

# On the existence of loss function for some useful classes of central bankers

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**Resumo:** Fornecemos vários tipos de preferências para um banqueiro central de acordo com suas escolhas de política monetária. Apesar de não exaustiva, nossa lista cobre desde comportamentos extremos (conservador e populista) a moderados, tal que é flexível com relação ao grau de conservadorismo, bem como em relação ao regime monetário. Também usamos o tradicional *approach* axiomático do estudo de representação de utilidade para investigar quais daquelas preferências admitem uma representação numérica. Nosso principal resultado estabelece que, o assim chamado, banqueiro central estabilizador é o único que admite representação através de utilidade. Apesar disso, mostramos que mesmo preferências sem representação por utilidade podem ser úteis como alternativas para estudos que propõem diferentes tipos de banco central.

**Palavras-chave:** banco central; preferências; política monetária.

**Abstract:** We provide several types of central banker's preferences according to their monetary policy choices. Although not exhaustive, our list covers from extreme (conservative and populist) to moderate behaviors, such that it is flexible in relation to the degree of conservativeness, as well as in relation to monetary regimes. We also use the traditional axiomatic approach for utility representation in order to investigate which of those preferences admit numerical representation. Our main finding states that so called stabilizer central bank is the only that admits representation through utility. Nevertheless, we show that even preferences without utility representation may be useful as alternatives for studies which propose different types of central bank.

**Keywords:** central bank; preferences; monetary policy.

**JEL:** E58; E52; D11.

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# 1 Introduction

Central bank's behavior has been studied both empirically and theoretically in last four decades. In particular, its preferences over inflation and output have been object of several researchers. The main reason of that interest is the potential inflation bias of the discretionary monetary policy. The standard approach of those studies assumes that central bank's preferences may be represented by a strictly convex loss function, such that its minimization problem has unique interior solution<sup>1</sup>. Seminal papers of Robert Barro and David Gordon ([Barro and Gordon, 1983a,b](#)) have formed the base of this literature by adopting a loss function quadratic in inflation and linear in output.

However, it is not clear that central bank preferences can be represented by a loss function. If, for the sake of simplicity, we assume that such preferences just express the underlying "tastes" of a central banker, we may wonder whether there is an utility (actually, disutility) function representing them. Adopting the axiomatic approach of decision theory, the answer of above question would be based on verifying, for example, completeness, transitivity, and continuity of those preferences. In this regard, the central banker would or would not satisfy such axioms depending on his particular preference over inflation and output. Henceforward we consider the central bank's loss function and the central banker's utility as synonymous.

If instead we assume that preferences of the central bank express those of the society or of a representative individual, the answer to the above question has been given by [Woodford \(2002\)](#). By assuming nominal price rigidities, that paper derives a loss function grounded in the welfare of private agents. Specifically, it is shown that loss function is a quadratic (second-order Taylor series) approximation to the level of expected utility of the representative household in the rational equilibrium. In this case, if the household's preferences can be represented by an utility function, those of the central bank can too. The functional form obtained in the Woodford's approximation is almost the same of [Barro and Gordon \(1983a\)](#), namely quadratic both in the inflation and in the output gap.

It is possible to observe different types of central bankers' preferences through the behavior of their monetary policies. For instance, a central banker may be more conservative by focusing his policy on fighting inflation, or more populist by trying to increase output over price stability. Yet the most usual behavior found both in practice and literature is one which concerning about those two key variables. Formally, such preferences usually are represented by a loss function like

$$L(\pi, y) = \frac{1}{2}(\pi - \pi^*)^2 + \frac{\lambda}{2}(y - y^*)^2, \quad (1)$$

where  $\pi$ ,  $\pi^*$ ,  $y$ ,  $y^*$  and  $\lambda$  are inflation, its target (which may be implicit), output, its target (potential output) and the relative weight given to output control, respectively. In this formulation, a higher (lower)  $\lambda$  indicates a more populist (conservative) central banker<sup>2</sup>.

In fact, almost all models about central bank's decision assume preferences like those expressed by (1). The vast literature related to asymmetric information problem in monetary policy is a good example of the theoretical limitation in choosing monetary authority's types (e. g. [Barro, 1986](#); [Canzoneri, 1985](#); [Cukierman and Liviatan, 1991](#); [Nolan and Schaling, 1996](#); [Walsh, 2000](#)). In general, these papers assume that the central bank may be either extremely conservative or an optimizer who always act to maximize its expected loss function. This is exactly the case presented by [Backus and Driffill \(1985\)](#), in one of the earliest reputational models of monetary policy. The first type is a single-minded inflation fighter while the second one wants minimize a function of type (1). The same

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<sup>1</sup>An excellent survey of models where central bank is an optimizer agent in the context of optimal monetary policy may be found in chapter 8 of [Walsh \(2010\)](#).

