility functions (power and habit) showing that consistent interpretations are possible when these new factors are introduced.

We show that open economy factors present a great sample variability when compared to aggregate durable and nondurable growth. As will be revealed for the case of the United States of America (USA), the variance of durable consumption growth is 44% higher than the variance of nondurable growth from 1967Q3 to 2012Q2. And the variance of imported consumption growth is 133% higher than the variance of durable consumer growth during the aforementioned period. Although our main focus is on consumption growth variances, we also show that these factors are strongly positively correlated to the USA business cycles, using both NBER recessions and the output gap as proxies. Finally, our model estimations and calibrations imply reasonable values for the share of domestic consumption goods (67%) and the parameter of risk aversion (9.69). It also implies reasonable levels for steady state risk, considering the habit persistence model, which ranges from the extremes -2.31% to 5.57% *per quarter*, but also reaches more modest numbers such as 0.31% *per quarter*, for instance. In summary, there is evidence supporting the claim that highly variable imported consumption growth, in addition to domestic growth, can be more appropriated stochastic discount factors for currency risk pricing than the usual nondurable/durable model of consumption growth.

This investigation is important for both researchers and policy makers. It is interesting to researchers not only because it complements the asset pricing literature, but also because it lies at the center of some of the most difficult unsolved puzzles in international macroeconomics, such as the uncovered interest parity empirical failure. For policy makers, the answer is important to understand the dynamics of real interest rates in an open economy, which is a vital issue for emerging countries with high average real interest rates.

The rest of the paper is organized as follows. The next section introduces the currency risk theory of asset pricing with domestic and imported consumption growth in the utility function. This is accomplished by using a simple model under no default risk and no state contingent assets. Section 3 explains the data sources and presents a descriptive and graphical analysis of the main growth variables discussing key empirical results.

[2]Earlier theoretical models have been explaining currency pricing using durable and non-durable consumption growth factors (see, for instance, Lustig & Verdelhan (2007)). Although, our monetary asset pricing model is based on the theoretical contribution of Ferreira & Moore (2015), the paper's approach is different in various and important ways. For instance, Ferreira & Moore (2015) completely focuses on estimating covariances between SDFs and excess returns in a log-linear factor model, whereas the present paper abstracts from covariances and focuses on precautionary and long run risks.

1 Currency Risk Theory

Consider two open economies, the "H-country" (H for home) and the "F-country" (F for foreign), populated by a representative household[3]. The period utility function of the representative agent in the home country is represented by $u(C_{yt}, C_{y^*t})$[4]. It follows that a domestic representative household can choose her optimal consumption allocation path deciding how much to buy of home and foreign goods as well as, from an inter-temporal dimension, deciding how much to hold of domestic and foreign bonds. Hence, the representative household in the domestic economy chooses to consume C_{yt} units of the home produced good and C_{y^*t} of the foreign produced good. Unless otherwise stated, foreign variables will be represented by an asterisk.

At the beginning of each period, indicated by the t subscript, the family receives an endowment, Y_t, in domestic currency and another, Y_t^*, in foreign currency. The state variables are the quantities B_t of domestic bonds and B_t^* of foreign bonds, each one paying one unit of its respective currency at $t+1$. E_t is the expectation conditional on all information available at t.

The problem of the representative agent is then

$$\max_{C_{ys}, C_{y^*s}} \left(E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_{ys}, C_{y^*s}) \right),$$

subject to

$$P_s C_{ys} + P_s^* C_{y^*s} S_s + B_s P_s^B + B_s^* S_s P_s^{B^*} - B_{s-1} - B_{s-1}^* S_s - P_s Y_s - P_s^* Y_s^* S_s = 0,$$

in addition to a given initial stock of wealth and a transversality condition, with $s = t, t+1, \ldots, \infty$. β is the subjective intertemporal discount factor; W_t is the wealth at the beginning of each period t;P_t is the price level of the domestic consumption basket and S_t is the nominal exchange rate, defined as the domestic price of foreign currency. The superscript refers to the country of origin of the good. Lowercase variables are in logarithms, Greek letters without subscripts are parameters. Incorporating the constraint into the maximization problem and optimizing using the Bellman equation below

$$V_t(B_{t-1}, B_{t-1}^*) = \max_{C_{yt}, C_{y^*t}} \left(u(C_{yt}, C_{y^*t}) + \beta E_t V_{t+1}(B_t, B_t^*) \right),$$

one can find the following Euler equations

$$u_1(C_{yt}, C_{y^*t}) \frac{P_t^B}{P_t} = \beta E_t \left(u_1(C_{yt+1}, C_{y^*t+1}) \frac{1}{P_{t+1}} \right), \tag{1}$$

$$u_1(C_{yt}, C_{y^*t}) \frac{S_t P_t^{B^*}}{P_t} = \beta E_t \left(u_1(C_{yt+1}, C_{y^*t+1}) \frac{S_{t+1}}{P_{t+1}} \right), \tag{2}$$

$$u_2(C_{yt}, C_{y^*t}) \frac{P_t^B}{S_t P_t^*} = \beta E_t \left(u_2(C_{yt+1}, C_{y^*t+1}) \frac{1}{S_{t+1} P_{t+1}^*} \right), \tag{3}$$

$$u_2(C_{yt}, C_{y^*t}) \frac{P_t^{B^*}}{P_t^*} = \beta E_t \left(u_2(C_{yt+1}, C_{y^*t+1}) \frac{1}{P_{t+1}^*} \right). \tag{4}$$

The left hand side corresponds to the loss in terms of marginal utility for the H-household of decreasing period t consumption in order to buy bonds. The right hand side is the expected marginal

[3]We will use the words country and economy, home and domestic as well as household and agent, respectively, as synonyms.

[4]We are also implicitly assuming a symmetric function for the F-household. However, for ease of exhibition, we will focus on the H-household.

benefit of the real consumption at $t+1$ discounted for period t. The cost of the home bond is P_t^B, which represents the nominal price of a zero-coupon discount bond that pays with certainty one unit of the domestic currency at $t+1$. An analogous definition applies for the foreign bond.

We will later analyze currency risk premium under both a power utility specification[5] and a habit based explanation. For the latter reason, we will compare our theoretical results with the ones from the papers of Verdelhan (2010) and Moore & Roche (2010) which are both based on the work of Campbell & Cochrane (1999). We will also discuss the implication of their empirical results on the light of the theory that is derived under these optimization conditions. Moore & Roche (2010) present a detailed monetary model following the two-country, two-good model framework of Lucas (1982) while Verdelhan (2010)'s simpler structure consider only a real economy - both under a habit consumption explanation. As the Euler equations above can be largely simplified by considering that some of the partial derivatives are equal, it becomes important to mention that the model presented in Verdelhan (2010) will appear as a special case of the one that is developed here.

Given the assumptions regarding bond prices, more specifically, given that the riskless bond pays one unit with certainty at time $t+1$ of the corresponding issuer-country currency, one can also write

$$P_t^B = \frac{1}{1+i_t},$$

$$P_t^{B*} = \frac{1}{1+i_t^*},$$

where i_t is the nominal interest rate (with $t+1$ maturity).

