Anticipations of the Kaldor-Pazos-Simonsen Mechanism¹

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ABSTRACT

This paper aims to contribute to the literature on the intellectual history of the economics discipline in Brazil and its place within the network of international transmission of economic ideas, by delving into the history of the Kaldor-Pazos-Simonsen mechanism, namely: the behavior of periodically-adjusted fixed nominal wages under persistent inflationary conditions (*the sawtooth wages model*). Ranging from the immediate post-War years to the end of the 1960s, our narrative reveals that prior to Nicholas Kaldor’s statement of the sawtooth model of real wages, other contributions sprung from various traditions. To this effect, we underline the discursive appraisal by Celso Furtado (1954) and the early neoclassical model by Bent Hansen (1951), only to fit both of them in a broader historical context of the economics discipline, whereby the emergence of formal mathematical methods allows for the connection between this debate and the Operations Research Program in the North-American universities in the early 1950s.

**Keywords:** chronic inflation, sawtooth model, formal mechanic models, Celso Furtado Operations Research Program

RESUMO

Este artigo busca contribuir com a literatura de história da ciência econômica no Brazil e seu lugar na rede internacional de transmissão das ideias econômicas, por meio de uma análise da história do mecanismo Kaldor-Pazos-Simonsen, qual seja: o comportamento de rendas nominais fixas com reajustamento periódico sob inflação crônica (ou modelo “dente-de-serra”). Nossa narrativa cobre o período do imediato pós-Guerra até finis da década de 1960, revelando contribuições que antecedem àquela de Nicholas Kaldor, tais como a abordagem discursiva de Furtado (1954) e o modelo neoclássico de Hansen (1951), enquadrando-as no contexto mais amplo da história da ciência econômica, em que a crescente adoção de métodos formais matemáticos permite a conexão entre esse debate e o programa de Pesquisa Operacional desenvolvidos na academia norte-americano no início da década de 1950.

**Palavras-Chave:** inflação crônica, modelo dente de serra, modelos mecânico-formais, Celso Furtado, Programa de Pesquisa Operacional

Classificação JEL: B22, B23, B31

ANPEC: Área 1 - História do Pensamento Econômico e Metodologia

¹This paper constitutes a larger research effort recently submitted as a PhD Thesis in Development Economics in the Economic Research Institute at the University of São Paulo (IPE-USP), under the title “The Conceptual Evolution of Inflation Inertia in Brazil”.

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Introduction

This work aims to contribute to the wider body of research in the intellectual history of the economics discipline in Brazil and its place within the network of international transmission of economic ideas. It attempts to understand one aspect of the evolving economics discipline, to which Brazilian economists have given a modest - albeit innovative – contribution, namely: the understanding of real wages under persistent inflationary conditions (the sawtooth model), one which has been overlooked in the literature on the history of economic ideas. The timespan of our speculations ranges from the immediate post-War years to the end of the 1960s.

To the Latin American audience, the theoretical framing of this particular regularity was conveyed in the “inflation feedback model”, which was developed independently by Mario Henrique Simonsen (1964, 1970) and by Felipe Pazos (1963, 1969, 1972). The model is an ingeniously simple way of portraying an economy with a persistently high inflation trend. The coupling together of inflation-correcting schemes, such as index-linking of contracts and prices, and a gradual approach to anti-inflation measures, sponsored by governments in Latin America, give nominal contractual incomes a visual pattern with a resemblance to a “saw-tooth”. Vera (2013) has found that Kaldor’s (1957c, 1957d) contributions to the stabilization debates in the 1950s involved this early representation of the pattern, thus claimed priority of Kaldor’s take over those by Simonsen (1964) and Pazos (1963).

Expanding on Vera’s paper, we find that not only had Furtado (1954) previously described the exact same phenomenon – albeit arguably drawing on a different tradition of economics -, but a similar portrait was put forth by Bent Hansen’s (1951) study of inflation theories. Harking further back to the past, we are able to connect this sawtooth-shaped diagram to the emerging Operations Research (OR) program, where mechanical models of inventory management held striking similarities to the visual representations of real incomes under sustained inflationary conditions. We claim that Kaldor’s input to this debate reflected a broader intellectual backdrop, in which the appearance of the first saw-tooth-shaped patterns reflected the rise of the mechanical models in economics, amongst which those pertaining to inflation. By doing so, we take a first step towards building a historical reconstruction of the sawtooth-shaped diagram, a visual representation of the self-replicating pattern of inflation, so widely known in Latin America in the 1980s.

Following this introduction, first section presents a brief overview of the postwar “eclipse of the goal of zero inflation” which sets out the backdrop to the recurrent erosion of purchasing power of fixed nominal incomes within periods of adjustment modeled thereafter. Section 2 tracks the anticipation of the seminal graphical representation by Nicholas Kaldor in the discursive appraisal offered by Celso Furtado, followed by Hansen’s institutions-laden neoclassical model of inflation (section 3) and, in section 4, by our working hypothesis that the OR program offered an analogous visual tool borrowed by economists to account for economic phenomena. Last section concludes.
1. Creeping inflation and the eclipse of the goal of zero inflation

Economists in the postwar period have singled out the existence of an inflationary bias to economies, both developed and underdeveloped. There are many perceptions and opinions regarding the underlying causes of this extensive upward bias in price level historical data in the postwar period. Is such a tendency a “natural” property of the capitalist system or is it simply bad government running bad policies? Or yet, is it a tolerable outcome or a morally reproachable evil? It is the profession’s general change of mind and heart regarding this question that this chapter is about. How did this inflationary bias come to be accepted within the economics profession? In what follows, we will paint with a rather broad brush this changing significance of inflation both in academic undertakings and in devising macroeconomic policy priorities.