<sup>2</sup>An important modification in the basic structure (1) is provided by [Nobay and Peel \(2003\)](#). This study models asymmetric preferences in which impacts of inflation deviations on the central bank's welfare depends on whether inflation is below or above its target.

may be found when we analyse studies which deals with the optimal degree of conservativeness of the central bank, like the seminal paper of Rogoff (1985). Even recent new keynesian DSGE models adopt loss functions like (1) (e. g. Clarida et al., 1999).

It seems very restrictive working with models where the objective function is always of the type (1). Consider literature related to games of asymmetric information between the central bank and the public, for example. There types are differentiated only by the value of  $\lambda$ . However, one may want to allow more extreme behaviors. In this context, we may want to model a game in which the public do not know whether the central banker is either very conservative (his only aim is to stabilize prices) or very populist (worries only about increase output). Note that even if  $\lambda$  is very high, (1) still give some weight to inflation control, such that formulation like (1) does not cover the behavior just cited.

Considering the above discussion, the aim of this paper is twofold. First, we provide several types of central banker's preferences according to their monetary policy choices. Although not exhaustive, our list covers from extreme (conservative and populist) to moderate behaviors such that it is flexible in relation to the degree of conservativeness, as well as in relation to monetary regimes. Secondly, we use the traditional axiomatic approach for utility representation in order to investigate which of those preferences admit numerical representation. Thus, as an intermediate step in this process, we need to study whether the central banker's type satisfy transitivity, completeness, and continuity. All our analysis assumes that the central banker's preferences are not necessarily the same of the society (or representative household), such that we want to answer a question different that posed by Woodford (2002).

Our results significantly contribute to all literature which models the central bank as a decision maker. In particular, models of monetary policy with microfoundations are suitable applications of our findings. Despite of having some desirable properties, like convexity, we found that several of central banker's preferences - mainly extreme behaviors - do not admit utility representation. Then, studies which follow the tradition of Barro and Gordon (1983a,b), and therefore use differential calculus tools, are restricted to adopt preferences like those expressed by (1). In fact, we show in our main result that so called stabilizer central bank is the only that admits numerical representation. Nevertheless, even preferences without utility representation may be useful as alternatives for studies which propose different types of central bank. The reason is that there is a multi-utility representation in those cases. For example, literature in monetary games with asymmetric information may consider our typology as a set of possible central banker's preferences.

This paper is divided in three sections besides this introduction. Section 2 introduces definitions of preferences and some of their properties, based on axiomatic approach of decision theory. That section also presents a typology of central bankers which covers from conservative to populist behavior. Section 3 investigates which of the presented preferences defined in the previous section admit utility representation. Finally, section 4 concludes by discussing about alternatives of representation for those preferences that do not admit it through utility. Appendix A presents proofs of our results omitted in the text.

## 2 A typology of central bankers

### 2.1 Preliminaries

We assume that central bankers has preferences over only two variables, namely inflation,  $\pi$ , and output,  $y$ . Let  $\Pi$  and  $Y$  be the subsets of  $\mathbb{R}$ , then  $(\pi, y) \in \Pi \times Y \subseteq \mathbb{R} \times \mathbb{R}$ . Despite of being the most usual case found in literature, the study of two dimension problem has the advantage of allowing analogies to the standard two goods consumer problem. Nevertheless, our results may be easily generalized to choice sets of any finite dimension. For example, they may apply to a central banker with preferences over inflation, output and exchange rate.

Definitions below are necessary in remaining of the text. In the following we closely follow notation adopted by [Debreu \(1954\)](#) and [Ok \(2010\)](#).

**Definition 2.1** A binary relation  $\succsim$  on  $\Pi \times Y$  is said to be:

- (i) reflexive if  $(\pi, y) \succsim (\pi, y)$  for every  $(\pi, y) \in \Pi \times Y$ ;
- (ii) complete if  $(\pi_1, y_1) \succsim (\pi_2, y_2)$  or  $(\pi_2, y_2) \succsim (\pi_1, y_1)$  for every  $(\pi_1, y_1), (\pi_2, y_2) \in \Pi \times Y$ ;
- (iii) transitive if  $(\pi_1, y_1) \succsim (\pi_2, y_2) \succsim (\pi_3, y_3)$  implies  $(\pi_1, y_1) \succsim (\pi_3, y_3)$  for every  $(\pi_1, y_1), (\pi_2, y_2), (\pi_3, y_3) \in \Pi \times Y$ .

**Definition 2.2** A preference relation  $\succsim$  on  $\Pi \times Y$  is a preorder on  $\Pi \times Y$ . In other words,  $\succsim$  is a binary relation that is transitive and reflexive.

As usual, definition 2.2 accounts to all the information that concerns how a central banker compares any two alternatives according to his “tastes”. We have chosen to define preferences as preorder rather than partial order because we wish to let choices free to be incomplete. Our aim is to allow behavior like the following. Consider two different scenarios: the first one presents high inflation and high output, while in the second there is low inflation and low output. Some type of central banker may find it hard to decide which situation is better.