In order to show the *ex ante* currency risk, let us concentrate on two of these Euler equations, equations (1) and (3). Since i_t, with $t+1$ maturity, is known at t, these Euler equations can be written as

$$1 = E_t\left(M_{t+1}(1+i_t)\frac{P_t}{P_{t+1}}\right), \tag{5}$$

$$1 = E_t\left(M_{t+1}^*(1+i_t)\frac{S_t P_t^*}{S_{t+1} P_{t+1}^*}\right), \tag{6}$$

where the stochastic discount factors (SDF) are defined as

$$M_{t+1} \equiv \beta\left(\frac{u_1(C_{yt+1}, C_{y^*t+1})}{u_1(C_{yt}, C_{y^*t})}\right), \tag{7}$$

$$M_{t+1}^* \equiv \beta\left(\frac{u_2(C_{yt+1}, C_{y^*t+1})}{u_2(C_{yt}, C_{y^*t})}\right). \tag{8}$$

Defining the *ex post* real interest rate as $1+r_{t+1} \equiv \frac{1+i_t}{1+\pi_{t+1}}$, with $1+\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$, and using this definition in (7) and (8) above, gives

$$1 = E_t\left(M_{t+1}R_{t+1}\right), \tag{9}$$

$$1 = E_t\left(M_{t+1}^* R_{t+1}\frac{Q_t}{Q_{t+1}}\right), \tag{10}$$

where the real return is $R_{t+1} \equiv (1+r_{t+1})$ and the real exchange rate is defined as $Q_t \equiv \frac{P_t^* S_t}{P_t}$.

[5] We will not delve into more sophisticated types of utility functions, such as Epstein & Zin (1989), due to their analytical complexity. If we decided to log-linearize them and find a "natural" point of expansion, we would collapse into a power utility specification.

Using (9) and (10), we have

$$E_t\left(M_{t+1}R_{t+1}\right) = E_t\left(M_{t+1}R^*_{t+1}\frac{Q_{t+1}}{Q_t}\right). \tag{11}$$

Ex ante currency risk premia is defined as

$$E_t(\rho^c_{t+1}) \equiv E_t(r_{t+1}) - E_t(r^*_{t+1}) - E_t(\Delta q_{t+1}), \tag{12}$$

where $E_t(r_{t+1}) \equiv E_t \ln(R_{t+1})$, $E_t(r^*_{t+1}) \equiv E_t \ln(R^*_{t+1})$, $q_t \equiv \ln(Q_t)$ and the first difference operator is represented by the Δ symbol[6].

It is convenient to express both foreign and domestic real interest rates as a function of the stochastic discount factors for the H-representative, $E_t(M_{t+1})$ and $E_t(M^*_{t+1})$ [as in Backus et al. (2001), for instance].

1.1 Precautionary Currency Risk

Let us consider that the covariance between these SDFs and the real return on the risk free rate are zero, which means that we are abstracting from the covariance between the SDFs and changes in inflation between t and $t+1$, the unknown variable for the expected real return at $t+1$ evaluated at time t. By doing this, we are focusing on risk (uncertainty regarding future marginal utility) derived from consumption growth variance and not on risk due to the covariance between excess returns and SDFs. We also departure from the It follows that equations (9) and (10), can be written as

$$E_t(R_{t+1}) = \frac{1}{E_t\left(M_{t+1}\right)}, \tag{13}$$

$$E_t(R^*_{t+1}) = \frac{1}{E_t\left(M^*_{t+1}\right)}. \tag{14}$$

Applying natural logs in both sides of (13) and (14) and considering that SDFs are lognormal:

$$E_t(r_{t+1}) = -\ln\left(E_t\left(M_{t+1}\right)\right) = -\ln\left(E_t\left(e^{m_{t+1}}\right)\right) = -E_t(m_{t+1}) - \frac{1}{2}\mathrm{Var}_t(m_{t+1}),$$

$$E_t(r^*_{t+1}) = -\ln\left(E_t\left(M^*_{t+1}\right)\right) = -\ln\left(E_t\left(e^{m^*_{t+1}}\right)\right) = -E_t(m^*_{t+1}) - \frac{1}{2}\mathrm{Var}_t(m^*_{t+1}).$$

Given (9) and (10), we can write the expected change in the exchange rate as

$$E_t(\Delta q_{t+1}) = E_t(m^*_{t+1}) - E_t(m_{t+1}). \tag{15}$$

We find that the *ex ante* currency risk premia is[7]

$$E_t(\rho^c_{t+1}) = \frac{\mathrm{Var}_t(m^*_{t+1}) - \mathrm{Var}_t(m_{t+1})}{2}. \tag{16}$$

The model earlier developed allows us to conclude that the risk factor that matters for the

[6]One might ask whether the definition above of "currency risk premia" makes sense, given that we mean "currency risk" for the domestic agent in the agent's own currency. There is an economic explanation for the above equation. Suppose that the H-country is the UK and the F-economy is the USA, for example. In addition, consider that the home representative agent borrows funds in the USA to buy bonds in UK, for instance. It follows that the expected excess return for this investor corresponds to the expected real return on the British bond minus the real cost of borrowing in the USA and minus the expected real depreciation of the British currency. If the pound Sterling depreciates, $\frac{Q_{t+1}}{Q_t}$ increases and the domestic agent who borrowed in dollars will loose.

[7]The currency risk in Verdelhan (2010) is $\frac{\mathrm{Var}_t(m^*_{t+1}) - \mathrm{Var}_t(m_{t+1})}{2}$, as the definition in this paper is $\Delta q_{t+1} + r^*_{t+1} - r_{t+1}$.

domestic household is the volatility of each of the SDFs which are closely related to the variance of H and F consumption growth shocks (i.e. overall consumption variance).

By observing that the SDFs are related to how agents feel regarding the consumption of both H and F goods, the economic interpretation of the previous equations becomes straightforward. SDFs will depend on the exogenously given consumption growth but will determine endogenously obtained bond prices. Hence, if the variance of the F-good's consumption is relatively higher than the variance of the H-good's consumption, *ceteris paribus*, domestic households will buy more of the foreign bond relative to the domestic bond increasing its relative price, $\frac{P_t^{B*}S_t}{P_t^B}$. In other words, the F-bond becomes more valuable since it insures the H-household against overall consumption fluctuations and, by consequence, marginal utility variations arising from higher F-good's consumption variance. This can be interpreted as precautionary savings in foreign currency because consumption growth variance is the unique force driving attitudes towards risk. The price of the F-bond in period t is thus higher than the H-bond when both assets are expressed in the same currency. As a consequence, the *ex ante* return on the F-bond is relatively lower, which explain an expected currency premium on the H-bond. In the next sub-sections, we assume specific functional forms for the period utility function and derive these relationships directly.

1.2 Power Utility

Consider that C_t is made of a composite of the two goods aggregated by a Cobb-Douglas function and that preferences are of the CRRA form

$$u(C_{yt}, C_{y^*t}) \equiv \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{17}$$

with

$$C_t \equiv C_{yt}^{\theta} C_{y^*t}^{1-\theta},$$

where $0 < \theta < 1$[8].

The marginal rates of substitution, equations (7) and (8), as a function of consumption are

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma} \left(\frac{C_{yt+1}}{C_{yt}}\right)^{-1},$$

$$M_{t+1}^* = \beta \left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma} \left(\frac{C_{y^*t+1}}{C_{y^*t}}\right)^{-1}.$$

The previous equations can be written as

$$M_{t+1} = \mathrm{e}^{\ln(\beta)}\, \mathrm{e}^{(1-\gamma)\Delta c_{t+1} - \Delta c_{yt+1}} = \mathrm{e}^{\ln(\beta)}\, \mathrm{e}^{[(1-\gamma)\theta - 1]\Delta c_{yt+1} + [(1-\gamma)(1-\theta)]\Delta c_{y^*t+1}}, \tag{18}$$

$$M_{t+1}^* = \mathrm{e}^{\ln(\beta)}\, \mathrm{e}^{(1-\gamma)\Delta c_{t+1} - \Delta c_{y^*t+1}} = \mathrm{e}^{\ln(\beta)}\, \mathrm{e}^{[(1-\gamma)\theta]\Delta c_{yt+1} + \{[(1-\gamma)(1-\theta)] - 1\}\Delta c_{y^*t+1}}. \tag{19}$$

Consider that consumption growth follows a random walk with drift

$$\Delta c_{yt+1} \equiv g_y + u_{yt+1},$$

$$\Delta c_{y^*t+1} \equiv g_{y^*} + u_{y^*t+1},$$

[8]Hence, we are excluding the cases in which the economy behaves as if it was closed, as the SDFs will be undetermined in our model.

where u_{yt+1} and u_{y^*t+1} are i.i.d process, $u_{yt} \sim \mathcal{N}(0, \sigma_u^2)$ and $u_{y^*t} \sim \mathcal{N}(0, \sigma_{u^*}^2)$, respectively.