As early as 1936, Keynes ([1936] 1964, p. 307] had already depicted an inflationary bias in economic trends, which owed much to the endogeneity of the money supply to the structural and institutional forces responsible for growth and innovation. Following the end of the World War II, Leeson (1997) has identified a gradual eclipse of the goal of zero inflation, as “employment” rose to the status of a primary policy goal, relegating “price stability” to a second tier. Between 1953 and 1959, the fact that inflation had become persistent in industrial countries reinforced this view. In turn, this prompted a reaction to these inflation-tolerant views during President Eisenhower’s term, when economists at the Joint Economic Committee recommended price stability be restored as the federal government’s primary policy aim, preferring high unemployment (a 6% rate seemed compatible with stability) to the unquestionable inflationary pressures. It was commonly held that creeping inflation (of 2-3% annual rate) would harm economic performance and would almost naturally degenerate into hyperinflation.5

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3 This section follows closely the approach set out by Leeson (1997) and Reddaway (1997).
4 In a discussion promoted in the year 1949 by the Review of Economics and Statistics, Seymour Harris (1949, p. 200) opened up his contribution to the series with the following statement: “Inflation is a worldwide phenomenon: in countries ravaged by war, and in those largely untouched; in countries blessed (?) by overemployment and in those suffering from unemployment; in countries favored by an excess of imports and those ‘injured by an excess of exports; in countries with budgetary surpluses and those with large deficits; in countries largely tethered to the principles of free enterprise and those relying on the planned economy and controls”. In his turn, William Brian Reddaway (1997, p. 32-33) dwelled with the more than apparent changing significance of the inflationary phenomenon in this period. He supports this claim with some simple empirical evidence on a handful of developed countries, in which the consumer price index have at least doubled in each one of them between 1962 and 1992 - Germany had a 184% increase, whereas Italy faced a 1300% increase.
5 For more details, see Leeson (1997, p. 454-456). In addition, subscribing to with this view, James Tobin (a key character in our narrative) was also a particularly notorious campaigning academic for the employment-bias within the Phillips curve trade-off, that an economy could live with inflation, but could hardly afford stagnation (Tobin & Ross 1971, p. 24). In Brazil, the notorious economist Eugênio Gudin (1965, p. 166) exposed some concerns over state-regulated prices and the ensuing downward rigidity they impose on the price level, thus engendering a systematically observed “gentle rise in the price level”. Gudin referred to Sumner Slichter, a famous Harvard professor at that time for his defense of the maintenance of a “creeping inflation” – ranging around a 2% constant yearly rate - directed at mitigating the harmful effects of union pressure for higher wages, usually above the rate allowed by the increases in productivity. Yet, Gudin claimed that there was no “automatic force acting in favor of the restoration of equilibrium”, once the inflationary process is triggered. According to his view, once it starts, the process tends to expand automatically, following a cumulative process à la Wicksell. As such, “in an inflationary regime, output production can only resume, at the same level, if the available amount of credits rise progressively”, which implies logically and immediately the setting of an escalating price-wage-price spiral (Gudin 1965, p. 183).
Moreover, this general stand was not confined to the United States. Kaldor’s memorandum to the Radcliffe Committee in the UK contained a section entitled “The Dangers of a Regime of Stable Prices”, in which he claimed that the low dynamism of the British economy should be countered by “doping” the economy with inflation. He also ascribed to “value-judgements and social preferences” the priority conferred to price stability or economic growth, and stated that “I would regard the sort of inflation we have been having, of the order of 3 to 4 percent a year, as a considerably lesser evil than economic stagnation or mass unemployment or instability in production”; in fact, he would sustain, “if we want to escape from economic stagnation, we must reconcile ourselves to rising prices” (Radcliffe et al. 1960, 715-17 quoted in Leeson, 1997, p. 455, n. 11).

The eclipse of the zero-inflation goal did not imply the complete withdrawal of inflation concerns from the policy agenda, although it was clearly relegated to a secondary position. Two points should have become quite clear by now: inflation was in the postwar years an empirically observable reality and economists, in general, have found it better to find ways to live with it than to unbendingly and unservingly keep it from happening on pain of higher unemployment and lower growth rates. The end result of was the transformation of the concept of price stability from nil to a non-zero positive rate.  

In this context, questions addressing the distributional effects were bound to arise. In the absence of any credible indication that prices will stop rising, economic agents set out to find ways protect the purchasing power of their incomes and the real value of their asset holdings. Once the ongoing inflation seems not to be going away, mechanisms are designed to cope with it. Sliding scales, escalator clauses, index-linked bonds and assets, indexation, monetary correction were all byproducts of the slow process of degeneration of unit-of-account property of money. Under the influence of such institutional adaptations, actual real world prices and wages are likely to behave quite mechanically (Heymann & Leijonhufvud 1995, p. 34); as history had it, so did the models that explained them.

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6 The creeping inflation that lasted a quarter of a century past the World War II prompted a change of feelings within the profession against the ongoing policy regime. Moderate commentators claimed, by 1971, that inflation was being greatly exaggerated as a menace (see, for instance, Tobin & Ross 1971). Soon, however, the price hikes in the first half of the decade would challenge such stands. The balance of perceptions began to tilt towards the monetarist side of the controversy. Even some sympathetic to Keynesian claims laid stress on the costs in terms of monetary authority’s credibility of seeking “the miracle” of a steady inflation, that is, a “target of stability in the first derivative of the value of the dollar rather than in its level” (see Okun 1973 and 1975). Furthermore, developed countries were then said to be undergoing a chronic inflationary problem (Okun & Perry 1978). To drive the point home, Leeson (1997, p. 482-483) reminds us that, in the 1960s, many warnings in this regard were issued in rather gloomy images; for instance, the view that a “Faustian pact” with the “inflation dragon” would engender the “Latin Americanisation of the world”, whereby a “cost inflation at a high rate has produced a ‘general crisis of capitalism’ and the stability of Communist prices shines like a good deed in a naughty world, yet economic theory has nothing to say”. In academia, Paul Samuelson echoed this altered sentiment toward inflation, concluding in the first edition of his textbook that: “if price increases could be held down to say less than 5 per cent per year, such a mild steady inflation need not cause too much concern” (Samuelson, quoted in Leeson, 1997 p. 455). This idea that would be repeated in a paper, he co-authored with Robert Solow (Samuelson & Solow 1960, p. 192), where both authors claimed that 4 or 5 percent inflation would be the necessary cost of enjoying high rates of employment and economic growth.