Another important definition is that of utility representation.

**Definition 2.3** Let  $\succsim$  be a preference relation on  $\Pi \times Y$ . We say that a function  $u : \Pi \times Y \rightarrow \mathbb{R}$  represents  $\succsim$  provided that

$$(\pi_1, y_1) \succsim (\pi_2, y_2) \text{ if and only if } u(\pi_1, y_1) \geq u(\pi_2, y_2)$$

for every  $(\pi_1, y_1), (\pi_2, y_2) \in \Pi \times Y$ . If such a function exists, we say that  $\succsim$  admits a utility representation, and refer to  $u$  as a utility function for  $\succsim$ .

As we have mentioned above, we consider the central bank’s loss function and the central banker’s utility as synonymous, such that function  $u$  in definition 2.3 is actually a disutility. Yet such fact does not change our analysis because a disutility function is just a negative of utility one, such that for the agent to maximize an utility function is equivalent to minimize a disutility one.

Finally, definitions of continuity and convexity will have a major role in our existence theorems.

**Definition 2.4** Let  $\Gamma^+(\hat{\pi}, \hat{y}) = \{(\pi, y) | (\pi, y) \succsim (\hat{\pi}, \hat{y})\}$  and  $\Gamma^-(\hat{\pi}, \hat{y}) = \{(\pi, y) | (\hat{\pi}, \hat{y}) \succsim (\pi, y)\}$  be the upper contour set and the lower contour set of central banker, respectively. A binary relation  $\succsim$  on  $\Pi \times Y$  is said to be continuous on  $\Pi \times Y$  if for all  $(\pi, y) \in \Pi \times Y$ , the upper contour set and lower contour set of  $(\pi, y)$  are closed.

**Definition 2.5** A binary relation  $\succsim$  on  $\Pi \times Y$  is said to be convex if for any  $(\pi_1, y_1), (\pi_2, y_2), (\pi_3, y_3) \in \Pi \times Y$ , where  $(\pi_1, y_1) \succsim (\pi_3, y_3)$  and  $(\pi_2, y_2) \succsim (\pi_3, y_3)$  is the case of

$$t(\pi_1, y_1) + (1 - t)(\pi_2, y_2) \succsim (\pi_3, y_3) \text{ for any } t \in [0, 1].$$

## 2.2 Central bankers’ types

We list several types of central bankers below.

1. **Conservative:** this central banker only aims to minimize inflation, such that output is considered in his choice between two alternative scenarios only when inflation is the same in both cases. Formally,  $(\pi_1, y_1) \succsim_{CON} (\pi_2, y_2)$  if and only if  $\pi_1 < \pi_2$  or  $\pi_1 = \pi_2$  and  $y_1 > y_2$ . Thus, this central banker has lexicographic preferences.

2. **Populist:** this central banker is the extreme opposite of conservative one. Now the only goal is to increase output and inflation is only considered in situations with two equal outputs. Again, preferences are lexicographic:  $(\pi_1, y_1) \succsim_{POP} (\pi_2, y_2)$  if and only if  $y_1 > y_2$  or  $y_1 = y_2$  and  $\pi_1 < \pi_2$ .
3. **Conservative with inflation target:** this central banker seeks to minimize the distance between observed inflation and its target level,  $\pi^*$ , and only consider output when this distance is the same in two alternative scenarios. Given a target level fixed, preferences are formally expressed by  $(\pi_1, y_1) \succsim_{CON^*} (\pi_2, y_2)$  if and only if  $|\pi_1 - \pi^*| < |\pi_2 - \pi^*|$  or  $|\pi_1 - \pi^*| = |\pi_2 - \pi^*|$  and  $y_1 > y_2$ .
4. **Stabilizer:** when there are targets of inflation and output<sup>3</sup>, the stabilizer central banker wants to approximate observed inflation and product to their targets. However the weight given to inflation stabilization may not be the same that given to output. Let  $\lambda$  be a parameter that measures the relative weight given to output and  $y^*$  be the output target, we define preferences of a stabilizer central banker as  $(\pi_1, y_1) \succsim_{STA} (\pi_2, y_2)$  if and only if  $|\pi_1 - \pi^*| + \lambda|y_1 - y^*| \leq |\pi_2 - \pi^*| + \lambda|y_2 - y^*|$ .
5. **Optimistic:** this central banker wants to minimize inflation and maximize output at the same time. A scenario is better than another only if it presents lower inflation and higher output. Formally,  $(\pi_1, y_1) \succsim_{OPT} (\pi_2, y_2)$  if and only if  $\pi_1 \leq \pi_2$  and  $y_1 \geq y_2$ . Note these preferences are equivalent to the product order.
6. **Asymmetric:** we say a central banker has asymmetric preferences if his choices change according to state of the economy. This kind of behavior assumes that inflation is a problem only when it is above its target. Therefore, it may be modeled by  $(\pi_1, y_1) \succsim_{ASY} (\pi_2, y_2)$  if and only if  $y_1 > y_2$  or  $y_1 = y_2$  and  $\pi_1 < \pi_2$ , if  $\pi_1, \pi_2 \leq \pi^*$ ; and  $(\pi_1, y_1) \succsim_{ASY} (\pi_2, y_2)$  if and only if  $\pi_1 < \pi_2$  or  $\pi_1 = \pi_2$  and  $y_1 > y_2$ , if  $\pi_1, \pi_2 > \pi^*$ .