For a normal variable z, $\boldsymbol{E}(\mathrm{e}^z) = \mathrm{e}^{\boldsymbol{E}(z)+\frac{1}{2}\mathrm{Var}(z)}$, thus

$$\boldsymbol{E}_t(M_{t+1}) = \mathrm{e}^{\ln(\beta)}\,\mathrm{e}^{[(1-\gamma)\theta-1]g_y+[(1-\gamma)(1-\theta)]g_{y^*}+\frac{1}{2}[(1-\gamma)\theta-1]^2\sigma_u^2+\frac{1}{2}[(1-\gamma)(1-\theta)]^2\sigma_{u^*}^2}, \tag{20}$$

$$\boldsymbol{E}_t(M^*_{t+1}) = \mathrm{e}^{\ln(\beta)}\,\mathrm{e}^{[(1-\gamma)\theta]g_y+\{[(1-\gamma)(1-\theta)]-1\}g_{y^*}+\frac{1}{2}[(1-\gamma)\theta]^2\sigma_u^2+\frac{1}{2}\{[(1-\gamma)(1-\theta)]-1\}^2\sigma_{u^*}^2}. \tag{21}$$

As can be seen above, $\boldsymbol{E}_t(M_{t+1})$ will be negatively correlated to the mean value of domestic consumption growth if $\gamma > -\left(\frac{1-\theta}{\theta}\right)$, which is the case for risk averse agents. In that case, it will also be positively correlated to the variance of consumption of both domestic and foreign goods as marginal utility depends on both types of goods (that is, overall consumption). The economic interpretation is that, the higher overall consumption volatility is, the higher bond prices are (as households will demand more of both bonds for precautionary motives), and the smaller their return. If $\theta \approx 1$, for instance, an increase in the volatility of the home produced good will affect the SDF, $\boldsymbol{E}_t(M_{t+1})$, increasing P_t^B and reducing the H-bond return (and $\sigma^2_{u_{t+1}*}$ will have no effect on P_t^B). For that reason, an increase in the volatility of the domestic good consumption will decrease the expected currency risk when $\theta \approx 1$. The expected currency risk, on the other hand will increase with the volatility of the domestic consumption of the foreign produced good, everything else remaining equal, even when $\theta \approx 1$ (and nearly nothing of the F-good is consumed) because the foreign bond will be more demanded.

Also, observe that

$$\mathrm{Var}(m_{t+1}) = [(1-\gamma)\theta - 1]^2\sigma_u^2 + [(1-\gamma)(1-\theta)]^2\sigma_{u^*}^2,$$

$$\mathrm{Var}(m^*_{t+1}) = [(1-\gamma)\theta]^2\sigma_u^2 + [(1-\gamma)(1-\theta)-1]^2\sigma_{u^*}^2.$$

The expected currency risk will thus be equal to

$$\boldsymbol{E}_t(\rho^c_{t+1}) = \left[(1-\gamma)\theta - \frac{1}{2}\right]\sigma_u^2 - \left[(1-\gamma)(1-\theta) - \frac{1}{2}\right]\sigma_{u^*}^2. \tag{22}$$

Given our assumption of symmetry between the H and the F economies, we can analyze currency risk both ways: from the perspective of the domestic agent and also from the point of view of the foreign household.

Assuming that the model parameters for the representative agent in the F-country are θ^F and γ^F, that is, the proportion of consumption of F-goods by the F-household is denoted by θ^F, an aggregator similar to (17), and the local curvature of the F-agent period utility function is γ^F, we symmetrically have

$$\boldsymbol{E}_t(\rho^c_{t+1}) = \left[(1-\gamma^F)(1-\theta^F) - \frac{1}{2}\right]\sigma_{uF*}^2 - \left[(1-\gamma^F)\theta^F - \frac{1}{2}\right]\sigma_{uF}^2. \tag{23}$$

where σ^2_{uF} and σ^2_{uF*} are shocks to the F-goods consumption growth and H-goods consumption growth shocks of the F-economy[9], respectively.

An economic explanation to these results is given below.

Currency Risk, Domestic and Imported Consumption

Let us analyze the excess return from the perspective of the F-household. First, note that the

[9]H-goods consumption growth can be understood in a two country world as F-economy imports of H-goods for internal consumption.

expected currency risk will be deterministic and a function of the model parameters and the variance of consumption shocks for H and F goods.

A numerical example can help understand the nature of the risk facing the F-agent. Suppose $\theta^F = 0.9$ and $\gamma^F = 2$. It follows that $\boldsymbol{E}_t(\rho^c_{t+1}) > 0$ whenever $\sigma^2_{uF} > \frac{3}{7}\sigma^2_{uF*}$. When the proportion of F-goods in the overall consumption of the F-household is very high, most of the shocks perturbing the overall consumption stream arise from F-goods consumption, so there is no need for the F-household to insure against H-goods shocks by buying too many of the H-bonds. It follows that H-bond prices will be relatively smaller than F-bonds, *ceteris paribus*, and returns on the H-bond will be higher. Thus, currency risk (as we defined) will be positive when F-households consume few goods from the H-economy, even if the variance of F-consumption is very small, relative to the H-consumption.

Also, note that the local coefficient of risk aversion or local curvature of the utility function is γ^F. It is interesting to note that

$$\frac{\partial \boldsymbol{E}_t(\rho^c_{t+1})}{\partial \gamma^F} = \theta^F \sigma^2_{uF} - (1-\theta^F)\sigma^2_{uF}, \tag{24}$$

Equation (24) shows that for a infinitesimal change in the local curvature parameter of the F-household utility function, the variation in currency risk will positively depend on the variance of F-goods consumption growth shocks and negatively on the H-goods consumption variance. It will also depend on the proportion of home and foreign goods on total consumption. For example, suppose that the proportions of F and H produced goods on total F-consumption are the same, that is $\theta^F = \frac{1}{2}$, then the effect is positive if $\sigma^2_{uF} > \sigma^2_{uF*}$. This *ceteris paribus* partial effect means that as agents become more risk averse, they give more importance to smoothing the consumption of the good that is more volatile, in that case, the F-good (demanding less of the H bonds, decreasing their price and increasing *ex ante* returns). Intuitively, if $\theta^F = \frac{1}{2}$ and variances are the same, the impact of an increase in the local curvature parameter is null. Finally, note that equations (22) and (23) must be equal with complete markets in optimizing equilibrium. This implies some consistency conditions for the model parameters and also for the variance of individual shocks. For example, consider for simplicity that $\gamma = \gamma^F = 2$ and $\theta = \theta^F = \frac{1}{2}$. The consistency condition will be

$$\sigma^2_{u^*} - \sigma^2_u = \sigma^2_{uF} - \sigma^2_{uF*}.$$

For instance if risk premium is positive $\sigma^2_{u^*} > \sigma^2_u$, as higher F-good consumption volatility induces H-agents to buy more of the F-bond. It must also be the case that the volatility of the F-goods consumption in the F country should also be higher to induce F-agents to buy relatively more of the F-bond (and prices will be higher and their *ex ante* return lower). There is no market clearing condition in this simple asset pricing model that will entail the condition above - recall that variances are exogenously given. It is beyond the objective of the present paper to model these market relationships as well as the underlying origin of such shocks and their propagation.