7 To cite a couple of books (see references therein for further literature), Hagger (1977) deploys an extensive survey up to 1976 and Okun & Perry (1978) compile the papers presented and discussed in a Conference held in Washington D.C. in the spring of 1978. To that effect, it suffices to show that in the 1960s only two fairly encompassing surveys on
2. Furtado’s anticipation of the Kaldor-Pazos-Simonsen Mechanism

Figure 1 below displays the model by whose dynamics a constant average real wage ($w^{ave}$, in the diagram) along the time periods of erosion by and adjustment according to the recorded inflation rate within each period.

![Diagram of the Sawtooth Model or The Simonsen Curve](image)

Figure 1 - Sawtooth Model or The Simonsen Curve. Source: Vera (2013, p. 269)

Vera (2013) has recently challenged the claim regarding the authorship of the model - commonly ascribed to Simonsen and Pazos – retrieving some overlooked documents that bear out Nicholas Kaldor’s (1957a, 1957b, 1957c) innovative input in the early debates concerning economic theories and policies of growth, distribution and inflation in Latin America. After his inflation theories were carried out (Bronfenbrenner & Holzman 1963; Brechling 1968), whilst a watershed of surveys and books entirely focused on inflation took over the profession in the 1970s. To keep our bibliographical references within bounds, we have been able to count at least six wide-ranging surveys along the mid-1970s (Laidler & Parkin 1975; Gordon 1976; Humphrey 1976; Frisch 1977; Harberger 1978 and Tobin 1980), as well as several books on the same topic of controversies and advances in inflation theories and their empirical support.

Regarding connections between Kaldor’s contribution and those by Felipe Pazos and M.H. Simonsen, we may entertain the following: during his stay in Chile and then in Brazil, especially at the Getúlio Vargas Foundation, where the International Economics Association Conference was held, Nicholas Kaldor most certainly had the opportunity to exchange ideas with both Felipe Pazos and Mario Henrique Simonsen, both of whom would later be responsible for publicizing, in greater detail, Kaldor’s 1956 sketch of the “saw-tooth” model. The records of the Conference were to no avail as of the writing of this thesis, but the fact that Cuban economists Felipe Pazos was one of the lecturers in the 1957 edition of the conference (see RBE 1957 and Kaldor, 1957b) suggests he may have attended Kaldor’s lectures in 1956. Even if he had not been personally present at the latter, Pazos’s connection with CEPAL’s economist Victor Urquidi is documented in their 1956 translation of Michal Kalecki’s (1954) Theory of Economic Dynamics: An essay on cyclical and long-run changes in the capitalist economy (Kalecki, 1956). Urquidi was a close participant in the first efforts to develop the Latin American structuralist theory of inflation and witnessed all the twists and turns of the early days of the new “inflation theory” (see Boianovskys, 2012; Palma & Marcel, 1989). Therefore, it is highly probable that Pazos had gotten word of the message conveyed by Kaldor. As regards Simonsen, he was trained as an engineer and an economist in the late 1950s and early 1960s and became the first director of the School of Graduate Studies in Economics (EPGE) at the Getúlio Vargas Foundation, Rio de Janeiro (Biderman, Cozac & Rego 1996, p. 189 and Boianovsky 2002, Note 1, p. 67). Kaldor visited Rio de Janeiro in October 1956, which allows for the possibility of
stay in Chile, Kaldor travelled to Brazil, in October 1956, where he attended a meeting of the International Economic Association and gave several lectures on the relationship between agriculture and industrialization in developing countries, which can be found in Kaldor (1957b). His analysis of the connection between inflation and development, Kaldor (1957c) set up a verbal model of conflicting claims that accounted for the self-sustaining pattern of inflation.

In evaluating wage dynamics, Kaldor (1957c, p. 75) assumes that nominal wages are periodically adjusted to cost-of-living indicators (p. 66-67), but readily devises an asymmetry between the frequency of price adjustments and those of wages: “while the adjustment of prices to changes in wages is gradual, the adjustment of wages to changes in prices tend to occur periodically considering, at one stroke, the accumulated change in the cost of living relative to some previous base date” (p. 75). He then draws out a straightforward regularity from the real wage behavior in an inflationary environment: “Real wages thus fluctuate around an average set by the extremes of the level of wages immediately prior to and immediately after a particular adjustment”. Next, Kaldor describes the visual pattern of real wages when the inflation rate picks up speed: “The characteristic of the wage/price spiral is that the zig-zag becomes increasingly compressed - the periods between subsequent wage adjustments tend to become successively shorter. This means, of course, a continued acceleration in the rate of increase in the general price level (Kaldor, 1957c, p. 75). This description is followed by the diagrammatic representation, in the Figure 2 below, of the process of erosion of the purchasing power of nominal wages under persistent inflation.

Figure 2 - Kaldor’s Real Wage “Zig-Zag” diagram. Source: Kaldor (1957c, p. 68).

Simonsen being present at Kaldor’s lectures or being motivated by others to read the transcripts of the latter. Besides, Simonsen must have assuredly gotten in touch with inventory models in his engineering undergraduate studies.

Henceforth, the translation of Kaldor’s (1957b) quoted excerpts is the one published in Vera (2013).
We agree with Vera (2013, p. 277) in that Kaldor’s relevant contribution to this debate extends beyond the mere graphic representation of the “saw-tooth model”: he should also be acknowledged for having anticipated the impact of endogenous variations in contract length in the acceleration process of inflation. Vera’s findings provide, on the one hand, further insight into the dissemination of scientific knowledge, but on the other, they raise further questions. In what follows, we take issue with this interpretation and provide further evidence of an even earlier recognition of the same pattern, in addition to Bent Hansen’s *en passant* depiction of the process in his 1951 book (see next section). Let us explore this maxim further taking heed - as Vera (2013) did - of Feiwel’s (1975) reminder that scientific work does not occur in a vacuum and, by this token, no discovery is totally new. We move back in time, to find anticipations of Kaldor’s masterly description in previous years.