List above does not cover all possible central banker's behavior. However, types presented are surely representative of the set of all preferences. First, list covers extreme behaviors, like conservative and populist types, for example. Moreover, it presents flexibility about the degree of conservativeness of the central bank. This is accounted by the stabilizer central banker, which is more or less conservative according to the value assumed by  $\lambda$ . Note that if  $\lambda = 0$ , for example, the central banker worries only about inflation<sup>4</sup>. Similarly, a more populist behavior may be obtained with higher  $\lambda$ . Further, few changes are necessary to define a populist with inflation (or product) target preference or even an alternative asymmetric type in which behavior changes according to output is above or below its target. Second, typology is flexible about monetary regimes, such that it covers preferences in an inflation targeting setting, for instance.

Regarding the nomenclature adopted, we closely follow the large literature of optimal monetary policy. For instance, the central banker which gives much more weight to inflation fight relative to output stabilization is called conservative by Rogoff (1985) and Barro (1986) among others. The name "populist" for the central banker mainly concerned with output growth, in its turn, is adopted by Guzzo and Velasco (1999) (see also Berger et al., 2001; Jerger, 2002; Lippi, 2002). Despite of dealing with another type of asymmetry, the names of our asymmetric central bankers are based on Nobay and Peel (2003) and Ruge-Murcia (2003). Further, we choose call "optimistic" the central banker who always wants more output and less inflation because of the great difficult of achieving such goal. To best of our knowledge, term "optimist" has never used in literature.

<sup>3</sup>Even if these targets are implicit our characterization is valid. For example, central bank may seek to achieve the potential output, namely a non explicit target.

<sup>4</sup>Such behavior, with  $\lambda = 0$ , is still different from that of conservative type. Observe that when both types face  $(\pi_1, y_1)$  and  $(\pi_1, y_2)$  with  $y_2 > y_1$ , conservative central banker prefers  $(\pi_1, y_2)$  while stabilizer ( $\lambda = 0$ ) is indifferent.

Example below illustrates differences between central banker's types and easily enable us to check some of above axioms.

**Example 2.6** *Lets consider the following “bundles” of inflation and output:  $(1, 0), (0, 1), (1, 1), (2, 2) \in \Pi \times Y$ . Further, suppose that  $\pi^* = 1$  and  $y^* = 1$ . Then we have:*

1.  $(0, 1) \succsim_{CON} (1, 1) \succsim_{CON} (1, 0) \succsim_{CON} (2, 2)$ ;
2.  $(2, 2) \succsim_{POP} (0, 1) \succsim_{POP} (1, 1) \succsim_{POP} (1, 0)$ ;
3.  $(1, 1) \succsim_{CON^*} (1, 0) \succsim_{CON^*} (2, 2) \succsim_{CON^*} (0, 1)$ ;
4. *here we have to analyse two cases:  $\lambda \geq 1$  and  $\lambda < 1$ . If  $\lambda \geq 1$ , so  $(1, 1) \succsim_{STA} (0, 1) \succsim_{STA} (1, 0) \succsim_{STA} (2, 2)$ . On the other hand, when  $\lambda < 1$ ,  $(1, 1) \succsim_{STA} (1, 0) \succsim_{STA} (0, 1) \succsim_{STA} (2, 2)$ ;*
5. *given the available alternatives, the only possible choice for the optimistic central banker is  $(0, 1) \succsim_{OPT} (1, 0)$ ;*
6. *the asymmetric central banker also is not able to choose between all alternatives, such that  $(0, 1) \succsim_{ASY} (1, 1) \succsim_{ASY} (1, 0)$ , but  $(2, 2)$  is not comparable with the other “bundles”.*

Binary relations  $\succsim_{CON}$ ,  $\succsim_{POP}$ ,  $\succsim_{CON^*}$  and  $\succsim_{ASY}$  are clearly complete. On the other hand, a central banker with  $\succsim_{OPT}$  is not able to choose between five of six possible pairs of inflation and output. For example, when the available choices are only  $(0, 1)$  and  $(2, 2)$ , an optimistic type would choose neither of them because the first one has lower inflation while the second has greater output. As we have defined  $\succsim_{STA}$ , it is also incomplete. The reason is that “bundles” above and below inflation target are not comparable. Yet we may change its definition by adding a criterium of decision for these situations - inflation below inflation target is preferred, for instance. Such change does not modify our result, such that we follow the original definition.