1.3 Long Run Risk

Consider the simple following stylized error structure for the consumption growth variance (following insights from Bansal & Yaron (2004))

$$u_{yt+1} = \nu_{t+1}\sqrt{\eta_{t+1}},$$

$$u_{y^*t+1} = \nu^*_{t+1}\sqrt{\eta^*_{t+1}},$$

with

$$\eta_{t+1} = \varphi_0 + \varphi_1 u^2_{yt},$$

$$\eta^*_{t+1} = \varphi^*_0 + \varphi^*_1 {u_{y^*t}}^2,$$

where ν_t and ν^*_t are white noise process, $\nu_t \sim \mathcal{N}(0,1)$ and $\nu^*_t \sim \mathcal{N}(0,1)$, for simplification. Using a similar reasoning applied to (20) an (21), we have

$$E_t(\rho^c_{t+1}) = \iota + \iota_1\varphi_1 u^2_{yt} - \iota_2\varphi^*_1 {u_{y^*t}}^2, \tag{25}$$

where $\iota \equiv (\iota_1\varphi_0 - \iota_2\varphi^*_0)$, $\iota_1 \equiv \left[(1-\gamma)\theta - \frac{1}{2}\right]$, $\iota_2 \equiv \left[(1-\gamma)(1-\theta) - \frac{1}{2}\right]$.

As shown, we are to model precautionary currency risk as a time-varying process, which solely depends on the parameters of (25) and time-varying consumption variances.

1.4 Currency Risk under Habit Formation

In the appendix we show how to obtain currency risk under a utility function that exhibits habit formation, which results in the equations below

$$\rho^c_{t+1} = \frac{1}{\bar{X}^2}\left[\frac{1}{2} - (x_t - \bar{x})\right](\sigma^2_{\varepsilon^*} - \sigma^2_{\varepsilon}), \tag{26}$$

where

$$\sigma^2_{\varepsilon} = \left(\frac{1}{1+\lambda(x_t)} + \gamma\theta\right)^2 \sigma^2_u + \gamma^2(1-\theta)^2\sigma^2_{u*},$$

$$\sigma^2_{\varepsilon^*} = \left[\frac{1}{1+\lambda(x_t)} + \gamma(1-\theta)\right]^2 \sigma^2_{u*} + \gamma^2\theta^2\sigma^2_u,$$

and, in terms of the individual shocks,

$$\rho^c_{t+1} = \left\{\frac{1}{2} + \gamma(1-\theta)\left[\bar{X}^{-1}\sqrt{1-2(x_t-\bar{x})}\right]\right\}\sigma^2_{u^*} - \left\{\frac{1}{2} + \gamma\theta\left[\bar{X}^{-1}\sqrt{1-2(x_t-\bar{x})}\right]\right\}\sigma^2_u. \tag{27}$$

where H_t is the subsistence consumption (or habits) of goods country H and X_t is the surplus consumption ratio defined as $X_t \equiv \frac{C_t - H_t}{C_t}$; the dynamics of the log surplus consumption rate, x_{t+1}, is assumed (following Campbell & Cochrane (1999)) to evolve according to

$$x_{t+1} \equiv (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - g) = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)[\theta u_{yt+1} + (1-\theta)u_{y^*t+1}],$$

where g is the deterministic part of the growth of overall consumption in the H-economy, $g = \theta g_y + (1-\theta)g_{y*}$; $\lambda(x_t)$ is the sensitivity function which determines the behavior of the surplus consumption ratio.

One way to interpret (27) is through the analysis of the consumption growth variances in (26). First, currency risk would be equal to zero in equation (26) above if $\sigma^2_{\varepsilon^*} = \sigma^2_{\varepsilon}$, as H-agents would be indifferent between holding H or F bonds. In order to understand this assertion, notice that the equality $\sigma^2_{\varepsilon^*} = \sigma^2_{\varepsilon}$, implies

$$[1 + 2\gamma(1-\theta)(1+\lambda(x_t))]\sigma^2_{u*} = [1 + 2\gamma\theta(1+\lambda(x_t))]\sigma^2_u.$$

The "non-trivial" cases in which the equality above hold occurs when $\theta = \frac{1}{2}$ and $\sigma^2_u = \sigma^2_{u*}$. It follows that, the composite of H and F shocks are such that they disturb the H consumption stream in the same way, thus domestic agents perceive both H and F bonds as being perfect substitutes.

In order to interpret equation (26), consider that the consumption surplus is above steady state,

$x_t - \bar{x} > \frac{1}{2}$, so that the local risk aversion is relatively low[10]. In that case, currency risk will be positive only if $\sigma^2_{\varepsilon*} > \sigma^2_{\varepsilon}$ which will happen when

$$[1+2\gamma(1-\theta)(1+\lambda(x_t))]\sigma^2_{u*} > [1+2\gamma\theta(1+\lambda(x_t))]\sigma^2_{u}.$$

For ease of exhibition, consider that $\theta = \frac{1}{2}$. The inequality above will hold if $\sigma^2_{u*} > \sigma^2_{u}$. The economic explanation is that, since shocks to the consumption of F-goods dominate the volatility of H-household overall consumption, risk averse agents will buy relatively more of the F-bond in order to be insured against that stronger source of utility fluctuation. It follows that the price of the F-bond will be higher than the H-bond, when both are denominated in the same currency, and hence real returns on the H-bond will be higher *ex ante*[11]. A similar explanation can also be done for the currency risk faced by the F-agent. Also, the condition that the risk expressions will be the same for both the H and F agent will hold for consistency reasons, as in the power utility case.

1.4.1 Currency Risk and Uncovered Interest Parity

The previous model also offer an explanation for the Uncovered Interest Parity puzzle: i.e., the strong theoretical foundations and poor empirical support. As previously mentioned, there is no distinction between the consumption of H and F goods in the model developed in Verdelhan (2010). It follows that *ex ante* currency risk from the perspective of the domestic agent will be a positive function of the consumption surplus differential, $s^*_t - s_t$[12]. *Ceteris paribus*, a positive surplus differential implies that *times are relatively bad* at home so that the agent feels relatively more risk averse, given that the set of parameters governing the domestic and foreign sensitivity function are the same. In that case, excess returns on the foreign bond will be positive *ex ante* since the domestic agent will have to be compensated by hers relatively higher risk aversion. It also follows that, everything else remaining constant, an increase in domestic consumption variance will have a positive effect on the expected excess returns to be obtained from the foreign bond (which is intuitive, since demand for foreign bonds should increase less than for domestic bonds). Finally, it must be stressed that systematic (persistent) positive currency risk premiums on emerging markets bonds would exist, for instance, only if the USA agents were relatively more in *bad times*, which is a strong hypothesis.

However, the aim of Verdelhan (2010)'s paper is to examine the motives behind the Uncovered Interest Parity failure under the habit formation set up and in a non-monetary structure (all variables are real). In his model, *a)* expected exchange rates are a function of both the real interest rate differentials and the variance of the SFDs[13] and *b)* risk free real interest rates in both countries will also be related to their respective SDFs. The solution to this system of equations will result in a reduced form model for the expected real exchange rate changes in which the coefficient on the real interest rate differential is not going to be exactly equal to one. In fact, the coefficient will be equal to one only in the trivial cases in which either $\gamma = 0$, so there is no risk aversion, or if the standard deviation of consumption shocks is null, $\sigma = 0$, i.e. there is no source of risk to be insured

[10]Since $u(C_t, H_t) = \frac{(X_t C_t)^{1-\gamma}}{1-\gamma}$ and it is common in this literature to treat the surplus consumption ratios as exogenous, the local curvature is $\frac{-C_t u''(C_t,H_t)}{u'(C_t,H_t)} = \frac{\gamma}{X_t}$.