The discursive representation of the saw-tooth pattern of real wages

In his *magnum opus*, *The Economic Growth of Brazil*, Furtado ([1959] 1963, chapter XXXV) had tackled the “two sides of the inflationary process”, that is, “the reason why prices rise on a persistent basis” and “the effects of such rise on the economic process”. Furtado starts out with a dynamic equilibrium concept of neutral inflation from the distributional standpoint. A neutral inflation implies that a distributional profile is left unaffected by the rise in the price level. As a dynamic concept, it is observed along a process of adjustment, e.g. one-year-long circuit. At any point during this period, “there is bound to be a group ahead of others in the struggle for the redistribution of income”. This group would benefit enormously from the stabilization of the price level at that point. Dynamically speaking, even if one could find an “average pattern of income distribution in a one-year stretch”, and that make prices stabilize around this pattern through sequential price and wage adjustments.

When all groups are institutionally and/or economically equipped to defend themselves against the erosion of their real income and have a clear notion of the position they occupy in that moment’s distributional profile of the economy, a successful stabilization becomes a troublesome task, for it is very unlikely that every group would be entirely satisfied with the end result (Furtado [1959] 1963, p. 239). To this effect, any stabilization program would face insurmountable hurdles due to this staggered structure of price and wage behavior. This self-reinforcing struggle leads to the “elevation of the price level goes on displacing the system from one unstable equilibrium position to the other” with no automatic – or even existent, to that effect - countervailing process to restore the system to a stable equilibrium position (p. 239).

For a history-minded structuralist economist, Furtado’s narrative is surprisingly mechanical in form.\footnote{For one, it is no wonder Furtado found such a phenomenon “of little interest to the economic analyst”. Apart from that, his description of the former reflects with astounding precision the later narrative entailed by the mathematical saw-tooth model. One cannot help but be struck by similarity in the discursive structure between Furtado’s take on neutral inflation and later representations by the economists at PUC-Rio, in the early 1980s (see Lopes & Williamson [1978] 1980 and Modiano 1988). At the risk underemphasizing the historical context of his thoughts on inflation, Furtado was prescient, as early as 1954 –, of the challenged posed to disinflation measures by the lack of synchronism among the adjustment frequencies of contracts and prices, a problem that would bedevil policymakers in charge of stabilization plans in the 1980s.} Despite Furtado’s lack of enthusiasm with this “neutral inflation”\footnote{For one, it is no wonder Furtado found such a phenomenon “of little interest to the economic analyst”. Apart from that, his description of the former reflects with astounding precision the later narrative entailed by the mathematical saw-tooth model. One cannot help but be struck by similarity in the discursive structure between Furtado’s take on neutral inflation and later representations by the economists at PUC-Rio, in the early 1980s (see Lopes & Williamson [1978] 1980 and Modiano 1988). At the risk underemphasizing the historical context of his thoughts on inflation, Furtado was prescient, as early as 1954 –, of the challenged posed to disinflation measures by the lack of synchronism among the adjustment frequencies of contracts and prices, a problem that would bedevil policymakers in charge of stabilization plans in the 1980s.}, this is the first real
theoretical assessment, as far as we know, of a fully indexed economy in Latin America. Although Noyola (1956a) had come to grips with a similar process, as Boianovsky (2012, p. 293-294) notes, this chapter of Furtado’s ([1959] 1963) book is but a literal copy of an excerpt (p. 174-187) of his 1954 book on the Brazilian Economy (Furtado 1954), thereby anticipating Noyola’s view on inflation as distributional struggle among income claimants and, possibly, as a neutral phenomenon from the distributive point of view.12

If we turn to empirics, although Furtado is clearly alluding to wages at large, available data on real minimum wage in Brazil is consistent Furtado’s basic dynamic framework. It would be a stretch to claim that Figure 2 below visually depicts the distributional dynamics Furtado had in mind. However, if we assume, for the sake of this counterfactual exercise, that all other income-recipients respond defensively to increases in minimum wage, the sawtooth shape of wage behavior can be easily noted. Even so, the data evokes not the “neutral” inflation, by which the peak real wage should be lined up at the same level through time.13

![Real Minimum Wage](image)

Figure 2 - Real Minimum Wage (July 1994 = 100) for Brazil: July 1950 – December 1960.

11 In light of this the points just raised, he did not believe “neutral inflation” to be of any significance to economists, for inflation would be a “sterile process” in the march toward development, for it entailed absolutely no real effects on the allocation of resources (Furtado, [1959] 1963, p. 239), a conclusion that Simonsen (1970, p. 208) would later agree with in its entirety. Later, in his Diagnosis of the Brazilian Crisis ([1962] 1965, p. 101-113), Furtado amended his earlier notion with an appraisal of the unbounded excitement of distributional struggles that eventually degenerates in an inflationary spiral. As inflation becomes an inseparable part of the growth process, its positive effects upon output growth cease and “the reaction of the working masses becomes increasingly rapid, and adjustments to earnings have to be made at increasingly shorter intervals”; this implies losses to wage-earners that are “probably greater than the gains still made from inflation by the capitalist class”. This is the “sterile phase of inflation” or the pointless “game of passing the buck” (p. 107). See also Noyola (1955a and 1955b) for a review of Furtado (1954) where the above point relating to neutral inflation is also overlooked.

12 It is worth stressing that Furtado’s reasoning on neutral inflation relies on a sophisticated and abstract conflicting claims approach that glaringly resembles the analysis by the French economist Aujac (1950) (see Craven 1994), which is telling of how far-reaching conflict theories were at the time.