Our first task is to answer whether all central banker's types listed above satisfy conditions of definition 2.2, that is, whether they all are preorders on  $\Pi \times Y$ . Proposition 2.7 answers that question.

**Proposition 2.7**  $\succsim_i$  is a preference on  $\Pi \times Y$  for  $i = \{CON, POP, CON^*, STA, OPT, ASY\}$ .

One interesting fact is that all preferences defined above have the desirable property of convexity. It turns out all central bankers analysed consider “averages better than the extremes”. As it is well known, this roughly corresponds to the concept of diminishing marginal utility (without requiring utility functions). Theorem 2.8 below states it formally.

**Theorem 2.8**  $\succsim_i$  is convex on  $\Pi \times Y$  for  $i = \{CON, POP, CON^*, STA, OPT, ASY\}$ .

### 3 Existence of loss function

Which of central banker's preferences defined in the section 2.2 admits an utility representation? There are several theorems on utility representation in literature (e.g. Eilenberg, 1941; Rader, 1963). Because the central banker's choice set is  $\Pi \times Y \subseteq \mathbb{R} \times \mathbb{R}$ , it suffices to consider those results which deal with separable metric spaces. Within such context, we based our analysis on the well known result due to Debreu (1954):

**Theorem 3.1** (Debreu, 1954) *Let  $X$  be a topological space with a countable basis, and  $\succsim$  a complete preference relation on  $X$ . If  $\succsim$  is continuous, then it can be represented by a continuous utility function.*

As we have mentioned above, the central banker's set of alternatives  $X = \Pi \times Y$  is separable, which implies it has a countable basis. Therefore, it satisfies one of the requirements of theorem 3.1. We must then verify which type of central banker has complete and continuous preferences in order to decide the applicability of Debreu's result. In fact, our main contribution in this section is a stronger result, given by theorem 3.2 below.

**Theorem 3.2** *The only central banker's type whose preference admits utility representation is the stabilizer ( $\succsim_{STA}$ ).*

Some comments are necessary about theorem 3.2. First, note that it implicitly makes two claims: stabilizer central banker does admit utility representation but all the remaining types do not. It implies we must divide strategy of proof in two parts. Concerning the existence part (stabilizer), it suffices to use theorem 3.1. However, Debreu's result enumerates only necessary conditions for utility existence. In fact, it does not exclude the possibility of discontinuous or incomplete preferences admit utility representation. Thus we must show non-existence part (other types) through other procedure.

Second comment is related to first one. Since all central banker's type except stabilizer admit utility representation, and considering necessary conditions given by theorem 3.1, we may ask which of such requirements are not satisfied by those preferences. First, observe that the optimist central banker has incomplete preferences<sup>5</sup>. In order to see that, consider  $\pi_1 < \pi_2$ ,  $y_1 < y_2$  and note he is not able to choose between  $(\pi_1, y_1)$  and  $(\pi_2, y_2)$ . In addition, it is well know that lexicographic preferences are not continuous, such that conservative and populist types clearly do not satisfy those conditions. Asymmetric preference does not satisfy them as well, because it is formed by two lexicographic segments (below and above  $\pi^*$ ). Proposition below summarizes the above discussion.

**Proposition 3.3**  $\succsim_i$  is not continuous on  $\Pi \times Y$  for  $i = \{CON, POP, CON^*, OPT, ASY\}$ .

## 4 Concluding Remarks

Although the only preference that admits utility representation be the stabilizer, it does not mean that the other types of central banker are not able to make a decision. In order to see that, consider one of those preferences of lexicographic type, say populist. Now suppose that a central bank chooses the most preferred "bundle" from a set of the form

$$B(m) = \{(\pi, y) | h(\pi, y; \mathbf{k}) = m\}$$

where  $\mathbf{k} \gg 0$  is a parameter vector,  $m$  is a scalar and function  $h(\cdot)$  is some policy constraint. If  $h(\cdot)$  is linear,  $h(\pi, y; \mathbf{k}) = k_1\pi + k_2y$ , for instance, then a central banker with decision function

$$\begin{aligned} v(B(m)) &= (0, \max \{y | h(\pi, y; \mathbf{k}) = m \text{ for some } \pi\}) \\ &= \left(0, \frac{m}{k_2}\right), \end{aligned}$$

will always choose the same pair  $(\pi, y)$  that a populist central banker constrained by  $B(m)$ . In this sense,  $v(\cdot)$  serves as an adequate behavior rule for preferences like populist one.

A more general result is provided by literature of representation of incomplete preferences (e. g. [Evren and Ok, 2011](#)).