[11]Consider that $\mathbf{E}_t(r_{t+1}) > \mathbf{E}_t(r^*_{t+1})$ and that expected real depreciation is zero, i.e. $\mathbf{E}_t(Q_{t+1}) = Q_t$ and $S_{t+1} = 1$. It follows that $\frac{1}{P^B_t(1+\mathbf{E}_t(\pi_{t+1}))} > \frac{1}{P^{B*}_t(1+\mathbf{E}_t(\pi^*_{t+1}))}$, $\frac{P^{B*}_t}{P^B_t} > \frac{1+\mathbf{E}_t(\pi_{t+1})}{1+\mathbf{E}_t(\pi^*_{t+1})}$ and $P^{B*}_t S_t > P^B_t$. The left hand side of the inequality is the price of the F bond in H monetary units. If one considers that $\mathbf{E}_t(S_{t+1}) = 1$, so both bonds are expected to pay one unit of the same currency at $t+1$, then the real expected return will be higher for the H bond only if the F-bond price denominated in H currency is higher, i.e. $P^{B*}_t S_t > P^B_t$.

[12]Recall that the *ex ante* excess return or *ex ante* currency risk is the sum between the foreign interest rate and expected real exchange rate changes minus the domestic rate and note, also, that the asterisk represent the foreign economy in his model.

[13]See, p. 129, the first unnumbered equation before equation 4.

from. A regression of the interest differential on the expected real exchange rate changes would not result in a unit coefficient unless one controls for *ex ante* currency risk - otherwise there will be an omitted variable (and consequent negative bias) that can be emulated from the model by proper parameterization.

In Moore & Roche (2010)'s model, a representative household decides how much to allocate between consumption (of home and foreign goods) and portfolio, given the existence of subsistence consumption and state contingent bonds. From the model's Euler equations, they obtain expressions for the real exchange rate and for the risk-free bond prices in both home and foreign currency. The real exchange rate that results from the model is a function of exogenously given: endowments, the surplus consumption ratio and the local curvature of the utility function. The bond-price also depends on the subjective inter-temporal discount factor and on the inflation of the respective country. The cash-in-advance constraint is further used to obtain, from the real exchange rate equation, a nominal exchange rate that is a function of exogenous relative money supplies instead of endogenous prices. After assuming that endowment growth and money growth are AR(1) processes, Moore & Roche (2010) calculate the variances of the terms of trade and nominal exchange rates. If the surplus consumption ratio is very persistent, its variance will be large and it will dominate the variance of the two other variables mentioned above. The assumption of a long memory process for consumption growth then offers an explanation for the exchange rate disconnect puzzle as in Meese & Rogoff (1983). Exchange rate forecasts will be inaccurate as long as the surplus consumption ratio is not accounted for in the estimations. The forward bias puzzle can arise in Moore & Roche (2010) from the same reason that was explained in Verdelhan (2010): the reduced form equation for nominal exchange changes will not necessarily have a unit coefficient on nominal interest rate differentials. Moore & Roche (2010) thus proceed by showing how the conditions for a negative coefficient can arise from the model equations.

As both domestic and imported growth variances were shown to be important for the theoretical and empirical understanding of currency risk, we present a detailed analysis of USA consumption growth data in the next section. Section 3 shows how the variance of imported consumption growth compares to the variance of domestic, nondurable and durable growth in the USA and how it relates, dynamically, to excess returns.

2 Empirics

A natural question which arises from the theory put forward in the first section regards its empirical suitability. After all, are open economy factors empirically relevant for currency risk pricing? This question will be answered in three parts. First, by means of a descriptive analysis of the second moment of the disaggregated USA consumption growth series, we will show that they are (at least) no less volatile than durable consumption growth. Second, we will reveal that these factors positively co-vary with respect to proxies for the USA business cycle and, third, we will provide supportive evidence that open economy factors are able to explain currency risk using simulations.

Our sample spans from 1950Q1 to 2012Q4 with the exception of the imported consumption (beginning in 1967Q1). Variables were constructed following the methodology of Yogo (2006) using USA consumption data from the Department of Commerce of the Bureau of Economic Analysis (BEA). "Nondurable consumption" was built using the Fisher index which corresponds to the geometric mean of the Laspeyres and Paasche for nondurable and consumption services[14]. The base year (equal to 100) is 2009, which, due to the construction of the series, generates a small difference with the quarterly growth rate series from Yogo (2006). The first quarter of 1950 was adopted as the initial capital stock for durable consumption. We assumed a value equal to the *per capita* stock of capital of the

[14]For simplicity, we used the term "nondurable" for this variable. However, it is important to notice that it also comprises the consumption of services.

year 1951, which is US$ 1,636.00 in 2009 prices[15]. The remainder of the series of durable goods was constructed according to the perpetual inventory method using a depreciation rate equal to 6% *per* quarter, an arbitrary value that is common in this literature.

The series of imported goods have some additional limitations. First, despite the prices of imported goods are available since 1950 or 1960, data on imported consumer spending begins only on 1967Q1. Secondly, it must be emphasized that imported consumption of both durable and nondurable goods excludes imports of automotive goods already in the original sources.

The initial capital stock for the series of imported durable goods was chosen arbitrarily to minimize the variance of the growth rate of the first thirty observations, which led to a value of US$ 684.3 for 1966Q3, in 2009 prices. This choice was subjective, but it is based on the fact that $(1-\delta)$ becomes small after 30 periods, reaching approximately the value of 0.156 when $\delta = 6\%$. That is, after thirty periods, the share of the initial capital stock in the total stock is reduced to 15.6%, making its importance relatively small[16]. The other variables were constructed following the same methodology adopted for the construction of the durable and nondurable consumption series as explained above.

The aggregation of nondurable, durable and services in total imports was also performed through a Fisher index. It is important to emphasize that we adopted the estimated value of US$ 2,326.00 in 1966Q4 for the initial capital stock of domestic durables, using data from BEA of the total durables in 1965. Domestic consumption series are constructed by subtracting the flows of imports from total consumption.

Descriptive Statistics

Table 1 presents some descriptive statistics for these consumption series. The average growth rate of durables is 91.3% higher than the average of the nondurables. Table 1 shows that the average growth rate of imported goods is 84% higher than the growth rate of consumer durables[17]. But it is worth noting that the share of imported goods is a small fraction of total consumption, even at the end of the sample period[18]. It can also be seen in Table 1 that imported durable goods present the highest growth rate, reaching an average value of 1.64% *per* quarter.

Table 1 shows that the variance of durable goods consumption is 44% larger than the variance of the consumption of nondurables. The variance of domestic consumption growth is close to the nondurables, 0.21 and 0.25, respectively. Finally, the variance of imported consumption is 133.3% greater than the consumption growth of durables and 236% greater than the consumption growth of nondurable goods. These quantitative differences remain in sub-samples. The most volatile component of imports is nondurable consumption growth, reaching a variance 19.36 times bigger than the durable ones. But it must be emphasized that the relative weight of this component is small: between 1967Q2 and 2012Q2, the average nondurable imported consumer spending represents 8.7% of the durable (also worth remembering that the automotive consumption data, which are essentially durable, are not computed).

The last two rows of Table 1 show that the statistics of the series constructed by Yogo (2006) for durable and nondurable and the ones used in this paper are similar, which is not a surprise as the same methodology was used. The assumption of normality of the series is strongly rejected for the entire period and for the period starting in 1973Q2. This is possibly due to the presence of extreme

[15]Calculated using the variable USA net stock of fixed assets - consumer durable goods and USA price chain -type index for pce, durable goods sadj, with the mnemonic USNSCDCA - annual series, in billions of dollars, without adjustment, end of period, whose availability was from 1950 to 2011.