13 Instead, under the strong assumptions above, it suggests an excited distributitional struggle between profits and wages, whereby the peak real minimum wage is being continuously pushed upwards by Getúlio Vargas’s (1951-54) and Juscelino Kubitschek’s (1956-1961) administrations.
It is somewhat striking that Furtado’s straightforward contribution to a dynamic understanding of inflation has been so widely overlooked (as in, for instance, Palma & Marcel 1989, p. 254), when its basic narrative is practically a verbatim reproduction of Kaldor’s (1957c) scheme.\footnote{14}

The fact that more than one economist was able to devise this pattern in seemingly independent fashion from each other, suggests that scattered (and unsubstantiated) notions regarding the phenomenon were probably “in the air”. We have show above that the models of first-generation structuralist were formalized in discursive form, that is, borrowing Morgan’s (2012) expression, in a “sea of words”. They upheld this core analytical thrust on a methodological basis. The contrast between this mode of reasoning and the indisputable mechanic form of the real wage pattern analyzed thus far provokes the suspicion that they shared a common cognitive procedure, at one level of abstraction higher. Next sections harkens further a few of years earlier than Furtado’s contribution to find this visual pattern of real incomes in its seemingly institutions-laden neoclassical framing.

3. The first Neoclassical moments of the sawtooth model

Bent Hansen has put forth an account of inflation theory in his 1951 book, titled A Study in the Theory of Inflation.\footnote{15} Hansen sets out to demonstrate that inflation may arise before full-employment is achieved launching a two-sector model comprised of a goods market and a factor market. When analyzing a policy of constant real wage (w/p), Hansen (1951) brought in an institutional factor of the interval in between adjustments of nominal wages, so that the above relation falls discontinuously along this interval and then is adjusted, pushing inflation up at discontinuous upward jerks. Since wages rise independently of supply and demand adjustment in the labor market (p. 16), a continuous “induced inflation”\footnote{16} ensues, which thus press upon prices in the commodities sector, resulting in a wage-price spiral. To illustrate this process, Hansen displays a set of diagrams that represent the time path of each set of variables; among them, the “saw-tooth” pattern of real wage behavior under constant inflation, as in the Error! Reference source not found. below. Hansen explores these possibilities allowing for the influence of monopoly (with a sort of mark-up pricing) and price controls on inflation dynamics. His implicit notion on an

\footnote{14} It is rather revealing of the intellectual landscape of those times that these saw-tooth-shaped wage dynamics were developed in synchrony with the full-blown adoption of government-sponsored import-substituting industrialization effort, that is, from 1951 forth; hence the idea that inflation is both cause and effect of distributional patterns along the cycle of adjustments to structural changes. Almost a decade after Furtado’s (1954) published his view of inflation, Pazos’s (1963) first writing on the oscillating pattern of real wages appeared in his book “Notas para un Estudio de la Espiral Inflacionaria”, published by the Mexican journal El Trimestre Economico in 1963 and also in the first volume of the “Centro de Estudios Monetarios Lationamericanos” – CEMLA with the support by The International Monetary Fund and the Interamerican Bank for Development (IBD), both from Washington D.C. Pazos (1963, p. 612) presents a table with nominal and real wages within a staggered structure of wage adjustments. Although it is still quite vague in its shape, the oscillating behavior of wages is taken into account, when he states that “real wages vary throughout the year and at any moment, except for a brief interval, the wages of each group of workers is above or below its annual average. This circumstance creates serious difficulties to the application of a wage freeze that proves fair and effective at the same time”(Pazos 1969, p.612) (my translation).

\footnote{15} See Day (1952) and Hagger (1961) for outlines of Hansen’s theory and further references therein.

\footnote{16} Hansen uses the term “induced” in the sense of an “autonomous” or “spontaneous” factor-related cost-inflation (See Hansen 1951, chapters 1 & 2).
inflationary monetary equilibrium is extensively informative (perhaps even prescient?) of the challenges that inflation would later impose on economic policymakers worldwide in the 1970s.

Figure 1 – Bent Hansen’s Dynamic Chart of Macroeconomic Impacts of Indexed-Wages. Source: Hansen (1951, p. 170, fig. 18).

This result comes out of the “price-inertia coefficients” (p. 227), a feature present in each and every sector of the economy, which is denoted by the reciprocal of constant \( k (>0) \), embedded in the price-reaction equations such as in the expression below:

\[
\frac{dp_t}{dt} = p_t \cdot k \cdot \frac{x_t}{q_t}
\]

where the time-variation of a specific price \( \frac{dp_t}{dt} \) is a function of itself multiplied by the price-inertia coefficient \( k \) and by the ratio of excess demand \( x_t \) to supply of commodity \( q_t \). Hansen then moves on to a “discontinuous analysis”, in which “the changes of prices occur only at the beginning of each period and are such that the excess demand occurring in the preceding period determines the marking-up of the price at the beginning of the period considered” (p. 224). Letting the length of the period be \( \Delta t = t_2 - t_1 \), and the excess demand at \( t_1 \) by \( x_1^1 \), we find the corresponding discrete relation to the above equation:

\[
\Delta p_t = p_1^1 \cdot k \cdot \frac{x_1^1}{q_1^1} \cdot \Delta t
\]

Thus the longer the period and the greater the value of the constant \( k \), the higher will be the variation of prices due to a lagged response of prices to excess demand. Hansen admits that, in reality, “it is not possible to reckon that \( k \) – of which the reciprocal can be taken to express inertia of
prices with respect to ‘induced’ changes – will in general be the same for all commodities and for all factors” (p. 225). The relevant price index, in the discontinuous case must have the form of

\[ \Sigma \left( \frac{1}{k_i} \right) \cdot p_i^2 \]

\[ \Sigma \left( \frac{1}{k_i} \right) \cdot p_i^1 \]

so that the weights are represented by the “price-inertia coefficients”.

Hence, Hansen anticipated the graphical dynamical representation of the direct inverse relationship between real wages and inflation in a system of periodically adjusted fixed nominal incomes, the very matter of later speculations on the determinants of inflation in Latin America. The relevance of Hansen’s contribution has been somewhat limited to punctuated references in macroeconomic history subsequent to the 1950s – being Simkin (1952) and Pitchford (1957) notable exceptions. Hansen’s inflation theory would – unexpectedly, one might add - influence Juan Noyola Vasquez (1956a; 1956b, p. 276) in his understanding of inflation. Interestingly, the theory Hansen puts forth cannot easily escape the typical CEPAL-based criticism of being essentially a-historical and essentially mechanical in nature, although many institutional aspects are considered throughout his book and his analytical framework is frequently liable to historical comparisons (see Hansen, 1951, p. 239-248).