**Definition 4.1** *Let  $\succsim$  be a preference relation on  $\Pi \times Y$ . We say that a nonempty set  $\mathcal{U}$  of real functions on  $\Pi \times Y$  represents  $\succsim$  provided that*

$$(\pi_1, y_1) \succsim (\pi_2, y_2) \text{ if and only if } u(\pi_1, y_1) \geq u(\pi_2, y_2) \text{ for each } u \in \mathcal{U}$$

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<sup>5</sup>In fact,  $\succsim_{OPT}$  is not continuous too, as proposition 3.3 states.

for every  $(\pi_1, y_1), (\pi_2, y_2) \in \Pi \times Y$ . If such a set  $\mathcal{U}$  exists, we say that  $\succsim$  admits a multi-utility representation.

**Proposition 4.2** *There exists a multi-utility representation for every preference relation.*

As an example of multi-utility representation, consider the optimistic central banker. Define  $u_1 : \Pi \times Y \rightarrow \mathbb{R}$  by  $u_1(\pi, y) := -\pi$  and  $u_2 : \Pi \times Y \rightarrow \mathbb{R}$  by  $u_2(\pi, y) := y$ . Then note that  $\{u_1, u_2\}$  represents  $\succsim_{OPT}$ . In fact, as proposition 4.2 makes clear, there are multi-utility representations for all preferences, such that it suffices to find which  $\mathcal{U}$  satisfies definition 4.1 in each case.

We conclude by recalling that always there is a loss by dealing with preferences without utility representation. The main reason is the impossibility of using standard optimization techniques. In this context, for models of central bank decision that follow traditional approach initiated in Barro and Gordon (1983a,b), our central result is a bad news. Those studies actually are restricted to assume stabilizer behavior for central banker and consequently adopt utility function like (1). However, we have shown that there is some flexibility if we are able to give up of differential calculus tools. Furthermore, all our analysis makes sense if preferences of the central banker are not necessarily the same of the representative household, because the case in that these preferences are equal has been addressed by Woodford (2002).

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## A Ommited proofs

**Proof. Proposition 2.7.** Clearly,  $\succsim_{CON}$  and  $\succsim_{POP}$  are lexicographic preferences, so they are preorders. Moreover,  $\succsim_{OPT}$  is a product order, such that it is a preorder as well. We must show that remaining types are preorders too.

Consider initially  $\succsim_{CON}^*$ . Define  $\hat{\pi}_j = |\pi_j - \pi^*|$ . Then note that  $(\pi_1, y_1) \succsim_{CON}^* (\pi_2, y_2)$  if and only if  $(\hat{\pi}_1, y_1) \succsim_{CON} (\hat{\pi}_2, y_2)$ . Therefore  $\succsim_{CON}^*$  is a preorder because so  $\succsim_{CON}$  is.

Now lets analyze  $\succsim_{STA}$ . Reflexivity is trivial. Suppose that  $(\pi_1, y_1) \succsim_{STA} (\pi_2, y_2)$  and  $(\pi_2, y_2) \succsim_{STA} (\pi_3, y_3)$ . Then  $|\pi_1 - \pi^*| + \lambda |y_1 - y^*| \leq |\pi_2 - \pi^*| + \lambda |y_2 - y^*| \leq |\pi_3 - \pi^*| + \lambda |y_3 - y^*|$ , what implies that  $(\pi_1, y_1) \succsim_{STA} (\pi_3, y_3)$ .

Finally, in order to show that  $\succsim_{ASY}$  is a preorder it suffices to note that if  $\pi_i > \pi^*$ , then  $\succsim_{ASY}$  is a lexicographic order, and if  $\pi_i \leq \pi^*$ ,  $\succsim_{ASY}$  is a lexicographic order as well. ■

**Proof. Proposition 2.8.** We have already seen that  $\succsim_i$  for  $i = CON, POP, CON^*, ASY$  is a lexicographic order, such that it suffices to show convexity for any  $i$ . Thus, lets consider conservative type and suppose that  $(\pi_1, y_1) \succsim_{CON} (\pi_3, y_3)$  and  $(\pi_2, y_2) \succsim_{CON} (\pi_3, y_3)$ . By definition of  $\succsim_{CON}$  we have  $\pi_1 < \pi_3$  or  $\pi_1 = \pi_3$  and  $y_1 > y_3$ , and  $\pi_2 < \pi_3$  or  $\pi_2 = \pi_3$  and  $y_2 > y_3$ . Now note that  $t\pi_1 + (1-t)\pi_2 \leq \max\{\pi_1, \pi_2\} \leq \pi_3$  for any  $t \in [0, 1]$ . Thus, two cases are possible. The first one is

$\max\{\pi_1, \pi_2\} < \pi_3$ , such that  $t(\pi_1, y_1) + (1-t)(\pi_2, y_2) \succsim (\pi_3, y_3)$ . The second is  $\max\{\pi_1, \pi_2\} = \pi_3$ , which implies  $\pi_1 = \pi_2$ . But since  $ty_1 + (1-t)y_2 \geq \min\{y_1, y_2\} \geq y_3$ , we also have  $t(\pi_1, y_1) + (1-t)(\pi_2, y_2) \succsim_{CON} (\pi_3, y_3)$ .