[16]Choosing the initial capital stock to minimize the variance of the series ensures that we are not assuming a very high or very low initial value, as implied by the perpetual inventory model.

[17]It is implicitly assumed in the utility function that households consume the services of durable goods proportionally to the stock and that the factor of proportion is equal to one.

[18]In 2013Q1, the share of imported stock ofr consumption durables in total stock of durable consumption reached 23.76%, while the nondurable was 9.43% of the total consumption of nondurables.

events, as can be seen from the reported maximum and minimum values.

Table 1: Consumption Growth Rate

	Start	End	Obs.	Mean	Variance	Min.	Max.	JB	Sig.
Nondurable. Yogo (2006)	1951Q1	2001Q2	204	0.51	0.29	-1.25	2.36	21.77	0.00
Durable Yogo (2006)	1951Q1	2001Q2	204	0.91	0.29	-0.49	2.03	6.29	0.04
Nondurable	1951Q1	2012Q2	248	0.46	0.25	-1.16	2.18	15.34	0.00
Durable	1951Q1	2012Q2	248	0.88	0.36	-0.58	2.10	9.49	0.01
Domestic	1967Q3	2012Q2	181	0.54	0.21	-0.64	1.60	9.82	0.01
Nondurable		–		0.41	0.20	-1.13	1.61	8.32	0.02
Durable		–		0.87	0.29	-0.47	1.95	6.49	0.04
Imported	1967Q3	2012Q2	181	1.62	0.84	-1.95	3.99	9.72	0.01
Nondurable		–		1.33	16.25	-15.48	19.42	194.10	0.00
Durable		–		1.64	0.75	-1.05	3.37	6.17	0.05

Source: own elaboration using data from Datastream. Notes: i) “JB” is the Jarque-Bera statistics and “Sig.” refers to statistical significance.

2.1 Consumption Factors and USA Business Cycles

The possibility of predicting stock market returns and asset prices using variables correlated with the business cycle is known [for an interesting summary of this evidence, see Nobel Prize Committee (2013)]. Thus, the risk premium increases as the economy moves toward the valley and decreases when it is going towards the peak. Asset pricing models such as the canonical Consumption Capital Asset Pricing Model fail to explain this countercyclical evidence both for the market and currency risk premium, mostly because there is not much variability in the aggregate consumption series. For this reason. disaggregated data on nondurable and durable consumption have been used in applied work (see Lustig & Verdelhan (2007)).

Yogo (2006) notes that the consumption of non-durable goods has lower volatility compared to stock returns, low autocorrelation, low correlation with the cycle. Yogo (2006) also shows that a pro-cyclical relationship can be observed with nondurable growth. Figure 1 presents “closed economy” consumption factors USA NBER recessions highlighted in the shaded areas. Comparison of our data with Yogo (2006), which is represented by the dotted lines in the graph, show that results presented in this paper are not drive by any particular feature of our sample. Figure 3 plots USA NBER recessions highlighted in the shaded areas and the growth of consumption factors against the output gap. As can be seen, open economy (domestic and imported consumption growth) seem to be “highly” pro-cyclical.

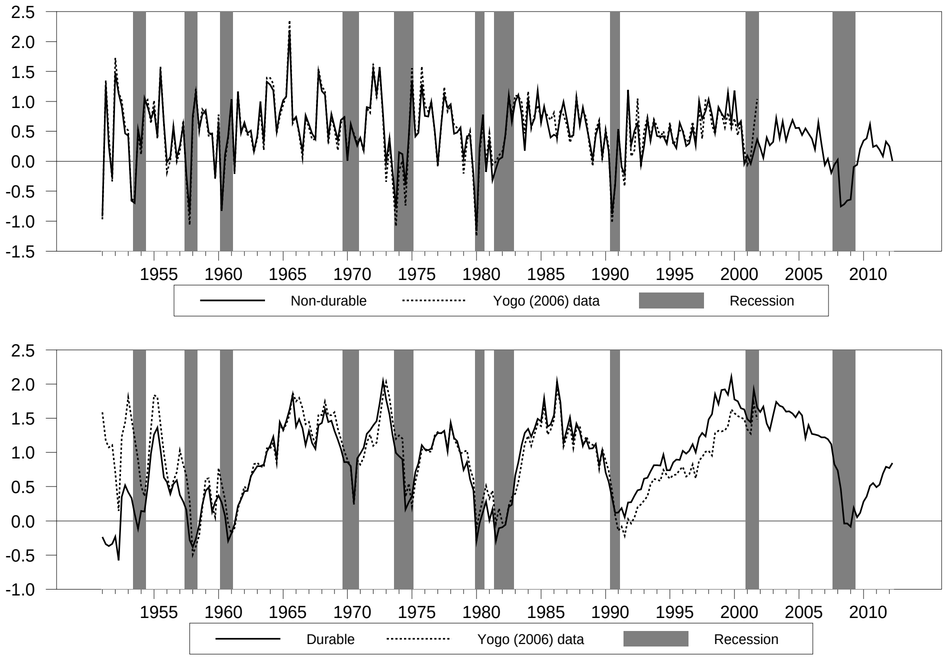
In order to formally evaluate the cyclical properties of these factors, we also ran the following regressions

$$\Delta c_{jt} = \tau_1 + \omega_j D_{NBER} + \mu_{1t}, \tag{28}$$

$$\Delta c_{jt} = \tau_2 + \gamma_j (y_t - \bar{y}_t) + \mu_{2t}, \tag{29}$$

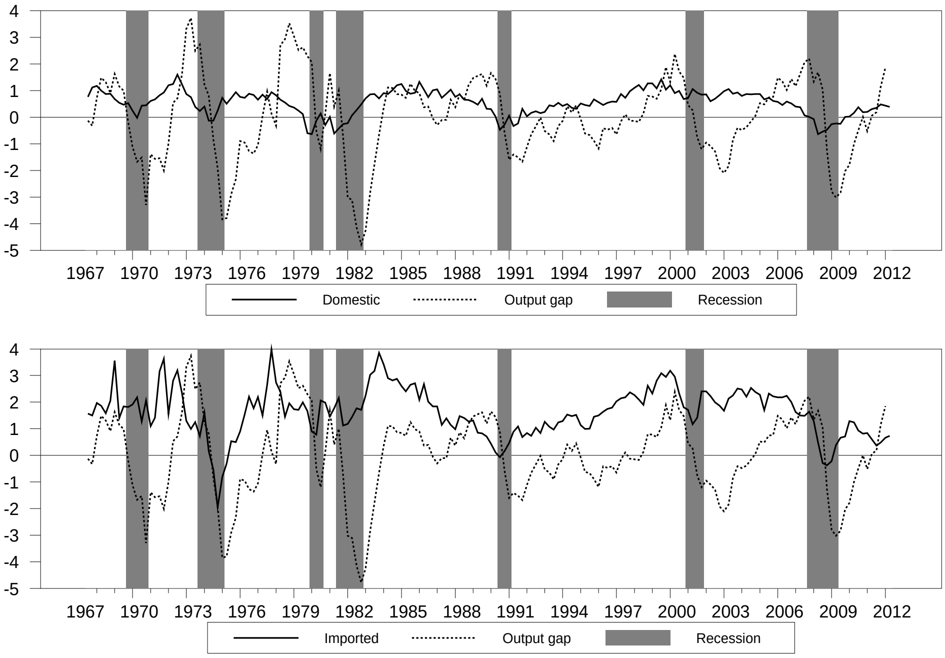
where Δc_{jt} refers to the consumption growth of either nondurable (Δc_{nt}), durable (Δc_{dt}), domestic (Δc_{yt})or imported goods (Δc_{y*t}), i.e., $j = n, d, y, y*$; μ_{1t} and μ_{2t} are Gaussian error terms, D_{NBER} is a dummy variable which receives a value equal to one during USA NBER recessions and zero otherwise, y_t is an index of real USA gross domestic product and $\bar{y}_t$ is the corresponding smoothed variable (through a Hodrick-Prescott filter) - in other words, $y_t - \bar{y}_t$ stands for a proxy of the output gap.