Hansen’s contribution can be construed as a symptom of an emergent tradition that gained momentum in the immediate post-war period and was further emboldened in the decades following, when the intellectual landscape of the economics profession grew more mechanically and mathematically oriented. For the reasons lined up above, it is not far-fetched to assume that

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17 This very fact is inkling of Hansen’s impact in the profession may have been more intense than we can infer from available documentation. Bent Hansen would later to inspire one of the threads of disequilibrium macroeconomics, namely, Jean-Pascal Bénassy’s model of three regimes (Backhouse & Boianovsky 2013, p. 73 and p.114-119), not mention the conflicting-claims-inspired approach to the new neoclassical synthesis carried by Carlin & Soskice (1990 and 2006), although no credit to Hansen is acknowledged by the authors. It would be out of place to try to foray into the reasons for this, but we suspect that Hansen’s had too conciliatory a tone for his own good when it came to joining separate lineages of models into one analytical construct. However, one should not fail to notice this, the fact that Hansen was up to date with mathematical modeling (linear difference and differential equations) so dear to the emerging Samuelsonian world may have facilitated his legacy to be preserved, albeit with extensive amputations.

18 To support this assertion we point to the rather peculiar overall “conflicting claims” vocabulary Hansen adopts. Concepts such as “Class struggle” and “Class warfare”, capitalists and workers with different propensities to consume are all part of the parlance. Also the definition of a wage-price spiral within a general equilibrium setting, whereby quasi-equilibria populated the expected results, and moving dynamic equilibria were achieve with the aid of adaptive and forward-looking expectations. For further details, see Hansen (1951, pages 17 and 246).

19 Even so, his commitment to a theory that considered explicitly the institutional features of modern capitalism should be enough to make his analysis stand out. The following excerpt can give an illustration of our point: “They way in which spontaneous changes can eventually be avoided will not be considered here. The causal factors involved belong for the greater part to the field of politics and social psychology (the ‘class struggle’), so that it is only possible to treat them economically to a minor degree. The problem is actually closely relaed to that of abolishing monopoly in all its aspects” (Hansen, 1951, p. 246). These aspects are highly influential of the results obtained in the model and are thus liable to a careful examination, although economics is, in Hansen’s view, quite limited in dealing with them.

20 Against this backdrop, Mario Henrique Simonsen would later set forth his rather eclectic approach to the dynamics of inflation in Brazil, as well as his graphic depiction of it (see section 1 above). As we have seen hitherto, Simonsen’s
Furtado had access to Hansen’s theory in 1951, but this is yet to be established in documental evidence. Given the timid reception of the Swedish economist’s book in the anglo-saxon economics quarters of the profession, it is more plausible to assume the Hansen’s influence worked through CEPAL – perhaps through his connection with Noyola - than through Kaldor. Apart from that, Hansen’s approach finds an analogue in the emerging Operations Research program, which provides a common ancestor for the analyses by Hansen, Furtado and Kaldor.

4. The general mechanics of the saw-tooth diagram: a working hypothesis

Our brief outline of Hansen’s inflation theory showed that such a visual representation was not entirely new to economists in early 1950s. It is another question entirely how wide was its use within the economics profession. Thus, in our seemingly archaeological retrieval of this sawtooth-shaped “object of economic reasoning” (Morgan 2012), Nicholas Kaldor and Bent Hansen appear to take on a significant role. But how did Kaldor and Hansen come across this particular use of this pattern? Information on Hansen’s life is scarce, for which reason we follow the more abundantly documented events in Kaldor’s life and work. Before we proceed, a note of caution is in order. The evidence on the connections we a trying to purport is still scant but a set of suggestive clues points us in the direction of our general claim.21

The intellectual environment of the wartime was highly conducive to the fertilization of economic ideas by modeling formats borrowed from mathematics and engineering. At the time, Operations Research was one of the central areas of investigation in the border among social and exact sciences (see Whitin 1952 and Mirrowski 2002).22 As it so happened, this intellectual environment is likely to have influenced Kaldor in depicting the way he did the problem of real wages under chronic inflation, as just seen above. But how so? Inventory management policy was among the areas covered by this institutional effort (see Metzler 1941 and Arrow, Harris & Scarf 1958, chapter 1). As it turns out, the visual pattern extracted from inventory theoretic approaches was revealed as quite influential on the economics profession of that time.

(1964, 1970) apparent innovative merits, as regards the saw-tooth model of real wages, should be restated as a masterly synthesis of what was already in the air for quite some time in those days.

21 Kaldor had relevant contributions in an extensive list of topics in economics, mostly concerned with “real world” problems of welfare (cost-benefit) analysis, development, growth and distribution and taxation policy. He also took part in several government positions and was a renowned economic advisor on matters development policy for governments all around the world. According to J.K Galbraith’s autobiography, by being such a star economist and by taking part in such high-profile dwellings, Kaldor was certainly in close contact with developments in cutting-edge economics of those times, given his conspicuous presence in Lionel Robbins’s world-class seminars at the London School of Economics (LSE) and, later, holding a teaching position at Cambridge University. In what follows, we line up a few pointers regarding the academic context of Kaldor’s contribution.

22 Mathematicians, Physicists and Engineers migrated into Economics joined economists in applying linear programming techniques into this wide-encompassing area of Operations Research, which sought to do away with the supposed lack of rigor of the social sciences, at least from the standpoint of those hard sciences. In this sense, the Cowles Commission stationed at the University of Chicago was crucial to the developments we are about to turn to. In 1948, when the RAND Corporation becomes the main patron of Cowles, a major shift in the research agenda leads concerns toward the axiomatization of choice theory, seeking to provide the Operations Research quarters optimal decision rules (Mirrowski 2002).
In the summer of 1950, a paper entitled *Optimal Inventory Policy* was presented by Kenneth Arrow, Theodore Harris and Jacob Marschak (1950) - henceforth, AHM - at the Logistics Conference organized by the RAND Corporation, in Santa Monica, California (see Arrow 2002). The article outlines a method for deriving optimal rules for inventory policy of finished goods. In the analysis of the “case of certainty”, a deterministic model is presented, which establishes a constant flow of demand for goods \( x \) that depletes the stock. When the inventory reaches a critical point, a purchase order is put forth in order to replenish the stock of goods. Time frame is set in discrete time with an \( i \)-th interval, given by \( \theta_i \). Under certain conditions, the optimal policy relates to an optimal time length between purchase orders. In the authors’ words: “the intervals, possibly excepting the first one, will have the same length and the same optimal highest and lowest stock levels”, as shown in “Panel 1b” in Figure 2 below.