For  $\succsim_{OPT}$ , assume again that  $(\pi_1, y_1) \succsim_{OPT} (\pi_3, y_3)$  and  $(\pi_2, y_2) \succsim_{OPT} (\pi_3, y_3)$ . Then we have  $\pi_1 \leq \pi_3$  and  $y_1 \geq y_3$ , and  $\pi_2 \leq \pi_3$  and  $y_2 \geq y_3$ . It follows that  $t\pi_1 + (1-t)\pi_2 \leq \max\{\pi_1, \pi_2\} \leq \pi_3$  and  $ty_1 + (1-t)y_2 \geq \min\{y_1, y_2\} \geq y_3$  for any  $t \in [0, 1]$ . We conclude therefore that  $t(\pi_1, y_1) + (1-t)(\pi_2, y_2) \succsim_{OPT} (\pi_3, y_3)$ .

Finally, suppose  $(\pi_1, y_1) \succsim_{STA} (\pi_3, y_3)$  and  $(\pi_2, y_2) \succsim_{STA} (\pi_3, y_3)$ . By definition of stabilizer central banker,

$$|\pi_1 - \pi^*| + \lambda |y_1 - y^*| \leq |\pi_3 - \pi^*| + \lambda |y_3 - y^*| \quad (2)$$

and

$$|\pi_2 - \pi^*| + \lambda |y_2 - y^*| \leq |\pi_3 - \pi^*| + \lambda |y_3 - y^*|. \quad (3)$$

Now note that

$$\begin{aligned} |(t\pi_1 + (1-t)\pi_2) - \pi^*| + \lambda |(ty_1 + (1-t)y_2) - y^*| &= |t(\pi_1 - \pi^*) + (1-t)(\pi_2 - \pi^*)| \\ &\quad + \lambda |t(y_1 - y^*) + (1-t)(y_2 - y^*)| \\ &\leq t(|\pi_1 - \pi^*| + \lambda |y_1 - y^*|) + (1-t)(|\pi_2 - \pi^*| + \lambda |y_2 - y^*|) \\ &\leq \max\{|\pi_1 - \pi^*| + \lambda |y_1 - y^*|, |\pi_2 - \pi^*| + \lambda |y_2 - y^*|\} \\ &\leq |\pi_3 - \pi^*| + \lambda |y_3 - y^*|, \end{aligned} \quad (4)$$

where we use expressions (2), (3) and the triangle inequality. Thus, by (4) we have  $t(\pi_1, y_1) + (1-t)(\pi_2, y_2) \succsim_{STA} (\pi_3, y_3)$ . ■

**Proposition A.1**  $\succsim_{STA}$  is continuous.

**Proof.** **Proposition A.1.** We must show  $\Gamma^+(\hat{\pi}, \hat{y}) = \{(\pi, y) | (\pi, y) \succsim_{STA} (\hat{\pi}, \hat{y})\}$  is a closed set. The same argument may be used to prove the closedness of  $\Gamma^-(\hat{\pi}, \hat{y})$ . For, consider any sequence  $\{(\pi_n, y_n)\} \in \Gamma^+(\hat{\pi}, \hat{y})$  with  $\{(\pi_n, y_n)\} \rightarrow (\pi, y)$ . Then, by the definition of stabilizer preference,

$$|\pi_n - \pi^*| + \lambda |y_n - y^*| \geq |\hat{\pi} - \pi^*| + \lambda |\hat{y} - y^*| \quad (5)$$

for every  $n \in \mathbb{N}$ .

Note that by applying triangle inequality in expression (5),

$$\begin{aligned} |\pi_n - \pi| + |\pi - \pi^*| + \lambda (|y_n - y| + |y - y^*|) &\geq |\pi_n - \pi^*| + \lambda |y_n - y^*| \\ &\geq |\hat{\pi} - \pi^*| + \lambda |\hat{y} - y^*|. \end{aligned} \quad (6)$$

Moreover, as  $\{(\pi_n, y_n)\} \rightarrow (\pi, y)$ , there exists  $M \in \mathbb{N}$  such that  $|\pi_n - \pi| < \epsilon_1$  and  $|y_n - y| < \epsilon_2$  for any  $n > M$ . Thus,

$$(\epsilon_1 + \lambda\epsilon_2) + (|\pi - \pi^*| + \lambda |y - y^*|) \geq |\hat{\pi} - \pi^*| + \lambda |\hat{y} - y^*|. \quad (7)$$

Now, it suffices to let  $\epsilon_1 \rightarrow 0$  and  $\epsilon_2 \rightarrow 0$  to see that  $(\pi, y) \in \Gamma^+(\hat{\pi}, \hat{y})$ . ■

**Proof.** **Theorem 3.2.** For the existence part, note that  $\succsim_{STA}$  is complete, because for every  $(\pi_1, y_1), (\pi_2, y_2) \in \Pi \times Y$  we have

$$|\pi_1 - \pi^*| + \lambda |y_1 - y^*| \geq |\pi_2 - \pi^*| + \lambda |y_2 - y^*|$$

or

$$|\pi_1 - \pi^*| + \lambda |y_1 - y^*| \leq |\pi_2 - \pi^*| + \lambda |y_2 - y^*|.$$

In addition, by proposition A.1,  $\succsim_{STA}$  is continuous. Then, by theorem 3.1,  $\succsim_{STA}$  admits a continuous utility representation.