Figure 1: Nondurable and Durable Consumption Growth and Cycles



Source: own elaboration using data from BEA and Yogo (2006).

Figure 2: Domestic and Imported Consumption Growth and Cycles



Source: own elaboration using data from BEA and IMF.

Table 2 presents these results. As can be seen, the estimated impacts of a recession on imported and durable growth are both significant at high confidence levels. However, a quarter of recession has a much bigger impact on the growth of imported consumption goods than to durable growth. Second part of 2 shows that durable, domestic and imported consumption factors are positively correlated with the cycle (an increase in the output gap in associated with a rise in consumption). Stronger statistical correlations, as implied by the point estimates and corresponding t-statistics, are for durable and imported consumption growth.

Table 2: The Cyclical Behavior of Consumption Factors

	Coefficient	Standard error	t- statistics	p-value	
Cycles measured by NBER recessions (ω_j)					
Δc_{nt}	0.10	0.0994	1.00	0.3151	
Δc_{dt}	0.33	0.1554	2.17	0.0306	
Δc_{yt}	0.08	0.1218	0.70	0.4846	
Δc_{y*t}	0.93	0.3122	2.99	0.0031	
Cycles measured by the output gap (γ_j)					
Δc_{nt} †	7.94	2.376	-3.34	0.0010	0.06
Δc_{dt} †	17.38	2.193	7.92	0.0000	0.26
Δc_{nt}	-4.89	2.045	-2.39	0.0177	0.03
Δc_{dt}	12.26	2.360	5.19	0.0000	0.11
Δc_{yt}	2.19	2.186	1.00	0.3172	0.01
Δc_{y*t}	10.10	4.291	2.35	0.0196	0.03

Note: † estimated using Yogo (2006)'s sample from 1951:Q2 to 2001:Q4; Source: own elaboration using data from BEA, IMF and NBER. The sample period goes from 1951:Q2 for closed economy factors; open economy factors' period spans from 1967:Q3 to 2012:Q2.

2.2 Long run and Habit Risks: estimation and calibration

We present two sets of results in this sub-section. The first relates to the estimation of the parameters that belong to the long run risk model, based on power utility in (25). For this purpose, we use data on currency excess returns. The second set relates to the implied steady state risk premium in the habit model, which is calibrated using parameters borrowed from Verdelhan (2010), Campbell & Cochrane (1999) and Wachter (2006), as shown later.

Firstly, it must be noted that our series of excess returns was constructed according to the following formula

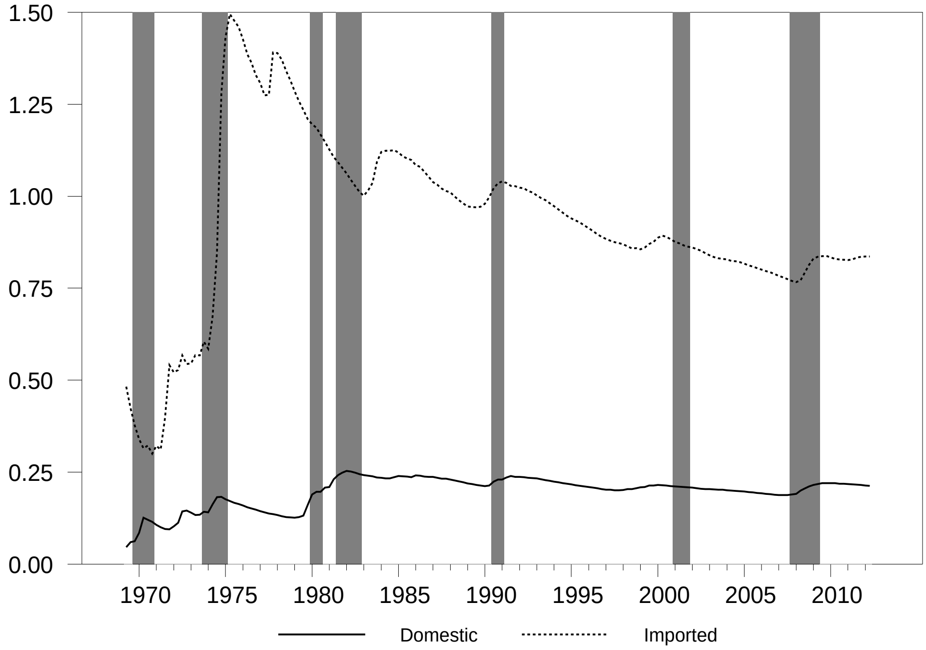
$$R^{je}_{t+k} \equiv \left\{ \left[(1+i^{j}_{t.k}) \left(S_t^{j/US\$} / S_{t+k}^{j/US\$} \right) - (1+i_{t.k}) \right] (P_t/P_{t+k}) \right\}, \tag{30}$$

where $i^{*}_{t.k}$ is the interest rate on a bond from the F economy, purchased at t with $t+k$ maturity; P_t is the USA CPI and $S_t^{j/US\$}$ is the exchange rate of the F economy, denominated as the country's F price of the USA dollar. R^{je}_{t+k} is the *ex post* excess real return obtained from buying a bond from country F with respect to the USA return, considering that they are both purchased at t, with the same maturity, $t+k$. We decided to use Japan as the foreign economy, since our sample data starts from 1967Q3 and Japan was the largest economy with data available since then.

We estimated φ_0, φ_1, φ_0^* and φ_1^* parameters via OLS, through an autoregressive (of order 1- AR(1)) model of the consumption growth variance. The first observation was obtained by taking the average of the last four quarters in a rolling manner. Figure 3 presents the time-varying growth variances and NBER recessions in the shaded areas. As can be seen, there appears to be a large increase in the imported growth variance in the second of the seventies' recession and a negative trend that followed, with no apparent regular episodes of ups and downs during the following cycles.

Domestic growth variance, on the other hand, seems to grow faster during NBER recessions. We then estimated (25) using excess returns on Japanese bonds obtained according to (30) and using IMF data (for the same time span of domestic and imported consumption growth). During the sample period, Japan presented a negative average excess return of -1.01 with a standard error equal to 3.17. Results are presented in Table 3 and 4. Two findings are worth noting, in particular. First, the implied share of consumption of domestic goods is 67%, which seems to be a reasonable value. Second, the implied coefficient of risk aversion is 9.69, which is considerably lower than what is suggested by the Mehra & Prescott (1985) puzzle, but much higher than what is thought to be a reasonable value - and that is also used for calibration purposes. Finally, Table 5 present simulation results of the implied steady state risk premium using the Habit Persistence Model. As can be seen, using parameters borrowed from Verdelhan (2010), Campbell & Cochrane (1999) and Wachter (2006), and different values for θ, we found that currency risk, from the perspective of a North American resident investing on Japanese bonds, ranges from the extremes -2.31% to 5.57% *per quarter*, but also reaches more modest values, such as 0.31% *per quarter*, for instance.

Figure 3: Time Varying Consumption Growth Variance and Cycles



Notes: *i*) variances were estimated using a rolling window from 1969Q2, i.e., the first observation computes the variance of the previous 4 quarters, the second computes the variance of the previous 5 quarters and so on; *i*) Shaded areas correspond to NBER recessions; Source: own elaboration.