Two years later, AHM’s model was translated into macroeconomics via an illustration of demand for cash balances with an interest rate opportunity cost. Baumol (1952) entitled his paper *The Transactions Demand for Cash: An Inventory Theoretic Approach*, in which the demand flow \( x \) in Arrow’s and his co-authors’ deterministic model is replaced by the interest rate that erodes the time-value of money holdings “in a steady stream”. The model thus aims at analyzing “transactions demand for cash dictated by rational behavior, which for our purposes means the holding of those cash balances that can do the job at minimum cost”. Quoting Baumol at a yet greater length:

*A stock of cash is its holder's inventory of the medium of exchange, and like an inventory of a commodity, cash is held because it can be given up at the appropriate moment, serving then as its possessor's part of the bargain in an exchange. We might consequently expect that inventory theory and monetary theory can learn from one another. This note attempts to apply one well-known result in inventory control analysis to the theory of Money* (Baumol 1952, p. 545 – emphasis added).

Baumol refers, in footnote 1 (1952 p. 545), to AHM’s model and expands on the fruitfulness of the this crossbreeding between inventory theory and economics and mentions, somewhat *en passant*, that this had been previously done in mid-1920s by “some half dozen writers”. He also
makes reference to Whitin (1952), where he took the basic framework for his inventory theoretic approach.  

The problem can be stated as follows: an individual that makes a “steady stream” of payments ($T \text{ dollars}$) in the course of a certain period. Cash is either obtained through borrowing or withdrawing it from an investment; in both cases, there is an interest (opportunity) cost ($i$) per period for holding the non-interest-bearing asset. Finally, the individual is set to withdraw cash in $C$-size amounts in evenly spaced periods throughout the year, which collects a “broker’s fee” ($b \text{ dollars}$) for each time she makes a withdrawal. The value of transactions $T$ is predetermined and $i$ and $b$ are assumed constant. The total number of withdrawals will be given by $(T/C)$ at a total broker’s fee of $(bT/C)$ over a one-year period. Additionally, since each time she withdraws $C$ she spends it in a steady stream and takes out a similar amount once it is gone, her average cash holdings are given by $C/2$ dollars. It is this latter steady stream definition of average cash balances that makes up for the sawtooth-shape of the later widely known diagrammatic representation of this model.

Therefore, under these assumptions, the individual will then incur in an opportunity cost of $(iC/2)$ over the period. The total cost of using cash to meet one’s transactions needs is constrained by rationality conditions that require minimizing the cost of holding cash. This minimization procedure was transposed almost *ipsis literis* from Whitin (1952, p. 505), where $C$ is the quantity purchased of a good that minimizes the combined ordering costs and carrying charges. Also in Whitin (1953, chapter 3), we find the same notation regarding the deterministic model of “economic purchase quantities” generating the saw-tooth pattern, as in Figure 3 below.

![Figure 3 – Ordering Cycle Systems. Source: Whitin (1953, p. 54)](image)

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23 Baumol’s (1952, p. 545) fully quoted footnote is as follows: “T. M. Whitin informs me that the result in question goes back to the middle of the 1920’s when it seems to have been arrived at independently by some half dozen writers (...). Its significant implications for the economic theory of inventory, particularly for business cycle theory, seem to have gone unrecognized until recently when Dr. Whitin analyzed them in his forthcoming Inventory Control and Economic Theory (...) which, incidentally, first suggested the subject of this note to me. See also, Dr. Whitin's "Inventory Control in Theory and Practice" (elsewhere in this issue, supra, p. 502), and Kenneth J. Arrow, Theodore Harris, and Jacob Marschak, "Optimal Inventory Policy," Econometrica, Vol. 19, July 1951, especially pp. 252-255. In addition to Dr. Whitin, I am heavily indebted to Professors Chandler, Coale, Gurley, Lutz, Mr. Turvey, and Professor Viner, and to the members of the graduate seminar at Harvard University, where much of this paper was first present".
A few years later, Tobin (1956) would pursue the same avenue of analysis, in his *The interest-elasticity of transactions demand for money*. He mentions in the first footnote (p. 241) that he had not read Baumol’s 1952 paper. Tobin makes no mention to the modeling format and its origin in the 1956 paper. Later on, in a 1961 article, he would acknowledge due credit to the borrowed inventory theory:

*The theory of optimal inventory holdings, for example, shows how transactions and delivery costs must be balanced against interest and carrying costs. Applied to inventories of cash, the theory gives precision to the relation of cash holdings to the volume of nonfinancial transactions, the costs of asset exchanges, and the yields available on alternative assets.*

(Tobin 1961, p. 25)

If it is beyond doubt that inventory theory was crucial to frame the ideas both authors set forth, it much less clear how the “saw-tooth pattern” came to be attached to the Allais-Baumol-Tobin connection, for neither the authors explicitly allude to this visual format. But once again, it is evident is that inventory theory has brought a significant inspiration to economists.

**Concluding Remarks**

The present paper attempted to shed light on the rich reservoir of overlooked contributions in the history of economic ideas. Our narrative enhances the old adage by which no scientific discovery is entirely new, granted that some prior input can always be found somewhere hidden in the past, albeit with a different purpose in mind or embedded in a distinct multitextual background. In order to tap this well of foregone contributions, we focused the supposedly early years of the sawtooth-shaped diagram, also dubbed by Vera (2013) Kaldor-Pazos-Simonsen mechanism.