Recall  $\succsim_{OPT}$  is incomplete, so obviously it does not admit utility representation. In order to show non existence for the remaining preferences we use a traditional proof by contradiction (e. g. Debreu, 1954; Ok, 2010). So suppose there exists  $u : \Pi \times Y \rightarrow \mathbb{R}$  which represents  $\succsim_i$  for  $i = CON, POP, CON^*, ASY$ , such that for every  $\pi_1$ ,  $(\pi_1, 1) \succsim_i (\pi_1, 0)$  if and only if  $u(\pi_1, 1) \geq u(\pi_1, 0)$ . Then, for each  $\pi_1$  we can assign a nondegenerate interval

$$R(\pi_1) = [u(\pi_1, 0), u(\pi_1, 1)].$$

Next, take  $\pi_2$  such that  $\pi_2 < \pi_1$ . Again, we have  $u(\pi_2, 1) > u(\pi_2, 0)$  and  $R(\pi_2) = [u(\pi_2, 0), u(\pi_2, 1)]$  is nondegenerate. Further,  $R(\pi_1) \cap R(\pi_2) = \emptyset$  because  $\pi_2 < \pi_1$  implies  $u(\pi_2, 0) > u(\pi_1, 1)$ . Observe that we can define  $R(\pi_j)$  for any  $\pi_j \in \Pi \subseteq \mathbb{R}$ , and such that all sets are disjoint from each other. Moreover, there are as many  $R(\pi_j)$  as the number of real numbers. Let  $\Phi(y) = \{R(\pi_j) | (\pi_j, y) \in \Pi \times Y\}$  the set of all those intervals. Thus, cardinality of  $\Phi(y)$  is  $2^{\aleph_0}$ .

Now, let  $Q(\pi_j) = \{q | q \in \mathbb{Q} \cap R(\pi_j)\}$  be the set of all rational numbers contained in  $R(\pi_j)$  for  $\pi_j \in \Pi \subseteq \mathbb{R}$ . Clearly,  $Q(\pi_1) \cap Q(\pi_2) = \emptyset$  for any  $\pi_1, \pi_2 \in \Pi$  with  $\pi_1 \neq \pi_2$ . Finally, if we define  $\Theta(y) = \{Q(\pi_j) | (\pi_j, y) \in \Pi \times Y\}$  as the set of all  $Q(\pi_j)$ , it is possible to note that there is a injective map from  $\Phi(y)$  into  $\Theta(y)$  because each  $R(\pi_j)$  contains  $Q(\pi_j)$ . Therefore, we conclude that cardinality of  $\Theta(y)$  is  $2^{\aleph_0}$ . But it is impossible given that  $\Theta(y) \subseteq \mathbb{Q}$ . ■

**Proof. Proposition 3.3.** It suffices to prove the result for one of the types, because the procedure is similar for  $i = CON, POP, CON^*, OPT, ASY$ . Lets show that conservative with inflation target preference is not continuous. In fact, as proof of proposition 2.7 makes clear,  $\succsim_{CON^*}$  is a kind of lexicographic order too. For, consider its upper contour set  $\Gamma^+(\hat{\pi}, \hat{y}) = \{(\pi, y) | (\pi, y) \succsim_{CON^*} (\hat{\pi}, \hat{y})\}$  and define a sequence  $(\pi_n, y_n) = (\hat{\pi} - \frac{1}{n}, \frac{\hat{y}}{2} + \frac{1}{n})$ . For the sake of simplicity, let  $\Pi \times Y \subseteq \mathbb{R}_+^2$ . Given  $|\pi_n - \pi^*| = |\hat{\pi} - \frac{1}{n} - \pi^*| < |\hat{\pi} - \pi^*|$ , we have  $(\pi_n, y_n) \subseteq \Gamma^+(\hat{\pi}, \hat{y})$ . However, observe that  $(\pi_n, y_n) \rightarrow (\hat{\pi}, \frac{\hat{y}}{2})$ . It turns out  $(\hat{\pi}, \frac{\hat{y}}{2}) \notin \Gamma^+(\hat{\pi}, \hat{y})$ , because  $|\hat{y} - y^*| > |\frac{\hat{y}}{2} - y^*|$ . Thus,  $\Gamma^+(\hat{\pi}, \hat{y})$  is not closed and then  $\succsim_{CON^*}$  is not continuous. ■