Table 3: Parameter Estimation

	Estimated Parameter	Std. Error	t-statistic	Significance
ι	-5.06	1.25	-4.04	0.000
$\iota_1\phi_1$	6.31	5.93	1.06	0.2887
$\iota_2\phi_1{}^*$	2.98	1.04	2.86	0.0047
ϕ_0	0.01	0.00	4.47	0.0000
ϕ_1	0.95	0.01	81.59	0.0000
$\phi_0{}^*$	0.03	0.01	1.97	0.0500
$\phi_1{}^*$	0.97	0.01	71.01	0.0000

Table 4: Model Calibration

	Parameter
ι_1	6.63
ι_2	3.06
θ	0.67
γ	9.69

Table 5: Simulation of the Habit Persistence Model

	Steady State Risk			
θ	0.70	0.75	0.80	0.85
Verdelhan (2010)	3.31	1.81	0.31	-1.18
Campbell & Cochrane (1999)	2.61	1.06	-0.48	-2.03
Wachter (2006)	5.56	2.94	0.31	-2.31
Common parameters	$\sigma_u^2 = 0.21$		$\sigma_{u^*}^2 = 0.84$	
	Other Parameters			
	γ	ϕ	$\bar{X}$	X_{max}
Example with Japan	9.7	0.97	0.06	0.09
Verdelhan (2010)	2	0.99	0.07	0.12
Campbell & Cochrane (1999)	2	0.97	0.06	0.09
Wachter (2006)	2	0.97	0.04	0.06

3 Concluding Remarks

This paper shows that there is a strong theoretical reason for keeping precautionary savings in foreign currency when foreign consumption matters for the utility of the representative agent. In an open economy, an increase in the volatility of imported consumption goods, keeping everything else constant, increases currency risk premium because it makes foreign bonds relatively more valuable than domestic ones. In regards to the correlation between consumption factors and cycles, the paper presents some evidence that domestic and imported consumption growth are highly correlated with the USA NBER recessions and a proxy for the business cycle. Our model estimations and calibrations imply reasonable values for the share of domestic consumption goods (67%) and the parameter of risk aversion (9.69). It also implies reasonable levels for steady state risk, considering the habit persistence model, which ranges from the extremes -2.31% to 5.57% *per quarter*, but also reaches more modest values, 0.31% *per quarter*, for instance. In summary, there is evidence supporting the claim that highly variable imported consumption growth, in addition to domestic growth, can be more appropriated stochastic discount factors for currency risk pricing than the usual nondurable/durable model of consumption growth.

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Appendix: Habit Formation Model

Following Moore & Roche (2010), we consider habits that are defined over types of goods

$$u(C_t, H_t) = u(C_{yt}, C_{y^*t}, H_t) = \frac{(C_t - H_t)^{1-\gamma}}{1-\gamma}, \tag{31}$$

where H_t is the subsistence consumption (or habits) of goods country H.

As already shown, the relation between habit and consumption is conveniently captured by the surplus consumption ratio, X_t, following Campbell & Cochrane (1999) and defined as below

$$X_t \equiv \frac{C_t - H_t}{C_t}.$$

The marginal rate of substitution for H and F goods as a function of consumption are

$$M_{t+1} = \beta \left(\frac{X_{t+1}}{X_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{C_{yt+1}}{C_{yt}} \right)^{-1},$$

$$M^*_{t+1} = \beta \left(\frac{X_{t+1}}{X_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{C_{y*t+1}}{C_{y*t}} \right)^{-1}.$$

Given that

$$\Delta c_t = \theta \Delta c_{yt} + (1-\theta) \Delta c_{y*t},$$

the equivalent of equations (18) and (19) can now be written as

$$M_{t+1} = e^{\ln(\beta)} e^{-\gamma(\Delta x_{t+1} + \Delta c_{t+1}) - \Delta c_{yt+1}} = e^{\ln(\beta)} e^{-\gamma \Delta x_{t+1} - (1+\gamma\theta)\Delta c_{yt+1} - \gamma(1-\theta)\Delta c_{y^*t+1}}, \tag{32}$$

$$M^*_{t+1} = e^{\ln(\beta)} e^{-\gamma(\Delta x_{t+1} + \Delta c_{t+1}) - \Delta c_{y^*t+1}} = e^{\ln(\beta)} e^{-\gamma \Delta x_{t+1} - \gamma\theta \Delta c_{yt+1} - [1+\gamma(1-\theta)]\Delta c_{y^*t+1}}. \tag{33}$$

Following Campbell & Cochrane (1999), we also assume that the dynamics of the log surplus consumption rate, x_{t+1}, evolves according to an AR(1) process with non-linear properties due to the function that governs the consumption shocks

$$x_{t+1} \equiv (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - g) = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)[\theta u_{yt+1} + (1-\theta) u_{y^*t+1}], \tag{34}$$

where g is the deterministic part of the growth of overall consumption in the H-economy, $g = \theta g_y + (1-\theta) g_{y*}$; $\lambda(x_t)$ is the sensitivity function which determines the behavior of the surplus consumption ratio:

$$\lambda(x_t) = \begin{cases} \bar{X}^{-1}\sqrt{1 - 2(x_t - \bar{x})} - 1 & \text{when } x \leq x_{\max}, \\ 0 & \text{elsewhere,} \end{cases} \tag{35}$$

where $\bar{X}$ and $s_{\max}$ are the steady state and upper bound value of X_t and x_t, respectively; It follows that

$$m_{t+1} = \alpha + \gamma(1-\phi)(x_t - \bar{x}) + (1 + \lambda(x_t))\varepsilon_{t+1}, \tag{36}$$

$$m^*_{t+1} = \alpha^* + \gamma(1-\phi)(x_t - \bar{x}) + (1 + \lambda(x_t))\varepsilon^*_{t+1}, \tag{37}$$

where

$$\alpha \equiv \ln(\beta) - (1+\gamma\theta) g_y - \gamma(1-\theta) g_{y^*},$$

$$\alpha^* \equiv \ln(\beta) - \gamma\theta g_y - [1+\gamma(1-\theta)]g_{y^*},$$

$$\varepsilon_{t+1} \equiv -\left[\frac{1}{1+\lambda(x_t)} + \gamma\theta\right] u_{yt+1} - \gamma(1-\theta)u_{y^*t+1},$$

$$\varepsilon^*_{t+1} \equiv -\left[\frac{1}{1+\lambda(x_t)} + \gamma(1-\theta)\right] u_{y^*t+1} - \gamma\theta u_{yt+1}.$$

Given that $\mathrm{Var}_t(m_{t+1}) = \boldsymbol{E}_t(m^2_{t+1}) - [\boldsymbol{E}_t(m_{t+1})]^2$, we have

$$\mathrm{Var}(m_{t+1}) = \frac{1}{\bar{X}^2}[1-2(x_t-\bar{x})]\sigma^2_{\varepsilon},$$

$$\mathrm{Var}(m^*_{t+1}) = \frac{1}{\bar{X}^2}[1-2(x_t-\bar{x})]\sigma^2_{\varepsilon^*}.$$

The currency risk facing the H household is

$$\rho^c_{t+1} = \frac{1}{\bar{X}^2}\left[\frac{1}{2} - (x_t-\bar{x})\right](\sigma^2_{\varepsilon^*} - \sigma^2_{\varepsilon}).$$

Given that

$$\sigma^2_{\varepsilon} = \left(\frac{1}{1+\lambda(x_t)} + \gamma\theta\right)^2 \sigma^2_u + \gamma^2(1-\theta)^2\sigma^2_{u*},$$

$$\sigma^2_{\varepsilon^*} = \left[\frac{1}{1+\lambda(x_t)} + \gamma(1-\theta)\right]^2 \sigma^2_{u*} + \gamma^2\theta^2\sigma^2_u.$$

Currency risk, in terms of the individual shocks, is thus written as in (27).