Looking beyond the strictures of modeling and into the historical context, both the emerging OR program and Bent Hansen’s inflation model with price-inertia indicate that Kaldor – and, for that sake, Furtado as well - borrowed this type of reasoning from the boiling intellectual landscape

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24 The introduction of Maurice Allais here honors historical accuracy for his alleged priority in deriving the mathematical basis for the optimization as it was adopted in the cash-balance theory. In a communication published in *Journal of Economic Literature*, at the occasion Allais was awarded the Nobel Prize, William Baumol and James Tobin recognized his early contribution as a forerunner of their own. In the authors’ words: “Maurice Allais’ well-deserved Nobel Prize fortuitously brought to our attention an injustice inadvertently done him, to which we were unknowing accessories. For years the literature has ascribed to us the parentage of the transactions-cost model of optimal cash balances, with its notorious square-root formula derived from inventory theory. Very recently, we found that its essence is contained in Allais’ 1947 *Economie et Intérêt* (pp. 238-41). As Jacob Viner used to say, no matter to what source the origin of an economic proposition is ascribed, someone is sure to come up with an earlier one” (Baumol & Tobin, 1989, p. 1160). See also Arrow (2002). Moreover, subsequent efforts, compiled in Arrow, Karlin & Scarf (1958), gave rise to the fruitful research program of (S,s) pricing policy models, as in Sheshinski & Weiss (1977 and 1983, p. 515). Caplin & Spulber (1987) would later apply the same inventory-theoretic - (S,S)-pricing policies - format to depict the behavior of “real prices” under inflationary conditions. I thank Prof. Kenneth Arrow for pointing out to these connections in an e-mail sent to the author.

25 Just to drive the point home, we can cite some later extensions of the contributions by Baumol and Tobin, which acknowledged them as “‘inventory’ models” (e.g., Javonovic 1982). In this same line, Romer (1986) displays the versatility of the Baumol-Tobin model by converting it into a general equilibrium framework. The purpose of Romer’s paper was to highlight the inflation-induced “shoe-leather” costs of frequent trips to the bank. Interestingly, he finds that in a model with money, inflation and a positive interest rate, a “sawtooth pattern” emerges when analyzing the consumption stream (Romer 1986, p. 680).
of the economics profession in the early post-War years. As regards Hansen, no evidence was found thus far to support this connection, whereas to the OR program, the first inkling is provided by one of Kaldor’s biographers, who reports that

In 1945 he was seconded to the United States Strategic Bombing Survey, where he worked with a talented team of economists that included John Kenneth Galbraith (the Survey’s director), Paul Baran, Edward Denison, Fritz Schumacher and Tibor Scitovsky (Galbraith 1979, pp. 219–20; 1981, p. 210). In the following year Kaldor worked briefly at the Ministry of Defence as adviser to the British Bombing Survey, and also advised the Hungarian government on postwar reconstruction (King 2009, p. 57).

Additionally, soon before coming to Latin America, Kaldor spent a few weeks in Stanford in April 1956, where he visited his close friend Tibor Scitovsky. Kenneth Arrow had been part of Stanford’s faculty since 1949, thus making it possible for Kaldor and Arrow to have met. However, in personal correspondence via electronic mail with the author, Prof. Kenneth Arrow stated having no recollection of Kaldor’s participating in any discussion on the former’s aforementioned 1951 article - on the optimal inventory policy - nor of his time in Stanford prior to Kaldor’s coming to Latin America. While further historical documentation is, as of this date, to no avail in supporting this avenue of investigation, one at the analytical level points yet again further into the past.

Kaldor could have easily derived the sawtooth-shaped diagram if not directly from the Allais-Baumol-Tobin apparatus, at least from an intuitive reading of Arrow, Harris and Marschak’s influential 1950 model. Instead of wealth and cash holding, a real magnitude such as the real wage would easily suffice. The stream of cash expenditure would give way to a steady inflation and, once a critical lower bound (the lower extreme) were achieved the restoration of the previous higher bound would ensue, restarting the cycle. Although it is not heavy-handed to assume that Kaldor had come across this type of modeling strategy and managed to generate an inflation-analogue to the inventory-theoretic reasoning, the transition from one application to the other seems to face a daunting analytical obstacles, such as the behavior of prices and profits, as well as rents and interest-based income. The history of this modeling-by-analogy may prove rewarding to the understanding of the distributional theories of inflation that arose in the post-War years following the groundbreaking contributions by J.M Keynes (1940) and Michal Kalecki (1941a, 1941b, 1943).

Finally, a word of caution is in order. Even though the timetable is consistent, this connection is, as it stands, simply a working hypothesis. The important message here alludes to one instance of how the inflation stabilization debates were affected that the overwhelming conquest of the mathematical sciences over economics in the post-War period, as documented by Mirowski (2002) and Weintraub (2002), among others. However, the saw-tooth model was not an immediately welcome contribution, for its heyday was yet to come a quarter of century later, to inform debates on anti-inflation measures in chronically inflationary economies, Brazil in particular. As the mathematical revolution slowly settled in economics, it was the structuralist quarters of the profession that would soon rise up to the challenge of mathematically formalizing the arguments conveyed by the sawtooth-shaped visual representation, as in Lopes & Williamson (1978), Bacha & Lopes (1983) and Taylor (1983), for instance. We leave this story though to another paper.
References


____. (1957c) “Inflação e desenvolvimento econômico”, Revista Brasileira de Economia, 11 (1), August. p. 55-82.
____. (1957d) “La inflación chilena y la estructura de la producción”, Panorama Económico, 180, November.
____. (1941b) Inflation, wages and rationing In: The Banker, October.
KEYNES, J.M. [1936 (1964)] The General theory of employment interest and money.
____. (1940) How to pay for the war: a radical plan for the Chancellor of the Exchequer. London: MACMILLAN AND CO., LIMITED.


_____. (1956b) *La evolución del pensamiento económico en el último cuarto de siglo y su influencia en la América Latina.* El Trimestre Económico. v. 23, n. 3, Julio-Septiembre, p. 269-283.


SAMUELSON, P. A. & SOLOW, R. M. (1960) *Analytical aspects of anti-inflation*


SIMONSEN, M.H.(1964) A experiência inflacionária no Brasil. Estudo IPES/GB.


